

ZERO-INTELLIGENCE ROBOTS AND THE DOUBLE AUCTION MARKET: A GRAPHICAL TOUR

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Experimental evidence has established that prices and volumes in markets populated by humans generally converge towards the outcome predicted by the competitive model. Earlier chapters in this text demonstrate this phenomenon at work over and over again. Yet, casual observations together with a variety of experiments suggest that humans are not perfect maximizers. This raises some interesting questions, such as:

- Is profit maximization really necessary for the functioning of such economic principles as the law of supply and demand?
- Could some lower form of “rationality,” such as agents who follow simple rules and/or stay within their budget constraint, lead to markets that converge to the competitive outcomes?

The pictures in this chapter illustrate the behavior of *Zero-Intelligence (ZI) Robots* in double auction markets and offer some comparison with humans. Later, we will add a little more intelligence to the ZI’s. As with the human markets described in earlier chapters, we will designate each robot as a buyer robot or a seller robot. The behavior of these robots is very simple: they simply choose a random bid or ask over a range of prices constrained by their budget and the market improvement rules.¹ This is made formal later in the descriptions below.

The goal of this chapter is to compare the behavior of markets populated with simple robots to markets populated by humans. As there is clearly a lot more that could be done, sufficient detail about the market, environment, and robots are provided so that an advanced undergraduate or graduate student could easily develop their own ZI robot and market software. After providing this detail, the literature surrounding robots in double auctions is reviewed.

A series of graphs summarizes some important features of various humans and robots in a variety of double auction environments. In particular, these graphs are an attempt to both show some of the more important phenomena reported in the several papers by [Gode and Sunder \(1993a, 1993b\)](#) as well as show new results where the ZI robots are pushed into environments that are quite different from [Gode and Sunder \(1993a, 1993b\)](#), and similar experimentalists. These environments involve flows (or “recycling”) of costs

¹ There is precedent for such random behavior in earlier work. The ZI robots can be seen as similar to the [Hurwicz, Radner, and Reiter \(1975a, 1975b\)](#) *B*-process, where agents negotiate by randomly choosing points that are jointly preferred.

1.4.1. Improvement rule

At any time, buyers may send a new bid to the market indicating the price they are willing to pay. Sellers may send a new ask to the market indicating the price they are willing to accept. To become the market bid or market ask the new bid or ask must be an improvement over any existing market bid/ask. That means bids must go up and asks must go down until a trade takes place. Trades reset the market, allowing any new bid or ask to become the market bid or market ask without regard to previous prices.

(Note: Bids and asks that are “beaten” are not saved for later use. Only the best bid and ask are retained by the market. Buyers and sellers may bid/ask again when they are beaten.)

1.4.2. Trading at ask price (new bid > market ask)

If a buyer sends a bid that is greater than the market ask price, the unit is traded at a price equal to the ask. The market bid and ask are reset to allow more trades.

1.4.3. Trading at bid price (new ask < current bid)

If a seller sends an ask that is less than the market bid price, the unit is traded at a price equal to the bid. The market bid and ask are reset to allow more trades.

1.4.4. Access to market

With humans, access to the market is generally first-come, first-served. Small groups (<100) of humans are usually, though not always, slower than the technology on which the markets are implemented, but computerized robots could overwhelm their market-place if left to act asynchronously. Typically with robots one uses a random number generator to decide which robot will act next in the market, with each robot having an equal chance.

1.5. Budget Constraints

Robots are not allowed to trade at a loss. In the market, this means that buyers may not bid above their value and sellers may not ask below their cost. In many trading experiments, humans can and do trade at a loss. Usually these are mistakes (e.g. hitting wrong keys on unfamiliar trading software a subject uses for the first time, confusion over bid vs ask).

1.6. Trades Involve Arbitrage; No Speculative Trades

The environment is basically one of arbitrage between one’s cost or value, which is private information, and the available market price, which is public information. Only

buyers may buy and only sellers may sell, no one may do both – this rules out many forms of speculation. Many, though not all, of the early trading experiments with humans are similarly structured.

2. Robot Agents

2.1. Zero Intelligence Robots – Trading Algorithm

The ZI robots basically act randomly within what their budget constraints and the market rules allow. In particular, the ZI's have no memory and do not mimic, anticipate or respond to other robots, strategize, or maximize anything. Their behavior is totally determined by their value or cost, the current market bid/ask, and the outcome of a random number generator.

