Chapter 51

COORDINATION FAILURE IN MARKET STATISTIC GAMES[1]

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1. Introduction

A central question in economics is how do markets coordinate the behavior of anonymous decision makers in a many person decentralized economy. Economic theory has traditionally addressed the question using the equilibrium method, which abstracts away from an important aspect of the general coordination problem, because it assumes an equilibrium. For abstract games, an equilibrium is defined as an assignment to each player of a strategy that is best for him when the others use the strategies assigned to them. The relevance of this abstract mutual consistency requirement for economic modeling is an open question, see Kreps (1990).

The requirement has two related problems: disequilibrium and coordination failure. First, the mutual consistency requirement of an equilibrium assignment is not an implication of individual rationality, but an additional strong assumption. Individual rationality means internal consistency and internally consistent beliefs and actions of different players may not be mutually consistent. In economies with stable and unique equilibrium points, the influence of inconsistent beliefs and actions would disappear over time, see Lucas (1987). The power of the equilibrium method derives from its ability to abstract from the complicated dynamic process that induces equilibrium and to abstract from the historical accident that initiated the process.

Second, there is often more than one equilibrium assignment. For example, multiple Pareto ranked equilibria arise in both macroeconomic models with production, search, or trading externalities and microeconomic models of monopolistic competition, technology adoption and diffusion, and manufacturing with non-convexities. These superficially dissimilar market and non-market models share the common property that a decision maker's best "level of effort" depends positively upon other decision makers' "level of effort." This property is called strategic complementarity in the coordination failure literature, see Cooper and John (1988). When these equilibria can be Pareto ranked it is possible for historical accident and dynamic process to lead to inefficient equilibria, that is, coordination failure. Consequently, understanding the origin of mutually consistent behavior is an essential complement to the theory of equilibrium points.

[1] Related research available at: http://econlab10.tamu.edu/JVH_gtee.

Handbook of Experimental Economics Results, Volume 1

DOI: 10.1016/S1574-0722(07)00051-0

The experimental method provides a tractable and constructive approach to the equilibrium selection problem. This chapter reviews experiments using a class of generic market statistic games with multiple equilibria, which are strictly Pareto ranked, and it reports experiments that provide evidence on how human subjects behave under conditions of strategic uncertainty. Strategic uncertainty exists when the players actions are not mutual knowledge.

A laboratory environment capturing the essential aspects of the mutual consistency problem in a many person decentralized economy must include three features: First, the environment must not assume away the problem by allowing an arbiter – or any other individual – to make common knowledge preplay assignments. Second, the environment must allow individuals little ability to unilaterally alter market outcomes. Finally, the environment must allow repeated interaction amongst the decision makers so that they have a chance to learn to coordinate.

For laboratory research, a tractable class of market processes with these features are market statistic games. Let x_{it} denote the action of player i in period t. An action combination is the vector of actions $x_t = (x_{1t}, \ldots, x_{nt})$ for the n players. A homogeneous action combination occurs when all players take the same action. An abstract market process is a mapping from the action space into a real number, the market outcome $y(x)$.

The market outcome could represent market thickness, industry production, average market price, aggregate demand, or aggregate supply. In the coordination failure literature, the mean of the players' actions is a common example of an abstract market process. As the number of players increases, the influence of an individual player on the mean goes to zero and in the limit an individual player cannot influence the market outcome.

Order statistics are an effective way to capture the anonymity of a many person economy without using enormous group sizes. The jth inclusive order statistic, m_j, is defined by $m_1 \leqslant m_2 \leqslant \cdots \leqslant m_n$, where the m_j are the x_i of action combination x arranged in increasing order. When $y(x) = m_j$ and $1 < j < n$, an actor contemplating defection from a homogeneous action combination cannot influence the market outcome.

Let OS$[n, j]$ denote the stage game of a finitely repeated order statistic game with n subjects and jth order statistic. Let the payoff function be such that an actor's unique best response to the market statistic $y(x)$ in the stage game is simply $x_i^* = y(x)$. This class of order statistic games has the property that any feasible homogeneous action combination is a strict equilibrium and depending on the payoff function these equilibria may or may not be ranked by efficiency.

