Chapter 63

THE EFFECT OF MESSAGE SPACE SIZE ON LEARNING AND OUTCOMES IN SENDER–RECEIVER GAMES

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1. Introduction

This chapter uses experiments to investigate learning and outcomes in sender–receiver games with imperfect incentive alignment. In sender–receiver games, the sender sends a message to a receiver who then takes an action that affects the payoffs of both players. Payoffs depend only on the sender's private information, represented by his type, and the receiver's action. We say that these games are of "partial common interest" if the sender wants to reveal some but not all of his private information. Using "a priori" meaningless messages in such games, we are interested in how agents attach meaning to these messages. Specifically, we ask: What are the outcomes? What is the learning process? Do outcomes and learning vary with the size of the message space?

Recent advances in evolutionary game theory allow us to address the interplay between private information, the size of the message space, and outcomes in sender–receiver games. In games with "a priori" meaningless messages, learning is necessary to reach those outcomes. We consider games of partial common interest because of the saliency of the message space size in such games.

2. The Games

Evolutionary game theory focuses on an environment in which there are repeated interactions among a population of agents. An individual game is played between a randomly paired informed sender and an uninformed receiver. The sender's private information is his type, t, where each type is equally likely. The sender sends a message m to the receiver, who responds with an action, a. Payoffs to both players depend on the sender's private information and the receiver's action, but not on the message sent. Repeated interactions occur by randomly pairing the sender and receiver populations to play an individual game in each iteration.

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	a_1	a_2
t_1	0, 0	700, 700
t_2	700, 700	0, 0

Figure 1. Game 1.

	a_1	a_2	a_3	a_4
t_1	0, 0	200, 700	400, 400	0, 0
t_2	200, 700	0, 0	400, 400	0, 0
t_3	0, 0	0, 0	0, 0	400, 400

Figure 2. Game 2.

A strategy for the sender maps types into messages; for the receiver, a strategy maps messages into actions. A strategy pair is a Nash equilibrium if the strategies are mutual best replies. An equilibrium is called "separating" if each sender type is identified through his message. In a pooling equilibrium, the equilibrium action does not depend on the sender's type. Such an equilibrium exists in every sender–receiver game.

Figure 1 presents the payoffs (sender, receiver) for a simple common interest game, Game 1, that is played by all agents in our experiments. The payoffs are a function of the sender's type and the receiver's action. An example of a separating equilibrium in this game is one where the sender sends m_1 if he is type t_1 and m_2 otherwise, and where the receiver takes action a_2 after message m_1 and a_1 otherwise. An example of a pooling equilibrium is one in which the sender, regardless of type, sends m_1 and the receiver always takes action a_1.

In Game 1, both senders and receivers prefer that all information be revealed. Thus, if players are repeatedly and randomly matched to play Game 1 with a message space of size two, $\#(M) = 2$, observed play should converge to an efficient separating equilibrium. This prediction is based on the intuition contained in the evolutionary approaches of Canning (1992), Nöldeke and Samuelson (1993), Wärneryd (1993), and Blume, Kim, and Sobel (1993).

In games where the sender wants to reveal some but not all of his information the size of the message space may affect the predicted equilibrium outcome. Consider the partial common interest game, Game 2, with payoffs shown in Figure 2.

While there is a fully separating equilibrium (with at least three messages), the sender prefers an equilibrium in which two of his types (t_1 and t_2) send a common message and the third type, t_3, separates. The treatment variable, "message space size," systematically changes the nature of the game as it varies from smaller, to equal to, and to greater than the size of the type space. With two messages there is a strict equilibrium; two types, t_1 and t_2, send a common message and the third type, t_3, separates. There is also another equilibrium component that is essentially strict; only the sender is indifferent in this equilibrium component and only for one of his types. Specifically, either t_1 or t_2 separates from the other two types, t_2 and t_3 or t_1 and t_3, in the game. With three messages there is a unique strict equilibrium; this equilibrium is fully separating. Finally, with four (or more) messages there is a plethora of equilibria and none are strict.

We appeal to the intuition from evolutionary game theory (e.g., Nöldeke and Samuelson, 1993; Wärneryd, 1993; and Blume, Kim, and Sobel, 1993) to hypothesize the following outcomes in Game 2: observed play will converge to the strict equilibrium with a message space of size two; observed play will converge to the unique strict, fully separating equilibrium with a message space of size 3; and, with a message space of size 4, observed play will converge to the partial pooling equilibrium with types t_1 and t_2 sending a common message and separating from type t_3.