ZI buyers: If redemption value $>$ market bid, send a new bid chosen from a uniform random distribution between the current market bid² and the buyer's redemption value. Otherwise, if redemption value \leq market bid, the robot is not allowed to send a bid because no profit is possible.

ZI sellers: If marginal cost $<$ market ask, send a new ask chosen from a uniform random distribution between the seller's marginal cost and the current market ask.³ Otherwise, if market ask \leq marginal cost, the robot is not allowed to send an ask because no profit is possible.

2.2. UNIT Robots – Trading Algorithm

The UNIT robots bid or ask at a price equal to, one unit above, or one unit below the previous transaction price, so long as this is within what their budget constraints allow. Like the ZI's, the UNIT's do not anticipate or respond to other robots, strategize, or maximize anything. Unlike the ZI's, they do have a memory and retain the previous transaction price. It is this link to previous transaction prices that results in market behavior that can be quite different from what is observed in markets with ZI robots.

UNIT buyers: Choose at random a bid B equal to one of 3 values $\{P - 1, P, P + 1\}$, where P is the previous⁴ transaction price in the market. Each choice has an equal (1/3) probability. If B is less than or equal to the robot's redemption value and greater than the current bid (if any), send the bid. Otherwise, do not send the bid – because it violates either the buyer's budget constraint or the market improvement rule.

² If there is no market bid then we use 0 as the lower limit.

³ If there is no market ask then we use a number well above the highest buyers' value for the maximum possible ask. We cannot use ∞ because the resulting uniform random distribution would be ill defined.

⁴ A pre-existing price P_0 must be chosen by the experimenter to seed the simulation. From the graphs we will see that in environments with continuously refreshed supply and demand, the effect of this choice of P_0 dies away as market prices converge towards instantaneous competitive equilibrium.

UNIT sellers: Choose at random an ask A equal to one of 3 values $\{P - 1, P, P + 1\}$, where P is the previous transaction price in the market. Each choice has an equal (1/3) probability. If A is greater than or equal to the robot's cost and less than the current ask (if any), send the bid. Otherwise, do not send the bid – because it violates the seller's budget constraint or the market improvement rule.

2.3. Kaplan's Parasitic Robots⁵ – Trading Algorithm

Kaplan's robots follow a 'steal the deal' strategy, waiting for a low bid–ask spread before accepting beneficial trades. Thus, they require a market populated by other participants (other kinds of robots or humans) who are actively submitting bids and asks.

Parasitic buyers: Wait until (current ask < redemption value) and (current ask – current bid) < target, then send a bid equal to the current ask. This causes a transaction.

Parasitic sellers: Wait until (current bid > cost) and (current ask – current bid) < target, then send an ask equal to the current bid. This causes a transaction.

3. Literature – Robots and the Double Auction

3.1. Types of Questions

Robot populated markets are interesting artifacts of study for many reasons. The range of questions that can be asked is broad. The list below is a humble attempt at some categorization of these questions. Necessarily incomplete, it does provide a starting point from which to understand the research.

- *Phenomenology* involves questions about 'How are market phenomena similar or different when the types of agents or other parameters of a market environment are changed?' In what ways are markets populated by ZI robots, for example, similar to markets populated by humans and in what ways are they different? The questions are about phenomena rather than about models. Of course such a question requires many details about the market institution and economic environments to be meaningful. Once the question is well defined, *similarities* can mean any of several things: humans and certain types of robots are exhibiting some similarity in behavior within some domain; institutional rules such as the double auction improvement rule or budget constraints filter different individual behaviors to reach common types of market behavior; other factors in the economic environment (existence of a Marshallian path) are causing the similar market behavior even though the agents' individual behaviors are different. Sorting out these different causes and effects requires a great deal of future work and could yield new insights to market

⁵ In the graphics section we do not show any examples of markets with Kaplan's robots, but they are discussed in the literature section below.

phenomena in general. As will be shown in the graphical section, similarities and differences in market behavior can exist at the same time – for instance closing prices might be similar but the path by which these prices are reached might be very different.

