Chapter 75

A SMART MARKET FOR THE SPOT PRICING AND PRICING OF TRANSMISSION THROUGH A POWER GRID

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This chapter illustrates a mechanism capable of competitively allocating power through an electricity network in which "loop flow" and the unusual economic phenomenon caused by loop flow are anticipated and integrated into the competitive process. At the base of the complexity is Kirchoff's law governing electricity flow through power grids. This physical law creates a form of externalities throughout a power transmission network and these externalities must be incorporated into any efficient and decentralized pricing mechanism. The mechanism developed here is a special, continuous double auction in which buyers and sellers are in essence buying and selling differentiated products in the sense that they wish to purchase and sell power at different locations, but the purchasing and selling activities are technically related through resource limitations and the physical law of electricity flow.

1. Kirchoff's Law and Resource Constraints

The easiest way to explain the process is by reference to the experiments that were actually conducted. Figure 1 represents a power grid with consumers and producers located at the nodes and the arcs connecting the nodes representing power transmissions lines with limited capacity. Consumers are located at Nodes 2 and 3, while producers are located at Nodes 1 and 2. In the example to be explored the consumers and producers located at Node 2 are marginal in the sense of low values and high marginal costs and would be excluded from the market if there are no transmission constraints. Kirchoff's law requires power to travel through all paths between the node of injection and the node of extraction in such as way that is (roughly speaking) inversely proportion to the impedance of the path. The power that can be transmitted across a line is limited by the capacity of the line. So, a single line with limited capacity can constrain the power that flows through the entire network even though that line connects no two nodes on a direct path between the source of injection and the location of extraction. In Figure 1 all lines have the same impedance so the impedance of a path is proportional to the sum of the lengths along a path. That is, if a megawatt is injected at Node 1 and extracted at Node 3, $2/3$ megawatts will travel along the line 1-3 and $1/3$ megawatt will travel along the lines 1-2 and 2-3. Constrained capacities along any line can thus constrain the transmission of power along all lines. Or, putting another way, in order to make the nature of the externality clear, transmission along any line has an influence on the feasible transmission along all other lines.

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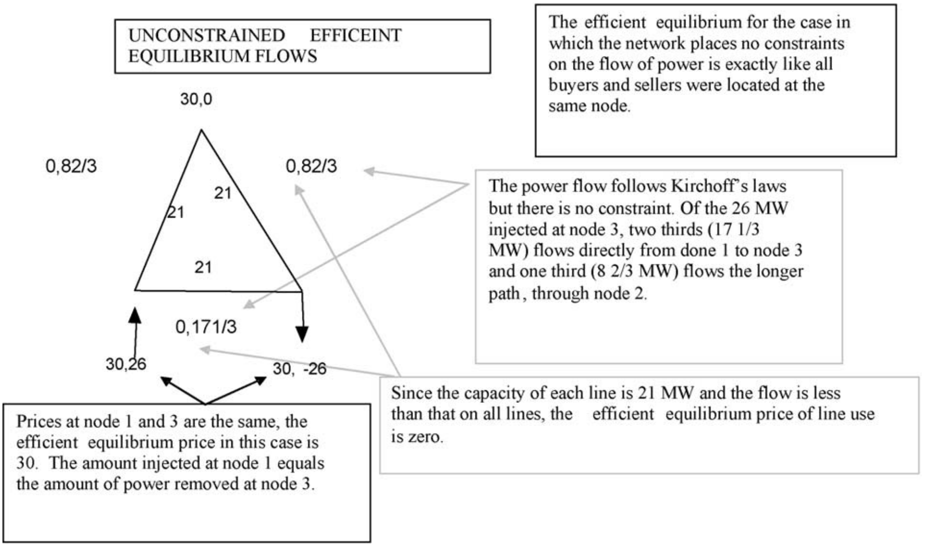


Figure 1. The unconstrained network and Kirchoff's laws as applied to the network.

2. The Mechanism

Power buyers submit bids for quantities of power demanded at a node. These bids are in the form of a price and a quantity that a bidder demands at that price or lower. The bidder can submit as many bids as desired. Sellers of power submit asks for power to be injected at a node. The asks are in the form of a price and a quantity that the seller is willing to sell at that price or higher. Sellers can submit as many bids as desired. Transmission line owners can submit asks for the capacity of the line that they will supply. Like suppliers of power, the asks are in the form of a price per megawatt that will travel through the line and a capacity quantity that the line owner is willing to supply at that price.