To ensure that initially there is no common understanding of messages among agents, each agent is endowed with his own representation of the message space. At the end of each iteration of the game, all agents receive information about sender types and all messages sent by the respective sender types. This information is displayed in terms of each agent's representation of the message space. From evolutionary theory, this is the population history needed by agents to assign meaning to the "a priori" meaningless messages.

We conduct three treatments, and each treatment consists of three replications. Further, each replication involves twelve players, six senders and six receivers. Players are randomly designated as either a sender or a receiver at the start of the experiment and keep their designation throughout. In each period of a game, senders and receivers are paired using a random matching procedure. Sender types are equally likely and independently and identically drawn in every period for each sender. Senders and receivers participate in two sessions. Session I is common to all treatments and replications; it consists of playing Game 1 for 20 periods with a message space of size two. Session II varies across the three treatments; it consists of playing Game 2 for 40 periods with a message space size of either two, three, or four.

3. Results

3.1. Game 1

Prior to playing Game 2, all participants play 20 periods of Game 1. Game 1 is included as part of the experimental design to familiarize participants with sender–receiver games and the learning environment. Our intention is to make the learning task more manageable in Game 2, where both the incentive structure and message space structure are more complex.

The essential properties of the Game 1 outcomes (type–action associations) are shown in Table 1. Notice from this table that for each of the nine replications of Game 1, the outcome is very close to full separation, and in four out of nine replications play over the final five periods is consistent with a fully separating equilibrium. Across the nine replications of Game 1, 89% of the type–action pairs in the last five periods are consistent with the separating equilibrium. In the final period alone this coordination proportion increases to 93%, and in six of the nine replications play in the final period is consistent with a fully separating equilibrium.

Figure 3 provides some insight about the dynamic (learning) process that led to the outcomes presented in Table 1. Notice from this figure that convergence to the efficient separating equilibrium is gradual, suggesting that it takes time for senders to learn to fully reveal their type through the message sent and receivers to respond to the information contained in senders' messages (see, e.g., Blume et al., 1998, 2001). This result is of particular interest because rational players could, in most instances, use play in the first period as a precedent that would allow them to perfectly coordinate in period 2. More specifically, the gradual adjustment process presented in Figure 3 is incompatible with Cournot adjustment and fictitious play (examples of adaptive dynamics) as well as optimal learning as in Crawford and Haller (1990) (an example of rational learning).

3.2. Game 2

We conduct three treatments of Game 2 where each treatment consists of three replications. The treatments involve having participants play Game 2 for 40 periods where the sender's message space is either of size two, #(M) = 2, three, #(M) = 3, or four, #(M) = 4.

The essential properties of the Game 2 outcomes (type–action associations) are presented in Table 2. Notice from this table that for each of the three replications of Game 2 with #(M) = 2, the outcome is most closely aligned with a partial pooling equilibrium (where t_1 and t_2 separate from t_3), and across the three replications, 73% of the type–action pairs in the last five periods are consistent with the partial pooling equilibrium. For both Game 2 with #(M) = 3 and #(M) = 4, the outcome is most closely aligned with a fully separating equilibrium, and across the three replications of each treatment, 59% and 60% of the type–action pairs in the last five periods are consistent with a fully separating equilibrium, respectively. However, notice that there is also a significant incidence of partial pooling play in both treatments; 46% (36%) for the four (three) message treatment. Finally, notice that in all three treatments of Game 2 there is a non-trivial proportion of non-equilibrium play in the final five periods.

Table 3 reports the messages that were sent by the various sender types over the last five periods for each replication of each treatment of Game 2. Consistent with the outcomes presented in Table 2, notice from Table 3 that across the three replications of Game 2 with #(M) = 2, sender types 1 and 2 essentially send a common message and sender type 3 separates by using the other message. Further notice that for both Game 2 with #(M) = 3 and #(M) = 4, 79% of senders are fully identified through the messages they send by the end of the game. This latter result, combined with the outcomes presented in Table 2, suggest that in the three and four message space treatments of Game 2, receivers had not fully responded to (or learned) the information contained in senders' messages.

