Econ241b: PS 3

Casey O'Hara

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1. Asymptotic Normality to Consistency

You undertake a research project on carbon sequestration with a colleague from the geology department. He defers to you as the expert on statistics.

(a) In one discussion, you note that the estimator you are using is consistent and asymptotically normal. He then asks the natural question, "What are the advantages of using an estimator that is consistent and asymptotically normal?" Please respond.

Consistency in a parameter estimator simply means that as you include more data points, you can be confident that the estimator more closely approaches the unknown population parameter. The logical extreme is when your n encompasses the entire population, and your estimate becomes the actual parameter.

Lack of consistency would imply that additional data will not improve your estimate. No matter how many samples we take, our estimator will never provide a good understanding of the population parameter. So consistency in an estimator is very important.

Asymptotically normal is less important, though convenient. As we collect more data, our sample distribution approaches a normal distribution, which allows us to do straightforward statistical tests using critical values. Other asymptotic distributions would probably still allow for statistical testing, though perhaps not quite as conveniently.

(b) Several weeks later, the geologist proposes an estimator that is common in his field, but with which you are not familiar. He proudly notes that the estimator, $\hat{\theta}_n$, is asymptotically normal (a concept he learned from you) and displays an equation he copied down:

$$\sqrt{n}(\hat{\theta}_n - \theta_0) \xrightarrow{d} N(0, \sigma^2)$$

He is puzzled, however, by the fact that he could not find out if the estimator is consistent for the population value θ_0. Please show why the asymptotic normality result does, or does not, establish consistency.

We know that $\sqrt{n}(\hat{\theta}_n - \theta_0)$ converges, in distribution, to a random variable $N(0, \sigma^2)$. One part of Slutsky's theorem tells us:

$$(X_n \xrightarrow{d} X, \bar{Y}_n \xrightarrow{p} 0) \Rightarrow \bar{Y}_n' X_n \xrightarrow{p} 0$$

Since $1/\sqrt{n} \to 0$ as n increases, then

$$\begin{aligned}
\frac{1}{\sqrt{n}} \times \sqrt{n}(\hat{\theta}_n - \theta_0) &\xrightarrow{p} 0 \\
(\hat{\theta}_n - \theta_0) &\xrightarrow{p} 0 \\
\hat{\theta}_n &\xrightarrow{p} \theta_0
\end{aligned}$$

Therefore, the asymptotic normality of this new estimator establishes its consistency.

(c) After you have obtained your results from $\hat{\theta}_n$, the geologist returns some time later with $\widetilde{\theta}_n$, which is another estimator of θ_0. He also found that

$$n(\widetilde{\theta}_n - \theta_0) \xrightarrow{d} N(0, \sigma^2)$$

Should the two of you present the results you have already obtained, or obtain new results from $\widetilde{\theta}_n$? Be sure to explain your answer clearly to your geologist coauthor.

This new estimator is asymptotically normal, and consistent. But in this case, if we follow the same steps as for $\hat{\theta}_n$ above, our $(\widetilde{\theta}_n - \theta_0)$ term will converge to zero as $1/n$ instead of $1/\sqrt{n}$. So for the same number of data points we have presumably already gathered, $\widetilde{\theta}_n$ will provide a much better estimate (much smaller variance) of θ_0 than would $\hat{\theta}_n$.

2. Asymptotic Convergence

You model the purchase of tickets in a lottery. There are n tickets sold and the winning ticket is selected at random. The holder of the winning ticket receives n^2 dollars, all others receive nothing. It costs $n/2$ dollars to purchase one ticket.

Let p_n represent the random payoff to an individual and let $w_n = p_n - n/2$ represent the random winnings for an individual from the lottery.

(a) If individuals are risk neutral, would they elect to purchase a lottery ticket?

Risk-neutral individuals will buy tickets if their expected payoff equals or exceeds the cost of the ticket, i.e. expected net payoff is positive:

$$\mathbb{E}[p_n] \geq n/2, \text{ or } \mathbb{E}[w_n] \geq 0$$

In this case, the lottery results in a payoff p_n of n^2 with probability $1/n$, and 0 with probability $(n-1)/n$. Therefore,

$$\begin{aligned}\mathbb{E}[w_n] &= \frac{1}{n}n^2 + \frac{n-1}{n}0 - \frac{n}{2} \\ &= n + 0 - \frac{n}{2} \\ &= \frac{n}{2} \geq 0\end{aligned}$$

Therefore, a risk-neutral individual will definitely purchase a ticket.

