

## PRINCIPLES OF MARKET ADJUSTMENT AND STABILITY

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Data produced in market experiments exhibit a process of convergence to near the levels predicted by the static competitive equilibrium model. The major result reported in this section is that classical models of market dynamics explain important features of that convergence process. That is, the (current) answer to the question of how markets manage to find the competitive equilibrium is that price movements are governed by a set of principles outlined by classical models of dynamics as opposed to technical aspects of the equilibrium itself.<sup>1</sup> Those principles tend to guide a process of equilibration to the competitive equilibrium.

Market stability can be used as a window to understand dynamics because the circumstances of stability and instability provide an opportunity to clearly separate the principles that govern convergence. Theoretical instances in which instability can be observed typically require some sort of “perverse” market conditions, such as a downward-sloping supply or an upward-sloping demand. Thus part of the experimental problem is the creation of circumstances in which such conditions exist.

### 1. Theory

Three distinct classical models of dynamics are found in the literature and these models make very sharp predictions about stability. By studying competing predictions about stability, one can perform a clear test between the models. Each of the three is illustrated in Figure 1. The models are all founded on a limited number of parameters relative to modern game theory. Basically, the only variables are demand price, demand quantity, supply price, supply quantity and time. The relationships assumed are of the form

$$D(P_d, Q_d) = 0 \quad \text{and} \quad S(P_s, Q_s) = 0, \quad (1)$$

which characterizes the behavior of the demand and supply sides of the market, respectively. The demand side of the market is characterized as a demand price,  $P_d$ , and a demand quantity,  $Q_d$ . The supply side has a similar characterization ( $P_s$ ,  $Q_s$ ). Time is

<sup>1</sup> For example, the equilibrium itself could be defined as the quantity demanded at a price equals quantity supplied (Walras) or demand price equals supply price (Marshall) or as the intersection of demand and supply correspondences (Debreu). The choice of which definition is used in a model might not be as important as the implications the definition holds for price movements.

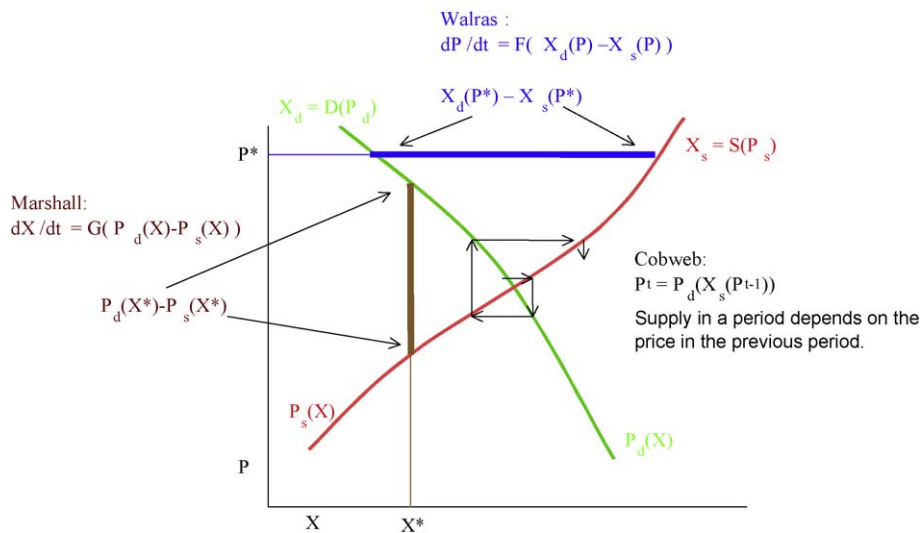


Figure 1.

not clearly defined and could be any of: clock time, time measured in trading periods, or time measured in number of trades. The functions themselves suffer from some ambiguity as to whether or not they characterize a stock or a flow. Similarly, prices are not defined operationally by the theory. In the experiments reported below, the assumptions about these variables are made both explicitly and implicitly in terms of the experimental procedures, parameters and measurements.

### 1.1. Cobweb Dynamics

The model assumes that only one price exists in a market, so demand price equals supply price equals  $P$ . However, the supply is not determined by the price in the current period. It is determined by the prices of the past, such as the previous period. Thus the equation defining the system balancing of demand and supply is

$$D(P_t) - S(P_{t-1}) = 0. \quad (2)$$

That is, the quantity demanded responds to the price at the current instant while the quantity supplied lags, dictated by the price of the previous period. According to the model the market is unstable if the slope of the demand curve is sufficiently greater than the slope of the supply curve. The possibility of instability is illustrated in Figure 1, in which initial prices move away from the equilibrium in a manner that can be interpreted as a spiral (prices cycle from high to low as periods progress).

