Chapter 2

FROM MARKET JAWS TO THE NEWTON METHOD: THE GEOMETRY OF HOW A MARKET CAN SOLVE SYSTEMS OF EQUATIONS

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Since market equilibrium can be interpreted as a solution to a system of equations, "price discovery," as it called in the language of market makers, can be viewed as having "found" the solution. Of course the information needed to even formulate the equations does not exist in one place so the idea that markets are "searching" for the solution to a system of equations as a numerical process would search, cannot be taken literally. Nevertheless, it is interesting that the language that has evolved from the world of practical markets has such an interpretation and curiosity alone makes one wonder how markets settle on the particular pattern of prices that solve a particular system of equations.

The substance of this note is to take seriously the analogy between market processes and the numerical methods of mathematics. The analogy is made in Figures 1 and 2. Figure 1 is a picture familiar to experimental economists. At any time, in many forms of continuous markets there exists an order book in which all standing orders to buy and sell are listed. If new orders are such that trades are possible, the trades are executed. In some cases the book is an "open book" in which all traders see the book and place orders with a full knowledge of existing orders. In other markets, the book is held by a specialist who operates under rules about what aspects of the book must be exposed to the market and how orders are executed.

It is known that in markets such as the multiple unit double auction, the shape of the book is a good indicator of the directions price changes are likely to take. In particular, when the sell order builds up, the sell order book extends beyond the buy order book and price movements down are likely. The opposite happens when the market goes up. The movement of the book when displayed (as in Figure 1) appear as "jaws" that move in a direction that anticipate which way the market is going to move.

The empirical foundation for jaws can be found in many forms.[1] When markets are on the way up the number of bids typically exceeds the number of asks. When markets are on the way down, the number of asks typically exceeds the number of bids. This is exactly the concept of "pushing" referred to Plott. When the speed of arrival of bids is greater than the speed of arrival of asks the buy order book builds and the lower jaw "juts out." When the sell orders arrive more rapidly, the sell order book builds and the upper jaw juts out. Thus the shape of the book reflects the underlying arrival rates of orders and the arrival rates in turn reflect an underlying excess demand or supply that will force price movement.

[1] The reader should refer to Plott (2008, this volume).

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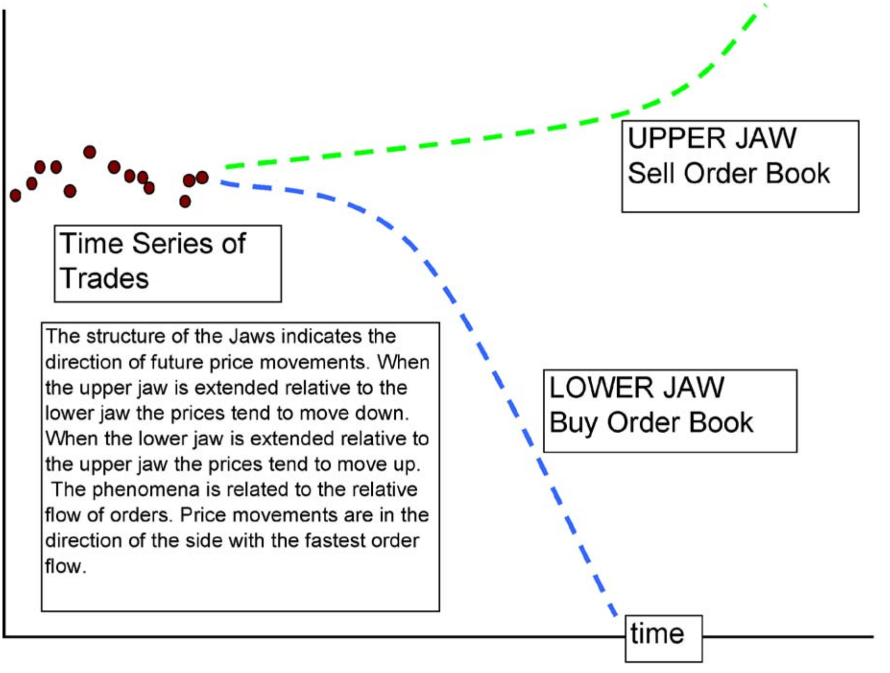

Figure 1. The market jaws can be used to predict future price changes.

