

Simulating a dark matter halo using a parallel grid method

Otto Hannuksela Janne Lampilahti

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1 INTRODUCTION

Dark matter appears to be the dominant form of matter in the universe. While no direct observations of dark matter exist, it has been observed indirectly through its gravitational interaction with visible matter and radiation (?). The effect of dark matter on structure formation in the universe is the subject of an increasing scientific interest and a useful tool in the search for possible candidates of dark matter. Current progress in the investigation of structure formation is mainly driven by advances in computational methods and capabilities (?).

In this work we simulate a dark matter halo, neglecting baryonic matter, by using a particle based method and solving the Poisson's equation of gravity. The software used is *Uintah* available from <http://uintah.utah.edu/>, a parallel grid framework with a support for particle interactions and adaptive mesh refinement and which is aimed at solving partial differential equations. A major part of the project was to learn how to use the tools provided by Uintah.

2 METHODS

2.1 Theory

A distribution of mass density ρ gives rise to a gravitational potential ϕ according to the Poisson's equation of gravity

$$\nabla^2 \phi = 4\pi G \rho, \tag{1}$$

where $G \approx 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ is the gravitational constant. In our simulation we normalize $G = 1$. The corresponding force field can be solved from the gradient

$$\mathbf{F} = -\nabla\phi. \quad (2)$$

In our simulation a distribution of massive particles create the mass density. A new position and velocity for the particles after a time step dt can be solved from the Newton's equation of motion.

$$\mathbf{v}(t + dt) = \int_t^{t+dt} \frac{\mathbf{F}(t')}{m} dt' + \mathbf{v}(t) \quad (3)$$

$$\mathbf{x}(t + dt) = \int_t^{t+dt} \mathbf{v}(t') dt' + \mathbf{x}(t) \quad (4)$$

The use of this theory assumes that we do not have relativistic speeds or masses and that the maximum grid size is small enough that expansion of the universe can be neglected.

One approach would be to try to calculate the particle-particle interactions but this is not computationally feasible since we would have 2^N interactions where N is the particle number and we are using thousands of particles.

2.2 Numerical methods

The simulation is set up with respect to a three dimensional grid that supports particles (Fig. 1).

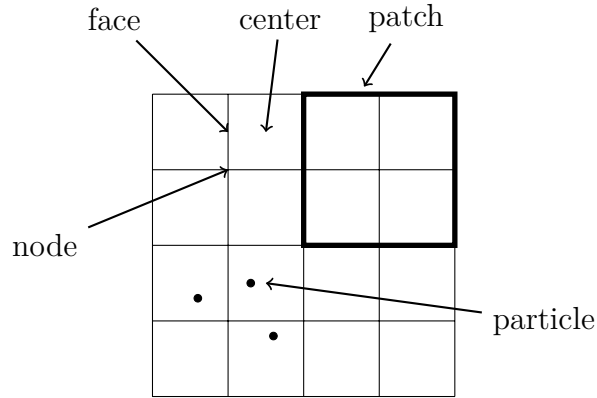


Figure 1. A two dimensional view of the grid with its various components and properties shown.

The main algorithm is expressed in Algorithm 1.

Algorithm 1. Main program.

```

set Dirichlet or periodic boundary conditions
set initial  $\mathbf{x}_p$ ,  $\mathbf{v}_p$  and  $m_p$  for all particles  $p$ 
interpolate initial  $\rho$  from particle mass  $m_p$ 
set initial guess for  $\phi$ 
loop
    solve  $\phi$  at the nodes using the SOR algorithm
    calculate  $\mathbf{F} = -\nabla\phi$  with respect to each particle  $p$ 
    for every particle  $p$  do
         $\mathbf{v}_p(t + \Delta t) = (\mathbf{F}_p/m)\Delta t + \mathbf{v}_p(t)$ 
         $\mathbf{x}_p(t + \Delta t) = \mathbf{v}_p(t)\Delta t + \mathbf{x}_p(t)$ 
    end for
    interpolate new  $\rho$  from particle mass  $m_p$ 
     $t \leftarrow t + \Delta t$ 
end loop

```

The Poisson's equation of gravity is discretized to

$$\begin{aligned}
4\pi G\rho_{i,j,k} = & \frac{\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}}{(\Delta x)^2} + \\
& \frac{\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}}{(\Delta y)^2} + \\
& \frac{\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}}{(\Delta z)^2}.
\end{aligned} \tag{5}$$

The gravitational potential ϕ is then solved at each node using the successive over-relaxation (SOR) algorithm (Algorithm 2). In the calculation of the potential gradient we obtain the potential values near the particles by interpolation and then calculate the gradient by numerical differentiation. For example at the p th particle the gradient is calculated in the x direction as

$$(\nabla\phi)(x_p) = \frac{\phi(x_p + dx) - \phi(x_p - dx)}{2dx}, \tag{6}$$

where x_p is the particle's x coordinate and dx is a small distance.

The particle velocity and position are evolved over a small time step dt , assuming that the force remains constant. Essentially this means calculating

for each particle

$$= \mathbf{v}_p(t + \Delta t) = (\mathbf{F}_p/m)\Delta t + \mathbf{v}_p(t) \quad (7)$$

$$= \mathbf{x}_p(t + \Delta t) = \mathbf{v}_p(t)\Delta t + \mathbf{x}_p(t). \quad (8)$$

After this using the new positions the particle masses are interpolated back to the nodes to obtain a new mass density ρ .

Algoritihm 2. Calculating potential ϕ using the SOR algorithm.

```

function SOR( $\phi$ , tolerance, max_iterations)
  for  $n = 0, 1, \dots$  max_iterations do
    error  $\sigma \leftarrow 0$ 
    for every node  $\phi_{i,j,k}$ 
       $\phi_{i,j,k}^{(n+1)} \leftarrow (1 - \omega)\phi_{i,j,k}^{(n)} + \frac{\omega}{6}(\phi_{i+1,j,k}^{(n)} + \phi_{i-1,j,k}^{(n)} +$ 
         $\phi_{i,j+1,k}^{(n)} + \phi_{i,j-1,k}^{(n)} + \phi_{i,j,k+1}^{(n)} + \phi_{i,j,k-1}^{(n)} +$ 
         $h^3 \rho_{i,j,k})$ 
      update  $\sigma$ 
    end for
    if  $\sigma \leq$  tolerance, break
  end for
  return  $\phi$ 
end function

```

2.3 Initialization of the simultaion

The specific case we want to study with our simulation is a dark matter halo. Dark matter halos are structures composed of dark matter, believed to envelope galaxy disks. We initialize the halo by letting a sphere of randomly distributed dark matter particles to collapse by the effect of gravity.

2.4 Verification of results

To verify the simulation results we test for energy conservation, compliance with the virial theorem

$$\langle T \rangle_t = -\frac{1}{2}\langle V \rangle_t. \quad (9)$$

Previous N -body simulations suggest that dark matter halos follow the

Navarro-Frenk-White radial mass distribution

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}, \quad (10)$$

where r is distance from the center of the halo and (ρ_0, R) are parameters. As part of our verification procedure we fit this distribution to the simulated dark matter halo.

3 IMPLEMENTATION

3.1 Uintah

3.2 The program

4 RESULTS

5 CONCLUSIONS