Jacob Gellman
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Problem Set 6 (Lecture Notes 10)

Problem 10.1

Suppose all members of a group are voting on a policy. Each will make a decision on how to vote. We can imagine that each member behaves strategically to get the best outcome for herself. Her *strategy* is her decision on how to vote.

Now suppose that the members of the group are asked to state their willingness to pay for a certain policy to be passed – say, a complete and across-the-board legalization of abortion, including late-term abortion. The legalization passes if the sum of the willingness to pay for those in favor exceeds the sum of the willingness to pay for those opposed.

We expect members to behave strategically. If we simply asked them how much they would be willing to pay, one person might have an incentive to lie and say he would be willing to pay an exorbitant amount, like \$1 billion, in order to tip the vote, even if his true willingness to pay is only \$100.

So instead we design a mechanism – a *pivotal mechanism*. Person i will pay for the change only if they are the *pivotal* voter. Consider again the person that bid \$1 billion, and suppose everyone else truthfully bid their values. Further assume that without his \$1 billion bid, the pro- side would lose, and so he is the pivotal voter. Then he would have a negative payoff of $\$100 - \$1$ billion. If he instead bid his own willingness to pay, he would have a payoff of $\$100 - \$100 = 0$, leaving him just as good off if the ban had passed as if it had not passed. Supposing person i has a utility function, their willingness to pay is given by V_i such that

$$U_i(X_i - V_i, 1) = U_i(X_i, 0), \tag{1}$$

where 1 denotes whether the policy has passed or not and X_i is a consumption good.

It turns out that, under the pivotal mechanism, it is a *weakly dominant* strategy to bid your true valuation. A strategy is weakly dominant if one cannot do better than to use this strategy, no matter what anybody else does. If I bid my true value V_i, but the policy does not pass, then I have $U_i(X_i, 0)$. If I bid my true value and I am the pivotal voter, then my utility is $U_i(X_i - V_i, 1)$. If I bid my true value, the policy passes, and I am not the pivotal voter, then my utility is $U_i(X_i, 1)$. If I underbid, there is a chance the policy won't pass. If I overbid, there is a chance I will be worse off if the policy passes and I am the pivotal voter. So my dominant strategy is to bid my true value – I can do not better than to use this strategy.

Problem 10.2

Consider a ban on handguns. We define X_i as the private good. Each person has a utility function giving $U_i(X_i, 1)$ if the ban is passed and $U_i(X_i, 0)$ if it is not passed. Define V_i as the willingness to pay for the ban in quantity of private goods, such that

$$U_i(X_i - V_i, 1) = U_i(X_i, 0). \tag{2}$$

So $V_i > 0$ for those in favor, $V_i < 0$ for those opposed. We ignore the case where $V_i = 0$.

Claim: A Pareto improvement can be achieved by passing the ban if and only if $\sum V_i > 0$, and could not be achieved if $\sum V_i < 0$.

Proof:

We begin by showing $\sum V_i > 0 \Rightarrow \exists$ side payments allowing Pareto improvement.

Let $\sum V_i > 0$.

Denote two types, Dudes that want the ban, and Toads that oppose the ban. A person's type $\theta_i \in \{D, T\}$, where $V_i > 0$ if $\theta_i = D$ and $\theta_i = T$ if $V_i < 0$.

Consider side payments made by all type D individuals in the amount of $V_i^D - \varepsilon$, conditional on the ban, where ε is some small number. Then

$$U_i^D = U_i^D(X_i - (V_i - \varepsilon), 1) > U_i^D(X_i - V_i, 1) = U_i^D(X_i, 0) \tag{3}$$

so the Dudes are better off with a side payment.

Since $\sum V_i > 0$, it is equivalent to saying that

$$\sum_{i:\ \theta_i = D} |V_i| > \sum_{j:\ \theta_j = T} |V_j|. \tag{4}$$

We define ε so that

$$\sum_{i:\ \theta_i = D} |V_i - \varepsilon| > \sum_{j:\ \theta_j = T} |V_j|. \tag{5}$$

Thus the total payments from the Dudes to the Toads amounts to $\sum\limits_{i:\ \theta_i = D} |V_i - \varepsilon|$.