### 1.2. The Walrasian (Hicks, Samuelson) Dynamics

The Walrasian model assumes that only one price exists in a market, so demand price equals supply price equals  $P$ . Equilibrium balancing occurs when the quantity demanded at a price equals the quantity supplied at that price. The dynamic of adjustment is characterized by the principle

$$dP/dt = F(D(P) - S(P)). \quad (3)$$

As illustrated in [Figure 1](#), the principle asserts that prices will adjust to reflect the difference between the quantity demanded at that price and the quantity supplied. The principle seems to assume the law of one price in a market; if the quantity demanded at a price equals the quantity supplied, then prices do not change.

In the cases in which the supply curve is downward sloping (and demand also downward sloping) the condition under which an equilibrium is stable is that the demand curve crosses the supply curve coming from below the supply curve. Similarly, if demand is upward sloping and supply is also upward sloping, the condition under which an equilibrium is stable is again that the demand curve cross the supply curve coming from below the supply curve. If an equilibrium is not stable, then it is unstable (neglecting boundary cases).

### 1.3. Marshallian Dynamics

The Marshallian model assumes that demand quantity equals supply quantity equals  $Q$ . Equilibrium is defined by the equating of demand price and supply price. The system dynamics is characterized by the principle

$$dQ/dt = F(P_d(Q) - P_s(Q)). \quad (4)$$

The principle asserts that markets adjust by changes in the quantity. Quantity in the market will adjust to reflect the profitability of the marginal unit. As illustrated in [Figure 1](#), if the demand price of the marginal unit is greater (less) than the supply price, then quantity will increase (decrease). As stated, the principle asserts nothing specific about prices but it does appear to assume that units are exchanged in the order of maximum surplus for otherwise the marginal units would not be well defined.

According to the model, if the demand curve crosses the supply curve coming from above the supply curve, then the equilibrium is stable. If the demand curve crosses the supply curve from below the supply curve, then the equilibrium is unstable. Notice that the conditions for stability can be exactly the opposite of those for the Walrasian model.

## 2. Experiments

### 2.1. *Instability does not Occur under Conditions Predicted by the Cobweb Model*

The experiments create a favorable condition for observing the type of instability predicted by the model by structuring an experimental market in which a lag could arise. The experiments reported in Figure 2 demonstrate that the cobweb model fails to make reliable predictions. The fact that market adjustments do not follow the lag assumed by the model was first demonstrated by Carlson (1967), so in essence, the experiments from Johnson and Plott (1989), referenced here, are replications and (substantial) extensions of Carlson's results. In the experiment suppliers made a decision on the quantity to supply before a period began and paid for the quantity chosen regardless of the number of units sold or the price they brought in the market. Thus, the supplier could lose money.

The first period the demand curve was at the lower of the two levels shown in Figure 2. After the first period, the demand curve shifted to the larger demand. This was implemented so that the suppliers were completely unaware of the shift. The shift was employed so the quantity supplied would be low (in the period just after the shift) and thereby producing prices above the equilibrium and starting a process of divergence according to the model. The solid green line is the competitive equilibrium predicted by the static model.

Two separate market institutions are studied. The first is the double auction and the second is the one-price sealed bid/sealed offer institution in which buyers and sellers submit orders. The orders are arrayed as in demand and supply the appropriate trades are made at the equilibrium price.

Typical results from the double auction are reproduced in the top panel. The red lines plot the competitive equilibrium given the production that took place in the period. The cobweb model predicts that the cycles will explode to a limit cycle. As can be seen the second period (the first period of high demand after the low demand experience) has only small supply. The supply is represented by the length of the red line. This small supply is exactly what was hoped for in the design. Prices move up rapidly, but not close to the competitive equilibrium given the supply. In the next period a large supply is forthcoming. Prices move up and then crash near the end of the period. In the third period, supplies are slightly above the static competitive equilibrium supply and prices hover near the static competitive equilibrium with sell-offs near the end of the period. As can be seen, supplies change over the succeeding periods but the general levels of activity are near the competitive equilibrium. The data are nearer to the static competitive equilibrium than the limit cycle predicted by the cobweb model.

The lower display in Figure 2 contains data from a sealed bid/offer process. The evidence of a cycle is greater under this institution than the double auction. However, the data do not explode to near the limit cycle predicted by the cobweb model. Indeed, the market activity is much closer to the static competitive equilibrium.