- *Simulation* involves questions such as ‘Can we use robot markets to simulate the behavior of human markets?’ ‘To what extent?’ and ‘How?’ Markets populated by robots are governed by some of the same natural principles that govern markets populated by humans. In some cases, it may be easier to calculate the competitive equilibria of complex systems of markets by robot simulation than by formally solving complex sets of equations. It might not be practical to even identify all the appropriate equations for fully rational agents in some theoretically complex game theoretic market setting, but – given some leeway with agent’s rationality – we might be able to easily set up a robot simulation. As a means of obtaining information about markets populated by agents who follow rules that can be formalized as a robot algorithm, computer simulation (as opposed to, say, explicit analysis of the math behind the behavior) is sensible and merely a means of bringing these robots to life so that economists can observe them. The extent to which robot simulations can answer questions about human markets is a matter of controversy to be resolved through future research.
- *Model building, testing and refinement* involves many kinds of questions, often controversial, that are related to models (e.g., specification, comparison, rejection, usefulness). Particular robot algorithms can be thought of as mathematical models of individual behavior. Through analysis or simulation one can determine the aggregate behavior of particular kinds of robots within a market setting. The results can be compared to the predictions of other mathematical models – whether they be from other types of robots or from more traditional sources such as classical economics or game theory, e.g., [Cason and Friedman \(1993, 1996\)](#). Robots allow us the flexibility to alter agents’ behavior in very exact and particular ways to examine various aspects of economic phenomena that might otherwise be difficult to probe.
- *Applied philosophy* involves questions such as ‘How much rationality is required for markets to converge to competitive equilibrium predictions?’ If the answer is “not much,” then this may have broad policy reaching implications about such issues as deregulation and consumer protection. Of course, this calls to issue such basic matters as ‘What is rationality?’ as well as what rationality means in a market context.
- *Tournaments* involve questions about comparative performance of different robot strategies in a particular market setting. Robots allow a formalization of strategies and computer simulation becomes an easy means of comparison. Of course, there are a number of important factors that could be varied in a tournament and that might affect the results – period length, distribution of costs and values, types of private and public knowledge, methods of reward in the tournament (e.g., genetic algorithms that reproduce profitable robots and kill those with low profits, cash to

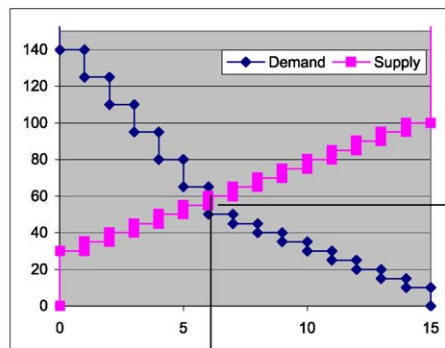
the author of the robot), etc. Tournaments can become interesting to economists when we try to explain the relative performance of strategies or what elements of strategies are important, because these issues can require pushing economic principles and ideas into new areas – sometimes beyond their normal realms of applicability.

3.2. Major Results from the Literature – A Chronology

The issues and questions which have motivated market research via robot simulation lie quite early in the literature and underpin some of the early laboratory experimental work as well. The reader should consult earlier chapters of this volume to get a fuller appreciation of the breadth of market phenomena.

