*Chapter 68*

# THE COMBINATORIAL AUCTION

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In the 1970s the airline industry was scheduled for deregulation. This meant that routes, flight schedules, and fares were to be freely chosen by individual airlines who were to be free to enter or exit any market for city pair service. Deregulating fares and the free movement of equipment among routes left open an important policy question: how should airport runway rights (time slots for the take-off or landing of aircraft to support airline flight schedules) be allocated? These rights have an important, and novel (but far from unique) characteristic: an airline's demand (willingness-to-pay) for a take-off slot is zero unless it is packaged with a landing slot at the flight destination airport. Moreover, a given flight may take off and land in a sequence of connected demand-interdependent segments that are required as a package to support a particular flight schedule.

Grether, Isaac, and Plott (1979, 1989) proposed a sequential procedure for using a primary sealed-bid auction, followed by an after-market, to achieve the allocation of slots. Under their proposal an independent primary market for slots at each airport would be organized as a sealed-bid competitive auction at timely intervals. Since this auction would allocate the primary resources, but did not make provision for package demand interdependence, a computerized version of the oral double auction, with block transaction capabilities, was proposed as an after market to allow the allocations to be adjusted for interdependent demand.

In this entry, we summarize a market mechanism for a "combinatorial" sealed-bid auction that was motivated by the airport slot problem under which bidders would submit bids for packages of slots that support their schedules. A more complete report is provided in Rassenti, Smith, and Bulfin (1982). Under this scheme, the elemental resources would be allocated only in the form of those combinations desired by the bidders. Under this scenario, the purpose of an after market would be to adjust for allocation errors in the primary market (the objective in the experiments reported below), or to adjust for post-primary market changes in demand. Although this form of combinatorial auction was never applied to airport runway rights, the issue of pricing runway slots has arisen in the new century as a means of managing airport congestion.

The combinatorial procedure below is not generally incentive compatible; i.e., if any bidder desires multiple units of either packages or elements, or multiple units of any element, then it may be to a bidder's advantage to strategically underbid the true value of a package. This has been known since Vickrey's (1961) remarkable paper, which provides the correct dominant strategy mechanism solution in which each bidder pays an amount equal to the opportunity cost(s) of the bidder(s) he/she displaces. Vickrey's solution is widely believed to be impractical for two reasons: (1) different agents pay

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different prices and it is thought that this creates a procedure too inscrutable to be acceptable to users; (2) in general, in two-sided versions, the budget is not balanced – sellers receive more than buyers pay.

The solution below, however, was simple and practical enough to be implemented in the laboratory, and yielded 98.5 to 99.3 percent of the maximum gains from exchange.

## 1. The Environment

We report experiments using both an "easy" and a "difficult" environment. Table 1 defines the "easy" combinatorial environment. (See Rassenti, Smith, and Bulfin, 1982, Appendix B.)

There are six agent bidders: 1, 2, ..., 6, and six elemental resource items, A, B, ..., F. Each row lists a package desired by some agent, with value to that agent listed in column 3 (1978 values in cents), and the composition of the package that consists of two or more of the elements. A package contains each element for which the integer, 1, appears in the column for that element. Thus, package 5, demanded by agent 2, has value 663, and consists of one each of the elements B and C. At the bottom of the table in the row for "# Units Demanded" appears the sum, across all 25 packages, of the maximum number of units of each resource item that are required to service the total package demand of all agents. Thus 15 units of item B would satisfy the maximum demand for item B from the 25 packages. The last row for "# Units Available" lists the total available supply of each resource item. Each item is scarce in the sense that more units are demanded than are in supply.

Table 1 is described as an "easy" combinatorial environment because (1) there is considerable replication of the demand for each package across subject agents; (2) there is considerable replication of the demand for the same elemental item across the packages of any one agent. Thus, only seven outlying '1s' are dispersed among items D, E. and F. Here Rassenti, Smith, and Bulfin (1982) followed Grether, Isaac, and Plott (1979, 1989) in introducing only a slight variation on an environment with a traditional non-combinatorial structure. The motivation is to create an environment that is favorable to the independent auction described below. Thus, a combinatorial auction may do worse (or at least no better) than an independent auction when applied to an environment with little combinatorial structure.

