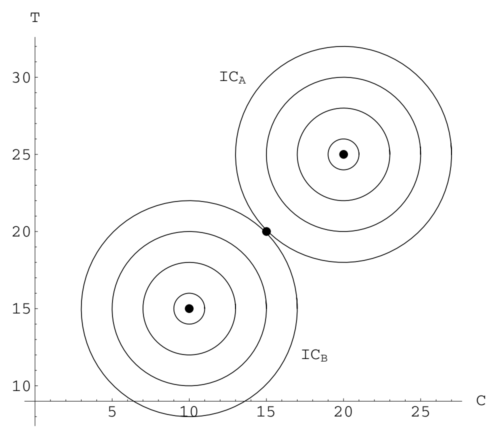
Solutions to the Exercises in Public Finance

Answers and Graphics supplied by Anita Gantner, UCSB

1 Chapter 1

1.1(a) The graph looks like this.



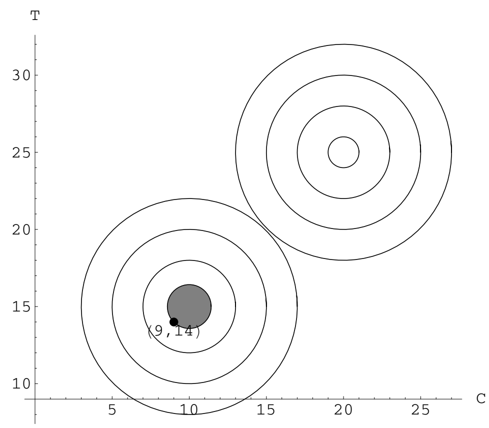
(b) The point (10,15) is Pareto optimal since it represents Bruce's bliss point. Any movement away from this point would make him worse off.

The P.O. set is the line segment $T = 5+C$, where $10 \leq C \leq 20$. All tangency points of the two circles around A and B must be on the straight line connecting A and B, however, only the points between A and B are P.O.

(c) The set of situations Pareto superior to (9,14) is the set of all C, T such that

$$-[(C-10)^2 + (T-15)^2] < 2$$

which is a circle around Bruce's bliss point of radius $\sqrt{2}$.



(d) For the indifference curves to be tangent, the condition $MRS^A = MRS^B$ must hold.

$$MRS^A = \frac{C-20}{T-25} = MRS^B = \frac{C-10}{T-15}$$

Consider the point (5,10), where $MRS^A = MRS^B$. At this point,

$$\tilde{U}^A(5,10) = -250$$

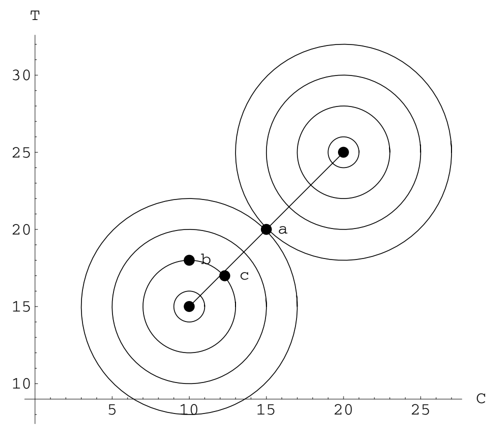
$$\tilde{U}^B(5,10) = -100$$

But at the point (10,15), we have

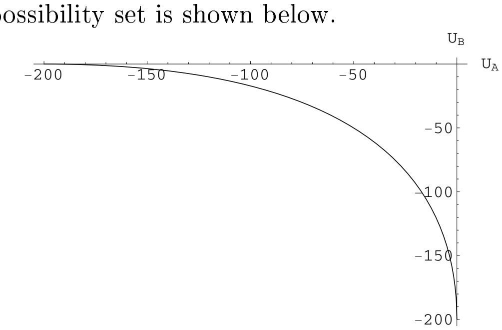
$$\begin{aligned} U^A(10,15) &= -200 > \tilde{U}^A \\ U^B(10,15) &= 0 > \tilde{U}^B \end{aligned}$$

Thus (5,10) is not P.O. To increase both $\tilde{U}^A$ and $\tilde{U}^B$, both T and C have to be increased. A necessary condition for P.O. is, however, that preferred directions of change be opposite. At (5,10) the indifference curves are tangent on the "wrong" side.

(e) In the graph below, suppose a is Pareto optimal. Consider the point b, which is not Pareto optimal, since there is c, which is Pareto superior to b. Both a and c are Pareto optimal and c is Pareto superior to b, but a is not Pareto superior to b, since moving from b to a would make B worse off.



