Econ 241A Probability, Statistics and Econometrics
Fall 2017

Problem Set 4

1. Assume X and Y are jointly distributed with pdf $f(x,y) > 0$ for $(x,y) \in \mathcal{W}$, $\mathcal{W} \subset \mathbb{R}^2$. The marginals of X and Y are given by $f(x)$ with support $\mathcal{X}$ and $f(y)$ with support $\mathcal{Y}$. Define $g(X)$ as a function only of X. Prove that $\mathbb{E}(g(x)) = \int_{x:x\in\mathcal{X}} g(x)f(x)dx$.

2. For the joint pmf in the table below:

	$x=1$	$x=2$	$x=3$
$y=0$	0.10	0.10	0.10
$y=1$	0.10	0.40	0.20

   (a) Find the conditional expectation function $\mathbb{E}(Y|X)$

   (b) Find the best linear predictor $\mathbb{E}^*(Y|X)$

   (c) Prepare a table that gives $\mathbb{E}(Y|x)$ and $\mathbb{E}^*(Y|x)$ for $x = 1, 2, 3$.

3. Assume X and Y are joinly distributed with pdf $f(x,y) = x + xy$, $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Define the bivariate random vector (U, V) as $U = X$ and $V = \sqrt{Y}$.

   (a) Are X and Y independent?

   (b) Are U and V independent?

   (c) Find the marginal pdf of V.

In addition, solve the following problems from Casella and Berger: 4.19 (a) (Hint: What is the distribution of the square of a standard normal rv (Ch 2)? Does this result surprise you given that X_1 and X_2 are iid?), 4.20, 4.22, 4.26, 4.30 (Hint for part b: does the pdf of $Y|x$ change for different values of x?) , 4.44, 4.47, 4.50 and 4.58 (a), (b) and (c).