A more difficult combinatorial environment is shown in Table 2. In particular note that there is relatively little replication of the same package across agents, or of the same elemental item across the packages of any one agent; i.e., the pattern of '1s' are well dispersed across rows and columns. This environment is conjectured to be challenging to both the independent and combinatorial auction, but one for which the latter is more clearly suited.

Table 1
Easy resource utilization design

| Agent | Package | Value | Item A | Item B | Item C | Item D | Item E | Item F |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 598 | 1 | 1 | | | | |
| 1 | 2 | 946 | 1 | 1 | 1 | | | |
| 1 | 3 | 517 | | 1 | 1 | | | |
| 2 | 4 | 632 | 1 | | | 1 | | |
| 2 | 5 | 663 | | 1 | 1 | | | |
| 2 | 6 | 951 | 1 | 1 | 1 | | | |
| 3 | 7 | 877 | 1 | 1 | | | | 1 |
| 3 | 8 | 595 | 1 | | 1 | | | |
| 3 | 9 | 515 | | 1 | 1 | | | |
| 3 | 10 | 885 | 1 | 1 | 1 | | | |
| 4 | 11 | 546 | 1 | | 1 | | | |
| 4 | 12 | 983 | 1 | | 1 | | | 1 |
| 4 | 13 | 569 | 1 | 1 | | | | |
| 4 | 14 | 603 | | 1 | 1 | | | |
| 5 | 15 | 642 | 1 | 1 | | | | |
| 5 | 16 | 450 | 1 | | | | 1 | |
| 5 | 17 | 498 | | 1 | 1 | | | |
| 5 | 18 | 913 | 1 | | 1 | | 1 | |
| 5 | 19 | 476 | 1 | | 1 | | | |
| 6 | 20 | 576 | 1 | | 1 | | | |
| 6 | 21 | 802 | | 1 | 1 | 1 | | |
| 6 | 22 | 439 | | 1 | 1 | | | |
| 6 | 23 | 945 | 1 | | 1 | | | 1 |
| 6 | 24 | 617 | 1 | | 1 | | | |
| 6 | 25 | 520 | 1 | 1 | | | | |
| Units demanded | | | 18 | 15 | 18 | 2 | 2 | 3 |
| Units available | | | 13 | 11 | 15 | 1 | 2 | 3 |

### *1.1. Two Market Mechanisms: The Independent Auction and the Combinatorial Auction*

We compare two market mechanisms.

An Independent Auction (IA). The first, introduced by Grether, Isaac, and Plott (1979, 1989), begins with a primary market consisting of six uniform price sealed-bid auctions run simultaneously, one for each of the six elemental resources with induced combinatorial values as shown in Tables 1 and 2. Thus, in Table 1, 13 units of A are offered for sale and the 13 units are allocated to the 13 highest bidders at the 13th highest bid price. Similarly for B, C, ..., F. Individual allocations are private. The clearing prices for each resource are made public.