(b) As the number of tickets sold increases, what does an individual's payoff converge to in probability?

We know that positive payout $P(p_n > 0) = \frac{1}{n}$, or $P(p_n > \epsilon) \leq \frac{1}{n}$. To look at convergence in probability, look at the limit of this as $n \to \infty$:

$$\lim_{n\to\infty} P(|p_n| > \epsilon) \leq \lim_{n\to\infty} \frac{1}{n}$$

But $\lim_{n\to\infty} \frac{1}{n} = 0$, so

$$\lim_{n\to\infty} P(|p_n| > \epsilon) = 0$$

The payoff converges to zero in probability.

(c) As the number of tickets sold increases, what does an individual's expected payoff converge to?

The expected payoff $\mathbb{E}[p_n] = n$, so as $n \to \infty$, $\mathbb{E}[p_n] \to \infty$. Expected payoff does not converge! Instead it gets larger as more people purchase tickets. In all cases, the expected payoff n exceeds the cost of the ticket $n/2$, i.e. $\mathbb{E}[w_n] > 0$.

(d) In light of your answers to parts b and c, would you be able to conclude that for large enough values of n individuals elect not to buy lottery tickets?

From part b, and basic logic of lotteries, we know that as more people buy tickets, the probability of actually winning decreases toward a probability of zero.

But this would not deter a risk-neutral lotto buff, because from part c, the expected payoff of a win increases without convergence, at a rate that exceeds the diminishing chance of winning. The positive expected net payoff $\mathbb{E}[w_n] = n/2 > 0$ means that a risk-neutral customer will always buy a ticket.

3. Let the estimator B be $\sqrt{n}$-consistent and asymptotically normal for β:

$$\sqrt{n}(B - \beta) \xrightarrow{d} N(0, \sigma^2)$$

where $\beta = 0$. Consider $G = 1/B$ as an estimator for $\gamma = 1/\beta$.

(a) Find the asymptotic distribution for G. Does it follow that G is a consistent estimator for γ?

Use the Delta method to determine the asymptotic distribution; from the given formula above, and a function $g(\cdot)$ with a non-zero first derivative $g'(\cdot)$ (NOTE: not using matrix formulation given in the notes, so g' is derivative, not transpose):

$$\sqrt{n}(g(B_n) - g(\beta)) \xrightarrow{d} N(0, g'(\beta)^2\sigma^2)$$

$$\begin{aligned} g(B) &= 1/B = G; g(\beta) = 1/\beta = \gamma \\ g'(\beta) &= -1/\beta^2 \\ \sqrt{n}(G - \gamma) &\xrightarrow{d} N(0, \left(-\frac{1}{\beta^2}\right)^2 \sigma^2) \\ &\xrightarrow{d} N(0, \frac{\sigma^2}{\beta^4}) \end{aligned}$$

Since $\sqrt{n}(G - \gamma)$ is asymptotically normal, by the same steps in question 1, we can show that $G \xrightarrow{p} \gamma$, therefore G is a consistent estimator for γ.

(b) From a sample of data with $n = 20$, the values $b = 6$ and $s^2 = 320$ are obtained. Find the associated estimate for γ along with the estimated "asymptotic" standard errors for B and G.

$g(b) = 1/b = 1/6$ is our estimator for γ.

Asymptotically, $\sqrt{n}(B - \beta) \sim N(0, \sigma^2)$, which can be rearranged as $(B - \beta) \sim N(0, \sigma^2/n)$. Standard error formula is $\sqrt{\frac{\sigma^2}{n}}$, so for our sample, our estimated standard error for B is:

$$\sqrt{\frac{s^2}{n}} = \sqrt{\frac{320}{20}} = 4$$

and for G:

$$\sqrt{\frac{s^2}{nb^4}} = \sqrt{\frac{320}{20 \times 6^4}} = \frac{1}{9}$$

(c) With the values from part b, test $H_0 : \beta = 2$ against $H_1 : \beta \neq 2$.

From our sample, $b = 6$, and standard error $= 4$. If we choose critical values within $6 \pm (1.96 \times 4)$, then 2 is within those bounds. We can't reject the null in this case.

(d) Recast the hypothesis test from part c in terms of γ and use the values from part b to test. Explain.