Table 1

This table reports, aggregated over the final five periods, which actions were taken by receivers conditional on sender type for each of the nine replications of Game 1. For example, under "Rep 2" we learn that in the final five periods of the second replication of Game 1, type t_1 was drawn 14 times and received the preferred action a_1 12 times. Notice that for each of the nine replications of Game 1 the outcome is very close to full separation, and in four of the nine replications play over the final five periods is consistent with a fully separating equilibrium

Game 1

Type-Action Associations for the Final Five Periods

Sender's Type

Receiver's Action

Replication #

	t_1		t_2	
	a_1	a_2	a_1	a_2
Rep 1	0	13	17	0
Rep 2	2	12	12	4
Rep 3	5	15	6	4
Rep 4	6	8	14	2
Rep 5	1	14	14	1
Rep 6	2	15	12	1
Rep 7	0	12	18	0
Rep 8	0	15	15	0
Rep 9	0	24	6	0
Total:	**16**	**128**	**114**	**12**

Notice that across the nine replications of Game 1, 89% of the type-action pairs in the last five periods are consistent with the separating equilibrium.

This common interest game, *Game 1*, is played by all participants in our experiments (prior to playing Game 2). Notice that both senders and receivers prefer that all information be revealed. Thus, if players are repeatedly and randomly matched to play this game where the sender's message space is of size two, #(M)=2, play should converge to an efficient separating equilibrium.

GAME 1

	a_1	a_2
t_1	0,0	700,700
t_2	700,700	0,0

Payoffs to the sender, receiver. Notice that payoffs are a function of the sender's type and the receiver's action.
Color Legend:
Blue = Separating Equilibrium.
Red = Other Outcomes.