2. Strategic Uncertainty and Coordination Failure

Van Huyck, Battalio, and Beil (1990) conducted an experiment based on Bryant (1983) Keynesian coordination game that systematically and consistently results in coordination failure. The period game was an OS[14 to 16, 1] defined by the following payoff

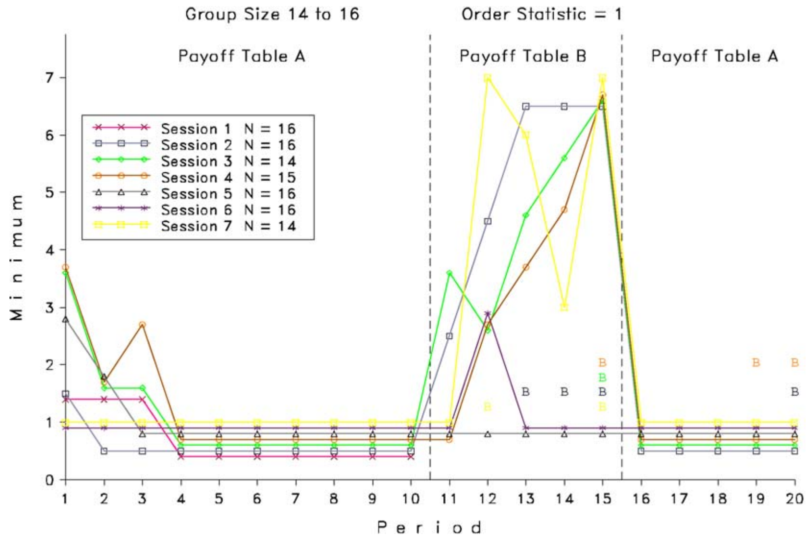

Figure 1. Results from Van Huyck, Battalio, and Beil (1990).

function and action space for each of 14 to 16 players:

$$\pi(x_i, m_1) = \$0.20m_1 - bx_i + \$0.60, \tag{1}$$

where m_1 is the inclusive minimum and b is a non-negative parameter less than \$0.20. Actions are restricted to the set of integers from 1 to 7. The players have complete information about the payoff function and strategy space and know that the payoff function and strategy space are common knowledge.

In treatment A and A', parameter b was set equal to \$0.10. Consequently, the payoff-dominant equilibrium $(7, \ldots, 7)$ paid \$1.30 while the secure equilibrium $(1, \ldots, 1)$ paid \$0.70 per subject per period. In treatment A, the period game was repeated ten times. The number of players varied between 14 and 16 subjects. Treatment A' designates the resumption of these conditions after treatment B.

In period one, the payoff-dominant action, 7, was chosen by 31 percent of the subjects and the secure action, 1, was chosen by 2 percent of the subjects. Neither payoff dominance nor security succeeds in organizing much of the data. The initial play of all seven sessions exhibit both disequilibrium and coordination failure. The minimum action for period one was never greater than 4, see Figure 1.

Repeating the period game does cause actions to converge to a stable outcome, but rather than converging to the payoff-dominant equilibrium or to the initial outcome of the treatment, the most inefficient market statistic obtains in all seven sessions, see Figure 1. By period ten 72 percent of the subjects adopt their secure action, 1, and the minimum for all seven experiments was a 1.

In treatment B, parameter b of Equation (1) was set equal to zero. Because a player's action is no longer penalized, the payoff-dominant action, 7, is a best response to all feasible minimums, that is, action 7 is a dominating strategy. Hence, treatment B tests equilibrium *refinements* based on the elimination of individually unreasonable actions. Strategic uncertainty should now cause an individually rational player to choose the payoff-dominant action, 7.

In period eleven, the payoff-dominant action, 7, was chosen by 84 percent of the subjects. However, the minimum in period eleven was never more than 4 and in sessions four, five, six, and seven it was a 1. Adaptive behavior converges to the efficient market statistic – the payoff-dominant equilibrium – in four of the six experiments. By period fifteen, 96 percent of the subjects chose the dominating and efficient action, 7.

Even in the sessions that obtained the efficient outcome, the B treatment was not sufficient to induce the groups to implement the payoff-dominant equilibrium in treatment A'. Returning to the original payoff table in period sixteen, 25 percent of the subjects chose the payoff-dominant action, 7. However, 37 percent chose the secure action, 1. This bi-modal distribution of actions suggests that play prior to period sixteen influenced subjects' behavior. However, the subjects exhibit a heterogeneous response to this history.

