Chapter 4

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Section 4.5 Covariance and Correlation

- **Covariance of X and Y Definition 4.5.1 - page 169**

$$Cov[X, Y] = E[(X - \mu_X)(Y - \mu_Y)]$$

- **Correlation of X and Y Definition 4.5.2 - page 169**

$$\rho_{XY} = \frac{Cov[X,Y]}{\sigma_X \sigma_Y}$$

- **Theorem 4.5.3 - page 170**

  For any random variable X and Y,

$$Cov[X, Y] = E[XY] - \mu_X \mu_Y$$

- **Theorem 4.5.5 - page 171**

  If X and Y are independent random variables, then $Cov[X, Y] = 0$ and $\rho_{XY} = 0$.

- **Theorem 4.5.6 - page 171**
  If X and Y are any two random variables and a and b are any two constants, then

$$Var[aX \pm bY] = a^2 Var[X] + b^2 Var[Y] \pm 2abCov[X, Y]$$

  If X and Y are independent then $Cov[X, Y] = 0$.

- **Theorem 4.5.7 - page 172**
  For any random variables X and Y,

  a) $-1 \leq \rho_{XY} \leq 1$

  b) $|\rho_{XY}| = 1$ if and only if there exist numbers $a \neq 0$ and b such that $P(Y = aX + b) = 1$. if $\rho_{XY} = 1$, then $a > 0$, and if $\rho_{XY} = -1$, then $a < 0$.

- **Definition 4.5.10 (Bivariate normal pdf) page 175**

Section 4.6 Multivariate Distributions

- **Multivariate Conditional PDF Form - page 178 - equation 4.6.6**

- **Using Multivariate PDFs - page 178 - example 4.6.1**

- **Definition 4.6.2 - Multinomial PDF - page 180**
  Similar to the Binomial except there are more than two distinct possible outcomes. Where Binomial is a sum of Bernoullis which each only have $n = 2$ possible outcomes, success or fail, the Multinomial is has $n > 2$ (e.g., rolling a die has $n = 6$).

- **Theorem 4.6.4 Multinomial Theorem - page 181**
  Let m and n be positive integers. Let $\mathcal{A}$ be the set of vectors $\vec{x} = (x1, ..., x_n)$ such that each x_i is a nonnegative integer and $\sum_{i=1}^{n} x_i = m$. Then, for any real numbers $p_1, ..., p_n$,

$$(p_1 + \cdots + p_n)^m = \sum_{x \in \mathcal{A}} \frac{m!}{x_1! \cdots x_n!} p_1^{x_1} \cdots p_n^{x_n}$$

- **Definition 4.6.5 - page 182 - Mutually Independent**

  Let $\vec{X}_1, \ldots, \vec{X}_n$ be random vectors with joint pdf or pmf $f(\vec{x_1}, \ldots, \vec{x_n})$. Let $f_{\vec{X}_i}(\vec{x_i})$ denote the marginal pdf or pmf of $\vec{X}_i$. Then $\vec{X}_1, \ldots, \vec{X}_n$ are called *mutually independent random vectors* if, for every $(\vec{x_1}, \ldots, \vec{x_n})$,

$$f(\vec{x_1}, \ldots, \vec{x_n}) = f_{\vec{X}_1}(\vec{x_1}) \cdots f_{\vec{X}_n}(\vec{x_n}) = \prod_{i=1}^{n} f_{\vec{X}_i}(\vec{x_i})$$

- **Theorem 4.6.6 through 4.6.12 - pages 183 to 185**
  Let $X_1, \ldots, X_n$ be mutually independent random **variables**.

  1. **Theorems 4.6.6 - page 183 (Generalization of THM 4.2.10)** Let $g_1, \ldots, g_n$ be real valued functions such that $g_i(x_i)$ is a function of only x_i then

  $$E[g_1(X_1) \cdots g_n(X_n)] = E[g_1(X_1)] \cdots E[g_n(X_n)]$$

  2. **Theorem 4.6.7 - page 183 (Generalization of THM 4.2.12)** Let mgfs of X_i's be $M_{X_i}(t)$ and let $Z = X_1 + \cdots + X_n$. then the mgf of Z is

  $$M_Z(t) = M_{X_1}(t) \cdots M_{X_n}(t)$$

  If $X_1, \ldots, X_n$ all have the same distribution with mgf $M_X(t)$, then

  $$M_Z(t) = [M_X(t)]^n$$

  3. **Corollary 4.6.9 - page 183** Gives the mgf of $Z = (a_1X_1 + b_1) + \cdots + (a_nX_n + b_n)$
  4. **Corollary 4.6.10 - page 184** Gives the distribution type of $Z = (a_1X_1 + b_1) + \cdots + (a_nX_n + b_n)$

  Now consider $\vec{X}_1, \ldots, \vec{X}_n$ to be random vectors

  1. **Theorem 4.6.11 - page 184** Generalization of Lemma 4.2.7
  $\vec{X}_1, \ldots, \vec{X}_n$ are mutually independent random vectors if and only if there exist functions $g_i(\vec{x_i})$, $i = 1, \ldots, n$, such that the joint pdf of pmf of $\vec{X}_1, \ldots, \vec{X}_n$ can be written as

  $$f(\vec{x_1}, \ldots, \vec{x_n}) = g_1(\vec{x_1}) \cdots g_n(\vec{x_n})$$

  2. **Theorem 4.6.12 - page 184** Generalization of Theorem 4.3.5
  Let $\vec{X}_1, \ldots, \vec{X}_n$ be independent random vectors. Let $g_i(x_i)$ be a function of only x_i. Then the random variables $U_i = g_i(\vec{X}_i)$, $i = 1, \ldots, n$, are mutually independent.

- **Inequalities start on page 186**