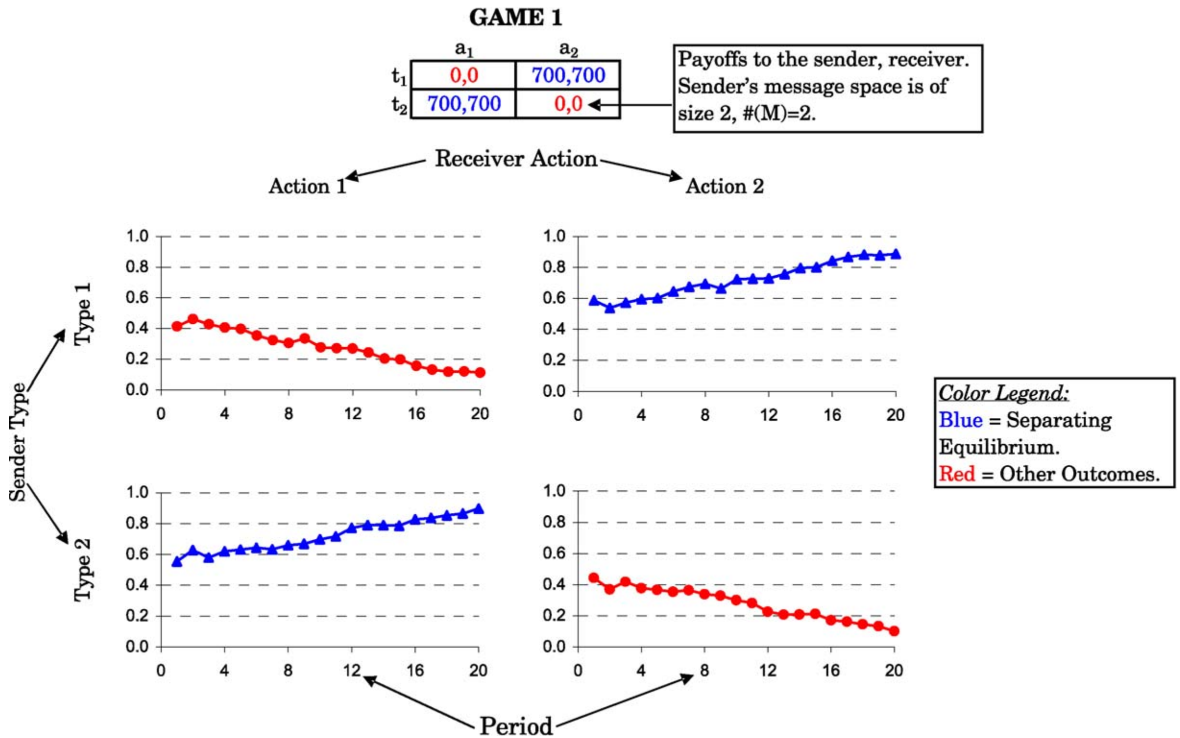

Figure 3. For the nine replications of Game 1 this figure presents the time paths that led to the outcomes presented in Table 1. Specifically, for each replication of Game 1 we first measure, over five-period intervals, the frequency of play (i.e., the # of a_i, t_i's) in each of the four cells of Game 1. Next, we divide each of these frequencies by their respective frequency of t_i's over these five-period intervals to measure, for each sender type, the proportion of play of each action. Finally, for purposes of parsimony and given the similarity of the outcomes over replications, these proportions are averaged across replications. Notice from this figure that convergence to the final separating outcome is a gradual process. This is particularly interesting because rational players could, in most instances, use play in the first period as a precedent that would allow them to perfectly coordinate in period 2.

Table 2

This table reports, aggregated over the final five periods, which actions were taken by receivers conditional on sender type for each replication of each Game 2 treatment. For example, under '#(M) = 2: Rep 1' we learn that in the final five periods of the first replication of Game 2 with two messages, type t_1 was drawn 6 times and received action a_3 4 times; type t_2 was drawn 11 times and received action a_3 7 times; and, type t_3 was drawn 13 times and received action a_4 13 times. Notice that for each of the three replications of Game 2 with #(M) = 2, the outcome is most closely aligned with a partial pooling equilibrium (where t_1 and t_2 separate from t_3). For Game 2 with #(M) = 3 and #(M) = 4, terminal play across the three replications of each treatment is most consistent with a separating outcome, 59% and 60%, respectively. However, also notice that the incidence of partial pooling play is 36% and 46% in the three and four message space treatments, respectively. Finally, notice that in all three treatments there is a non-trivial proportion of non-equilibrium play in the final five periods

Game 2

Type-Action Associations for the Final Five Periods

Sender's type

Receiver's action

Size of the sender's message space.

Replication #

	t_1				t_2				t_3			
	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
#(M)=2												
Rep 1	1	1	4	0	1	3	7	0	0	0	0	13
Rep 2	0	0	8	0	1	1	8	1	0	1	2	8
Rep 3	0	1	6	1	2	0	6	2	1	0	5	6
Total:	**1**	**2**	**18**	**1**	**4**	**4**	**21**	**3**	**1**	**1**	**7**	**27**
#(M)=3												
Rep 1	4	3	0	2	5	2	0	2	1	4	2	5
Rep 2	1	6	2	1	1	1	5	1	0	1	4	7
Rep 3	0	7	1	0	10	0	3	0	0	0	0	9
Total:	**5**	**16**	**3**	**3**	**16**	**3**	**8**	**3**	**1**	**5**	**6**	**21**
#(M)=4												
Rep 1	4	5	2	0	4	4	2	0	0	0	0	9
Rep 2	3	6	5	0	3	4	1	0	0	0	1	7
Rep 3	1	3	3	0	9	2	4	0	0	0	0	8
Total:	**8**	**14**	**10**	**0**	**16**	**10**	**7**	**0**	**0**	**0**	**1**	**24**

This partial common interest game, *Game 2*, is played by all participants in our experiments (after playing Game 1). We conduct three treatments of Game 2 where the sender's message space is either of size 2, 3, or 4. Predictions regarding equilibrium outcomes vary depending on the size of the sender's message space.

GAME 2

	a_1	a_2	a_3	a_4
t_1	0,0	200,700	400,400	0,0
t_2	200,700	0,0	400,400	0,0
t_3	0,0	0,0	0,0	400,400

Payoffs to the sender, receiver. Notice that payoffs are a function of the sender's type and the receiver's action.

Color Legend:

Blue + Purple = Separating Equilibrium.

Green + Purple = Partial Pooling Equilibrium.

Red = Other Outcomes.