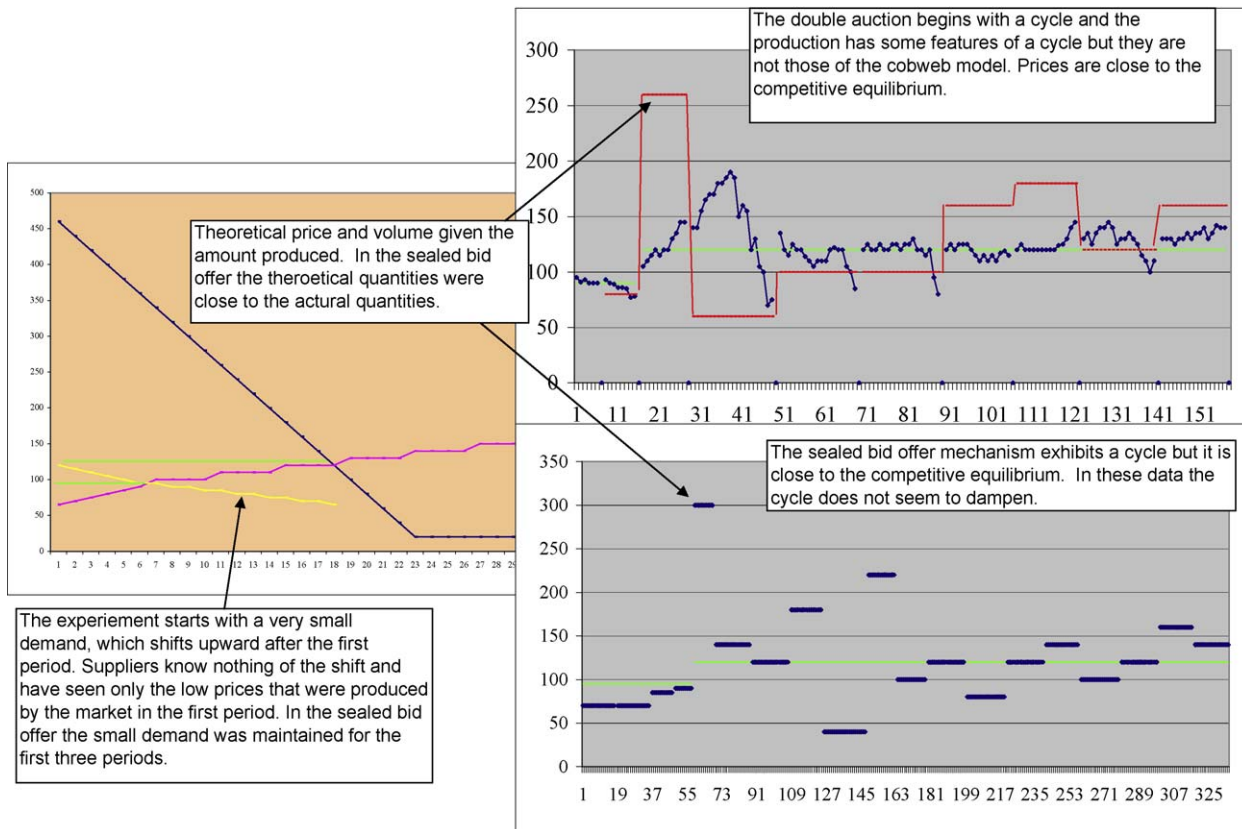


Figure 2. Both the double auction and the sealed bid offer mechanism converge to the competitive equilibrium price. Some small cycles are evident but the tendency is convergence.

## 2.2. Walrasian Dynamics and not Marshallian Dynamics Capture the Backward-Bending Case

The classic case of the backward-bending supply has been studied in [Plott \(2000\)](#) and the major result is represented here. Preferences of a subject with a capacity to supply  $x$  are induced using incentives roughly like those in the left panel of [Figure 3](#). Those who could supply sold units of  $x$  and were paid in francs. Prices in the market were prices per unit of  $x$ . Where  $f$  is the total franc income of a supplier and  $x$  is the number of units supplied, the incentives given suppliers were of the form  $U(f, x) = \$$ . That is, the dollars earned by a subject were a function of the amount of  $x$  he/she supplied and the total francs received from the sale. The level surfaces (indifference curves) were similar to those in the left panel. Application of the competitive model produced a backward-bending supply curve like the one shown in the right panel.