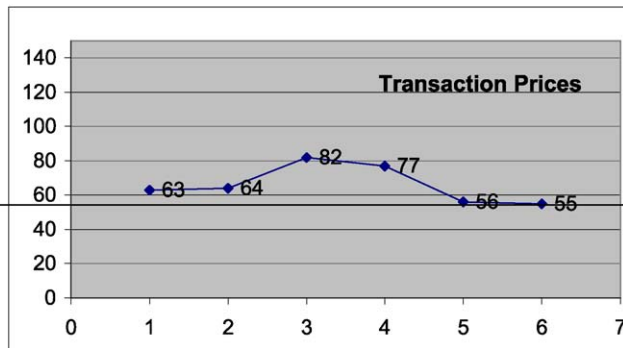
- [Becker \(1962\)](#). *Downward sloping demand can result from purely random behavior limited by budget constraints*. Theoretical work showing that rationality at the level of utility maximizing agents is not required for the demand side of markets to function as generally expected.
- [Hurwicz, Radner and Reiter \(1975a, 1975b\)](#). *A particular resource allocation process, called the B-process which is stochastic in nature will almost surely result in Pareto-optimal outcomes*. This theoretical paper can be seen as a precursor to the [Gode and Sunder \(1993a, 1993b\)](#) work below. The *B*-process operates by iterative improvements: in each iteration, agents reveal subsets of allocations preferred to the current status quo and a market-maker chooses randomly over any possible Pareto-improvements.
- [Gode and Sunder \(1993a\)](#). *'Zero-intelligence' robots that behave randomly within their budget constraint when placed in a double auction environment reach high trading efficiencies and exhibit prices converging towards competitive equilibrium outcomes*. This was the first journal publication of computer simulations with ZI robots. It is a combination of the private costs/values environments of experiments, the double auction trading institution, and ideas about randomness inspired by Becker, and shows that rationality on the order of human rationality is not necessary for markets to converge to an equilibrium: Randomness constrained by budgets and market rules will suffice. Some of the results of this study are replicated in [Figure 1](#) of this chapter.
- [Gode and Sunder \(1993a\)](#). *The marketplace can be seen as a partial substitute for individual rationality*. This is a matter of philosophy advocated by [Gode and Sunder \(1993b\)](#) that is still important today. If purely random behavior under a budget constraint yields the same market behavior in terms of competitive prices and allocations that would occur under ideal rationality and foresight, then this has important implications for public policy regarding matters such as deregulation and consumer protection. It also suggests that economists need not worry so much about individual behavior and how it differs from ideal rationality – in a market setting the differences may not really matter in the aggregate.

Initial Demand and Supply Parameters

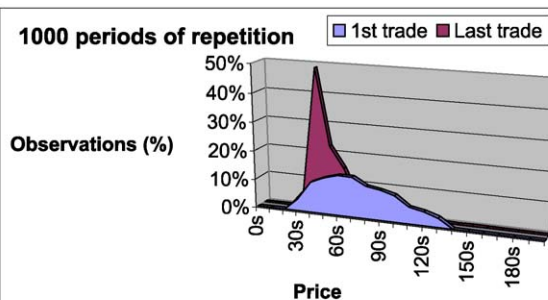


CE quantity = 6

A typical ZI robot trading period



Here we demonstrate Gode and Sunder's [1993a] results: that the prices and quantities in a sample trading period with the Zero Intelligence [ZI] Robots appear to correspond to the predictions of competitive equilibrium. The implication is that intelligence may not be very important for price convergence.



Repeating the period 1000 times reveals an important feature of markets populated by ZI robots: the first trade can be almost anywhere, while the last trade in a period is likely to be much closer to competitive equilibrium. This is driven purely by randomness and the increasing scarcity of trading partners as the market progresses. Essentially ZI's trade on a noisy *Marshallian* path from left to right on the S/D curves. The first few trades are not very price sensitive but at the end we are likely to be left with traders whose costs and values on the S/D curves are close to the S/D intersection. They will randomly try again and again to trade until they trade at a mutually agreeable price -- which must be near the equilibrium. But is this all there is to price discovery? Do Humans do more or behave differently?

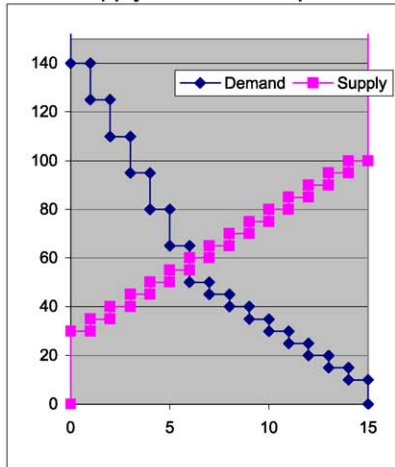
Figure 1.