Table 3

This table reports, aggregated over the final five periods, which messages were sent by various sender types for each replication of each Game 2 treatment. Messages are presented in terms of the underlying message space used to construct each player's representation of the message space. For example, under '#(M) = 2: Rep 1' we learn that in the final five periods of the first replication of Game 2 with two messages, type t_1 was drawn 6 times and sent the message 'B' 6 times; type t_2 was drawn 11 times and sent the message 'B' 11 times; and, type t_3 was drawn 13 times and sent the message 'A' 13 times. Notice that for each of the three replications of Game 2 with #(M) = 2, t_1 and t_2 separate from t_3 by using a common message distinct from sender type 3's message. For Game 2 with #(M) = 3 and #(M) = 4, notice that across the three replications of each treatment, 79% of senders are fully identified through the messages they send

Game 2

Type-Message Associations for the Final Five Periods

Sender's type →	t_1				t_2				t_3			
Sender's message →	A	B	C	D	A	B	C	D	A	B	C	D
#(M)=2												
Rep 1	0	6	*	*	0	11	*	*	13	0	*	*
Rep 2	0	8	*	*	1	10	*	*	8	3	*	*
Rep 3	6	2	*	*	5	5	*	*	3	9	*	*
#(M)=3												
Rep 1	5	1	3	*	0	4	5	*	2	0	10	*
Rep 2	6	2	2	*	1	5	2	*	2	0	10	*
Rep 3	8	0	0	*	0	13	0	*	0	0	9	*
#(M)=4												
Rep 1	7	3	0	1	4	3	0	3	0	0	9	0
Rep 2	0	0	2	12	4	0	2	2	0	8	0	0
Rep 3	1	0	0	6	14	0	0	1	0	8	0	0

Size of the sender's message space.

Replication #

* = Message not available in this treatment of Game 2

Messages are colored according to the receiver's best response to the posterior distribution derived from the message frequencies over the last five periods.

This partial common interest game, *Game 2* , is played by all participants in our experiments (after playing Game 1). We conduct three treatments of Game 2 where the sender's message space is either of size 2, 3, or 4.

GAME 2

	a_1	a_2	a_3	a_4
t_1	0,0	200,700	400,400	0,0
t_2	200,700	0,0	400,400	0,0
t_3	0,0	0,0	0,0	400,400

Payoffs to the sender, receiver. Notice that payoffs are a function of the sender's type and the receiver's action.

Color Legend:

Blue + Purple = Separating Equilibrium.

Green + Purple = Partial Pooling Equilibrium.

Red = Other Outcomes.

Figures 4, 5, and 6 provide some insight about the learning process that led to the outcomes presented in Table 2. Notice from these figures that convergence to the observed equilibrium, across each treatment of Game 2, is both gradual and incomplete. For example, in Game 2 with #(M) $=$ 3 and #(M) $=$ 4, one might suspect that t_3, a_4 coordination would be quick because incentives are aligned for this type and because the cardinality of the message space is at least as large as the cardinality of the type space. Accordingly, senders could use one available message to identify t_3 and receivers, knowing this, should respond optimally with a_4. However, as the figures show, this convergence is by no means instantaneous. Second, notice from Figures 4–6 that, compared to Game 1, there is more noise in the Game 2 data, particularly with regard to play involving sender types 1 and 2, suggesting that resolving the conflict between senders and receivers in this part of the game is not a trivial matter.

To summarize, our Game 2 results provide some evidence that the size of the message space influences equilibrium selection. Specifically, we find that with a message space of size two, observed play is most closely aligned with a partial pooling equilibrium (where t_1 and t_2 separate from t_3), but with a message space size of three or four, observed play is most closely aligned with a fully separating equilibrium. Further, in both the three and four message treatments of Game 2, we observe a substantial amount of partial pooling play, with the four message treatment exhibiting a higher frequency of partial pooling play than the three message treatment (as suggested by theory).