Table 2
Difficult resource utilization design

| Agent | Package | Value | Item A | Item B | Item C | Item D | Item E | Item F |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 627 | 1 | 1 | | | | |
| 1 | 2 | 577 | | | 1 | 1 | | |
| 1 | 3 | 506 | 1 | | | | 1 | |
| 1 | 4 | 825 | 1 | | 1 | | | 1 |
| 1 | 5 | 834 | | 1 | 1 | | 1 | |
| 2 | 6 | 531 | 1 | 1 | | | | |
| 2 | 7 | 556 | | | 1 | 1 | | |
| 2 | 8 | 576 | 1 | | 1 | | | |
| 2 | 9 | 644 | | 1 | | | 1 | |
| 2 | 10 | 584 | | | 1 | | 1 | |
| 2 | 11 | 886 | 1 | | | | 1 | 1 |
| 3 | 12 | 517 | 1 | 1 | | | | |
| 3 | 13 | 576 | | | 1 | 1 | | |
| 3 | 14 | 887 | 1 | | 1 | | 1 | |
| 3 | 15 | 940 | | 1 | 1 | | | 1 |
| 4 | 16 | 598 | 1 | 1 | | | | |
| 4 | 17 | 627 | | | 1 | 1 | | |
| 4 | 18 | 578 | 1 | | | | | 1 |
| 4 | 19 | 578 | | 1 | | 1 | | |
| 4 | 20 | 556 | | | | 1 | | 1 |
| 4 | 21 | 861 | | 1 | | | 1 | 1 |
| 5 | 22 | 560 | 1 | 1 | | | | |
| 5 | 23 | 582 | | | 1 | 1 | | |
| 5 | 24 | 565 | | 1 | | | | 1 |
| 5 | 25 | 834 | | 1 | | 1 | 1 | |
| 5 | 26 | 782 | 1 | | | 1 | | 1 |
| 6 | 27 | 507 | 1 | 1 | | | | |
| 6 | 28 | 565 | | | 1 | 1 | | |
| 6 | 29 | 833 | | 1 | | 1 | | 1 |
| 6 | 30 | 959 | 1 | | | 1 | 1 | |
| # Units demanded | | | 14 | 14 | 12 | 12 | 9 | 9 |
| # Units available | | | 7 | 7 | 7 | 7 | 7 | 7 |

A Combinatorial Auction (CA). Each agent submits bids of the form $(c_j; x_j)$ one for each package, $j$ that is desired, where $c_j$ is the bid price to be paid for package $j$, and $x_j = (a_{1j}, a_{2j}, \ldots, a_{mj})$, $a_{ij} \varepsilon \{0, 1\}$, where $m$ is the number of resources. Thus, in Table 1, bidder 1 might submit $(c_1; a_1, \ldots, a_6) = (\$5; 1, 1, 0, 0, 0, 0)$: a bid of \$5 for one unit each of A and B for package 1.

All such agent bids are then processed using algorithms that solve the following integer programming problem, $P$:

$$\begin{aligned}
&\text{Maximize} && \sum_j c_j x_j \\
& && \sum_j a_{ij} x_j \leqslant b_i \quad \forall i \\
&\text{Subject to} && \sum_j d_{kj} x_j \leqslant e_k \quad \forall k \\
& && x_j \varepsilon \{0, 1\},
\end{aligned}$$

where $i = 1, \ldots, m$, subscripts an elemental resource item ($m = 6$ in Tables 1 and 2) $j = 1, \ldots, n$, subscripts a package of resource items ($n = 30$ in Table 2), $k = 1, \ldots, \ell$, subscripts a logical constraint imposed on a set of packages by some bidder (see below).

$$a_{ij} = \begin{cases} 1 & \text{if package } j \text{ includes item } i, \\ 0 & \text{otherwise;} \end{cases}$$

$$d_{kj} = \begin{cases} 1 & \text{if package } j \text{ is in logical constraint } k \\ 0 & \text{otherwise;} \end{cases}$$

$e_k =$ some integer $\geqslant 1$,

$c_j =$ the bid for package $j$ by some buyer.

The logical constraints indicated above are included to illustrate the generality of the combinatorial process; they were not implemented in the experiments reported here. The logical constraints represent any restriction on an agent's bids that can be expressed in linear form. One class of examples is "accept no more than $p$ of the following packages." Thus, suppose an agent bids for $C_a$ on package $a$, and $C_b$ on $b$, and specifies either $a$, or $b$, but not both. Then the added logical constraint is $x_a + x_b \leqslant 1$.

Another class of logical constraints is of the form "$b$ only if $a$." This is satisfied by creating a new package as defined, $C_{ab} = C_a + C_b$, which replaces package $b$ in the submission set, and the logical constraint is $x_a + x_{ab} \leqslant 1$. Another logical constraint could be "accept any or all of the following packages but do not spend more than $e_1$ dollars."