In treatment A', the minimum in all periods of all six experiments was 1. By period twenty, 84 percent of the subjects chose the secure action, 1, and 94 percent chose an action less than or equal to 2. Sessions two and four even satisfy the mutual best response property of an equilibrium by period twenty. Obtaining the efficient outcome in treatment B failed to reverse the observed coordination failure. Like the A treatment, the most inefficient outcome obtained.

This experiment provides an interesting example of coordination failure. The minimum was never above four in period one and all seven experiments converged to a minimum of one within four periods. Since the payoff-dominant equilibrium would have paid all subjects \$19.50 in the A and A' treatments and the average earnings were only \$8.80, the observed coordination failure cost the average subject \$10.70 in lost earnings.

3. The Influence of Out-of-Equilibrium Payoffs

Van Huyck, Battalio, and Beil (1991) report an experiment that replicates the coordination failure result of their large cohort minimum rule paper for smaller cohorts and a best response determined by the median, which they call average opinion games. The aspect of their experiment that we emphasize here is the influence of out-of-equilibrium payoffs on the frequency of observed coordination failure.

The period game was an OS[9, 5] defined by the following payoff function and action space for each of 9 decision makers indexed by i:

$$\pi(x_i, m_5) = \$0.10 m_5 - \$0.05[m_5 - x_i]^2 + \$0.60, \tag{2}$$

where $x_i \in \{1, 2, \ldots, 7\}$. A player's payoff is decreasing in the distance between the player's choice, x_i, and the inclusive median, m_5, and is increasing in the median, m_5.

All symmetric action combinations are equilibria and can be Pareto ranked. Like Van Huyck, Battalio, and Beil (1990), payoffs range from \$1.30 in the payoff dominant equilibrium $(7, \ldots, 7)$, to \$0.70 in the most inefficient equilibrium $(1, \ldots, 1)$. The secure equilibrium is $(3, \ldots, 3)$, which pays \$0.90 in equilibrium and insures a payoff of at least \$0.50.

Game Ω differs from Γ only in that disequilibrium outcomes result in a zero payoff. Unlike game, all actions are equally secure in game Ω, because they all insure a payoff of zero. Hence, security cannot be a salient equilibrium selection principle for game Ω, but payoff-dominance uniquely selects $(7, \ldots, 7)$ and, hence, is potentially salient.

Of the six cohorts, three cohorts had an initial median of 4 and three had an initial median of 5. The median never changed over the ten periods of treatment for any cohort. Behavior converged to a mutual best response outcome in five of six cohorts. The historical accident of the initial median selected the equilibrium and this equilibrium was never efficient.

Of the three Ω cohorts, two cohorts had an initial median of 7 and one had an initial median of 5. Again, the historical accident of the initial median selected the equilibrium, but now two of three cohorts coordinate on the payoff-dominant equilibrium. Setting out-of-equilibrium payoffs to zero increased the psychological salience of payoff-dominance.

4. The Influence of Group Size, Grid Size, and Order Statistic

Van Huyck, Battalio, and Rankin (1997) study coordination failure in market statistic games holding the payoff function constant but changing group size, grid size, and order statistic. Group size was 5 or 7. The order statistic was 2 or 4. Crossing these parameters gives four stage games: OS[5, 2], OS[5, 4], OS[7, 2], and OS[7, 4].

Equation (2), the payoff function in Van Huyck, Battalio, and Beil's (1991) Γ treatment, was used in all treatments. Since the sessions were designed to last forty rather than fifteen periods, the constant was reduced to \$0.20. The grid size was increased from the 7 actions in treatment Γ to 101 actions. This much finer grid was obtained by using their "blue box" interface, which allows a subject to quickly search the payoff space with a mouse. The action space was $e \in \{0, 1, \ldots, 100\}$ and the map used to determine payoffs was

$$x = 1 + 0.06e. \tag{3}$$

All homogeneous action combinations are strict equilibria and the equilibria can be Pareto ranked. In the payoff dominant equilibrium $(100, \ldots, 100)$ each player earns \$0.90 per period. In the secure equilibrium $(33, \ldots, 33)$ each player earns \$0.50. And in the least efficient equilibrium $(0, \ldots, 0)$ each player earns \$0.30.