- **Cason and Friedman (1993, 1996).** *Laboratory investigations with human-populated markets show negative autocorrelation of returns similar to those observed in ZI-populated markets.* This is the first paper comparing ZI robots as a model of human behavior against other more traditional models from the literature. **Cason and Friedman (1993)** examine the trade-by-trade returns $R_t = P_t - P_{t-1}$ from human traders, ZI traders, and two alternative models of market behavior obtained from game theory. They acknowledge that the price distribution for the ZI traders will change from one trade to the next, but claim that it can be approximated by independent draws. In the case of independent draws, it is easy to show that the autocorrelation of returns would be -0.5 .⁶ While prices are clearly not IID (e.g., see the first set of graphs here and note the differences in probability distribution of prices for initial and final trades), autocorrelations of returns are typically negative in ZI simulations. Laboratory data from humans also shows a negative autocorrelation of returns, which agrees with the ZI simulations and disagrees with the other alternatives examined by Cason and Friedman.
- **Gode and Sunder (1993b, 1997a).** *Double auction trading reaches high efficiencies of allocation because it creates a tradeoff between the size vs probability of inefficiencies.* Using ZI traders in a simple private values environment of 1 buyer, 1 inframarginal seller and N extramarginal sellers, it is shown via analytical methods that the probability of an inefficient trade involving an extramarginal seller goes down as the amount of lost surplus increases. The relationship between the shape of the extramarginal portions of the supply and demand curves to inefficient trading is also explored.
- **Rust, Miller and Palmer (1993).** *Markets populated by heterogeneous collections of robots exhibit price convergence to competitive equilibrium and high efficiency of allocations.* *Kaplan's simple parasitic robots are highly effective even against very sophisticated learning algorithms.* Reports the results of a double auction based robot-trading tournament held in 1990 at the Santa Fe Institute. The first result can be seen as support for the idea that the environment, budget constraints and market rules inherent in the double auction institution are more important for determining outcomes than individual agent behavior. The second result caused some concern: Kaplan's very simple design, illustrated in the robots section above, outperformed submissions from a number of prominent researchers in artificial

⁶ Following an argument given by **Cason and Friedman (1993)**, suppose transaction prices are IID with finite positive variance V and mean M . Thus $E(P_{t+k}^2) = V$ and $E(P_{t+j}P_{t+k}) = 0$ for $j \neq k$. Since prices are IID, all autocorrelation coefficients for the prices themselves are zero. For returns, or price differences, this is not true. The first-order autocorrelation coefficient for returns is $\rho_1 = E(R_{t+1}R_t)/E(R_t^2)$ where $R_t = P_t - P_{t-1}$. We can reduce the numerator: $E(R_{t+1}R_t) = E((P_{t+1} - P_t)(P_t - P_{t-1})) = E(P_{t+1}P_t - P_{t+1}P_{t-1} - P_t^2 + P_tP_{t-1}) = E(P_{t+1}P_t) - E(P_{t+1}P_{t-1}) - E(P_t^2) + E(P_tP_{t-1}) = 0 - V - 0 + 0 = -V$. Similarly the denominator is $E(R_t^2) = E((P_{t+1} - P_t)(P_{t+1} - P_t)) = 2V$. Thus, for any finite V and M , the autocorrelation $\rho_1 = -V/2V = -0.5$. For $k > 1$, $\rho_k = 0$ since there will be no overlapping P terms in the numerator.

Continuously Refreshing the Demand and Supply Parameters Confuses the Ro

Initial Supply and Demand parameters

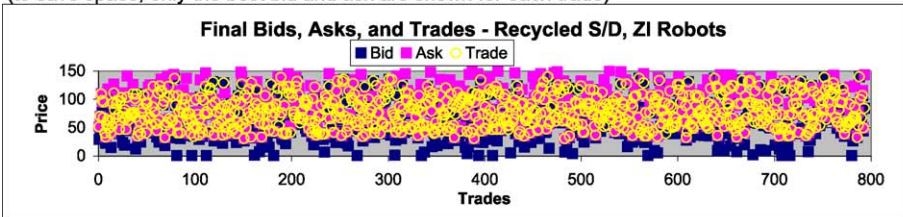


What is continuously refreshed S/D?

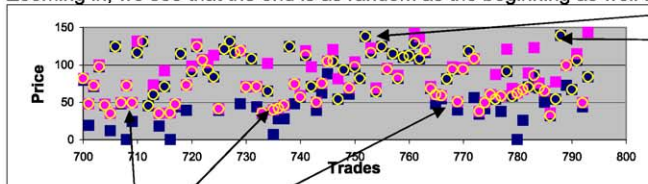
Standard S/D - When a buyer and a seller trade, their redemption values and costs are used up. A seller must pay a higher cost for the next unit. A buyer's next unit is worth less to him. These diminishing returns to scale result in an upward sloping market supply and downward sloping demand that *initially* looks like the figure at left but shrinks back over time as agents trade. Gains from trade are finite.