Having solved $P$ at the end of a bidding period, what information does the mechanism report back to the agents in preparation for the next bid period? In the experiments reported below, we used the following procedure:

1. Each subject was informed as to his/her submitted bids that were accepted, and which were rejected.

In addition two pseudo-dual programs of problem $P$ were solved to determined two sets, low and high, of elemental resource prices that would allow each package to be priced as follows:

2. Each accepted bid represents the purchase of one package at a price equal to the sum of its low item resource values. The purchase price is always less than or equal

to the bid price of an accepted bid, because any bid less than the sum of its low item prices is definitely rejected. Any bid greater than the sum of its high item prices is definitely accepted. Bids in between these low and high prices may or may not be accepted depending upon whether they provide an integer fit with other package bids in satisfying all constraints. The resulting low and high package prices are reported to each subject and provides a set of best, worst and problematic replies to all other bids.

Here is an example. Assume an agent has values, and submits bids, for three packages as follows: $(A, F) = 72$; $(A, F) = 48$; $(A, C) = 37$. Let the set of low and high computed dual prices for $A, C$, and $F$ be $(A, C, F)_L = (25, 10, 32)$; $(A, C, F)_H = (25, 16, 34)$. Then the bid $(A, F) = 72$ is definitely accepted, since $72 > 25+34 = 59$, and the market price paid is $25 + 32 = 57$. The package bid $(A, F) = 48$ is definitely rejected since $48 < 57$. But the bid $(A, C) = 37$ might be rejected or accepted since $25 + 10 = 35 < 37 < 25 + 16 = 41$. Note that it is hazardous to bid less than one's value for a package, while bidding above value risks winning the package but paying more than it is worth and losing money on it.

## 2. The After Market

In each experiment after the completion of the primary market, whether run as an IA or as a CA, an open book two-sided auction was conducted. Any subject was free to announce a price and corresponding element or set of elements at which the subject was willing to either buy or sell the indicated element(s). The announcement was posted on the blackboard and remained standing until modified or accepted. Thus, a bid to buy might be Bid ($5; 1, 1, 0, 0, 0, 1): buy one unit each of A, B, and F for $5. A seller might post Ask ($6; 0, 0, 1, 1, 0, 0): asking $6 for one C and one D. This allowed missing elements in packages that were bid in the primary auction to be obtained in an open two-sided bid/ask auction. The secondary market was essential if efficient combinations were to be had in the IA, and allowed missed packages in the CA to have a second chance to be filled.

## 3. Results

We report data on market efficiency and agent profitability in eight experiments comparing: (i) inexperienced with experienced subjects, (ii) IA with CA, and (iii) easy with difficult environment.

In Figure 1 market efficiency is plotted in each trading period for four different experiments using the easy environment. Efficiency following the primary market is plotted in red, while efficiency achieved following the after market is plotted in green. Of course the after market can only improve on the primary market if agents prefer more money to less. The top panel, for inexperienced subjects, illustrates a problem that surfaced with inexperience: a tendency to use the primary market to acquire resources or packages beyond what is demanded by the agent in an attempt to profit from resale to others in the after market. Thus in IA, upper left panel of Figure 1, the primary market efficiency in periods, 2, 3, 4 and 5 was actually less than it was in period 1, but generally recovered in the after market as subjects found buyers. In the CA such speculative attempts were largely absent except in the final period, when efficiency was not improved by the after market. Generally, in the easy environment of Figure 1, efficiency was higher in the after market under CA than IA in 4 of 5 periods with inexperienced subjects, and in 2 of 4 periods with experienced subjects. In the easy environment CA provided most of its advantage over IA when subjects were inexperienced. Experienced subjects handle the easy environment quite well without combinatorial aid.

In contrast, as shown in Figure 2 for the difficult environment, CA dominates IA in both the primary and after markets in every period. In complex combinatorial environments, IA causes severe coordination problems even with experienced subjects. In fact the CA does so well in achieving high efficiencies in the primary market (85% to over 99% across experience levels) that improvements in the after market are almost impossible to discover in the open auction.