The data in period 1 are particularly interesting, because they provide evidence on the salience of payoff-dominance and security. The median choice overall was 55. The median choice overall in VHBB's baseline treatment was 4.5 or 58.3. The mean action overall was 62 in both OS$[n, j]$ and VHBB's baseline treatment. So there exists a sense in which they succeeded in replicating the baseline conditions observed in VHBB despite a much finer action grid, smaller group sizes, and very different experimental methods. However, an analysis of variance conditioned on OS$[n, j]$ rejects the null hypothesis of equal treatment means at conventional levels of statistical significance. The group size and order statistic had a small influence on initial behavior.

Figure 2 reports the order statistic by treatment for periods 1 to 20. A significant number of cohorts converge to the efficient equilibrium, which was never observed using a coarser grid. The most striking examples occur in the average opinion treatment: OS[7, 4]. Using a finer grid reduces the salience of the initial market statistic. Now only half the OS[7, 4] cohorts coordinate on the initial market statistic. Session 8A is remarkable in that subjects coordinate on a time dependent play path. (This phenomena was also observed in four sessions during the crossover treatment.)

Adaptive behavior converges to a mutual best response outcome in 56 percent of the initial cohorts. Comparing terminal outcomes Van Huyck, Battalio, and Rankin (1997) found statistically significant differences in the empirically distribution function between OS[7, 4] and OS[7, 2] at the one percent level, between OS[5, 4] and OS[5, 2] at the five percent level, and between OS[5, 2] and OS[7, 2] at the ten percent level of statistical significance. They conclude that strategic uncertainty interacting with the order statistic had a larger effect on behavior than did group size.

The subjects' inability to solve the strategy coordination problem results in significant inefficiencies. Cohorts in treatment OS[5, 4] realized 85 percent of the efficient earnings, OS[7, 4] realized 76 percent, OS[5, 2] realized 65 percent, and OS[7, 2] realized 58 percent. So it is not true that a much finer approximation of a continuous action space eliminates coordination failure.

5. The Separatrix

All of the cohorts up to this point played in coordination games with the property that any homogeneous action combination was an equilibrium. Van Huyck, Cook, and Battalio (1997) investigate a market statistic game in which there only exist two symmetric equilibria: a high equilibrium $(12, \ldots, 12)$ and a low equilibrium $(3, \ldots, 3)$. The group size was 7 and the order statistic was 4. The separatrix between the two symmetric equilibria derived under a best response based dynamic is illustrated in Figure 3.

All ten cohorts started with an initial median contained in the set $\{7, 8, 9, 10, 11\}$; after period six none of the observed medians were contained in the set $\{7, 8, 9, 10, 11\}$; moreover, the median never crossed the separatrix, that is, subjects trapped in the low equilibrium's basin of attraction never escaped, see Figure 3. While best response based