Our null hypothesis will compare $H_0 : \gamma = 1/2$ and $H_1 : \gamma \neq 1/2$. From our sample, our estimate of $\gamma = 1/6$, and standard error $= \frac{1}{9}$. In this case, we would choose critical values within $\frac{1}{6} \pm (1.96 \times \frac{1}{9})$. This gives a range of $[.0511, .3844]$, so we reject our null. Thus, in this case our estimator for γ seems to be more precise than our estimator for β - seems like the standard error is proportionally smaller.

4. Computational Exercise

   Return to the paper by Charness and Kuhn listed on the syllabus. Write programs in both Matlab and Stata (the results from each program should match) that estimate the models in columns (6) and (7) of Table 3 of Charness and Kuhn. Calculate classic standard errors (the authors report cluster-robust standard errors, so your estimated standard errors will not match those in the table). Test the hypothesis that all four slope coefficients are equal and provide the p-value for the estimated test statistic.

Stata output:

Note: manually deleted the 0.w1 rows for clarity... note also the discrepancy in column 7, worker type 2 wage == 1 (paper says .229, my calcs say .259).

Table 1: B: Type 1 workers

	(1)	(2)
	e1	e1
1.w1	.499	.42
	(.0843)	(.07098)
2.w1	1.278	1.119
	(.1116)	(.0947)
3.w1	1.544	1.308
	(.2936)	(.2324)
rel_w1_low	-.02431	.01501
	(.03485)	(.02947)
N	555	555
R^2	0.295	0.604

Standard errors in parentheses

Table 2: B: Type 1 workers

	(1)	(2)
	e2	e2
1.w2	.3467	.2594
	(.1447)	(.1151)
2.w2	1.045	.9421
	(.1386)	(.1118)
3.w2	1.718	1.5
	(.1597)	(.1303)
4.w2	1.488	1.575
	(.2226)	(.177)
rel_w2_low	.05617	.009565
	(.1081)	(.08327)
N	555	555
R^2	0.318	0.654

Standard errors in parentheses

Test if coefficients all equal:

column 6a:

```
test _Iw1_1 = _Iw1_2 = _Iw1_3

 ( 1)  _Iw2_1 - _Iw2_2 = 0
 ( 2)  _Iw2_1 - _Iw2_3 = 0

       F(  2,   521) =   39.25
            Prob > F =    0.0000
```

column 7a:

```
test _Iw1_1 = _Iw1_2 = _Iw1_3

 ( 1)  _Iw2_1 - _Iw2_2 = 0
 ( 2)  _Iw2_1 - _Iw2_3 = 0

      F(  2,   486) =   45.90
            Prob > F =    0.0000
```

column 6b:

```
test _Iw2_1 = _Iw2_2 = _Iw2_3 = _Iw2_4

 ( 1)  _Iw2_1 - _Iw2_2 = 0
 ( 2)  _Iw2_1 - _Iw2_3 = 0
 ( 3)  _Iw2_1 - _Iw2_4 = 0

       F(  3,   520) =   39.23
            Prob > F =    0.0000
```

column 7b:

```
test _Iw2_1 = _Iw2_2 = _Iw2_3 = _Iw2_4

 ( 1)  _Iw2_1 - _Iw2_2 = 0
 ( 2)  _Iw2_1 - _Iw2_3 = 0
 ( 3)  _Iw2_1 - _Iw2_4 = 0

       F(  3,   485) =   58.28
            Prob > F =    0.0000
```

