5 Chapter 5

5.1

(a) see graph

(b) At an interior solution: $MRS_E = MRS_F$ From their respective utility functions, we solve for S:

$$S = \sqrt{\frac{B_E}{2}}$$

Thus, the set of interior P.O. allocations is

$$\{B_E > 0, B_F > 0, S = S = \sqrt{\frac{B_E}{2}}\}.$$

The corner solutions are

$$\{B_E = 16, B_F = 0, 2\sqrt{2} < S < 4\} \quad \text{and} \quad \{B_E = 0, B_F = 16, S = 0\}$$

(c) From (b) we have the condition $B_E = 2S^2$, plugging this into $U_E = B_E S$, we get

$$U_E = 2S^3$$

From $U_F = B_F - S^2$, since $B_F = 16 - B_E$, we get

$$U_F = 16 - 3S^2,$$

then

$$U_F = 6 - 3\left(\frac{U_E}{2}\right)^{\frac{2}{3}}$$

see graph

(d) initial property rights forbid smoking

$$p_F = -p_E$$

The budget constraints are

$$\begin{aligned} B_E + p_E S &\leq W_E \\ (16 - B_E) - p_E S &\leq 16 - W_E \end{aligned}$$

or

$$W_E \leq B_E + p_E S$$

Setting up the Lagrangean for both Fiona and Ed

$$\begin{aligned} L_E &= B_E S + \lambda(W_E - B_E - p_E S) \\ L_F &= B_F - S^2 + \lambda(W_F - B_F + p_F S) \end{aligned}$$

and solving simultaneously for Lindahl equilibrium prices, we get

$$p_E = \sqrt{W_E}$$

$$p_F = -\sqrt{W_E}$$

and the efficient amount of smoking is

$$S = \frac{\sqrt{W_E}}{2}$$

(e) If Ed can smoke as much as he wishes, there will be only one price which Fiona pays Ed to smoke less. Setting up the Lagrangean for Ed and Fiona:

$$\begin{aligned} L_E &= B_E S + \lambda_1(W_E + pS - B_E) + \lambda_2(4 - S) \\ L_F &= 16 - B_E - S^2 + \lambda_3(B_E - W_E - pS) \\ \frac{\partial L_E}{\partial B_E} &= S - \lambda_1 = 0 \Rightarrow S = \lambda_1 && (1) \\ \frac{\partial L_F}{\partial B_E} &= -1 + \lambda_3 = 0 \Rightarrow \lambda_3 = 1 && (2) \\ \frac{\partial L_E}{\partial S} &= B_E + \lambda_1 p - \lambda_2 = 0 \Rightarrow \lambda_2 = B_E + p && (3) \\ \frac{\partial L_F}{\partial S} &= -2S - p\lambda = 0 \Rightarrow p = -2S && (4) \\ \frac{\partial L_E}{\partial \lambda_1} &= W_E + pS - B_E = 0 \Rightarrow B_E = W_E - 2S^2 && (5) \\ \frac{\partial L_E}{\partial \lambda_2} &= 4 - S \geq 0 && (6) \\ \frac{\partial L_E}{\partial \lambda_3} &= B_E - W_E - pS = 0 && (7) \\ & && (8) \end{aligned}$$

From condition (5) we have two possibilities:

If $S = 4$ then $\lambda_2 > 0$, then condition (4) gives $p = -8$ and condition (5) gives $B_E = W_E - 32$, which is not possible.

If $S < 4$ then $\lambda_2 = 0$ and we can solve for B_E using (3) and (5):

$$B_E = 2S = -2S^2 + W_E$$

$$2S^2 + 2S - W_E = 0 \Rightarrow S = \sqrt{\frac{1}{4} + \frac{1}{2}W_E}$$

and $p = -2S = -2\sqrt{\frac{1}{4} + \frac{1}{2}W_E}$.

5.2

(a) To solve for the P.O. set, we can maximize the sum of Jim and Tammy's utilities subject to their budget constraints. We get $x = 10$ and $y = \frac{25}{6}$ as optimal level of activities.

(b) $I^J = I^T = 500,000$ and no bargaining. Now we maximize each person's individual utility subject to his/her budget constraint, taking the other person's activity as given:

$$\begin{aligned} \max c_J &+ 500 \ln x - 20\bar{y} + \lambda(500,000 - c_J - 40x) \\ \max c_T &+ 500 \ln y - 10\bar{x} + \lambda(500,000 - c_T - 100y) \end{aligned}$$

Solving for the optimal amounts of x and y, we get $x = 12.5$ and $y = 5$.

(d) No x and no y allowed without the other's consent. Then the budget constraints are:

$$\begin{aligned} c_J + (40 + p_x)x + p_y y &\leq 500,000 \\ c_T + p_x x + (100 + p_y)y &\leq 500,000 \end{aligned}$$

where p_x is the price Jim has to pay Tammy for xing and p_y is the price Tammy has to pay Jim for ying. Setting up the individual maximization problems for each of them, and solving simultaneously for Lindahl prices and optimal quantities, we get $p_y = 20$, $p_x = 10$, $x = 10$ and $y = \frac{25}{6}$.