1.2 The utility possibility set is shown below.



1.3 (a) No, they cannot improve on the sure thing P.O. allocations by gambling, since we are in the negative quadrant and making the utility possibility set a convex set by allowing for gambling does not increase their utilities.

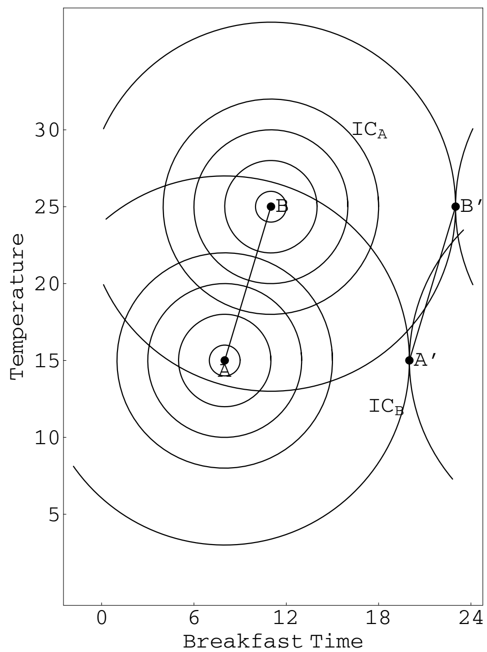
(b)

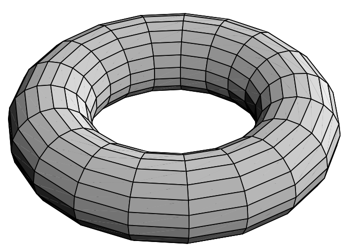
$$
\begin{aligned}
U_A &= \frac{1}{(C-20)^2 + (T-25)^2} \\
U_B &= \frac{1}{(C-10)^2 + (T-15)^2}
\end{aligned}
$$

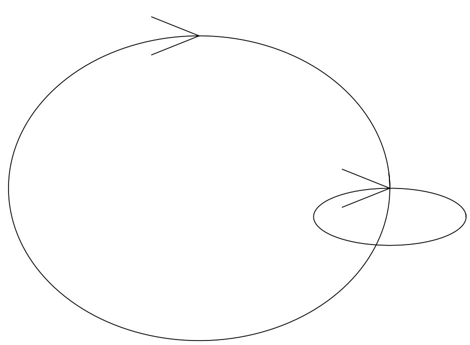
1.4 compensated by more T or vice versa. They prefer much more playing in colder temperature to playing just a little more with a little higher temperature. The more "sloped" the indifference curves are, the more extreme is this trade off.

1.5 The boundary of the utility possibility set slopes uphill to the west of point B because these points are representations of the "bad" tangency points. Moving away from these points, i.e. increasing U^A, also means increasing U^B.

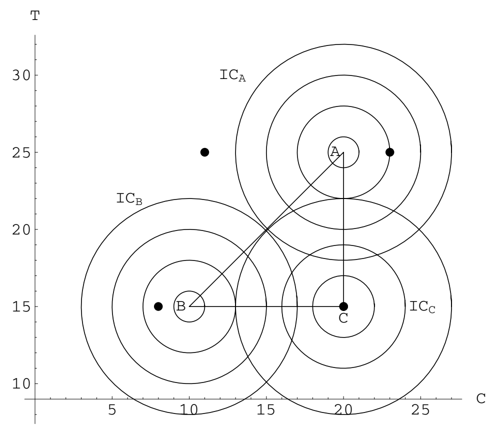
1.6 Have a look at these graphs.







1.7 Here is the graph.



The Pareto set is the triangle connecting the bliss points A, B and C. Set up the Lagrangean:

$$U^A(T,C) + \lambda(U^B(T,C) - \bar{U}^B) + \mu(U^C(T,C) - \bar{U}^C)$$

$$\begin{aligned}
\frac{\partial L}{\partial C} &= -2(C-20) - 2\lambda(C-10) - 2\mu(C-20) \\
\frac{\partial L}{\partial T} &= -2(T-25) - 2\lambda(T-15) - 2\mu(T-15) \\
\frac{\partial L}{\partial \lambda} &= U^B(T,C) \geq \bar{U}^B, > if \lambda = 0 \\
\frac{\partial L}{\partial \mu} &= U^C(T,C) \geq \bar{U}^C, > if \mu = 0
\end{aligned}$$

The Lagrange multiplier method applies here and it gives us a perfectly good necessary conditions. The only "oddity" is that there are is a two-dimensional set of solutions rather than just a one-dimensional set as in the case of Anne and Bruce.