Thus, while our results suggest that, at least for the message space sizes considered, the equilibrium most frequently selected is not affected by the size of the message space (as long as the size of the message space is at least as large as the size of the type space), this finding should be interpreted with some caution for a number of reasons. First, the presence of Game 1 may have prodded players towards a higher frequency of separation in Game 2 with #(M) $=$ 3 and #(M) $=$ 4. Specifically, because players experienced separation in Game 1, they may have carried over their efforts to separate in Game 2 when the size of the message space allowed them to do so. Second, in another paper, Blume et al. (1998) find that in games of divergent interest, the size of the message space influences whether the final outcome is pooling or separating. Essentially, they show that when the size of the message space equals (exceeds) the size of the type space, the final outcome is more likely to be separating (pooling). Accordingly, the current results may only generalize to games of partial common interest. In other words, because senders in our partial common interest game, Game 2, want to experience separation for one of their types (type 3), experiencing separation with this type may prod players to pursue separation with the other types as long as the size of the message space allows them to fully separate. Finally, compared to the divergent interest games considered by Blume et al. (1998), the partial common interest game we consider is more complex (i.e., there are more types and actions as well as equilibria) and, in addition, because both games were conducted over the same number of periods, we have fewer observations per type than they had. As a result, it is possible that extending the number of periods participants play might affect equilibrium selection in the direction suggested by theory.

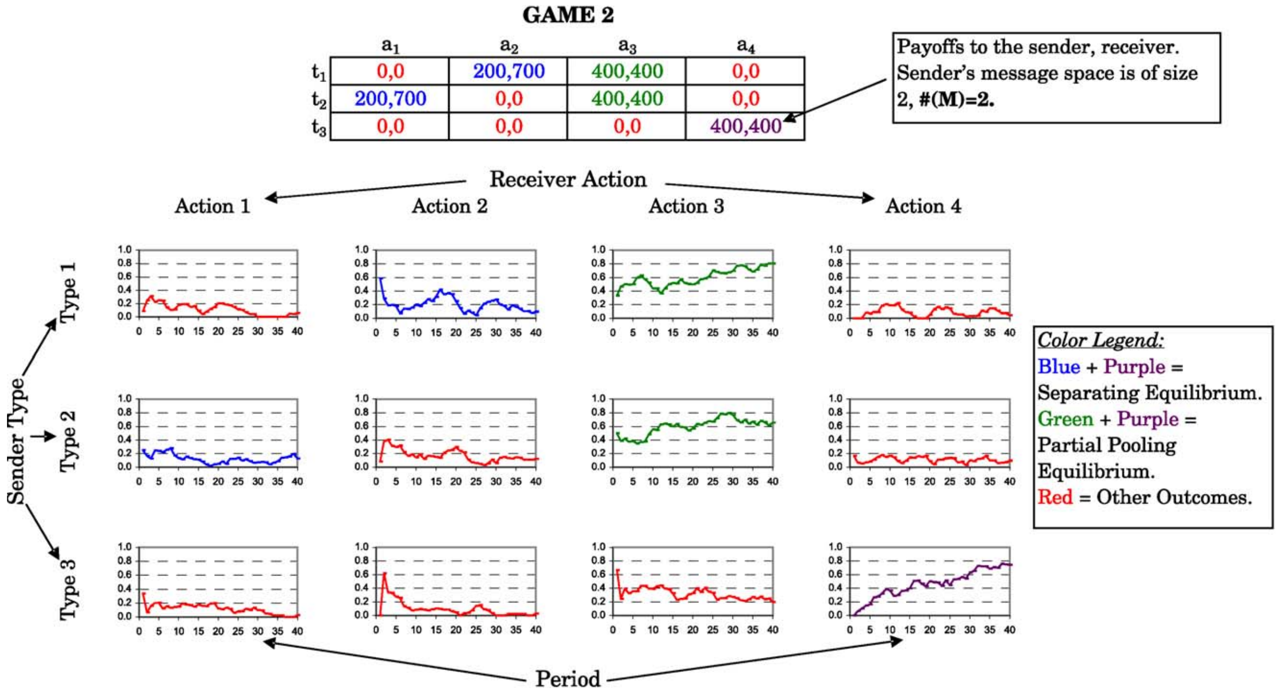

Figure 4. For the three replications of Game 2 with a message space of size 2, this figure presents the time paths that led to the outcomes presented in Table 2. Specifically, for each replication of Game 2 with #(M) = 2, we first measure, over five-period intervals, the frequency of play (i.e., the # of a_i, t_i's) in each of the twelve cells of Game 2. Next, we divide each of these frequencies by their respective frequency of t_i's over these five-period intervals to measure, for each sender type, the proportion of play of each action. Finally, for purposes of parsimony and given the general similarity of the outcomes over replications, these proportions are averaged across replications. Notice from this figure that while behavior is most consistent with partial pooling (where types t_1 and t_2 separate from t_3) there is persistent non-equilibrium play (i.e., convergence to this equilibrium is not complete). Further notice that convergence to the partial pooling equilibrium is gradual.