Stata code

```
set more off
clear

// Set working directory

cd "~/github/econ_courses/econ241b/assts/asst4"

/*
The key variables:
    e1 = effort of type 1 workers
    w1 = wage of type 1 workers
    w2 = wage of type 2 workers
    pubwage = flag for public-wage regime
*/
clear all
use "prob_set_4.dta"

// drop non-public wages
drop if pubwage != 1

// drop unused columns
keep wrk1id wrk2id period e1 e2 w1 w2

///////////////////////////////////////////////
// Part A: Type 1 Workers (low productivity)
///////////////////////////////////////////////

// create relative wage variable
gen rel_w1 = w1 - w2

// create indicator for wage 1 <= wage 2
gen w1_low = (w1 <= w2)

// calculate rel wage * dummy
gen rel_w1_low = rel_w1 * w1_low


// regress variables for public wages for low productivity workers (1).
// Col 6: effort vs wage and relative wage (< 0, asymmetric model)
//   with fixed effects of period
// Col 7: effort vs wage and relative wage (< 0, asymmetric model)
//   with fixed effects of worker and period
eststo clear

// For fixed effects, we use xtreg instead of reg; use xtset to set the
// panel variable(s)
/*
xtset period
xtreg e1 i.w1 rel_w1_low, fe
*/
xi: reg e1 i.w1 rel_w1_low i.period
```

```
eststo col_a6


test _Iw1_1 = _Iw1_2 = _Iw1_3
/*
xtset wrk1id period
xtreg e1 i.w1 rel_w1_low i.period, fe
*/
xi: reg e1 i.w1 rel_w1_low i.period i.wrk1id
eststo col_a7
test _Iw1_1 = _Iw1_2 = _Iw1_3


esttab col_a6 col_a7 using table3a67.tex, ///
    drop(*wrk1id* *period* *cons*) title(B: Type 1 workers) noconst b(%10.4g) ///
    se r2 nostar replace


///////////////////////////////////////////////
// Part B: Type 2 Workers (high productivity)
///////////////////////////////////////////////

// create relative wage variable
gen rel_w2 = w2 - w1

// create indicator for wage 1 <= wage 2
gen w2_low = (w2 <= w1)

// calculate rel wage * dummy
gen rel_w2_low = rel_w2 * w2_low

// regress variables for public wages, for high productivity workers (2).
// Col 1: effort vs wage
// Col 2: effort vs wage and relative wage (symmetric model)
// Col 3: effort vs wage and relative wage (< 0, asymmetric model)
eststo clear

xi: reg e2 i.w2 rel_w2_low i.period
eststo col_b6
test _Iw2_1 = _Iw2_2 = _Iw2_3 = _Iw2_4

/*
xtset wrk1id period
xtreg e1 i.w1 rel_w1_low i.period, fe
*/
xi: reg e2 i.w2 rel_w2_low i.period i.wrk2id
eststo col_b7
test _Iw2_1 = _Iw2_2 = _Iw2_3 = _Iw2_4


esttab col_b6 col_b7 using table3b67.tex, ///
    drop(*wrk2id* *period* *cons*) title(B: Type 1 workers) noconst b(%10.4g) ///
    se r2 nostar replace
```

Matlab output:

Note, all the regression coefficients match the table in the paper, with the same exception noted for the Stata outputs. My F statistics are close but not identical to those output by Stata (my manual ones are slightly lower), though I can't see where the problem is being introduced.

```
Matrix calculated coefficients for table 3, column 6a
    0.4990    1.2778    1.5445   -0.0243
Regress() calculated coefficients for table 3, column 6a
    0.4990    1.2778    1.5445   -0.0243
Regress() calculated R^2, F, p val, error var
    0.2949    6.6031    0.0000    0.5810
wald_6a = 36.9544
p_val = 9.9920e-16

Matrix calculated coefficients for table 3, column 7a
    0.4200    1.1188    1.3080    0.0150
Regress() calculated coefficients for table 3, column 7a
    0.4200    1.1188    1.3080    0.0150
Regress() calculated R^2, F, p val, error var
    0.6039   10.8984    0.0000    0.3498
wald_7a = 44.4233
p_val = 0

Matrix calculated coefficients for table 3, column 6b
    0.3467    1.0450    1.7184    1.4877    0.0562
Regress() calculated coefficients for table 3, column 6b
    0.3467    1.0450    1.7184    1.4877    0.0562
Regress() calculated R^2, F, p val, error var
    0.3178    7.1245    0.0000    0.8583
wald_6b = 37.2247
p_val = 0

Matrix calculated coefficients for table 3, column 7b
    0.2594    0.9421    1.5005    1.5753    0.0096
Regress() calculated coefficients for table 3, column 7b
    0.2594    0.9421    1.5005    1.5753    0.0096
Regress() calculated R^2, F, p val, error var
    0.6537   13.2698    0.0000    0.4671
wald_7b = 53.7397
p_val = 0
```