The efficiency comparisons in Figures 1 and 2 provide no insight as to the profit consequences of the coordination properties of the two auction mechanisms. Data on the percentage of agents across all periods who ended with a deficit (profit loss) in a period is plotted in the bar charts of Figure 3 for each experimental condition. Hence, in the upper left panel, corresponding to the IA primary market in the easy environment using inexperienced subjects, there were a total of 30 observations on agent-period profits (5 subjects $\times$ 6 periods each); in 14 of these cases (46.7%) a subject ended the period with a deficit, as indicated by the red bar. This deficit rate for agents declines to 36.7% (11 of 30) following the after market as shown by the red bar in the upper right panel for inexperienced subjects. The corresponding results for experienced subjects are 37.5% and 8.3%, respectively. For the CA the agent deficit rate is never over 6.7% and is zero for experienced subjects following the after market shown plotted as green bars. The deficit rate contrast between CA and IA is much more pronounced in the difficult environment shown in the lower panel. For experienced subjects there are no deficits following the primary market, for CA, and hence none following the after market. But 22.2% of the IA cases show a deficit for experienced subjects following the after market.

Profitable, efficient allocation is just not achievable in difficult environments using IA. But why is efficiency for the CA mechanism higher in the difficult than in the easy environment? Our explanation is that the greater replication of elements and packages across agents in the easy environment invites speculative purchases in the primary market for resale in the after market, introducing some deficits in the primary market, and reducing efficiency. Thus, easy environments appear to invite manipulations that are avoided in difficult environments because they are perplexing to subjects and hazardous to manipulate.

