Chapter 11

FIRST PRICE INDEPENDENT PRIVATE VALUES AUCTIONS

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The first price sealed bid auction is the market institution in which the high bidder acquires ownership of the auctioned item and pays a price equal to the amount of the highest bid. This market institution is distinguished from the second price sealed bid auction in which the high bidder obtains the auctioned item and pays an amount equal to the second highest bid. Bids in sealed bid auctions are often literally sealed in envelopes but need not be; the essential distinction is from a real time (or "oral") auction in which the time at which bids are submitted during the auction is an essential feature of the market institution. For example, in the English "oral" (or progressive or increasing-price) auction, a bid is admissable only if it is higher than the standing bid, and the last bid is the winning bid. In the Dutch "oral" (or decreasing-price) auction, the first and only bid is the winning bid.

The independent, private-values information environment is the one in which each bidder knows with certainty his own value for the auctioned item but knows only the probability distribution from which each of the other bidders' values are independently drawn. Vickrey (1961) first derived Nash equilibrium bid functions for the first price, second price, English, and Dutch auctions for bidders with independent private values of the auctioned item.

We are here concerned with experimental tests, using independent private values, of the consistency of bidding behavior in the first price auction with three nested Nash equilibrium bidding models. The risk neutral model (RNM) was developed by Vickrey (1961). The constant relative risk averse model (CRRAM) was developed by Cox, Roberson, and Smith (1982) and Cox, Smith, and Walker (1982). The log concave model (LCM) was developed by Cox, Smith, and Walker (1988) and Cox and Oaxaca (1996). These models are "nested," in that LCM contains CRRAM as a special case and CRRAM contains RNM as a special case. Of course a more specialized model places more restrictions on data than does a more general model; thus tests for consistency of the nested models with bidding in first price auctions are concerned with identifying those theoretical restrictions that are consistent with the patterns of empirical regularity in the auction data. Such tests have been conducted with data for market prices, subject payoffs, and individual bids and values.

1. Tests of the RNM with Market Prices

The model of the first price auction developed by Vickrey is based on the following set of assumptions: (a) the bidders are risk neutral, expected utility maximizers (and

Handbook of Experimental Economics Results, Volume 1

DOI: 10.1016/S1574-0722(07)00011-X

Table 1

Market price tests (5% significance). Three tests with auction market prices imply rejection of the risk neutral model in favor of the risk averse alternative for all numbers of bidders and experiment series except one series of three-bidder experiments

Number of bidders	Experiment series	Number of auctions	t-Test	Binomial test	Kolmogorov–Smirnov test
3	1	70	Accept	Accept	Accept
3	$1'$	100	Reject	–	–
4	2	60	Reject	Reject	Reject
4	3	30	Reject	–	–
4	4	250	Reject	–	–
5	5	60	Reject	Reject	Reject
5	6	30	Reject	–	–
6	7	60	Reject	Reject	Reject
9	8	30	Reject	Reject	Reject

they all know this); and (b) the bidders' values for the auctioned item are independently drawn from the uniform distribution on the $[v_\ell, v_h]$ interval (and they all know this). These assumptions imply that the Nash equilibrium bid function is

$$b_i = v_\ell + \frac{n-1}{n}(v_i - v_\ell), \tag{1}$$

where b_i is the amount bid by bidder i when her value for the auctioned item is v_i.

Bid function (1) immediately implies that the auction market price, p, is

$$p = v_\ell + \frac{n-1}{n}\max_i(v_i - v_\ell). \tag{2}$$

Vickrey also first demonstrated that if all bidders are identically risk averse then the probability distribution of market prices (first-order) stochastically dominates the market price distribution of the risk neutral model.

Table 1 presents market price tests reported in Cox, Roberson, and Smith (1982) and Cox, Smith, and Walker (1988). Three types of tests are reported: t-tests comparing mean market prices with the RNM's predicted mean prices; Kolmogorov–Smirnov tests comparing the cumulative distributions of market prices with the cumulative distributions implied by the RNM; and binomial tests of the differences between market prices and prices predicted by the RNM conditional on auctioned item values. All three one-sided tests imply the same conclusion: except for one series of three bidder auctions, the hypothesis that market prices are the same as the RNM's predicted prices is rejected in favor of the hypothesis that they are higher. This result is consistent with bidding theory for risk-averse bidders.

2. Tests of the RNM with Subject Payoff Data

Harrison (1989) argued that researchers should test bidding models with subject payoff data, not market price or individual subject bid and value data. He applied “metrics” to (only) the median payoffs in experiments. Friedman (1992) developed a loss function that uses all of the subject payoff data to answer the question of whether the actual payoffs in an experiment differ significantly from the payoffs that would result from bidding according to a theoretical model.