The suggestion of Figure 2 is that the information in market jaws has important aspects of the information needed to apply the Newton method of solving equations. It is known that, under a wide range of conditions, the Newton method will converge to the solution to a system of equations. In general to find the value of z that solves the equation $G(z) = 0$ one needs to start at some value of z and then make small changes in z that satisfy the equation

$$\mathrm{d}z/\mathrm{d}t = -G'(z)^{-1}G(z).$$

The information needed is $G(z)$ and the derivative of G at the point z.

Notice that if we let $z = P$, and if $G(z) = D(P) - S(P)$, then the problem of equilibration is to find the solution of the equation

$$D(P) - S(P) = 0.$$

To apply Newton's method one needs to know for some P^*, $D(P^*) - S(P^*)$ and the value of the derivatives. Notice further that approximations of these magnitudes are given by the market jaws.

It is interesting to compare this formulation with the classical notion of Walrasian adjustment in which

$$\mathrm{d}P/\mathrm{d}t = A\big[D(P) - S(P)\big],$$

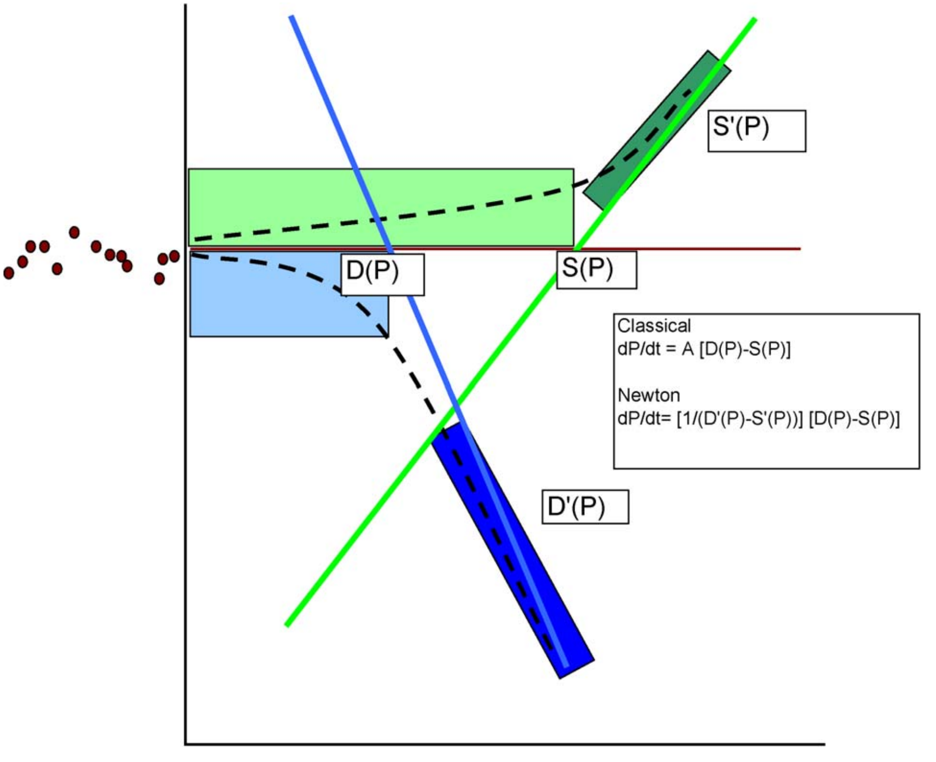

Figure 2. The relationship between the Newton method and the market jaws is that the latter is the source of information that is needed by the former.

where A is a positive constant. Thus to connect the models one only needs to recognize that the variable A needs to be the inverse of the derivative of the function. In the classical theory of price adjustment, this derivative is absent and in the classical metaphor about tatonnement, the information is not needed.

Does such a model of price adjustment work? That it does is exactly the result reported in Asparouhova, Bossaerts and Plott (2003).

References

Asparouhova, Elena, Bossaerts, Peter, Plott, Charles (2003). "Excess demand and equilibration in multi-security financial markets: The empirical evidence". Journal of Financial Markets 6 (1), 1–21.

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