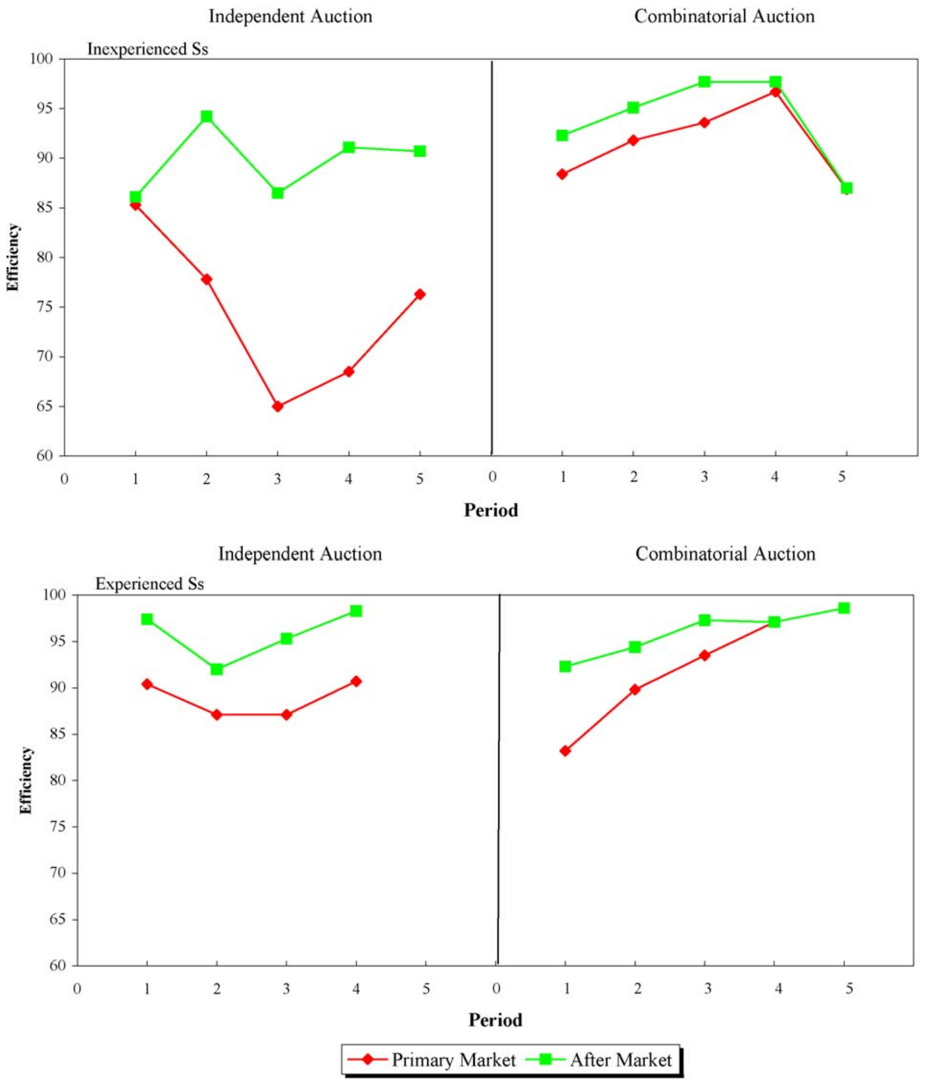


Figure 1. For the easy combinatorial environment, each of the four panels plots efficiency by period in the primary market (red), and efficiency for each period following the after market (green). For inexperienced subjects in the top panels, the declines shown in periods 1 to 3 for the independent auction (IA), and in period 5 for the combinatorial auction (CA) are the result of speculation: subjects purchase items in excess of their own demands in an attempt to profit from resale in the after market. These speculations caused losses (see Figure 3) and use of this behavioral strategy was much reduced when subjects become experienced. Overall the CA provided only slightly improved after market efficiency relative to IA in this simple environment. With inexperienced subjects, CA efficiency was higher in periods 1 to 4 than in IA, but below it is period 5. With experienced subjects in periods 1 and 4 IA efficiency was higher than CA, while in periods 2 and 3 this ranking was reversed.

