Economics 241B Exercise 1
Conditional Expectation Functions and Specification of Conditional Expectation Functions

1.

a. If $\mathbb{E}(y|x) = \beta_1 x + \beta_0$, find $\mathbb{E}(yx)$ as a function of the moments of x.

b. Suppose the random variables y and x take only the values 0 and 1 and have the following joint probability distribution

	$x=0$	$x=1$
$y=0$	a	c
$y=1$	b	d

To satisfy the properties of a joint distribution, what must be true of (a, b, c, d)? Find $\mathbb{E}(y|x)$, $\mathbb{E}(y^2|x)$, and $Var(y|x)$ for $x = 0$ and $x = 1$.

2. Assume $\mathbb{E}|g(x)y| < \infty$.

Prove

$$\mathbb{E}(g(x)y|x) = g(x)\mathbb{E}(y|x).$$

3. If $y = x\beta + u$, $x \in \mathbb{R}$, then for each of the following statements either establish that they are true or provide a counterexample:

i) $\mathbb{E}(u|x) = 0$ implies $\mathbb{E}(x^2 u) = 0$,

ii) $\mathbb{E}(xu) = 0$ implies $\mathbb{E}(x^2 u) = 0$,

iii) $\mathbb{E}(u|x) = 0$ implies $\mathbb{E}(y|x) = x\beta$,

iv) $\mathbb{E}(xu) = 0$ implies $\mathbb{E}(y|x) = x\beta$,

4. Recall that the conditional variance is $\sigma^2(x) = Var(y|x) = \mathbb{E}\left((y - \mathbb{E}(y|x))^2 |x\right)$. Show that the conditional variance can be written as

$$\sigma^2(x) = \mathbb{E}\left(y^2|x\right) - (\mathbb{E}(y|x))^2.$$