Cox and Oaxaca interpreted Friedman’s loss function in the context of first price auctions as follows. If all $n - 1$ of a representative bidder’s rivals bid according to the risk neutral bid function in Equation (1), then the probability that a bid in amount b by the representative bidder will be a winning bid is given by the composition of the inverse bid function and the cumulative distribution function for the $(n - 1)$st order statistic for a sample of size $n - 1$ from the uniform distribution on $[v_\ell, v_h]$, which is $[n(b-v_\ell)/(n-1)(v_h-v_\ell)]^{n-1}$. Multiplying the preceding expression by the difference between the value of the auctioned item and the amount of the bid gives one the expected payoff from bidding the amount b. The loss function then evaluates the expected payoff implications of unilateral deviation from the RNM bids to the observed bids. Thus the loss function is

$$L_{it} = \left[\frac{n}{(n-1)(v_h - v_\ell)}\right]^{n-1} \times \left[\left(b_{it}^{\mathrm{RN}} - v_\ell\right)^{n-1}\left(v_{it} - b_{it}^{\mathrm{RN}}\right) - (b_{it} - v_\ell)^{n-1}(v_{it} - b_{it})\right], \tag{3}$$

where b_{it} and v_{it} are the observed bid and value for subject i in auction t and b_{it}^{RN} is the bid given by the risk neutral bid function, Equation (1), for the observed value. The test statistic is

$$x_{it} = Z_{it} L_{it}, \tag{4}$$

where

$$Z_{it} = \begin{cases} +1 & \text{for } b_{it} > b_{it}^{\mathrm{RN}}, \\ 0 & \text{for } b_{it} = b_{it}^{\mathrm{RN}}, \\ -1 & \text{for } b_{it} < b_{it}^{\mathrm{RN}}. \end{cases} \tag{5}$$

Cox and Oaxaca’s results from tests with (their interpretation of) Friedman’s loss function are exhibited in Figure 1. The furthest-right bar in Figure 1 shows that 90% of the subjects had significantly positive foregone expected earnings from submitting bids higher than RNM bids. Thus this test based on foregone expected earnings leads to the same conclusion as the three tests based on market prices: the risk neutral model is rejected.