(e) Any amount of x and y allowed The the budget constraints are

$$\begin{aligned} c_J + (40 + p_x)x + p_y(y^{max} - y) &\leq 500,000 \\ c_T + +(100 + p_y)y + p_x(x^{max} - x) &\leq 500,000 \end{aligned}$$

where $y^{max}(x^{max})$ is the maximum amount Tammy(Jim) can y(x).

We get the same FOCs as in (d), since the utility functions are quasilinear in c_J and c_T. i.e., we get the same results for the Lindahl prices and the optimal amounts of xing and ying, no matter how the initial property rights are assigned.

5.3

(a)

$$U(c = 1, l = 1) = 0$$

(b)

$$U(c = \frac{3}{4}, l = \frac{3}{4}) = \frac{3}{16}$$

(c) In this case $c = l$, thus we have to solve

$$\max U = c - c^2$$

and get $c = \frac{1}{2}$.

(d) Suppose we look at some of the cottagers (without loss of generality)who live in a circle as in the following graph:

see graph

Take two cottagers L and M. Their utility is

$$\begin{aligned} U_L &= c - L - l_L L^2 \\ U_M &= c_M - {l_L}^2 \end{aligned}$$

Since cottager LL consumes 1 unit, L's utility is $U_L = c_L - 1$. Thus, in order for him to be better off, c_L must be greater than 1. To make M better off, we have the condition

$$\begin{aligned} U_M &= c_M - {c_L}^2 > 0 \\ c_M &> {c_L}^2 \end{aligned}$$

Since $c_L > 1$, so is c_M. On the other hand, $c_L + c_M \leq 2$, and we get a contradiction. Thus, they cannot both be better off by redistributing consumption.

(e) Take three cottagers L, M and R. From their utility functions we get the following conditions for their consumption:

$$\begin{aligned} U_L &= c_L - 1 \Rightarrow c_L > 1 \\ U_M &= c_M - {c_L}^2 \Rightarrow c_M > 1 \\ U_R &= c_R - {c_M}^2 \Rightarrow c_R > 1 \end{aligned}$$

But $c_L + c_M + c_R > 3$ is not feasible for this group of three people. Thus they cannot be made better off.

(f) It must be that all of the 100 cottagers cooperate in order to be better off, since, as shown in (d) and (e) as long as the circle is not closed, no subgroup can improve upon.

5.4

(a) If $\alpha = \frac{2}{3}$ then

$$U_R = {S_R}^{\frac{2}{3}} {S_J}^{\frac{1}{3}}, U_J = {S_R}^{\frac{1}{3}} {S_J}^{\frac{2}{3}}$$

If Romeo maximized his own utility, the problem would be

$$\max {S_R}^{\frac{2}{3}} {S_J}^{\frac{1}{3}}$$

subject to $S_R + S_J = 24$. From the Cobb-Douglas utility function, we know the demand functions:

$$S_R = \frac{2}{3} \cdot 24 = 16, S_J = \frac{1}{3} \cdot 24 = 8$$

Similarly, if Juliet maximized her own utility, we get $S_J = 16, S_R = 8$.

(b) The Pareto optimal allocations are all combinations of S_R and S_J between their individually optimal consumption points R^* and J^*, i.e. all points such that $8 \leq S_R \leq 16$ and $S_R + S_J = 24$.

(c) see graph

(d) By analogy to (a), if $\alpha = \frac{1}{3}$, we get $S_R = 8$ and $S_J = 16$ for Romeo maximizing his utility, and $S_R = 16$ and $S_J = 8$ when Juliet is maximizing her utility. In other words, Romeo wants Juliet to consume more than himself since he regards her consumption higher than his own, and vice versa.

5.5

(a) Denote the price that Romeo pays per unit of Juliet's consumption by p_{RJ}.

$$p_{RJ} + p_{JJ} = 1$$

Romeo's maximization problem:

$$\max {S_R}^a {S_J}^{1-a} + \lambda(18 - p_{RJ} S_J - p_{RR} S_R)$$

From the Cobb-Douglas utility function, we have the demands

$$S_R = \frac{a}{p_{RR}} \cdot 18, S_J = \frac{1-a}{p_{RJ}} \cdot 18$$

and from Juliet's maximization problem we get

$$S_R = \frac{1-a}{p_{JR}} \cdot 6, S_J = \frac{a}{p_{JJ}} \cdot 6$$

Since $p_{JJ} + p_{RJ} = 1 \Rightarrow p_{JJ} = 1 - p_{RJ}$. Since there can be only one quantity of S_J and one for S_R, we can solve for the prices:

$$\frac{6a}{p_J J} = \frac{18(1-a)}{1 - p_{JJ}}$$

$$\begin{aligned} p_{JJ} &= \frac{a}{3-2a} \\ p_{RJ} &= \frac{3(1-a)}{3-2a} \\ S_J &= 6(3-2a) \end{aligned}$$

Similarly for S_R:

$$\frac{18a}{p_{RR}} = \frac{6(1-a)}{1-p_{RR}}$$

$$\begin{aligned} p_{RR} &= \frac{3a}{2a+1} \\ p_{JR} &= \frac{a+1}{2a+1} \\ S_R &= 6(2a+1) \end{aligned}$$