The demand curve was structured to yield several equilibria which differed in stability properties. A stylized demand curve is shown in [Figure 3](#) and the actual parameters used in experiments are shown in [Figure 4](#). The experiment called for initial periods to operate under the parameters in the left panel of [Figure 4](#). After equilibration, or sufficient movement for purposes of the experiment, the demand curve was shifted so the stability properties of all equilibria were reversed. Thus, if the data moved to an equilibrium before the shift then it became unstable after the shift according to the same theory that predicted it would go there in the first place. Such an experiment demonstrates that the model predicts movements both to and from an equilibrium.

The data from a typical experiment are in [Figure 5](#). From the discussion and the preferences in [Figure 3](#), the reader can imagine how differently shaped indifference curves can be induced and, in particular, can imagine how shapes can be chosen such that they produce the theoretical supply curve found in [Figure 4](#). The experiment begins with the parameters represented by the supply curve  $S$  and the demand curve  $D_1$ . Then, after several periods the demand curve is changed to  $D_2$  with the supply curve remaining at  $S$ . After a few more periods, the demand curve shifts back to  $D_1$  and the supply curve still remains at  $S$ . As these shifts are made, all equilibria shift between stability and instability.

Notice first that there are several equilibria. On the left, points  $a$ ,  $b$  and  $d$  are all equilibria but the stability properties differ. Under conditions  $D_1$ , points  $a$  and  $d$  are Walrasian stable and Marshall unstable. Point  $b$  is Walrasian unstable and Marshallian stable. By contrast, under conditions  $D_2$  the equilibria are points  $a$ ,  $b$  and  $c$ . Points  $b$  and  $c$  are Walrasian stable and Marshallian unstable. Point  $a$  is Walrasian unstable and Marshallian stable. Thus, for each of these equilibria the two models always give exactly the opposite predictions about stability. Other equilibria exist on the boundaries but these will not be discussed.

The nature of the exercise is to start with the parameters  $D_1$  and  $S$ . If the market converges to one of the equilibria it will be a stable equilibrium according to one of the models. Prices should only converge to a stable equilibrium and thus convergence to one of the points will lend support for one of the models and serve as evidence against

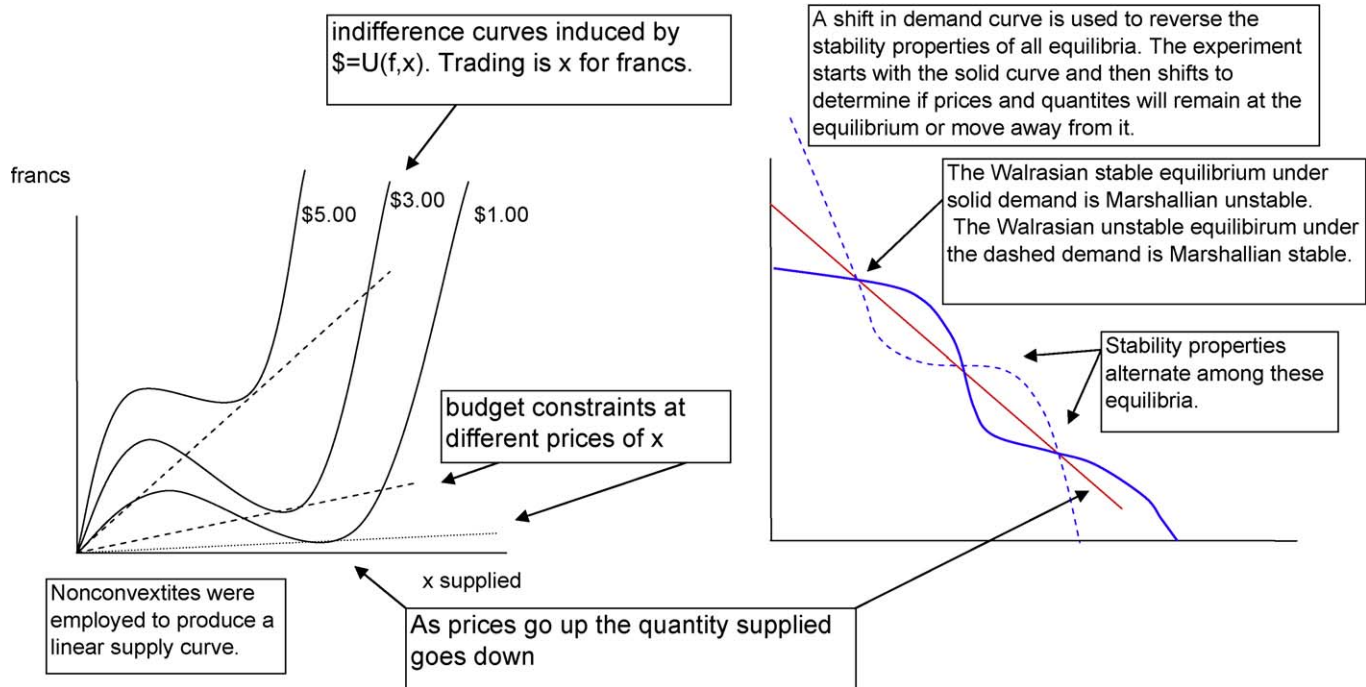


Figure 3.