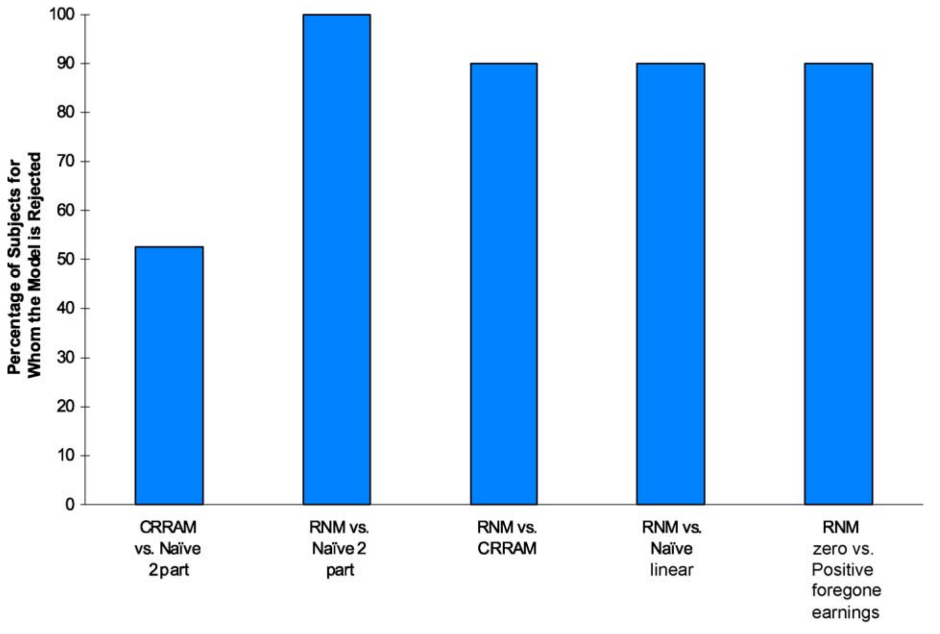

Figure 1. This figure reports tests using bid data of Nash equilibrium bid function parameter restrictions for the constant relative risk averse model (CRRAM) and the risk neutral model (RNM). It also reports a test of RNN using subjects' expected foregone earnings. The bar graph shows the rejection rates at 10% significance. CRRAM is rejected for 52.5% of the subjects in favor of a naïve two-part bid function that places no restrictions on the bid function slope and does not impose consistency of the slope with the location of the knot joining the two parts of the bid function. RNM is rejected in favor of the naïve two-part bid function for 100% of the subjects. RNM is rejected in favor of CRRAM for 90% of the subjects. RNM is rejected for 90% of the subjects in favor of a naïve linear bid function that places no restrictions on the bid function slope and intercept. Zero foregone expected earnings from bidding higher than RNM bids is rejected in favor of positive foregone earnings for 90% of the subjects. Thus, CRAMM is rejected for slightly more than one-half of the subjects by tests using bid data. RNN is rejected for 90–100% of the subjects by tests using either bid data or earnings data.

3. Tests of the CRRAM and the RNM with Individual Bid Data

The theory can be generalized to the environment where bidders can have preferences that exhibit any degree of constant relative risk aversion (and/or constant relative risk preference) and the bidders are not required to have the same preferences (Cox, Roberson, and Smith, 1982; Cox, Smith, and Walker, 1982). Let bidder i have power function utility for monetary payoff, y_i, such that $u(y_i) = y_i^{r_i}$, for $i = 1, 2, \ldots, n$. The r_i are independently drawn from a probability distribution with integrable *cdf* $\Psi(\cdot)$ on $(0, r_h]$. Note that if $r_h \geqslant 1$ then the risk neutral model is the special case where $\Psi(\cdot)$ places all of its probability mass at $r = 1$.

If auctioned item values are drawn from the uniform distribution on $[v_\ell, v_h]$ then the Nash equilibrium bid function for this model has two parts that are joined together at a "knot," v_i^*. The linear part of the bid function is

$$b_i = v_\ell + \frac{n-1}{n-1+r_i}(v_i - v_\ell) \quad \text{for } v_i \in [v_\ell, v_i^*], \tag{6}$$

and the knot is

$$v_i^* = v_\ell + \frac{n-1+r_i}{n-1+r_h}(v_h - v_\ell). \tag{7}$$

The upper part of the bid function, with domain $(v_i^*, v_h]$, is strictly concave but does not have a closed form.

Note that both Equations (6) and (7) contain r_i and n. If we assume that the least risk averse bidder in the population is risk neutral, then $r_h = 1$, and Equations (6) and (7) imply

$$b_i = v_\ell + \frac{(n-1)(v_h - v_\ell)}{n(v_i^* - v_\ell)}(v_i - v_\ell) \quad \text{for } v_i \in [v_\ell, v_i^*]. \tag{8}$$

As explained in Cox and Oaxaca (1996), the stochastic version of the two part bid function can be estimated by searching for the value of the knot and the parameters of a polynomial approximation of the upper part that minimize the sum of squared residuals. Data limitations made it impossible to use a polynomial of higher order than the quadratic.

Figure 1 exhibits test results reported by Cox and Oaxaca (1996) using data from experiments reported in Cox, Smith, and Walker (1988). The left-most bar in Figure 1 shows the results from testing the parameter restrictions implied by CRRAM against an unrestricted (or naive) two-part bid function. CRRAM is rejected for 52.5% of the subjects by this test. The second-to-the-left bar in Figure 1 reports results from applying this same test to RNM. We observe that RNM is rejected for 100% of the subjects by this test. The middle bar in Figure 1 shows results from testing RNM against CRRAM. Note that RNM is rejected in favor of CRRAM for 90% of the subjects. The second-from-the right bar in Figure 1 shows results from testing RNM against an unrestricted (or naïve) linear bid function. By this test, RNM is rejected for 90% of the subjects.

4. Tests of the LCM with Individual Bid Data

The theory can be generalized further (Cox, Smith, and Walker, 1988) by letting a bidder's utility for monetary income, y_i, depend on an $(M-1)$-dimensional characteristic vector, θ_i, that is drawn from a probability distribution with integrable *cdf*, Φ, on a convex set, Θ. Bidders' preferences are required to be strictly log-concave but not necessarily concave; that is, $\ln u(y_i, \theta_i)$ must be strictly concave in monetary income, y_i, for all $\theta_i \in \Theta$. This means that different bidders can be risk averse, risk neutral, or risk