As bids and asks are tendered, the mechanism computes winning bids and prices in either real time or in rounds and publicly announces them. That is, the mechanism computes the allocations and payments that will take place and publicly announces them. The allocation and prices are called the "provisional winners" in the sense that if the mechanism stops, the provisional winners are those that have the contracts as specified by the provisional winning allocation and prices. If the process does not stop, bids and asks not involved as provisional winners can be revised but provisional winners cannot be revised. The system stops if no new bids or asks are submitted but if the process is taking "too long" a random stopping process is employed with an increasing probability that the mechanism will stop.

The computation of the provisional winners and prices takes place in a "smart market" optimization technique. The allocation problem is formulated as a linear (integer) programming problem and the multipliers of the problem (the dual) are the prices. The objective function of the problem reflects "consumer's surplus" as embodied in bids and the constraints of the problem are those imposed by the physical properties of the network and the capacities supplied by those who own the lines. The prices themselves become the competitive equilibrium prices if there were no externalities in the system so the general behavioral postulate, since there are no monopolists in the study, is captured by the same principles of competitive behavior that exist in the competitive equilibrium model.

Technically, the mechanism works as follows. We apply an algorithm to compute provisional allocations and prices in either a continuous or a multiple round auction of electricity and transmission rights. Tender orders can be submitted during each round or instant of time. Each eligible tender will be indexed by l. Suppose that there are n nodes in an electric network and T trading periods in a day. There are potentially $n(n-1)$ directed links, each of which is associated with a set of transmission capacity rights. Each tender is represented generically as an n^2+1 vector, (V_l, q_l, T_l), where $V_l \in R$ represents the value of an offer (+) or bid (−), $q_{il} \in R^n$ represents the quantities of power injected (+) or extracted (−) at node $i(=1,\ldots,n)$, and $T \in R^{n\times n-1}$ represents the quantities of transmission capacity rights offered (+) or demanded (−).

The components of a tender offer can take on either positive or negative values. We adopt the convention that a positive value of V_l signifies payment (for a purchase) and a negative value signifies receipt (from a sale). Similarly, a positive value of q_{lk} signifies power injection, whereas a negative value signifies power extraction. This vector representation is flexible. For instance, we allow a portfolio tender which involves simultaneous trades at multiple nodes over multiple periods.

For simplicity, we assume that each tender can be accepted fractionally, and the net trade vector will therefore be scaled in proportion to the accepted fraction. When the tendered quantities take on integer values, the algorithm could always yield integer solutions. The following linear program is solved each round.

2.1. Notation

V_l: the value of the lth tender
x_l: the fraction of tender l accepted
q_{lkt}: the amount of electricity traded at node k in period t by tender l
Q_{kt}: net power injection at node k in period t
β_{ij}^k: the loading factor of power flow on link (i,j) for injection at node k
K_{ij}: the transmission capacity on link (i,j)

$$\underset{x,Q}{\text{Maximize}} \sum_l V_l x_l \tag{1}$$

subject to:

$$Q_{kt} - \sum_{l} x_l q_{lkt} = 0, \quad \text{for } k = 1, \ldots, n \text{ and } t = 1, \ldots, T, \tag{2}$$

$$\sum_{k=1}^{n} Q_{kt} = 0, \quad \text{for } t = 1, \ldots, T, \tag{3}$$

$$\sum_{k=1}^{n-1} \beta_{ij}^{k} Q_{kt} \leqslant \sum_{l} x_l T_{ij}^{l}, \quad \text{for all } i, j, \quad 0 \leqslant x_l \leqslant 1 \quad \text{for all } l. \tag{4}$$

Tender orders that are accepted with positive fractions, $x_l > 0$, will be accepted. The above problem will be solved repeatedly during each round. The condition in (4) requires that the power flows associated with each trade must be covered by an adequate amount of transmission capacity rights.

Since no trade (i.e., $x \equiv 0$) is always a feasible solution to (2)–(4), and thus the objective value should always be nonnegative. In other words, the trade surplus is always nonnegative. When multiple solutions exist, the convention adopted as an equitable way to resolve the indeterminacy is to set a priority of acceptance based on the chorological order of tender submission.

For reference, the corresponding dual linear program that calculates the prices is described as follows.