We can specify that each Toad receives his WTP plus a portion of the remaining payments. Arbitrarily, the remaining payments will be divided up equally among the Toads. Mathematically, each j where $\theta_j = T$ will receive a payment of $|V_j| + \frac{1}{N_T}\Big[\Big(\sum\limits_{i:\ \theta_i = D} |V_i - \varepsilon|\Big) - \Big(\sum\limits_{j:\ \theta_j = T} |V_j|\Big)\Big]$.

By inspection, $|V_j| + \frac{1}{N_T}\Big[\Big(\sum\limits_{i:\ \theta_i = D} |V_i - \varepsilon|\Big) - \Big(\sum\limits_{j:\ \theta_j = T} |V_j|\Big)\Big] > |V_j|$.

Then we have

$$U_j^T\Big(X_j + |V_j| + \Big[\Big(\sum_{i:\ \theta_i = D} |V_i - \varepsilon|\Big) - \Big(\sum_{j:\ \theta_j = T} |V_j|\Big)\Big], 1\Big) > U_j^T(X_j - V_j, 1) = U_j^T(X_j, 0) \tag{6}$$

so the Toads are better off with the side payment. Thus, by equations (3) and (6), we have shown that if $\sum V_i > 0$, there exist side payments such that everyone is better off with the ban.

We will continue by showing the reverse direction of the proof. That is, if there exist side payments to produce a Pareto improvement by passing the ban, then $\sum V_i > 0$.

Let there be side payments producing a Pareto improvement. The Dudes, where $\theta_i = D$, make payments p_i, and each Toad with $\theta_j = T$ will receive some payment p_j.

Since the side payments produce a Pareto improvement, $U_i^D(X_i - p_i, 1) \geq U_i^D(X_i - V_i, 1) = U_i^D(X_i, 0)\ \forall\ i \ni \theta_i = D$, with strict inequality for some i.

This implies that $X_i - p_i \geq X_i - V_i\ \forall\ i \ni \theta_i = D$, with strict inequality for some. In turn, we conclude that $V_i \geq p_i\ \forall\ i$, with strict inequality for some. So it must be that

$$\sum_{i:\ \theta_i = D} V_i > \sum_{i:\ \theta_i = D} p_i \tag{7}$$

Note that the sum of payments made must equal the sum of payments received:

$$\sum_{i:\ \theta_i = D} p_i = \sum_{j:\ \theta_j = T} p_j \tag{8}$$

Since the side payments produce a Pareto improvement, $U_j^T(X_j + p_j, 1) \geq U_j^T(X_j - V_j, 1) = U_j^T(X_j, 0)\ \forall\ j \ni \theta_j = T$, with strict inequality for some j.

This implies that $X_j + p_j \geq X_j - V_j\ \forall\ j \ni \theta_j = T$, with strict inequality for some. In turn, we conclude that $p_j \geq -V_j\ \forall\ j$, with strict inequality for some. So it must be that

$$\sum_{j:\ \theta_j = T} p_j > \sum_{j:\ \theta_j = T} |V_j| \tag{9}$$

Combining equations (7) to (9),

$$\sum_{i:\ \theta_i = D} V_i > \sum_{i:\ \theta_i = D} p_i = \sum_{j:\ \theta_j = T} p_j > \sum_{j:\ \theta_j = T} |V_j| \tag{10}$$

$$\Rightarrow \sum_{i:\ \theta_i = D} V_i > \sum_{j:\ \theta_j = T} |V_j| \tag{11}$$

$$\Rightarrow \sum_{i:\ \theta_i \in \{D,T\}} V_i > 0. \tag{12}$$

Finally, we prove that a Pareto improvement is not possible if $\sum V_i < 0$.

We previously showed that ($\exists$ side payments for a Pareto improvement) $\Rightarrow \sum V_i > 0$.

The contrapositive is $\neg(\sum V_i > 0) \Rightarrow \neg$ ($\exists$ side payments for a Pareto improvement).

Then $\sum V_i < 0 \Rightarrow$ a Pareto improvement cannot be achieved with side payments.

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