RESEARCH ASSISTANT - CRYPTO

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Finance Section

2. (Please show your workings). Yara Inc is listed on the NYSE with a stock price of $40 - the company is not known to pay dividends. We need to price a call option with a strike of $45 maturing in 4 months. The continuously-compounded risk-free rate is 3%/year, the mean return on the stock is 7%/year, and the standard deviation of the stock return is 40%/year. What is the Black-Scholes call price?

Solution

The Black-Scholes call price is given by

$$C(S,T) = SN(x_1) - BN(x_2)$$

S is the current stock price

B is the bond price given by Xe^{-Tr_f} , where r_f is the continuously-compounded risk free rate,

N(x) is the cumulative normal distribution

$$x_1 = \frac{log(\frac{S}{B})}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$$

and

$$x_2 = \frac{log(\frac{S}{B})}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}$$

σ is the standard deviation of stock returns

T is the time-to-maturity

log is the natural logarithm

From question, S = $40

$B = 45e^{-.03(1/3)}$ ≈ **44.55224**

Where ⅓ is the time to maturity (4/12 months)

$$x_1 = \frac{log(\frac{40}{44.55224})}{0.4\sqrt{1/3}} + \frac{1}{2}0.4\sqrt{1/3}$$

x_1 **≈ -0.3512442**

$$x_2 = \frac{log(\frac{40}{44.55224})}{0.4\sqrt{1/3}} - \frac{1}{2}0.4\sqrt{1/3}$$

x_2 **≈ -0.5821843**

To determine $N(X_1)$ and $N(X_2)$, we use the standard normal distribution table.

N(-0.3512442) and N(-0.5821843) cannot be gotten directly from the table, hence we need to interpolate.
For $N(X_1)$, we interpolate between -0.35 and -0.36
For $N(X_2)$, we interpolate between -0.58 and -0.59

Using the interpolation formula

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{x_2 - x_1}$$

Where y = N(x)

So, for $N(-0.3512442)$,

y_1 = N(-0.35) = 0.36317 (from standard normal distribution tables)

y_2 = N(-0.36)= 0.35942 (from standard normal distribution tables)

x_1= 0.35

x_2= 0.36

x= -0.3512442

Substituting into the interpolation formula gives

$$N(-0.3512442) = 0.36317 + \frac{(-0.3512442-(-0.35))*(0.35942-0.36317)}{-0.36-(-0.35)}$$

y = **0.362703**

for $N(-0.5821843)$,

y_1 = N(-0.58) = 0.28096 (from standard normal distribution tables)

y_2 = N(-0.59)= 0.27760 (from standard normal distribution tables)

x_1= 0.58

x_2= 0.59

x= -0.5821843

Substituting into the interpolation formula gives

$$N(-0.5821843) = 0.28096 + \frac{(-0.5821843-(-0.58))*(0.27760-0.28096)}{-0.59-(-0.58)}$$

y**= 0.280226**

$N(x_1)$ = 0.362703

$N(x_2)$= 0.280226

To find the Black-Scholes call price , we evaluate

$$C(S,T) = SN(x_1) - BN(x_2)$$

$$C(S,T) \approx (40 \times 0.362703) - (44.55224 \times 0.280226)$$

$$C(S,T) \approx \$2.023$$

The Black-Scholes call price is $2.023