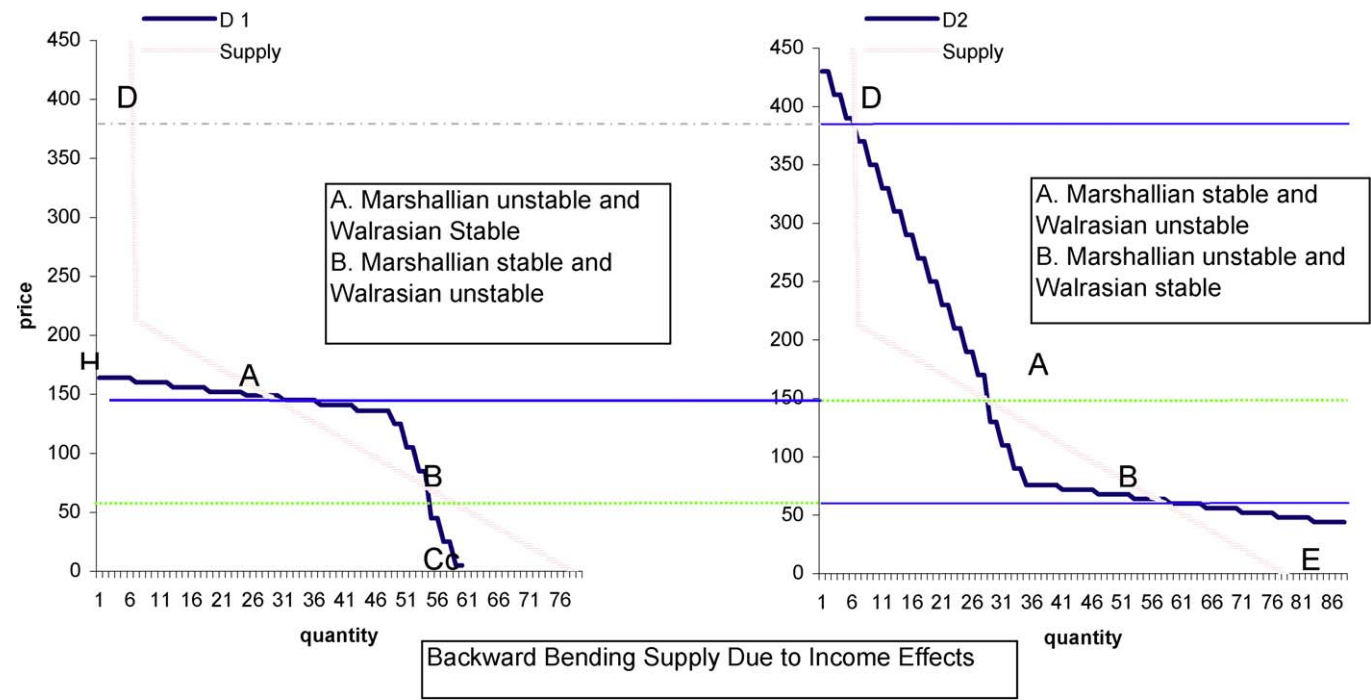


Figure 4.



