Not p-values, Said a little differently

Is it a fair coin?

- Evidence: 26 heads out of 64 tosses

- Frequentist:
  $$p(Data|Hypothesis)$$
- Bayesian
  $$p(Hypothesis|Data)$$

Frequentist

- p-value

$$F_B(h, n|\mu = .5) + \left(1 - F_B(n - h, n|\mu = 0.5)\right)$$

where F_B is binomial cdf

$$n = 64, h = 26 \Rightarrow p\text{−value} = .08$$

Wald with normal approximation

$$\hat{\mu} = \frac{26}{64}$$

$$var(\hat{\mu}) \approx \frac{\mu(1-\mu)}{n}$$

$stddev(\hat{\mu}|\mu_0 = .5) = 0.0625, \mathrm{Z} = -1.50,$
$p\text{-Value} = .134$

$stddev\left(\hat{\mu}\middle|\hat{\mu} = \frac{26}{64}\right) = 0.0614, \mathrm{Z} = -1.53,$
$p\text{−Value} = .126$

LR

$$\mathcal{L}_{mle} = \log(f_B(h, n|\mu = \hat{\mu})) = -2.29$$
$$\mathcal{L}^* = \log(f_B(h, n|\mu = .5)) = -3.42$$

$$LR = -2(\mathcal{L}^* - \mathcal{L}_{mle}) = 2.26 \overset{A}{\sim} \chi_1^2$$
$$p\text{—value} = 0.1325$$

Bayesian Approach

$$P(\mu = 0.5|h, n) = \frac{f_B(h, n|\mu = 0.5) \times P(\mu = 0.5)}{\int_{-\infty}^{\infty} f_B(h, n|\mu = 0.5) \times P(\mu) d\mu}$$

Prior and posterior

$$\pi \equiv P(\mu = 0.5) = 0.5$$
$$P(\mu) = \begin{cases} 1 - \pi, 0 \le \mu < .5, .5 < \mu \le 1 \\ 0, \text{otherwise} \end{cases}$$

$$P(\mu = 0.5|H, n) = \frac{f_B(h, n|\mu = 0.5) \times \pi}{\int_0^1 f_B(h, n|\mu = 0.5) d\mu \times (1 - \pi) + f_B(h, n|\mu = 0.5) \times \pi}$$

- $P(\mu = 0.5|H, n) = .69$