Continuously Refreshed S/D - When a buyer and a seller trade, their redemption values and costs are recycled to other agents, who may trade again. This results in a *stationary instantaneous supply and demand* that looks like the figure at left and never shrinks back. Its unusual properties include infinite gains from trade and a constant instantaneous clearing price.

The ZI robots behave as if the first trade is being repeated over and over again.
(to save space, only the best bid and ask are shown for each trade)



Zooming in, we see that the end is as random as the beginning as well as some interesting details.



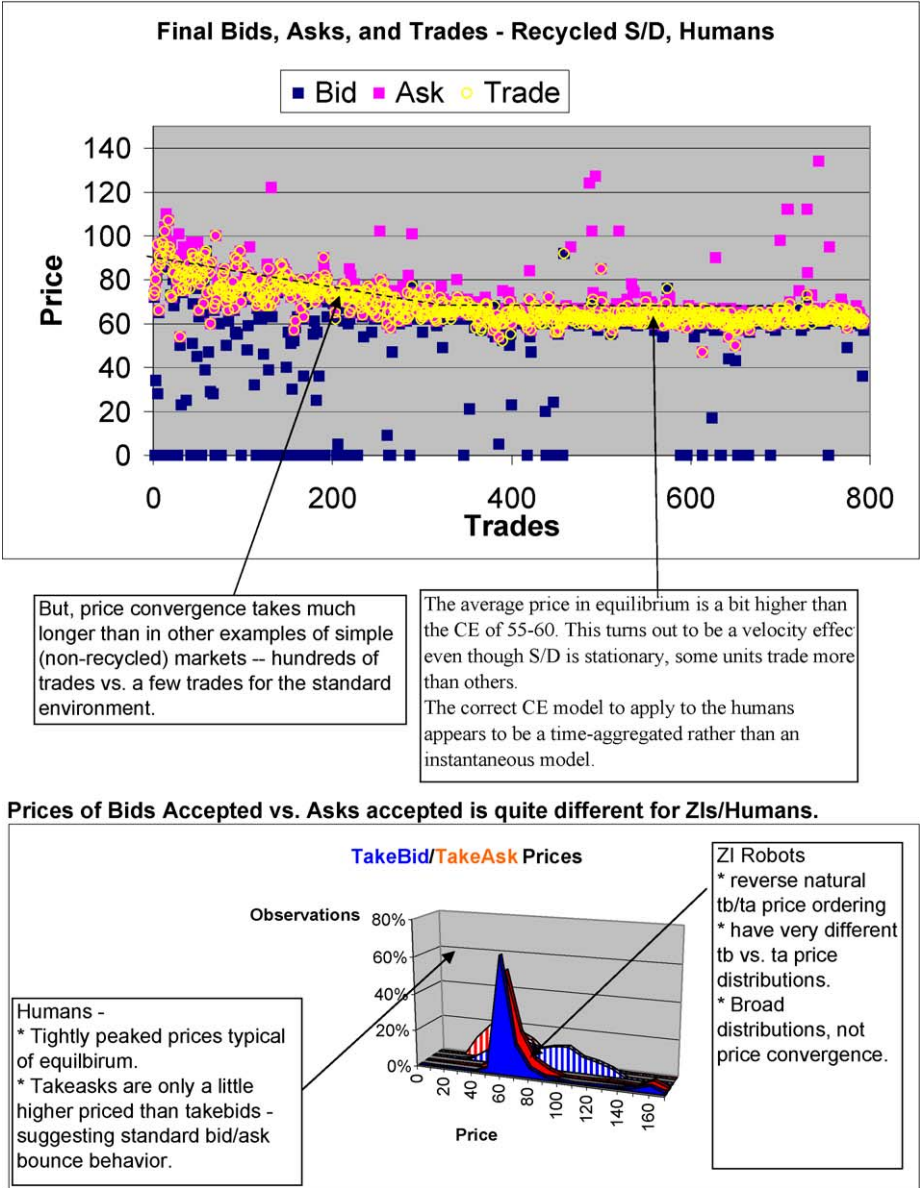
Low prices are often acceptances of low asks.

High prices are often acceptances of high bids.

