Causal Effects

Econometrics II

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Overview

Reference: B. Hansen Econometrics Chapter 2.30

- interest in determining casual effects
  - causal effects are individual specific
  - causal effects are unobserved
  - hence, individual causal effects cannot be measured
- aggregate causal effects can be measured
  - focus on average causal effect
  - under conditional independence assumption
    - conditional mean derivative equals the average causal effect
    - i.e. conditional mean has causal implications

What are Causal Effects?

- Casual effects are: $\nabla_x y$
  - individual specific (not $\nabla_x \mathbb{E}(y|x)$ nor $\nabla_x \mathcal{P}(y|x)$)
  - unobserved
- Examples
  - effect of class size on student test score
  - effect of years of schooling on wage
  - consequences of medical procedure for health
- consider schooling on wage
  - $\nabla_x y$ is the actual difference in wages a person would receive if we could change the number of years of schooling, holding all else constant
    - specific to each individual
    - can't observe counterfactual (only observe actual wages and schooling$\nabla_x y$)

Example

- 2 individuals: Jennifer and George
  - both have possibility of college graduation (otherwise HS graduates)
  - both would receive wages that differ over their level of education

	Wage if HS	Wage if College	Causal Effect
Jennifer	$\$10/hr$	$\$20/hr$	$\$10/hr$
George	$\$8/hr$	$\$12/hr$	$\$4/hr$

  - causal effect is individual specific
  - causal effect is unobserved
    - either a HS graduate or a college graduate, but not both

General Model

- $y = h(x_1, x_2, u) \quad (x_1, x_2)$ observed u unobserved $\quad (\ell \times 1)$
  - special case: random coefficients model $h(x_1, x_2, u) = x^{\mathrm{T}}\eta$
- causal effect of x_1 on y
  - x_1 has a causal effect on response y if a change in x_1, holding all other inputs constant, leads to a change in y
  - in other words: the change in y due to a change in x_1, holding x_2 and u constant

$$C(x_1, x_2, u) = \nabla_1 h(x_1, x_2, u)$$

- this is a definition
  - doesn't necessarily describe causality in a fundamental or experimental sense
  - perhaps should be called a structural effect
    - ★ the effect within the structural model $h(x_1, x_2, u)$

Average Causal Effects

because casual effects vary over individuals and are not observable, they cannot be measured at the individual level

- only aggregate causal effects can be measured
  - we focus on the average causal effect
- the average causal effect of x_1 on y conditional on x_2 is

$$\begin{aligned} ACE\left(x_1, x_2\right) &= \mathbb{E}\left(C\left(x_1, x_2, u\right) | x_1, x_2\right) \\ &= \int_{\mathbb{R}^\ell} \nabla_1 h\left(x_1, x_2, u\right) f\left(u | x_1, x_2\right) du \end{aligned}$$

  - $ACE\left(x_1, x_2\right)$ is the average of the causal effects across all individuals in the general population
  - not an individual causal effect (not a function of u)
- the goal is to learn the average causal effect

ACE and the Conditonal Expectation Function

What is the derivative of the conditional mean?

- let $m(x_1, x_2)$ represent $\mathbb{E}(y|x_1, x_2)$

$$\begin{aligned} m(x_1, x_2) &= \mathbb{E}(h(x_1, x_2, u)|x_1, x_2) \\ &= \int_{\mathbb{R}^\ell} h(x_1, x_2, u) f(u|x_1, x_2)\, du \end{aligned}$$

- $\nabla_1 \mathbb{E}(y|x_1, x_2)$ is

$$\begin{aligned} \nabla_1 m(x_1, x_2) &= \int_{\mathbb{R}^\ell} \nabla_1 h(x_1, x_2, u) f(u|x_1, x_2)\, du \\ &\quad + \int_{\mathbb{R}^\ell} h(x_1, x_2, u) \nabla_1 f(u|x_1, x_2)\, du \end{aligned}$$

$$= ACE(x_1, x_2) + \int_{\mathbb{R}^\ell} h(x_1, x_2, u) \nabla_1 f(u|x_1, x_2)\, du$$

Conditional Independence Assumption

- $\nabla_1 \mathbb{E}\left(y|x_1, x_2\right) = ACE\left(x_1, x_2\right)$ only in the special case

$$\nabla_1 f\left(u|x_1, x_2\right) = 0$$

- Conditional Independence Assumption: $f\left(u|x_1, x_2\right) = f\left(u|x_2\right)$ does not depend on x_1, implying

$$\nabla_1 f\left(u|x_1, x_2\right) = 0$$

- CIA is weaker than full independence of u from $\left(x_1, x_2\right)$
  - full independence would imply that each regression derivative equals an average causal effect
  - CIA is all that is needed to causally interpret only a subset of the covariates

CIA and Average Causal Effects

- Theorem: In the structural model $y = h(x_1, x_2, u)$, the Conditional Independence Assumption implies

$$\nabla_1 \mathbb{E}(y | x_1, x_2) = ACE(x_1, x_2)$$

  - x_1 can be binary or continuous
- powerful result
  - whenever the unobservable is independent of the treatment variable (after conditioning on appropriate covariates), the conditional mean derivative equals the average causal effect
  - hence the CEF has causal meaning