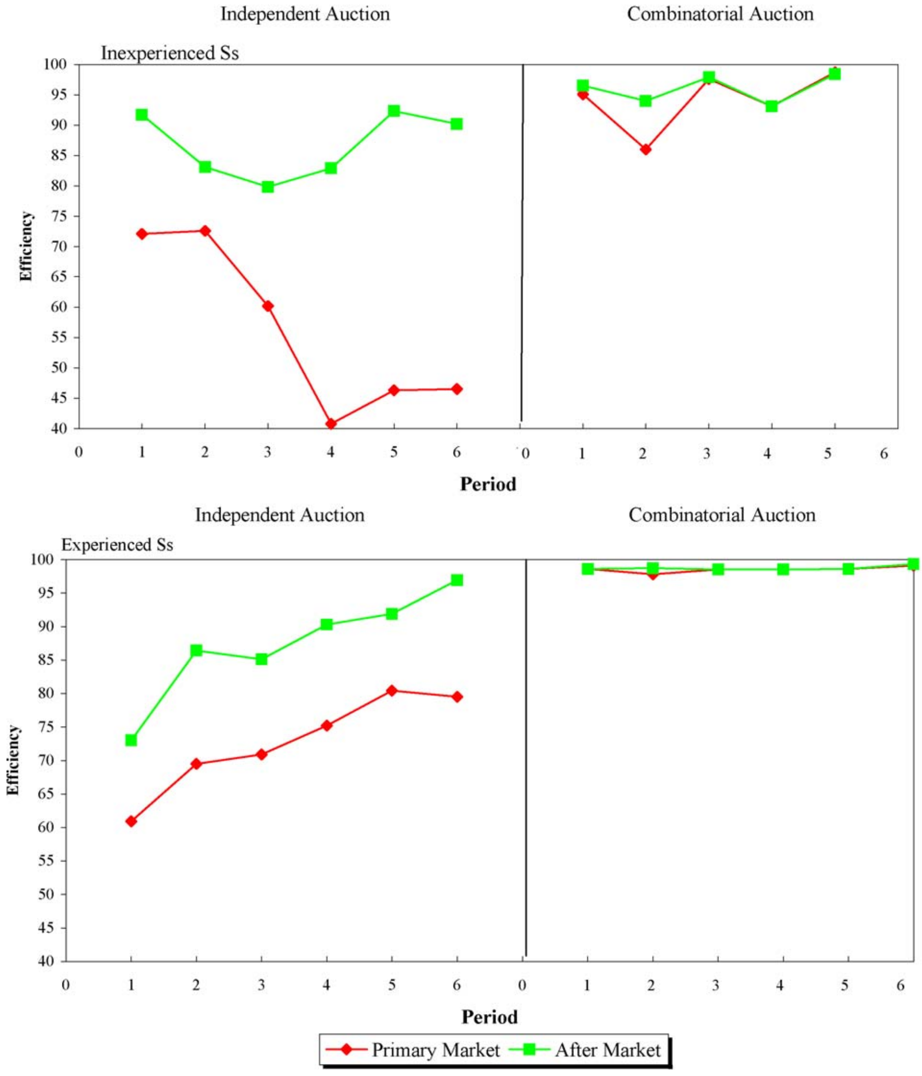


Figure 2. Efficiency is plotted by period for each treatment using the difficult combinatorial environment: primary market efficiency is in red, the after market in green. Note that the improvement in efficiency going from the primary market to the after market is much more pronounced in IA than CA. Even for inexperienced subjects the primary market CA easily solves the coordination problem in a complex environment, leaving little room for efficiency improvement in an after market. In IA the low primary market efficiencies were a consequence of speculative attempts by subjects to buy elements in excess of their individual demand needs in the hope of reselling at a profit in the after market. This was a much less severe problem in CA where the complexity of the environment led subjects to reply on the support of computer coordination in the primary market and to avoid using the after market to piece together valuable packages.

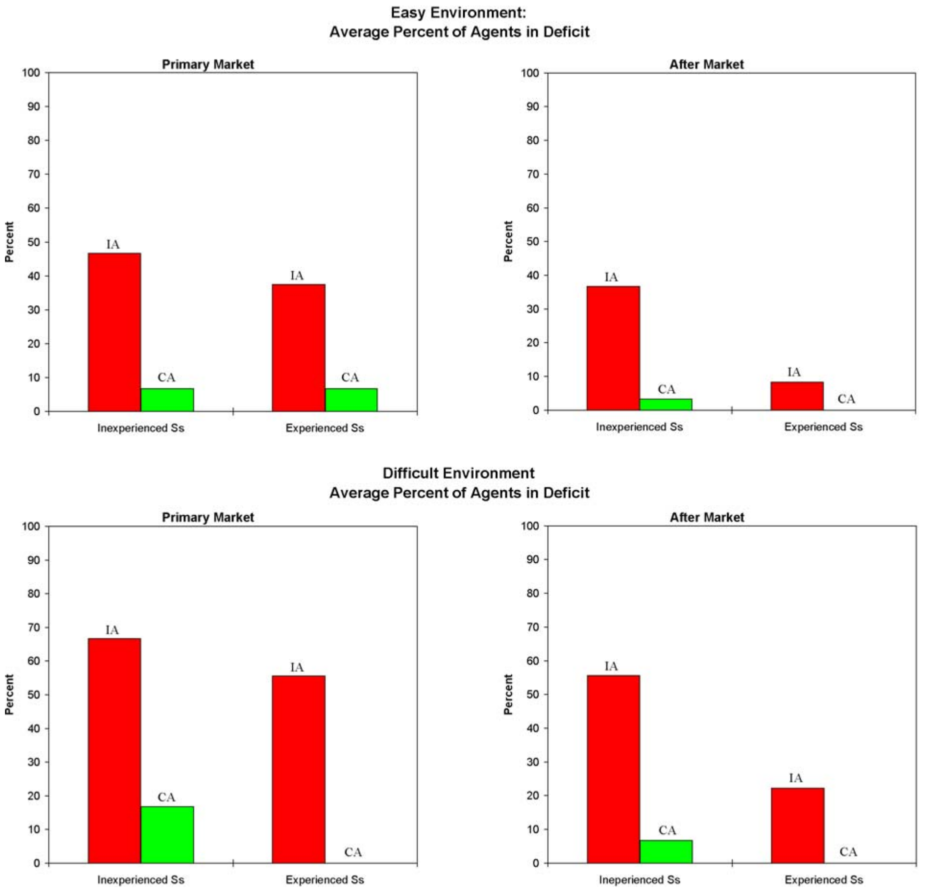


Figure 3. The average percent of subject agents incurring a deficit across all periods is shown in red for the independent auctions (IA) and green for the combinatorial auction (CA). Deficits are common in the IA primary market, even with experienced subjects, and persist into the after market, especially in the difficult combinatorial environment shown in the lower panels. Thus over 20% of agent periods end in deficit with experienced subjects in the IA. (Lower right panel.) Such deficit problems are minor in the CA with inexperienced subjects, and disappear entirely with experience in both the easy and difficult environments.

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