The market seems driven by some robots ("fools") who make mistakes and other robots who take advantage of the mistakes ("thieves"). But there is no purposeful exploitation as the ZI robots behave randomly rather than with any specific goal or purpose. Next: Do humans behave this way too?

Figure 2.

Human Markets Equilibrate, even with continuously refreshed supply and demand.
Human intelligence adds a robustness to the dynamics that the ZIs do not possess.



Can we make the ZI robots "smarter" like the Humans?

Yes -- by having them randomize their bids/asks about previous prices!

We will call these new agents "UNIT robots" for reasons that are made clear in the key.

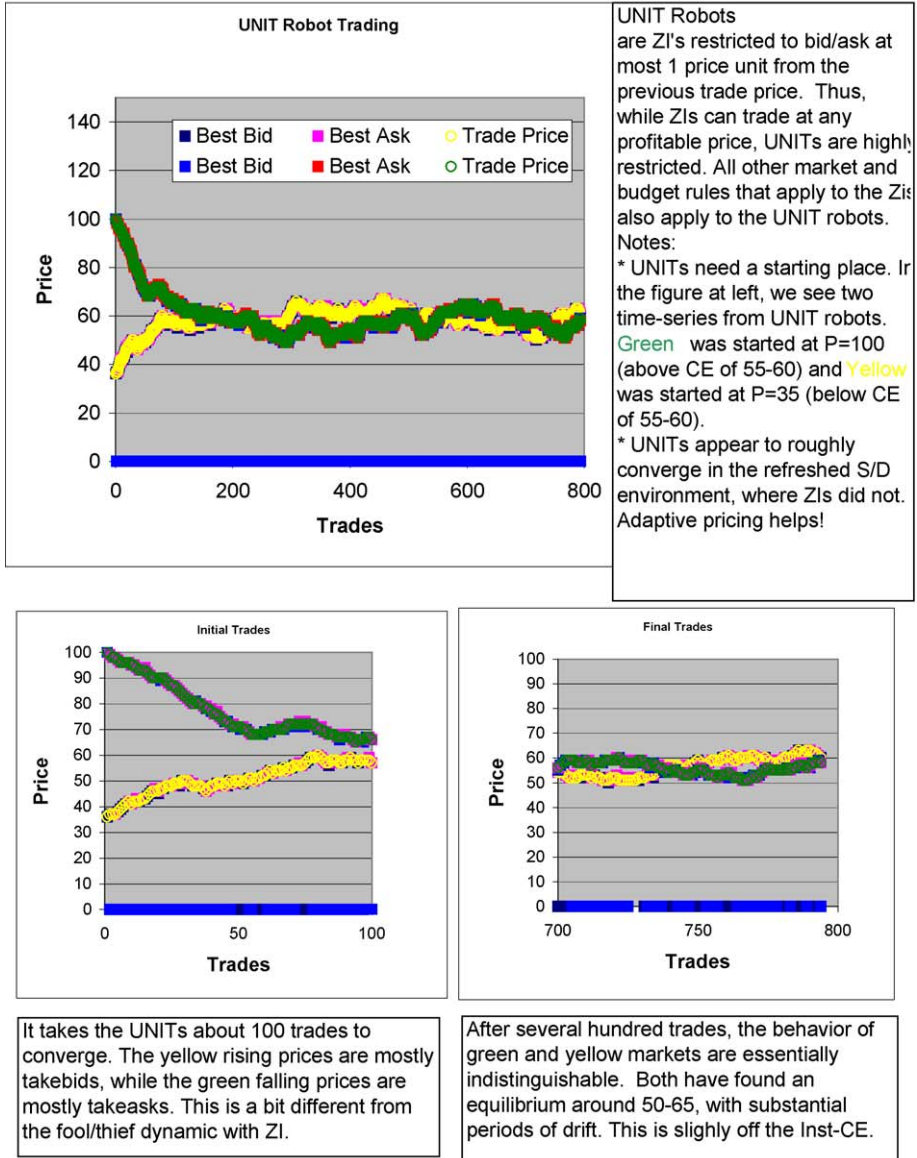
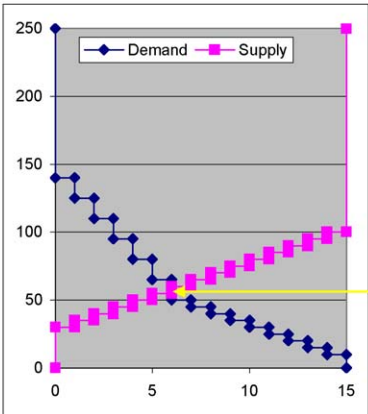