Potential Outcome Function

Binary Covariate

- if x_1 is binary (structural effect is called treatment effect)
  - $x_1 = 1$ treatment
  - $x_1 = 0$ non-treatment
- replace $h(x_1, x_2, u)$ with $y(x_1)$
  - $y(x_1)$ implies holding x_2 and u constant
  - latent outcomes
    - ★ $h(1, x_2, u) = y(1)$ (outcome if "treated")
    - ★ $h(0, x_2, u) = y(0)$ (outcome if not "treated")
- causal effect

$$C(x_2, u) = y(1) - y(0)$$

  - random (it is a function of (x_2, u)), as both potential outcomes differ across individuals

Aggregate Causal Effects

Potential Outcome Function

- causal effect depends on both $y(1)$ and $y(0)$
  - we observe only the realized value $y = \begin{cases} y(1) & \text{if } x_1 = 1 \\ y(0) & \text{if } x_1 = 0 \end{cases}$
  - cannot measure individual causal effect
- we focus on the average causal effect
  - $$ACE(x_1, x_2) = \mathbb{E}\left(y(1) - y(0) \,|x_1, x_2\right)$$
- the average causal effect is the best we can hope to learn

Wage, Schooling Example

- $y = wage$ and x_1 is binary
  - $x_1 = 1$ treatment (college graduate)
  - $x_1 = 0$ non-treatment (HS graduate)
- population
  - 50% are Jennifers: (\$10 if HS \$20 if college)
  - 50% are Georges: (\$8 if HS \$12 if college)

$$ACE(x_1) = \frac{1}{2}(10) + \frac{1}{2}(4) = 7$$

  - given data only on education and wages, the most we could hope to learn is the average causal effect of \$7

Analysis

- collect wage and education data for 32 randomly sampled individuals

	\$8	\$10	\$12	\$20	Mean
HS	10	6	0	0	8.75
College	0	0	6	10	17.00

- because the only covariate is an indicator
  - $\mathbb{E}\left(y|x_1\right) = \beta_1 x_1 + \beta_2$
- with the data at hand
  - $\mathbb{E}\left(wage|\,\mathrm{col}\right) = 8.25\,\mathrm{col} + 8.75$
- the regression derivative, \$8.25, is larger than the ACE, \$7
  - the CIA must not be satisfied

Failure of the CIA

- $wage = h(x_1, x_2, u)$
  - u is individual type (i.e. Jennifer or George)
- because type Jennifer is more likely to go to college than type George, u is not independent of x_1

$$\int_{\mathbb{R}^\ell} h(x_1, u)\, \nabla_1 f(u|x_1)\, du \neq 0$$

  - recall $\nabla_1 m(x_1) = ACE(x_1) + \int_{\mathbb{R}^\ell} h(x_1, u)\, \nabla_1 f(u|x_1)\, du$
- \$8.25 is not the average benefit of college attendance, rather it is the observed difference in realized wages in a population whose decision to attend college is correlated with their individual causal effect

Selection into College

- in high school, all students take an aptitude test
  - score is recorded as high (H) or low (L)
    - $\mathbb{P}\left(\text{college}|H\right) = 3/4 \quad \mathbb{P}\left(\text{college}|L\right) = 1/4$
- the two types of students do not perform the same on the test
  - Jennifers get a high score $3/4$ of the time
  - Georges get a high score $1/4$ of the time
- probability of enrollment in college
  - for Jennifers: 62.5% $\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2$
  - for Georges: 37.5% $2 \cdot \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)$

Restoration of the CIA

- we need to find a variable, x_2, such that u and x_1 are independent conditional on x_2
- decision to attend college is based on aptitude test score $(x_2 = 1\{H\})$
  - education and type are independent, conditional on test score
    - i.e. given test score, Jennifers and Georges are equally likely to attend college
    - $f(u|x_1, x_2) = f(u|x_2)$ does not depend on x_1
- including test score, alters the ACE, which is now a function of x_2
  - $ACE(x_1, x_2 = H) = \frac{3}{4}(10) + \frac{1}{4}(4) = 8.50$
  - $ACE(x_1, x_2 = L) = \frac{1}{4}(10) + \frac{3}{4}(4) = 5.50$

Analysis

- collect wage and education data for 32 randomly sampled individuals

	\$8	\$10	\$12	\$20	Mean
HS + H	1	3	0	0	9.50
College + H	0	0	3	9	18.00
HS + L	9	3	0	0	8.50
College + L	0	0	3	1	14.00

- because the only 2 covariates are indicators
  - $\mathbb{E}\left(y|x_1, x_2\right) = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4$
- with the data at hand
  - $\mathbb{E}\left(wage|\,\mathrm{col}, hscore\right) = 5.50\,\mathrm{col} + 1.00 hscore + 3.00\,\mathrm{col} * hscore + 8.50$
  - $\nabla_1 m\left(\mathrm{col}, score = H\right) = 8.50 = ACE\left(x_1, x_2 = H\right)$
  - $\nabla_1 m\left(\mathrm{col}, score = L\right) = 5.50 = ACE\left(x_1, x_2 = L\right)$

Review

- What are individual causal effects?
- $\nabla_x y$

Can individual causal effects be measured?

- No, because they require a counterfactual

What aggregate causal effect do we focus on?

- average causal effects

What is the first requirement for β_1 to equal the average causal effect?

- $\mathbb{E}\left(y|x_1, x_2\right) = x_1^{\mathrm{T}}\beta_1 + x_2^{\mathrm{T}}\beta_2$ so that $\beta_1 = \nabla_1 \mathbb{E}\left(y|x_1, x_2\right)$

What is the second requirement for β_1 to equal the average causal effect?

- Conditional Independence Assumption

$$f\left(u|x_1, x_2\right) = f\left(u|x_2\right)$$