Economics 241B
Exercise 4

1. Asymptotic Normality to Consistency

You undertake a research project on carbon sequestration with a colleague from the geology department. He defers to you as the expert on statistics.

a. In one discussion, you note that the estimator you are using is consistent and asymptotically normal. He then asks the natural question, "What are the advantages of using an estimator that is consistent and asymptotically normal?" Please respond.

b. Several weeks later, the geologist proposes an estimator that is common in his field, but with which you are not familiar. He proudly notes that the estimator, $\hat{\theta}_n$, is asymptotically normal (a concept he learned from you) and displays an equation he copied down

$$\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) \xrightarrow{d} N\left(0, \sigma^2\right).$$

He is puzzled, however, by the fact that he could not find out if the estimator is consistent for the population value θ_0. Please show why the asymptotic normality result does, or does not, establish consistency.

c. After you have obtained your results from $\hat{\theta}_n$, the geologist returns some time later with $\tilde{\theta}_n$, which is another estimator of θ_0. He also found that

$$n\left(\tilde{\theta}_n - \theta_0\right) \xrightarrow{d} N\left(0, \sigma^2\right).$$

Should the two of you present the results you have already obtained, or obtain new results from $\tilde{\theta}_n$? Be sure to explain your answer clearly to your geologist coauthor.

2. Asymptotic Convergence

You model the purchase of tickets in a lottery. There are n tickets sold and the winning ticket is selected at random. The holder of the winning ticket receives n^2 dollars, all others receive nothing. It costs $\frac{n}{2}$ dollars to purchase one ticket. Let p_n represent the random payoff to an individual and let $w_n = p_n - \frac{n}{2}$ represent the random winnings for an individual from the lottery.

a. If individuals are risk neutral, would they elect to purchase a lottery ticket?

b. As the number of tickets sold increases, what does an individual's payoff converge to in probability?

c. As the number of tickets sold increases, what does an individual's expected payoff converge to?

d. In light of your answers to parts *b* and *c*, would you be able to conclude that for large enough values of n individuals elect not to buy lottery tickets?

3. Let the estimator B be $\sqrt{n}$-consistent and asymptotically normal for β:

$$\sqrt{n}\,(B-\beta) \xrightarrow{D} N\left(0,\sigma^2\right),$$

where $\beta \neq 0$. Consider $G = \frac{1}{B}$ as an estimator for $\gamma = \frac{1}{\beta}$.

a. Find the asymptotic distribution for G. Does it follow that G is a consistent estimator for γ?

b. From a sample of data with $n = 20$, the values $b = 6$ and $s^2 = 320$ are obtained. Find the associated estimate for γ along with the estimated "asymptotic" standard errors for B and G.

c. With the values from part b, test $H_0 : \beta = 2$ against $H_1 : \beta \neq 2$.

d. Recast the hypothesis test from part c in terms of γ and use the values from part b to test. Explain.

4. Computational Exercise

Return to the paper by Charness and Kuhn listed on the syllabus. Write programs in both Matlab and Stata (the results from each program should match) that estimate the models in columns (6) and (7) of Table 3 of Charness and Kuhn. Calculate classic standard errors (the authors report cluster-robust standard errors, so your estimated standard errors will not match those in the table). Test the hypothesis that all four slope coefficients are equal and provide the p-value for the estimated test statistic.