Econ 241 Probability, Statistics and Econometrics

Jaime Ramirez-Cuellar

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1. Let $Y \sim exp(\lambda)$ (i.e. $f_Y = \lambda exp(-\lambda y)$, $y > 0$).
   (a) What are $E[Y]$ and $Var(Y)$
   (b) Now assume that $\lambda^{-1} \sim Poisson(\mu)$. What are $E[Y]$ and $Var(Y)$?
2. Let $X \sim Pareto(\alpha, \beta)$ (i.e. $f_X(x) = \frac{\beta \alpha^\beta}{x^{\beta+1}}$, $y > 0$).
   (a) Verify that $f_X(x)$ is pdf.
   (b) What are $E[Y]$ and $Var(Y)$.
   (c) Prove that the variance does not exist if $\beta \leq 2$.
   (d) Prove that $E[X^r]$ does not exist if $\beta \leq r$ for $r > 0$.
3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Gamma distribution with parameters α and β with pdf given by

$$f(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad 0 \leq x < \infty, \quad \alpha, \beta > 0$$

   (a) Find the method of moments estimator for α and β.
   (b) Find the asymptotic distribution of $(\alpha_{MME}, \beta_{MME})$
4. (Hayashi, p. 74) Consider the restricted least squares regression

$$\begin{aligned} \min_{\boldsymbol{\beta}} \quad & (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})'(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \\ \text{s. t.} \quad & \boldsymbol{R}\boldsymbol{\beta} = \boldsymbol{r} \end{aligned}$$

   (a) Find the restricted OLS estimator $\tilde{\boldsymbol{\beta}}$ and a vector of Lagrange multipliers $\boldsymbol{\lambda}$.
   (b) Show that $SSR_R - SSR_U = \tilde{\boldsymbol{\varepsilon}}\boldsymbol{P}\tilde{\boldsymbol{\varepsilon}}$, where SSR_R and SSR_U are the sum of squared residuals of the restricted and unrestricted models, $\tilde{\boldsymbol{\varepsilon}}$ are the residuals from the restricted model and $\boldsymbol{P}$ is the projection matrix.
   (c) Show that the F statistic can be computed using the following

$$F = \frac{(SSR_R - SSR_U)/r}{SSR_U/(n-K)}$$

   where r is the number of linear restrictions in the restricted least squares, n is the length of $\boldsymbol{y}$, and K is the number of coefficients including the intercept.
5. (Hayashi, p. 71) Prove that $\boldsymbol{\beta}_{OLS}$ minimizes SSR.