1. Ch. 5

   (a) Section 5.2

      i. Theorem 5.2.4 (p.212)
      Let $x_1, \dots, x_n$ be any numbers and $\bar{x} = \frac{x_1+\cdots+x_n}{n}$. Then:

      - $min_a \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$
      - $(n-1)s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

      ii. Lemma 5.2.5 (p.213)
      Let $X_1, \dots, X_n$ be a random sample from a population and let g(x) be a function such that $\mathbb{E}[X_1]$ and $Var(g(X_1))$ exist. Then:

      - $\mathbb{E}[\sum_{i=1}^{n} g(X_i)] = n(\mathbb{E}[g(X_1)])$
      - $Var(\sum_{i=1}^{n} g(X_i)) = n(Var(g(X_1)))$

      iii. Theorem 5.2.6 (p.213)
      Let $X_1, \dots, X_n$ be a random sample from a population with mean μ and variance $\sigma^2 < \infty$. Then:

      - $\mathbb{E}[\bar{X}] = \mu$
      - $Var(\bar{X}) = \frac{\sigma^2}{n}$
      - $\mathbb{E}[S^2] = \sigma^2$

      iv. Theorem 5.2.7 (p.215)
      Let $X_1, \dots, X_n$ be a random sample from a population with mgf $M_X(t)$. Then the mgf of the sample mean is:

      - $M_{\bar{X}}(t) = [M_X(t/n)^n]$

      v. Theorem 5.2.9 (p.215)

      - If X and Y are independent continuous random variables with pdfs $f_X(x)$ and $f_Y(y)$, then the pdf of $Z = X + Y$ is:

        $f_Z(z) = \int_{-\infty}^{\infty} f_X(w) f_Y(z-w)dw.$

      vi. Theorem 5.2.11(p.217)
      Suppose $X_1, \dots, X_n$ is a random sample from a pdf or pmf $f(x \mid \theta)$, where:

      $f(x \mid \theta) = h(x)c(\theta)exp(\sum_{i=1}^{k} w_i(\theta)t_i(x))$

      is a member of the exponential family. Define statistics $T_1, \dots, T_k$ by:

$T_i(X_1, \dots, X_n) = \sum_{j=1}^{n} t_i(X_j)$, $i = 1, \dots, k$.

If the set $\{(w_1(\theta), w_2(\theta), \dots, w_k(\theta)), \theta \in \Theta\}$ contains an open subset of $\mathbb{R}^k$, then the distribution of $(T_1, \dots, T_k)$ is an exponential family of the form:

$f_T(u_1, \dots, u_k \mid \theta) = H(u_1, \dots, u_k)[c(\theta)]^n exp(\sum_{i=1}^{k} w_i(\theta) u_i)$.