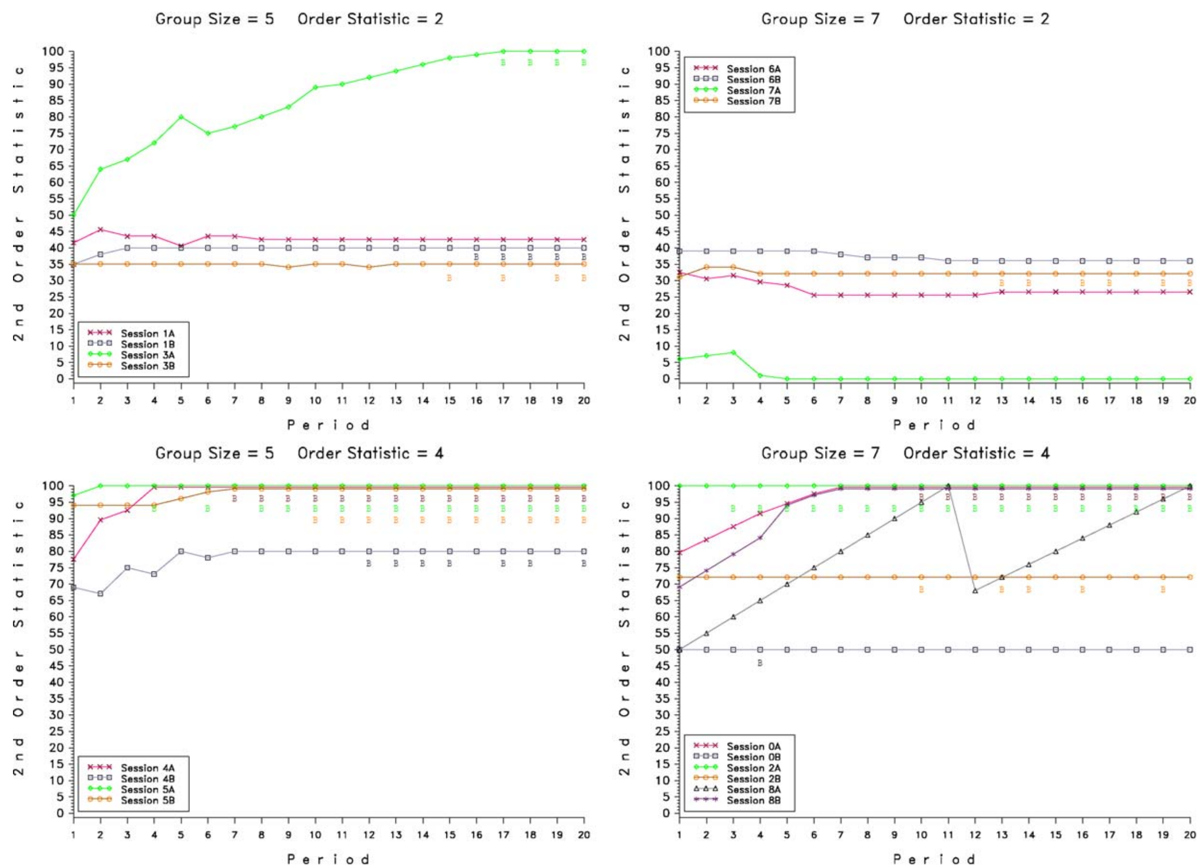

Figure 2. Results from Van Huyck, Battalio, and Rankin (1997).

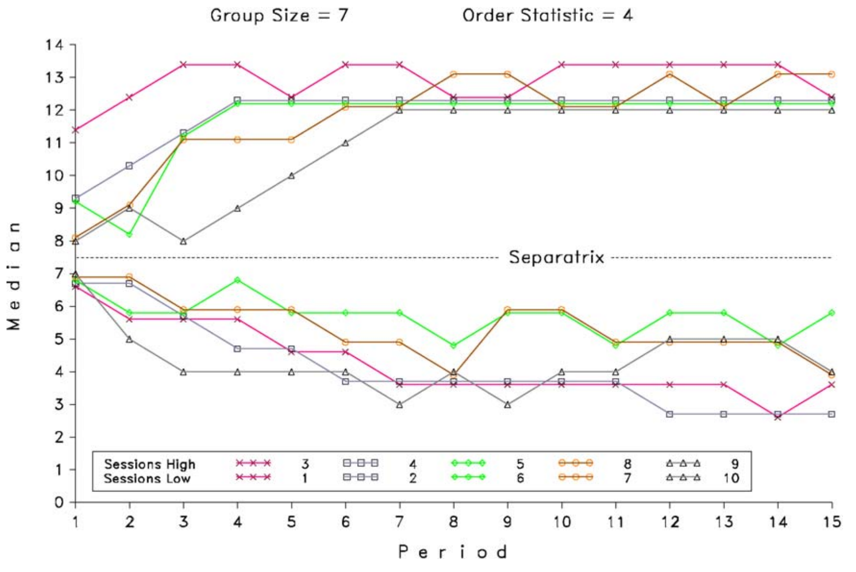

Figure 3. Results from Van Huyck, Cook, and Battalio (1997).

dynamics are symmetric around the separatrix, adaptive behavior is biased towards efficiency, that is, cohorts move to more efficient outcomes quickly, but resist the adaptive dynamics when moving to more inefficient outcomes.

The average subject in the first fifteen periods of the five low sessions earned $9.71. The average subject in the first fifteen periods of the five high sessions earned $15.57. The small differences in the distribution of subjects' period one choices result in large differences in average earnings. Specifically, the average subject in a high session earns about $6 (or 60 percent) more than the average subject in a low session.

Acknowledgements

The National Science Foundation and Texas Advanced Research Program provided financial support. A faculty development leave at the University of Pittsburgh provided time for Van Huyck to write this paper.

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