Matlab code:

```
%% Import data from text file.
clear all

filename = '~/github/econ_courses/econ241b/assts/asst4/prob_set4.csv';

raw = readtable(filename);
% NOTE: this is the same data as for asst 3; I did not save the Stata
% edits made for assignment 4.

%% set up data
% keep just public wages and drop the pubwage column
wage = raw(raw.pubwage == 1, :);
wage.pubwage = [];

% add in w11-w14, w21-w24 columns
wage.w11 = wage.w1 == 1;
wage.w12 = wage.w1 == 2;
wage.w13 = wage.w1 == 3;
wage.w14 = wage.w1 == 4;
wage.w21 = wage.w2 == 1;
wage.w22 = wage.w2 == 2;
wage.w23 = wage.w2 == 3;
wage.w24 = wage.w2 == 4;

nrows = length(wage.w1);
const = ones(nrows, 1);


%% TYPE 1 WORKER
%% Effort vs wages and rel wages, type 1 worker
% set up X2 to be a constant plus own wage regressor plus relative wage
wage.rel_w1 = wage.w1 - wage.w2;
wage.rel_w1_low = wage.rel_w1 .* (wage.rel_w1 < 0);

%% manual regression with period fixed effects

% Dummy variable expansion for period variable, then cut one column to
% avoid singular matrix at the end
d_period = dummyvar(wage.period);
d_period(:, 1) = [];

% Set up X matrix including constant, regressors, and fixed effects matrix
X6a = [const wage.w11 wage.w12 wage.w13 wage.rel_w1_low d_period];
y  = wage.e1;

disp('Matrix calculated coefficients for table 3, column 6a')
b6a = (X6a' * X6a)^-1 * (X6a' * y);
disp(b6a(2:5)') %%% these are the coefficients on wage 1, 2, 3, and difference
% 0.4990    1.2778    1.5445   -0.0243

disp('Regress() calculated coefficients for table 3, column 6a')
```

```
[B6a, BINT, R, RINT, STATS] = regress(y, X6a);
disp(B6a(2:5)') %%% regression coefficients in table 6a
% 0.4990    1.2778    1.5445   -0.0243
disp('Regress() calculated R^2, F, p val, error var')
disp(STATS) %%% R-square statistic, the F statistic and p value for the full
      %%% model, and an estimate of the error variance
%  0.2949    6.6031    0.0000    0.5810

%% Try Wald test to check whether coefficients are equal
% H_0: B1 - B2 = 0, B1 - B3 = 0

R = [ones(2, 1) -eye(2) zeros(2, 31)];
r = zeros(2, 1);

e_i = y - X6a * B6a;

ssr = sum(e_i.^2);
dof = length(y) - length(B6a);
s2 = ssr / dof;

cov_mat = (X6a' * X6a)^(-1) * s2;

[n, k] = size(X6a);

% [h, pValue, stat, cValue] = waldtest(r,R,cov_mat) % throws an error
wald_6a = (R*B6a - r)'*(R * cov_mat * R')^-1 * (R*B6a - r) / 2

p_val = 1 - fcdf(wald_6a, 2, n - k)

%% Manual regression: fixed effects on period and worker ID

% Dummy variable expansion for worker 1 variable, then cut two columns to
% avoid singular matrix at the end.
d_wrk1id = dummyvar(wage.wrk1id);
sum_test = sum(d_wrk1id) ~= 0;
d_wrk1id = d_wrk1id(:, sum_test);
d_wrk1id(:, 37) = [];
d_wrk1id(:, 1) = [];
%%
% Set up X matrix including constant, regressors, and fixed effects matrix
X7a = [const wage.w11 wage.w12 wage.w13 wage.rel_w1_low d_period d_wrk1id];
y  = wage.e1;

disp('Matrix calculated coefficients for table 3, column 7a')
b7a = (X7a' * X7a)^-1 * (X7a' * y);
disp(b7a(2:5)') %%% these are the coefficients on wage 1, 2, 3, and difference
% 0.4990    1.2778    1.5445   -0.0243

disp('Regress() calculated coefficients for table 3, column 7a')
[B7a, BINT, R, RINT, STATS] = regress(y, X7a);
disp(B7a(2:5)') %%% regression coefficients in table 6
%    0.4200    1.1188    1.3080    0.0150
disp('Regress() calculated R^2, F, p val, error var')
```

```
disp(STATS) %%% R-square statistic, the F statistic and p value for the full
      %%% model, and an estimate of the error variance
%    0.6039   10.8984    0.0000    0.3498


%% Try Wald test to check whether coefficients are equal
% H_0: B1 - B2 = 0, B1 - B3 = 0