2.2. Notation

p_{kt}: Shadow price of electricity at node k in period t
π_{ij}^{t}: Shadow price of transmission link (i, j) in period t
μ_l: Shadow price of the constraint $x_l \leqslant 1$, which measures the benefit of the bid

2.3. Dual Linear Program for Continuous-time Double Auction

$$\underset{\pi,\mu,p}{\text{Minimize}} \sum_{l} \mu_l \tag{5}$$

Subject to:

$$\sum_{ij} \pi_{ij}^{t} \beta_{ij}^{k} + p_{kt} - p_{nt} = 0, \tag{6}$$

$$-\sum_{k=1}^{n} p_{kt} q_{lkt} - \sum_{i,j} \pi_{ij}^{t} T_{ij}^{l} + \mu_l \geqslant V_l, \quad \mu_l \geqslant 0. \tag{7}$$

Equation (6) indicates that the sum of the transmission prices weighted by the loading factors equals to the nodal price difference between the injection node and the hub. Condition (7) implies that the shadow price of each accepted tender minus the energy and transmission costs must exceed the value of the tender.

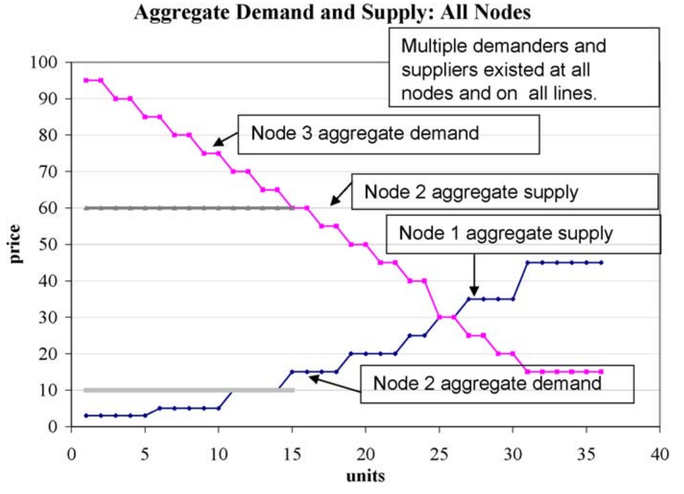


Figure 2. Demand and supplies of power at nodes. The costs of lines to line suppliers are not shown.

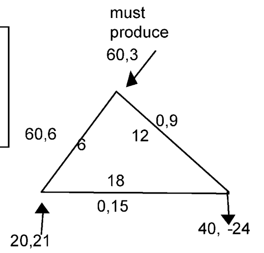
3. Parameter and the Testbed

Figure 2 contains the demand and supply for power at the various nodes. If the transmission system places no constraints on deliveries, as is the case as represented in Figure 1, then electricity would have one price that is the same at all nodes and the price of transmission would be zero. The electricity price would simply be that which equates demand and supply in Figure 2.

Figures 1, 3 and 4 illustrate the configuration of power flows and the capacities of the lines at the efficient allocation. The figure also contains the competitive equilibrium prices that are predicted by the behavioral model employed. The testing strategy was as follows. The first experiments were conducted in an unconstrained system with the parameters of Figure 1. This helped train subjects in the operation of the system and also provided an "easy test" of the system performance. The test then moved to more difficult environments in which economically surprising phenomena might emerge. These cases are the "must produce" and "must take" cases illustrated in Figure 3. The final test parameters represented in Figure 4, had multiple constraints satisfied or similar difficulties that had multiple prices as possible equilibria. The issue was whether or not the multiple prices would themselves be a source of difficulty in the allocation process.

Three experiments were conducted under each condition except the unconstrained case, for which four were conducted. The subjects and general circumstances were the same for all experiments.

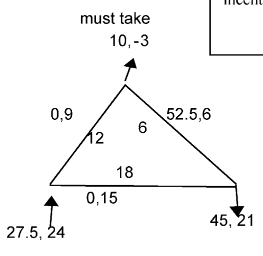
MUST PRODUCE: Efficiency demands that power be produced by the inefficient producer and injected at node 2. The (competitive equilibrium) prices appear odd: 20 at node 1, 60 at node 2 and those at node 3 pay 40. Notice that the amounts produced at nodes 1 and 2 just equals the amount consumed at node 3 (24 MW).



The odd pattern of efficient prices is caused by the limited capacity between nodes 1 and 2. The requirement to use integer values in the experiment compromises Kirchoff's Law somewhat but the problem should be clear. Efficiency demands 24 MW consumption at node 3 but the maximum that can be transported from node 1 is 21 MW, since approximately 1/3 of it must go through node 2 and is a constraint on that line of 6 MW. The demand at node 3 is met by the high cost producer at node 2.

MUST TAKE: PRICES ENCOURAGE POWER REMOVEAL TO AVOID DOWN STREAM CONSTRAINTS

The MUST TAKE case is an example of low price but it is possible that consumers could be paid to consume the power at a particular node. In this example it happens at node 2, where power is removed from the system. At node 1 24 MW are produced and at node 3 the consumption is at 21MW. At node 2 the consumption is 3 MW and the incentive compatible price at node 2 is only a fourth of the price at node 3.



