Math Camp: PS 1 Logic and Sets

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1. **Using truth tables, prove both of DeMorgan's Laws for logical connectives.**
   (a) $\neg(P \wedge Q)$ **is logically equivalent to** $\neg P \vee \neg Q$

P	Q	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	**F**	**F**
T	F	F	T	**T**	**T**
F	T	F	F	**T**	**T**
F	F	F	T	**T**	**T**

   The bolded columns are identical, thus $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$.

   (b) $\neg(P \vee Q)$ **is logically equivalent to** $\neg P \wedge \neg Q$

P	Q	$\neg P$	$\neg Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
T	T	F	F	**F**	**F**
T	F	F	T	**F**	**F**
F	T	F	F	**F**	**F**
F	F	F	T	**T**	**T**

   The bolded columns are identical, thus $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$.

2. **Find the contrapositive and converse of each of the following statements:**
   (a) **"If squares have four sides, then triangles have four sides."**
   - Contrapositive: "If triangles do not have four sides, then squares do not have four sides."
   - Converse: "If triangles have four sides, then squares have four sides."

   (b) **"A sequence a is bounded whenever a is convergent."**
   - Rewritten in $p \Rightarrow q$ form: "If a sequence a is convergent, then a is bounded."
   - Contrapositive: "If sequence a is not bounded, then a is not convergent."
   - Converse: "If sequence a is bounded, then a is convergent."

   (c) **"The differentiability of a function f is sufficient for f to be continuous."**
   - Rewritten in $p \Rightarrow q$ form: "If function f is differentiable, then f is continuous."
   - Contrapositive: "If function f is not continuous, then f is not differentiable."
   - Converse: "If function f is continuous, then f is differentiable."

3. **Let x and y be integers. Prove that if x and y are even, then $x + y$ is even.**
   To show: $x + y$ is even.

Proof:

Let x and y be even integers.	(by hypothesis)
$\Rightarrow \exists i, j \in \mathbb{Z} \ni x = 2i \wedge y = 2j$	(by definition of even)
$\Rightarrow x + y = 2i + 2j$	(substitution)
$\Rightarrow x + y = 2(i + j)$	(distributivity)
$\Rightarrow \exists k \in \mathbb{Z} \ni k = i + j$	(closure of integer addition)
$\Rightarrow x + y = 2k$	(substitution)
$\Rightarrow x + y$ is even $\blacksquare$	(by definition of even)

4. **Let A and B be sets. Prove that $A \subset B$ if and only if $A - B = \emptyset$.** By contrapositive, $A - B \neq \emptyset \iff A \not\subset B$
Proof by contraposition, to show: $A \not\subset B$, and then back to $A - B \neq \emptyset$.
Proof:

Let A and B be sets such that $A - B \neq \emptyset$	(toward contrapositive)
$\iff \exists C \neq \emptyset \ni A - B = C$	(definition of not empty set)
$\iff \exists x \in C \ni x \in A \wedge x \notin B$	(definition of set subtraction)
$\iff A \not\subset B \blacksquare$	(by definition of subset)

While I believe each of these steps should be biconditional, it seems like by creating a set C to contain an element x (on the way toward $A \not\subset B$) may not be so reversible, since moving upward I would invoke $\exists x \in C$ before $\exists C$. Maybe I can skip invoking set C entirely? Looking forward to the answer key!