R = [ones(2, 1) -eye(2) zeros(2, 66)];
r = zeros(2, 1);

e_i = y - X7a * B7a;

ssr = sum(e_i.^2);
dof = length(y) - length(B7a);
s2 = ssr / dof;

cov_mat = (X7a' * X7a)^(-1) * s2;

[n, k] = size(X7a);

% [h, pValue, stat, cValue] = waldtest(r,R,cov_mat) % throws an error
wald_7a = (R*B7a - r)'*(R * cov_mat * R')^-1 * (R*B7a - r) / 2

p_val = 1-fcdf(wald_7a, 2, n - k)



%% TYPE 2 WORKER
%% Effort vs wages and rel wages, type 1 worker
% set up X2 to be a constant plus own wage regressor plus relative wage
wage.rel_w2 = wage.w2 - wage.w1;
wage.rel_w2_low = wage.rel_w2 .* (wage.rel_w2 < 0);

%% manual regression

% Set up X matrix including constant, regressors, and fixed effects matrix
X6b = [const wage.w21 wage.w22 wage.w23 wage.w24 wage.rel_w2_low d_period];
y  = wage.e2;

disp('Matrix calculated coefficients for table 3, column 6b')
b6b = (X6b' * X6b)^-1 * (X6b' * y);
disp(b6b(2:6)') %%% these are the coefficients on wage 1, 2, 3, and difference
%  0.3467    1.0450    1.7184    1.4877    0.0562

disp('Regress() calculated coefficients for table 3, column 6b')
[B6b, BINT, R, RINT, STATS] = regress(y, X6b);
disp(B6b(2:6)') %%% regression coefficients in table 6
%  0.3467    1.0450    1.7184    1.4877    0.0562
disp('Regress() calculated R^2, F, p val, error var')
disp(STATS) %%% R-square statistic, the F statistic and p value for the full
      %%% model, and an estimate of the error variance
%  0.3178    7.1245    0.0000    0.8583
```

```
%% Try Wald test to check whether coefficients are equal
% H_0: B1 - B2 = 0, B1 - B3 = 0, B1 - B4 = 0

R = [ones(3, 1) -eye(3) zeros(3, 31)];
r = zeros(3, 1);

e_i = y - X6b * B6b;

ssr = sum(e_i.^2);
dof = length(y) - length(B6b);
s2 = ssr / dof;

cov_mat = (X6b' * X6b)^(-1) * s2;

[n, k] = size(X6b);

% [h, pValue, stat, cValue] = waldtest(r,R,cov_mat) % throws an error
wald_6b = (R * B6b - r)'*(R * cov_mat * R')^-1 * (R*B6b - r) / 3

p_val = 1-fcdf(wald_6b, 3, n - k)


%% Manual regression: fixed effects on period and worker ID

% Dummy variable expansion for worker 2 variable, then cut two columns to
% avoid singular matrix at the end.
d_wrk2id = dummyvar(wage.wrk2id);
sum_test = sum(d_wrk2id) ~= 0;
d_wrk2id = d_wrk2id(:, sum_test);
d_wrk2id(:, 37) = [];
d_wrk2id(:, 1) = [];
%
% Set up X matrix including constant, regressors, and fixed effects matrix
X7b = [const wage.w21 wage.w22 wage.w23 wage.w24 wage.rel_w2_low d_period d_wrk2id];
y  = wage.e2;

disp('Matrix calculated coefficients for table 3, column 7b')
b7b = (X7b' * X7b)^-1 * (X7b' * y);
disp(b7b(2:6)') %%% these are the coefficients on wage 1, 2, 3, and difference
%    0.2594    0.9421    1.5005    1.5753    0.0096

disp('Regress() calculated coefficients for table 3, column 7b')
[B7b, BINT, R, RINT, STATS] = regress(y, X7b);
disp(B7b(2:6)') %%% regression coefficients in table 6
%    0.2594    0.9421    1.5005    1.5753    0.0096
disp('Regress() calculated R^2, F, p val, error var')
disp(STATS) %%% R-square statistic, the F statistic and p value for the full
      %%% model, and an estimate of the error variance
%    0.6537   13.2698    0.0000    0.4671

%% Try Wald test to check whether coefficients are equal
% H_0: B1 - B2 = 0, B1 - B3 = 0, B1 - B4 = 0
```

```
R = [ones(3, 1) -eye(3) zeros(3, 66)];
r = zeros(3, 1);

e_i = y - X7b * B7b;

ssr = sum(e_i.^2);
dof = length(y) - length(B7b);
s2 = ssr / dof;

cov_mat = (X7b' * X7b)^(-1) * s2;

[n, k] = size(X7b);

% [h, pValue, stat, cValue] = waldtest(r,R,cov_mat) % throws an error
wald_7b = (R * B7b - r)'*(R * cov_mat * R')^-1 * (R*B7b - r) / 3

p_val = 1-fcdf(wald_7b, 3, n - k)
```