In the "must take" case the problem is the binding constraint of the line connecting node 2 with node 3. In this case production at node 1 is cheap relative to demand at node 3. It maintains the 21 units consumption at node 3 by producing 24 units of which 15 MW go directly to node 3 and 9 MW (approximately 1/3) goes through node 2. At node 2 a low price encourages the removal of 3 MW so the quantity that travels further to node 3 satisfies the line capacity constraint of 6 MW.

Figure 3. Parameters for the must produce case-inefficient producers supply the demand not met by efficient producers due to transmission constraints.

4. Performance

How one might evaluate the performance of a new mechanism are outlined in Plott (1994) as: (1) Does the mechanism do what it is supposed to do? (2) Does it do it for understandable (the correct) reasons? In the context of this mechanism the objective is to achieve an efficient allocation of resources in light of the fact that no economic features, costs or benefits, are known to the mechanism except as might be reflected by behavior. So, if the system achieves near 100% efficiency as calculated from information known only to the experimenter but not the mechanism, it is doing what it is supposed to do. In addition it should do that job in a "smooth" and "rapid" manner. The second criterion is captured by the prices. Are the prices the "competitive equilibrium" prices as based on the information unknown to the mechanism but are known to the experimenter?

As an auction process the mechanism worked quickly, with many auctions taking place in a two hour period. Prices developed smoothly and while termination of the

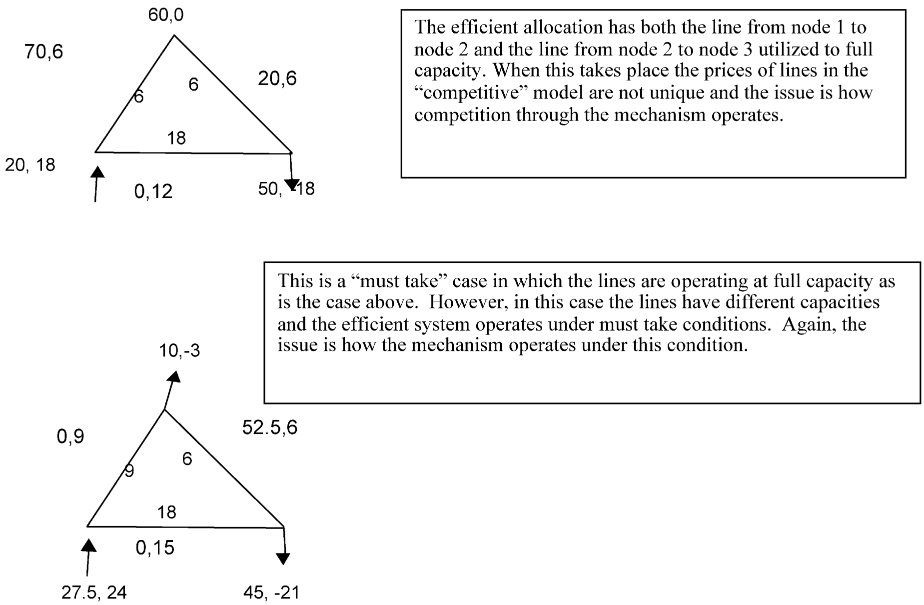


Figure 4. Non-unique price examples – extreme cases to test the system operation.

process sometimes required the use of the random stopping rule, it generally converged in an asymptotic manner. More modern versions of this mechanism employ the use of various types of clocks that force the bidding process. The general impression is that continuous processes work much better than processes that employ rounds.

Does the mechanism do what it is supposed to do? In almost all cases the efficiency levels are in the high 90% and close to 100%. From an efficiency point of view the mechanism works exceedingly well.

Does it perform well-understandable (the right) reasons? Prices and allocations are near the predicted competitive equilibrium levels as shown in Figures 5 and 6, respectively. As shown in Figure 5, prices tend to be slightly higher than the competitive equilibrium, especially when the equilibrium prices are zero. This upward tendency is most pronounced in the unconstrained case, perhaps reflecting the tendency of prices to converge from above when the consumer surplus exceeds producer surplus as it does in these parameters. Thus, there is a bias in the incomes away from the competitive model but at this point there is no reason to assume that the bias is pronounced relative to what is ordinarily observed in experimental markets. In general, the prices are highly correlated with the competitive equilibrium.

The allocations are illustrated in Figure 6. Allocations tend to be distributed around the competitive equilibrium quantities. The figure contains the frequency with which the allocations deviated from the efficient allocations that are predicted by the competitive model. As is clear from the figure, the mass of data are centered around zero deviations.