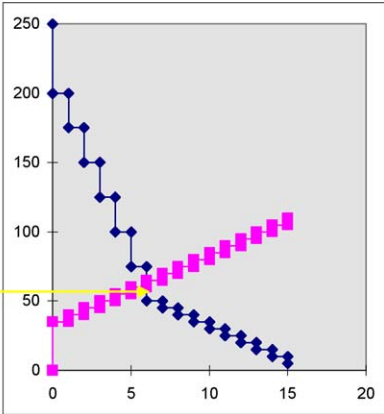
Figure 4.

But there are still differences...

Consider the following two choices of continuously refreshed supply and demand environments
Environment A



Environment B



same
instantaneous
competitive
equilibrium
[55,60]

But different types of market behavior depending on what kind of agents populate the market...

(averages over 500 or more trades in each case; source: Brewer, Huang, Nelson, Plott 1999 and recent simulation)

Average Trading Prices in Environment A		Average Trading Prices in Environment B	
63.4	Humans	75.9	
78.3	ZI Robots	95.0	
56.9	UNIT robots	57.4	

Even though the parameters in the left and right panes result in the same instantaneous competitive equilibrium they elicit different market behaviors that are sensitive to agent type. (We summarize and provide average price here rather than provide a set of six additional graphs). First, notice that the markets populated by UNIT robots are not affected and appear to trade near the instantaneous CE. The markets populated by humans or zi robots both show a higher average price for environment B. In the human markets, we think there is a velocity effect in the price convergence that can be reconciled with velocity adjusted supply and demand as described by Brewer, et.al. 1999. With the ZIs, there is no price convergence in this continuously refreshed environment but there is shift in the stationary IID probability distribution of prices favoring higher prices.

Another place to look for differences is in autocorrelations of trading prices - 1 parameter set is enough
Environment "A" parameters (left pane)

1st order autocorrelation of Prices		1st order autocorrelation of Returns	
	Corr(Pn,Pn+1)		Corr(Pn+1-Pn, Pn-Pn-1)
ZI robots	-0.015		-0.494
UNIT robots	0.988		-0.06
Humans	0.722		-0.468

Here we see the ZI robots and the UNIT robots are on different "corners" of Price vs. Return autocorrelation. The ZI robots show return autocorrelation but no price autocorrelation as prices are IID. The UNIT robots show no return autocorrelation, by their Martingale-like design, but significant price autocorrelation. Humans show both kinds of autocorrelations: a ZI-like autocorrelation of returns together with a level of price autocorrelation that is a bit less than the UNIT robots.

Figure 5.

intelligence, economics, psychology, and the physical sciences. By trading only at low bid–ask spreads, Kaplan’s robots avoided making mistakes that were possible in other robot designs.

- **Gode and Sunder (1997b).** *The effects of non-binding price ceilings and floors that occur in human markets can also be observed in markets populated by ZI robots.* The effect of non-binding price ceilings and floors was first discovered experimentally by **Isaac and Plott (1981)**. A price ceiling is an upper limit price for allowable bids, asks, and trades. Similarly, a price floor is a lower limit. These limits are called ‘non-binding’ when they do not preclude trades at the competitive equilibrium price. While non-binding, these floors and ceilings do affect the process of market negotiation because they affect the prices at which bids and asks may be made – an effect that can apparently be studied and observed with either human agents or the ZI robots.
- **Brewer et al. (1999).** *In environments where supply and demand is continuously refreshed, as in a flow, ZI robot traders will not exhibit price convergence but instead transaction prices will be independent, identically distributed draws from a random distribution. In contrast, markets populated by human traders will converge.* This is the first paper to introduce a continuously refreshed double auction environment where instantaneous supply and demand curves are held stationary by giving buyers and sellers new costs and redemption values as they trade. Velocity of units affects the price towards which the transaction prices seem to converge – suggesting that the equilibrium model for these environments may be a time-aggregated concept of equilibrium rather than an instantaneous concept of equilibrium. **Figures 2–5** show some of the data from this research along with recent extensions of the research obtained through additional robot simulations.

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