Risk Lovingness over Losses

Prop 3 says risk-loving over losses.

From K&T 1979

4000, .8	3000	−4000, .8	−3000
20	80	92	8
4000, .2	3000, .25	−4000, .2	−3000, .25
65	35	42	52
3000, .9	6000, .45	−3000, .9	−6000, .45
86	14	8	92
3000, .002	6000, .001	−3000, .002	−6000, .001
27	73	70	30

The Reference Point

Up until now we have taken the reference point as fixed (and usually normalized to 0).
But the reference point may

- depend upon past behaviour
- depend upon expectations about present and future behaviour

In an influential set of papers, Koszegi and Rabin (2006, 2007) suggest that reference "point" may

- be a lottery
- be determined by rational expectations

Personal Equilibrium (Koszegi and Rabin 2006, 2007

- Let $Z = \{z_1, \ldots, z_N\}$ be a set of money prizes
- $L = (p_1, \ldots, p_N) \in \Delta(Z)$ is a lottery over prizes.
- Define $u(z|r) = m(z) + v(m(z) - m(r))$, the utility of outcome z from reference point r, where $m(.)$ is standard "consumption utility"
- This differs from KT (1979) in two ways:
  1. The domain of $v(.)$ is utils and not money
  2. Utility is consumption utility plus loss aversion term (rather than utility of reference point plus loss aversion term)
- Take $m(z) = z$ and $v(x) = \eta x$ for $x > 0$ and $v(x) = \lambda\eta x$ for $x < 0$, where $\lambda\eta > \eta > 0$, i.e., piecewise-linear value function

Preferences over Lotteries

Define preferences over lotteries $L = (p_1, \ldots, p_N)$ given reference lottery $L' = (p'_1, \ldots, p'_N)$ by

$$U(L|L') = \sum_{z_i \in Z} \left(\sum_{z_j \in Z} p'_j u(z_i|z_j) \right) p_i$$

We say that L'' is preferred to L' from reference lottery L if $U(L''|L) \geq U(L'|L)$.

Definition

L is a *(unacclimating) personal equilibrium UPE* for choice set $C \subset \Delta(Z)$ if $\forall L' \in C, U(L|L) \geq U(L'|L)$.

L is a UPE if someone expecting to choose it from C likes L at least as much as every lottery in C

Observations

$$U(L|L') = \sum_{z_i \in Z} \left(\sum_{z_j \in Z} p'_j u(z_i|z_j) \right) p_i$$

- When C is convex and compact, a UPE exists.
- For any $L, L', L'' \in C$ such that L' first-order stochastically dominates L'',

$$U(L'|L) \geq U(L''|L)$$

Sydnor Insurance

Example

Buying the (extra) house insurance from reduced deductible with wealth w gives $w - 100$ with probability one. Rejecting gives the lottery that gives w with probability 0.95 and $w - 500$ with probability 0.05 (written $(w, w - 500; 0.95, 0.05)$). Someone who expects to insure prefers to insure iff

$$U([w-100]|[w-100]) \geq U((w, w-500; 0.95, 0.05)|[w-100])$$

$$\Leftrightarrow w - 100 + v(0) \geq w - 25 + 0.95v(w - (w - 100)) + 0.05v(w - 500 - (w - 100))$$

$$\Leftrightarrow w - 100 \geq w - 25 + 0.95\eta(100) + 0.05\lambda\eta(-400)$$

$$20\eta\lambda \geq 95\eta + 75$$

Sydnor Insurance (Continued)

Example

Someone who expects to reject the insurance prefers to buy the insurance iff

$$
\begin{aligned}
& U([w-100] | (w, w-500; 0.95, 0.05)) \\
& \geq U((w, w-500; 0.95, 0.05) | (w, w-500; 0.95, 0.05)) \\
& \Leftrightarrow w-100+(0.95)v(w-100-w)+(0.05)v(w-100-(w-500)) \\
& \geq w-25+(0.95)(0.95v(0)+0.05v(w-(w-500))) \\
& +(0.05)(0.95v(w-500-w)+0.05v(0)) \\
& \Leftrightarrow -95\eta\lambda+20\eta \geq 75+25(0.95)\eta(1-\lambda),
\end{aligned}
$$

which cannot hold for any $\eta > 0, \lambda > 1$ (since $25(0.95)\eta(1-\lambda) > 20\eta - 25\lambda\eta > -95\eta\lambda + 20$).

Hence, someone who expects to reject insurance prefers to do so.

This demonstrates the existence of multiple equilibria with strict preference: someone who expects to insure may strictly prefer to insure, and someone who expects not to insure may strictly prefer not to insure.

Preferred Personal Equilibrium

Definition

L is a *preferred personal equilibrium (PPE)* for choice set $C \subset \Delta(Z)$ if for each personal equilibrium $L' \in C, U(L|L) \geq U(L'|L')$.