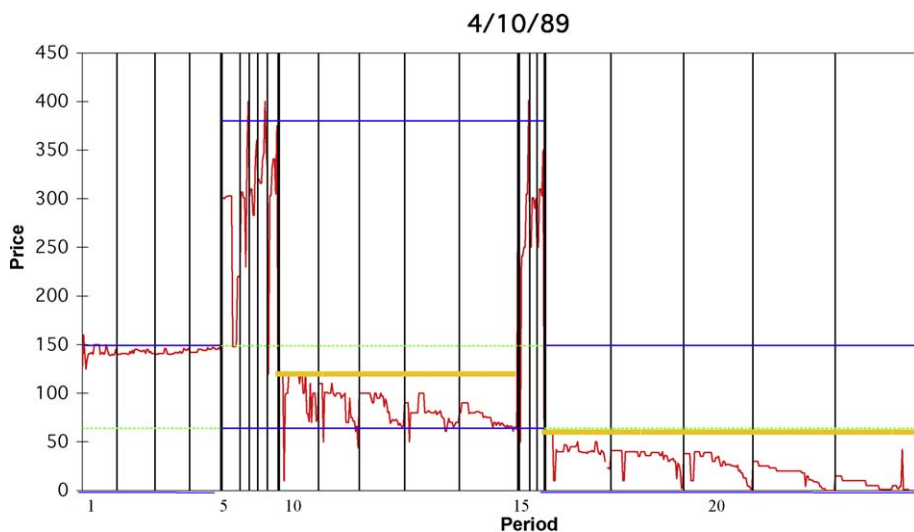


Figure 5.

the others. After the shift, the stability properties of all equilibria are reversed so the equilibrium where the prices exist will become unstable according to the model that had previously been supported. Thus, if the principles of the model were active then one would expect the prices to move away from the equilibrium toward one of the other stable equilibria of the model.

The basic result is that the data from experiments support the Walrasian model and not the Marshallian model. As can be seen in Figure 5, when the markets begin under demand condition  $D_1$ , prices converge to point  $a$ , an equilibrium that is stable according to the Walrasian model and unstable according to the Marshallian model. When the demand shifts to  $D_2$  and point  $a$  becomes an unstable equilibrium according to Walras (stable according to Marshall) and the data move rapidly upward toward the stable Walrasian equilibrium that exists at point  $c$ . Such exercises produce strong evidence for the Walrasian model.

Additional exercises represented in the figure give additional support for Walras. About period 12 under demand conditions  $D_2$  a price ceiling was imposed on the system just below the Walrasian unstable equilibrium at  $a$  and as can be seen prices bumped against the ceiling and then immediately fell to the nearest Walrasian stable equilibrium at  $b$ . The price ceiling was removed during period 16 and prices jumped over the unstable equilibrium at  $a$  and moved toward the stable equilibrium at  $c$ . To complete the demonstration, the demand parameters were returned to  $D_1$  and a price ceiling was imposed just below the resulting unstable Walrasian equilibrium at  $b$ . The result was that prices fell to the Walrasian stable equilibrium at  $d$ . Removal of the price ceiling in the final period resulted in prices jumping away from the Walrasian stable  $d$  equilibrium to the stable  $a$  equilibrium. Thus the behavior is rather unintuitive, since high prices

forced down by a price ceiling and without and demand or supply change resulted in prices falling still further. When the (non-binding) ceiling was removed, prices jumped up,<sup>2</sup> moving toward and then through an unstable equilibrium and then on to a different stable equilibrium.

Such an exercise demonstrates three points. First, market equilibria can exhibit instability. Second, the nature of the instability and stability in this type of environment is captured by Walrasian adjustment as opposed to Marshallian. Third, there is something wrong with the theory because jumps across unstable equilibria should not occur. But, such jumps are observed. Evidently the principles of dynamics are not restricted to “local” dynamics. Thus, the theory is partially misspecified.

### 2.3. *The Marshallian Model and not the Walrasian Model Best Describes Market Behavior in the Case of a Marshallian Externality or a “Fad”*

In order to study the second environment that leads to “perversely shaped” curves, an externality must be introduced. In the case of a fad this is accomplished in [Plott and Smith \(1999\)](#) by letting the redemption values of the individual buyers be a function of own purchases as well as the total volume of purchases of others. Of course, the supply case could be studied just as well as it was in [Plott and George \(1992\)](#). Let  $X_i$  be the consumption of individual  $i$  and let  $X_{-i}$  be the total consumption of agents other than individual  $i$ . Redemption values are of the form  $R(X_i, X_{-i})$  and let  $X_{-i}^e$  be the expectations of individual  $i$  about the consumption of others.

According to theory market demand is derived from the incentives  $R(X_i, X_{-i})$  through the decision problem defined by (5) and (6).

$$\max R(X_i, X_{-i}^e) - PX, \quad (5)$$

$$X_{-i}^e = X_{-i}. \quad (6)$$

The problem defined by (5) says that the individual attempts to maximize money income based on the redemption values and the expectations of the consumption of others. Equation (6) is a form of rational expectations, which says that the expectations of individuals are correct. From these equations an upward sloping demand can be computed as shown in [Figure 6](#), although this demand curve is a much more complex theoretical construction than the ordinary demand curve since it has an element of rational expectations. Indeed it is more of an equilibrium curve than a demand curve in the usual sense.