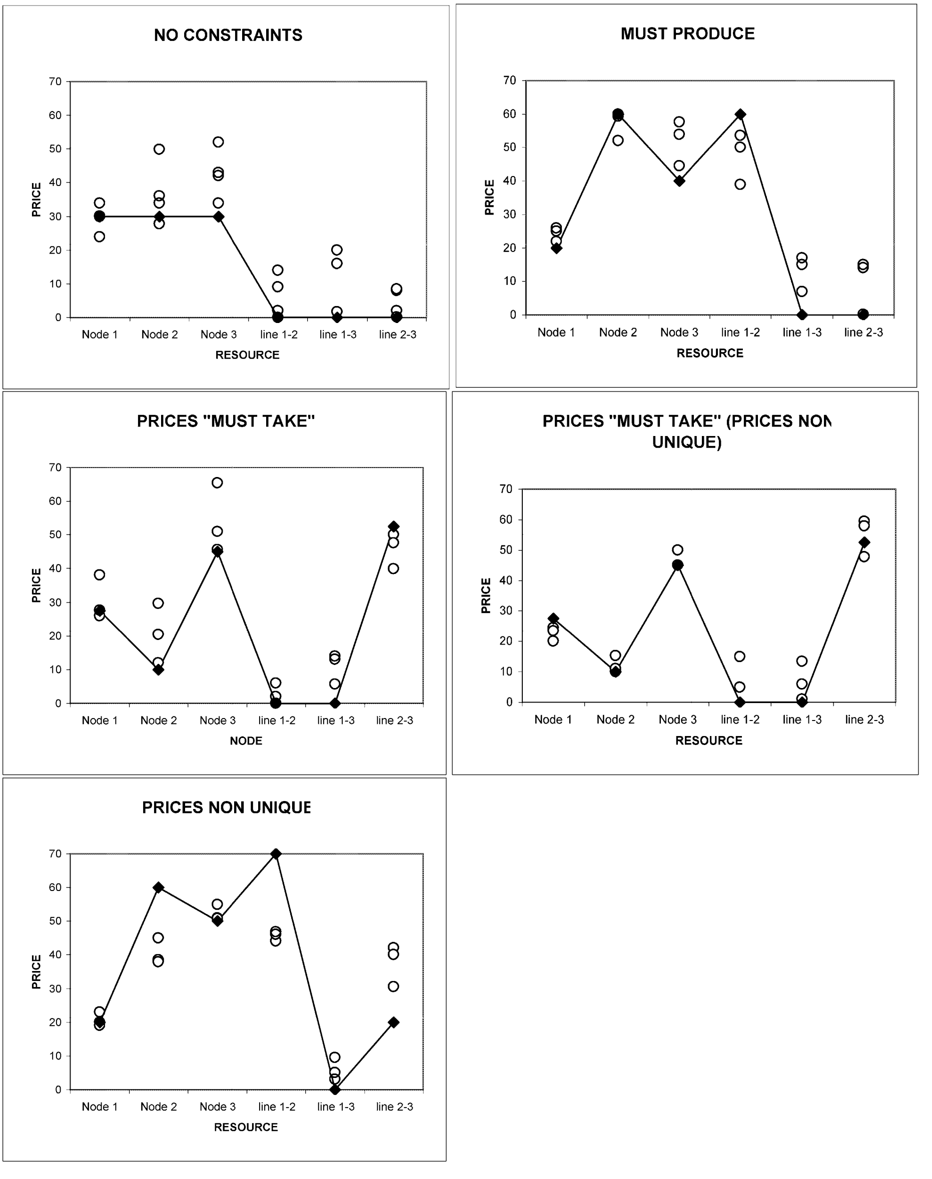


Figure 5. Prices of power and lines under all experimental conditions.

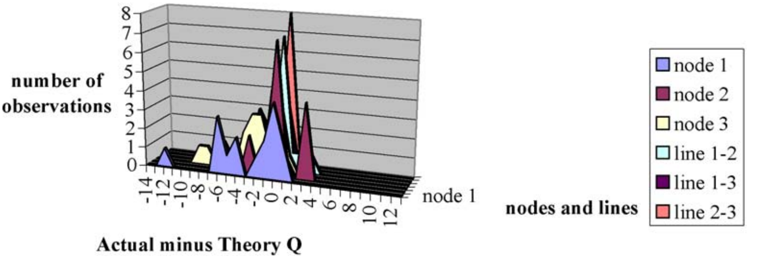


Figure 6. Actual Q minus competitive equilibrium Q.

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