Someone able to make any plans she wishes and follow through on her plans should choose a PPE.

Observations

- When C is convex and compact, a PPE exists.
- For any $L, L' \in C$ such that L' first-order stochastically dominates L'', L' is not a PPE.

How do stochastic reference points affect risk attitudes?

Let $L + L'$ be the lottery whose prizes are the sum of one prize from L and one from L' (and not the probabilistic mixture)

Proposition

Suppose that v is piecewise linear. For any wealth w and lotteries $L, L', L'' \in \Delta(Z)$, $U(w + L|w) \geq U(w|w)$ implies $U(L' + L|L'') \geq U(L'|L'')$.

Proof.

(Rough) Let L_+ be the expected gains from L and L_- be its expected losses. Then $U(w + L|w) > U(w|w)$ iff $L_+(1 + \eta) > (1 + \eta\lambda)L_-$. But now combined with L', losses from L sometimes offset gains from L' and therefore are evaluated less negatively (by factor $1 + \eta$ rather than $1 + \eta\lambda$); gains in L sometimes offset losses in L' and are evaluated more positively (by factor $1 + \eta\lambda$ rather than $1 + \eta$). □

Stochastic reference points reduce loss aversion.

We saw that loss aversion can produce "first-order risk aversion", the unwillingness to accept even an arbitrarily small position in some better-than-fair bets.

Our example for last time had a fixed reference point. Is the same true for the stochastic reference points of personal equilibrium?

Proposition

For any lottery L with positive expected value, there exist $\epsilon, A > 0$ such that if the lottery L'' satisfies $Pr_{L''}[w \in (k - A, k + A)] < \epsilon$ for each k, then $V(L' + L|L'') > V(L'|L'') > V(L' - L|L'')$ for any lottery L'.

As the background lottery becomes sufficiently dispersed, risk aversion vanishes. Since life is one big stochastic lottery, this suggests people should not be too risk averse.

However, in contrast to the model, people tend to *bracket narrowly*, focusing on one decision at a time, and do not integrate the outcomes of an individual lottery with other risks they face. A richer model would include this.

Choice-Acclimating Personal Equilibrium (CPE)

Definition

L is a *(choice-acclimating) personal equilibrium CPE* for choice set $C \subset \Delta(Z)$ if $\forall L' \in C, U(L|L) \geq U(L'|L')$.

Unlike with UPE, CPE models settings in which a person's reference point adjusts to her choice, for example if consequences of choice are not realized until long after that choice

Observations

- A CPE need not be a PPE (or even UPE).
- CPE does respect first-order stochastic dominance.
  Consider choosing between 1 for sure, and L that gives 1 with probability $1-p$ and 2 with probability p.
  $U(1|1)=1$, and

$$\begin{aligned} U(L|L) = & \quad 1+p+(1-p)\left((1-p)v(1-1)+pv(1-2)\right) \\ + & \quad p\left((1-p)v(2-1)+pv(2-2)\right) \\ = & \quad 1+p+(1-p)p\eta(1-\lambda) \end{aligned}$$

where is less than 1 for $(1-p)\eta(1-\lambda) < -1$, e.g., if η or λ large

Reference-Dependent Consumption

A person consumes mugs (c) and money (d). Her utility is

$$U(c, d; r_d, r_d) = m(c) + d + v(m(c) - m(r_c)) + v(d - r_d)$$

where r_c is her reference point over mugs and r_d her reference point over money.

- For simplicity, take $m(c) = mc$, where $c \in \{0, 1\}$.
- Take $v(x) = \eta x$ for $x > 0$ and $v(x) = \lambda\eta x$ for $x < 0$, where $\lambda\eta > \eta > 0$, i.e., piecewise-linear value function.
- Assume that reference points match equilibrium behavior.

Purchaser Preferences

Consider someone who expects to buy a mug at price p. Her utility from buying is

$$U(1, -p; 1, -p) = m - p + v(m - m) + v(-p - (-p)) = m - p$$

Her utility from not buying is

$$U(0, 0; 1, -p) = 0 + 0 + v(0 - m) + v(0 - (-p)) = -\eta\lambda m + \eta p = \eta(p - \lambda m)$$

She buys whenever $m(1 + \eta\lambda) \geq p(1 + \eta)$. Note that consumer willing to pay $p > m$.

Sales

Suppose that $m(1 + \eta\lambda) = p(1 + \eta)$ and now mug sometimes goes on sale for $p' < p$.

It is not a UPE to buy at p' and at p.

Intuition: If it were, then when price p, would compare price paid to $p' < p$. Since consumer indifferent between buying and not buying when comparing to p, consumer now experiences loss when using p' as reference point, and is no longer willing to buy at p.