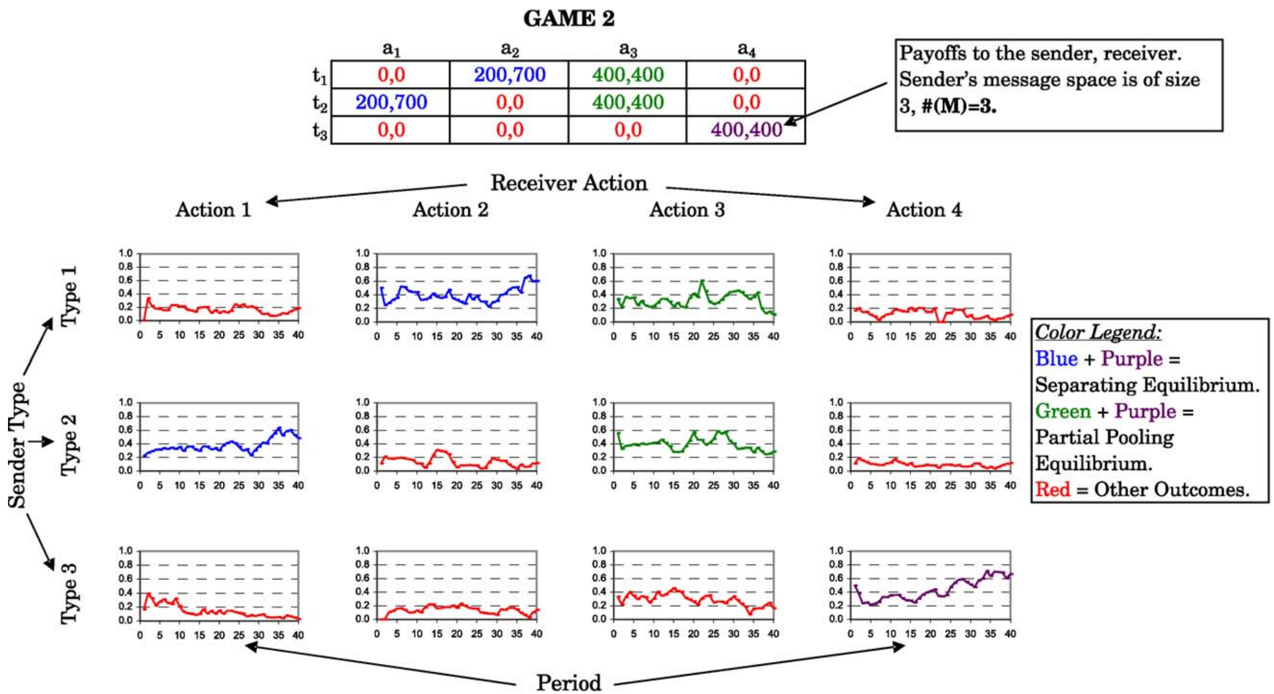

Figure 5. For the three replications of Game 2 with a message space of size 3, this figure presents the time paths that led to the outcomes presented in Table 2. Specifically, for each replication of Game 2 with #(M) = 3, we first measure, over five-period intervals, the frequency of play (i.e., the # of a_i, t_i's) in each of the twelve cells of Game 2. Next, we divide each of these frequencies by their respective frequency of t_i's over these five-period intervals to measure, for each sender type, the proportion of play of each action. Finally, for purposes of parsimony and given the rough similarity of the outcomes over replications, these proportions are averaged across replications. Notice from this figure that while terminal behavior is most consistent with separation, there is both a fairly high incidence of partial pooling play and some persistent non-equilibrium play. Compared to Game 2 with #(M) = 2, however, observed separating (partial pooling) play is significantly higher (lower) in Game 2 with #(M) = 3.