All experiments with the upward-sloping supply curve are with the double auction. The downward-sloping supply curve with a Marshallian externality was studied in [Plott](#)

<sup>2</sup> [Isaac and Plott \(1981\)](#) were the first to report the fact that a non-binding price control can have surprising effects on a market. The fact that removal of the non-binding controls could cause a switch in the equilibrium selected was new with the demonstration shown in the figure.

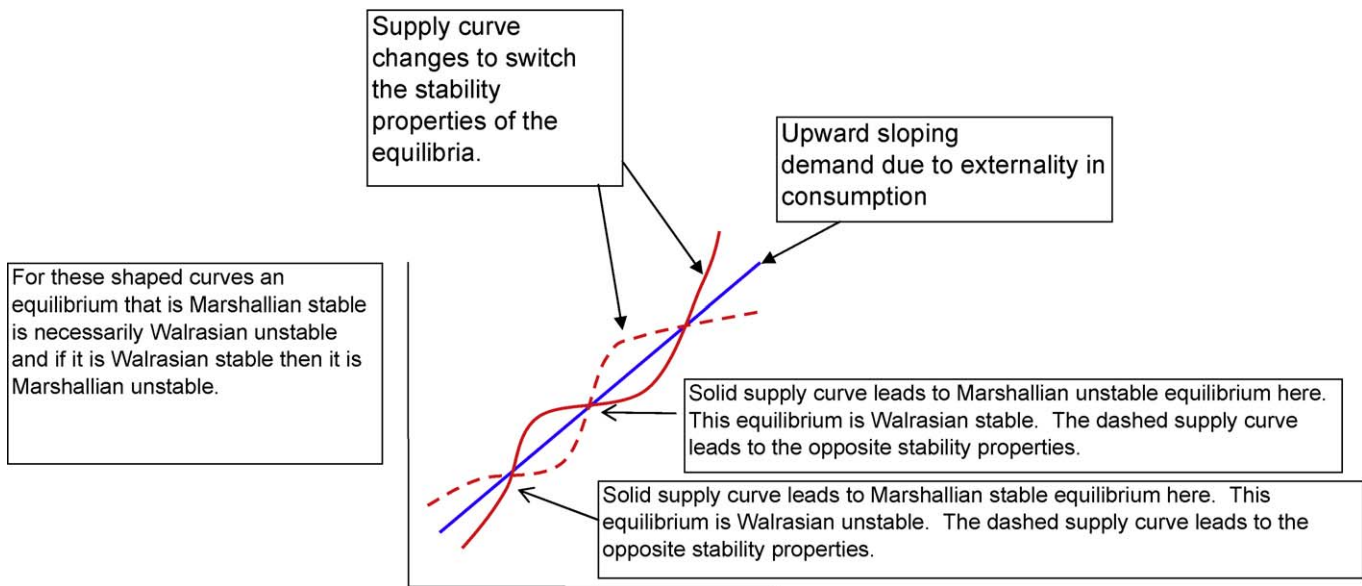


Figure 6. Market demand and supply for the case of a fad or other externality that causes an upward slope demand.