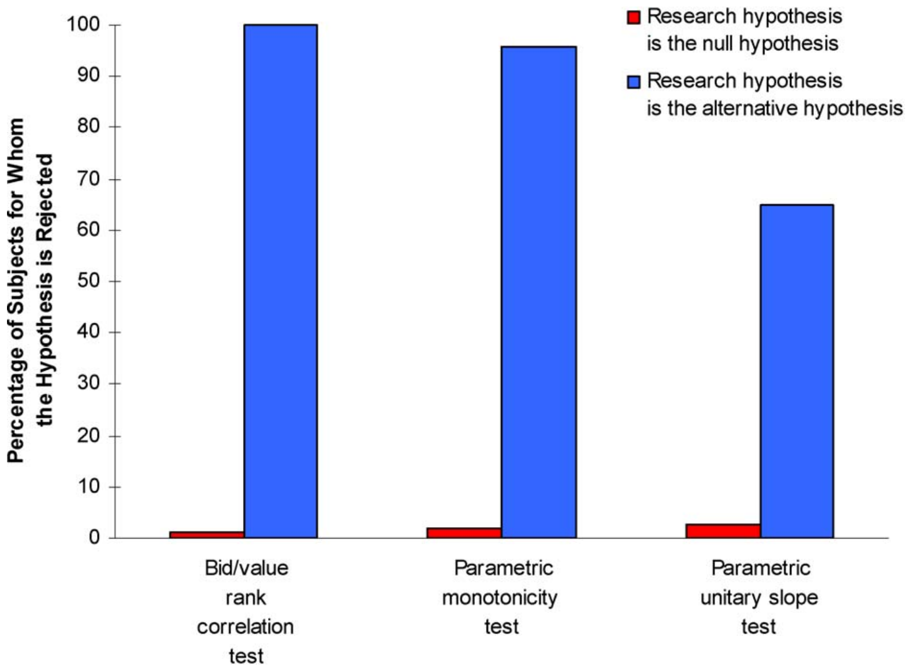

Figure 2. This figure reports tests using observed bids of Nash equilibrium bid function parameter restrictions for the log-concave model. The bar graph shows the rejection rates at 10% significance. Three tests imply very low rejection rates of 0%, 0.1%, and 2.6% when the log-concave model's properties ("the research hypotheses") are the null hypotheses of the tests. Three tests imply very high rejection rates of 100%, 95.7%, and 64.9% of the null hypotheses when the research hypotheses are the alternative hypotheses of the tests. The tests are for positive or negative rank correlation of bids and values, positive or negative slope of the estimated cubic bid function, and estimated bid function slope less than or greater than 1.

preferring. Furthermore, an individual bidder can be risk neutral in some parts of the payoff space, risk averse in some other parts, and/or risk-preferring in other parts. The only restriction on risk preferences is that $u(y_i, \theta_i)$ be everywhere less convex than the exponential function of y_i.

The Nash equilibrium bid function for LCM, $b(v_i, \theta_i)$, does not have a closed form. However, it does have testable implications. Cox, Smith, and Walker (1988) demonstrate that more risk averse bidders will bid more than less risk averse bidders with the same item value. Cox and Oaxaca (1996) demonstrate that the slope of the bid function with respect to v_i is everywhere contained in the interval (0, 1); that is, bids must be monotonically increasing in the value of the auctioned item but can never increase faster than that value.

Figure 2 shows results from tests of LCM reported by Cox and Oaxaca. The red bars show results from tests that use the research hypothesis as the null hypothesis while the blue bars are for tests where the research hypothesis is the alternative hypothesis.

The left-most pair of bars show the results from rank correlation tests of the positive monotonicity property of the LCM. Positive rank correlation of bids and values is not rejected for any of the subjects whereas negative rank correlation is rejected for 100% of the subjects. The middle pair of bars shows results from parametric monotonicity tests. Positive monotonicity of the estimated bid function slopes can be rejected for only 0.1% of the subjects. Non-positive monotonicity of bid function slopes can be rejected for 95.7% of the subjects. The right-most bars in Figure 2 show results from tests of the unitary slope property of LCM. Note that the restriction that the bid function slope be everywhere less than 1 can be rejected for only 2.6% of the subjects. The hypothesis that the bid function slope is everywhere greater than 1 can be rejected for 64.9% of the subjects.

5. Summary of the Test Results

The reported tests have the following implications for the three nested models. Depending on which test is used, data for only 0–10% of the subjects are consistent with the risk neutral model. This conclusion is the same regardless of whether one conducts the tests with market prices, individual subjects' bids and values, or subjects' expected foregone earnings. Tests with individual subjects' bids and values indicate that data for about 48% of the subjects are consistent with the constant relative risk averse model and data for almost all subjects are consistent with the log-concave model.

Acknowledgement

Todd Swarthout provided valuable assistance by preparing the table and figures.

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