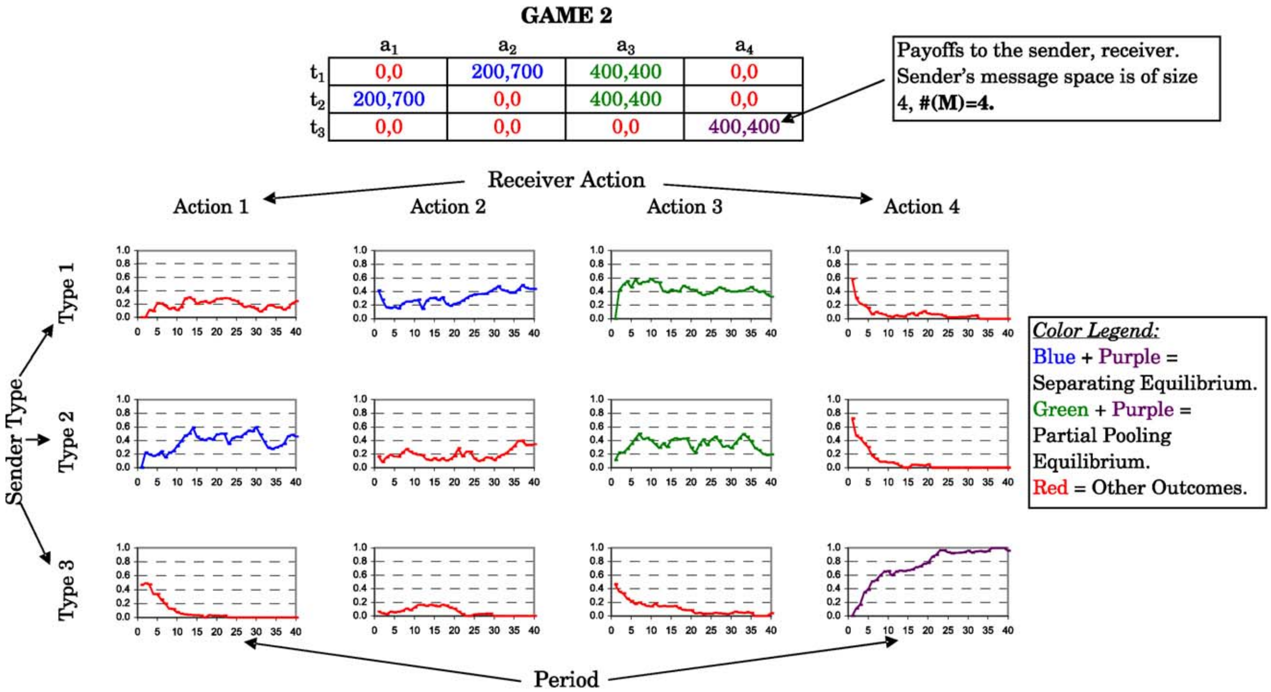

Figure 6. For the three replications of Game 2 with a message space of size 4, this figure presents the time paths that led to the outcomes presented in Table 2. Specifically, for each replication of Game 2 with #(M) = 4, we first measure, over five-period intervals, the frequency of play (i.e., the # of a_i, t_i's) in each of the twelve cells of Game 2. Next, we divide each of these frequencies by their respective frequency of t_i's over these five-period intervals to measure, for each sender type, the proportion of play of each action. Finally, for purposes of parsimony and given the rough similarity of the outcomes over replications, these proportions are averaged across replications. Notice from this figure that the frequency of terminal play is higher for separation than partial pooling, but the frequency of partial pooling play is high. Compared to Game 2 with #(M) = 3, the separating (partial pooling) tendencies appear to be slightly lower (higher) in Game 2 with #(M) = 4.

4. Summary

As documented in the dynamics (figures) and outcomes (tables), agents learn to attach meaning to "a priori" meaningless messages. For the common interest game, Game 1, we observe the efficient separating outcome, although the learning process is gradual. For the partial common interest game, Game 2, the outcomes and dynamics vary with the size of the message space. With two messages, we observe a partial pooling outcome with a minimal amount of non-equilibrium play. Again, the dynamic adjustment (learning) process is gradual. For three and four messages, the highest frequency of play is the fully separating equilibrium, although there is a significant amount of partial pooling play in both treatments and, consistent with theory, there is more partial pooling play in the four message treatment than in the three message treatment. Finally, while there is less non-equilibrium play in the four message treatment than in the three message treatment, the dynamic adjustment (learning) process in both treatments is gradual and incomplete.

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