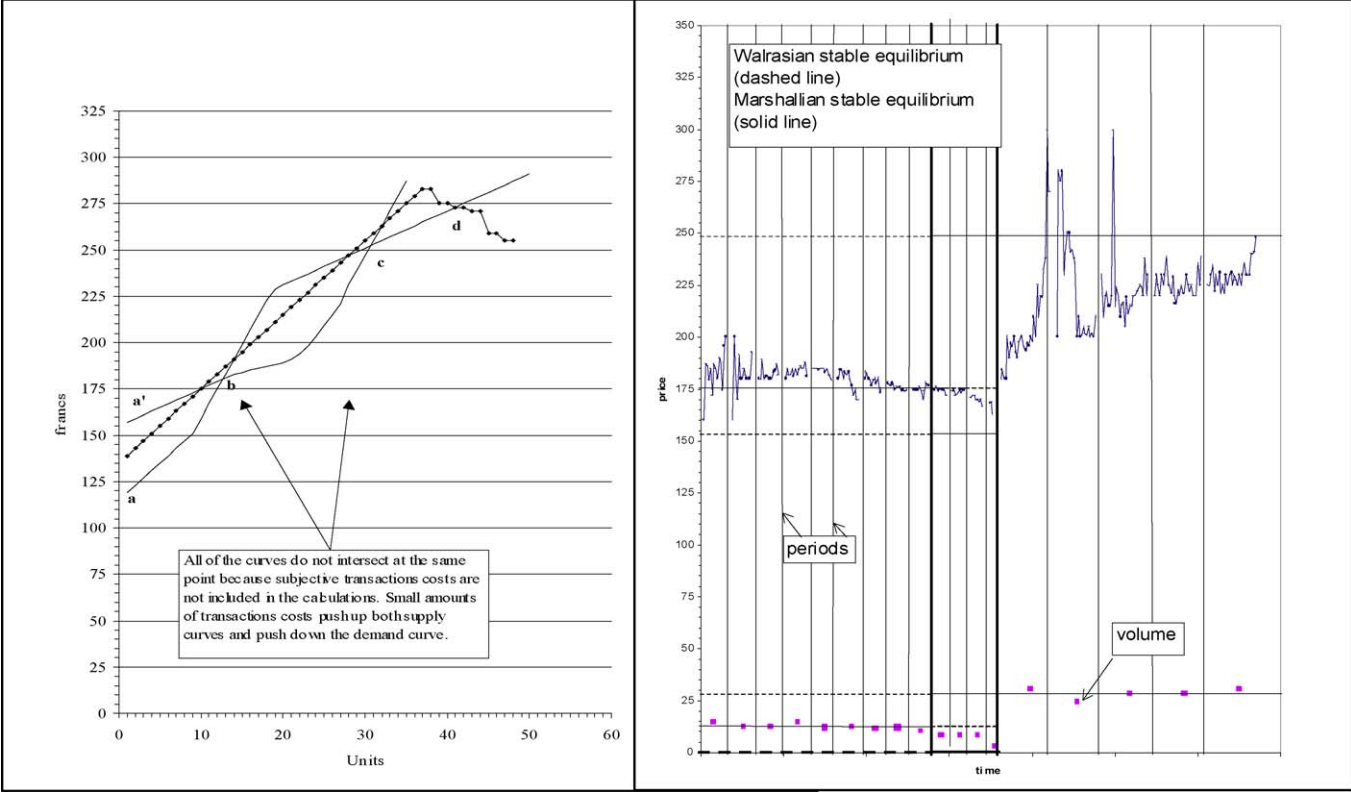


Figure 7.

and George under the double auction, the sealed bid/offer and a form of tatonnement with results similar to those reported here.

The structure of the experimental environment is shown in Figure 7. The demand curve is held constant over periods while the supply curve is shifted to create different stability conditions of the equilibria.

The results of three different conditions are reproduced in the right hand panel of Figure 7. First, the markets were opened under supply conditions such that the equilibrium at point  $b$  is stable according to Marshall but is unstable according to Walras. The supply conditions are then changed so that the stability conditions are reversed. Finally, a “volume guarantee” is added so the individuals get redemption values as if the volume was slightly above the volume at point  $b$  even though the volume might have been less. This started the system to the “right” of the equilibrium volume at  $b$ , which is an unstable equilibrium according to Marshall.

As can be seen in Figure 7, the data move toward the stable Marshallian (unstable Walrasian) equilibrium at point  $b$ . When point  $b$  becomes an unstable Marshallian (stable Walrasian) equilibrium, the market immediately moves to one of the nearby stable Marshallian equilibria (at the boundary). The volume completely dries up to zero. Then, when the process is started above the unstable equilibrium at  $b$ , the market immediately moves toward the nearest stable Marshallian (unstable Walrasian) equilibrium above  $b$ .

### 3. Summary

First, market prices can exhibit the type of instability predicted by classical dynamic models.<sup>3</sup> Second, the conditions under which instability is observed are not captured by the cobweb model but such conditions are captured by models of the form developed by Marshall and Walras in which the market has “perversely shaped” curves, such as an upward-sloped demand or downward-sloped supply. Third, the appropriate model, Marshall or Walras, depends on properties of the underlying demand and supply. If the special shape is due to the existence of an externality such as a fad or a Marshallian external economy, then experiments have demonstrated that the Marshallian model is the appropriate model. If the special shape is due to income effects, then the appropriate model is the Walrasian model and not the Marshallian model. That is, if the special shape is due to income effects such as Giffen goods or a backward-bending supply of labor, then the Walrasian model reflects the appropriate principles. In summary, the mystery of the price discovery process is solved, in part, by classical models of adjustment.

<sup>3</sup> In a very early paper Smith (1965) conducted experiments on the nature of dynamic adjustment but his design was inadequate for separating the competing theories and his econometric analysis misled him about the phenomena. He mistakenly rejected the Walrasian model.

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