Required Problems

1. Using truth tables, prove both of DeMorgan's Laws for logical connectives.

   (a) $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$

   (b) $\neg(P \vee Q)$ is logically equivalent to $\neg P \wedge \neg Q$

2. Find the contrapositive and converse of each of the following statements:

   (a) "If squares have four sides, then triangles have four sides."
   (b) "A sequence a is bounded whenever a is convergent."
   (c) "The differentiability of a function f is sufficient for f to be continuous."

3. Let x and y be integers. Prove that if x and y are even, then $x + y$ is even.

4. Let A and B be sets. Prove that $A \subset B$ if and only if $A - B = \emptyset$.

Practice Problems

5. If P, Q, and R are true while S and T are false, which of the following are true?

   (a) $Q \wedge (R \wedge S)$
   (b) $(P \vee Q) \wedge (R \vee S)$
   (c) $(P \vee S) \wedge (P \vee T)$

6. Make truth tables for these propositional forms:

   (a) $P \implies (Q \wedge P)$
   (b) $(\neg P \implies Q) \vee (Q \implies P)$
   (c) $\neg Q \implies (Q \iff P)$

7. Rewrite each of the following sentences to be symbolic sentences using logical connectives.

   (a) If $x = 1$ or $x = -1$, then $|x| = 1$.
   (b) B is invertible is a necessary and sufficient condition for $|B| \neq 0$.
   (c) $6 \geq n - 3$ only if $n > 4$ or $n > 10$.
   (d) S is compact iff S is closed and bounded.

8. Rewrite each of the following sentences to be symbolic sentences using quantifiers. The universe of discourse for each is given in paratheses.

   (a) Every nonzero real number is positive or negative. (Real Numbers)
   (b) Every integer is greater than some integer. (Integers)
   (c) There is a smallest positive real number. (Real Numbers)

9. The qualifier $\exists!$ is defined as follows:

$$\exists! x \ni A(x) \iff \big(\exists x \ni A(x)\big) \wedge \big(\forall y \wedge \forall z, A(y) \wedge A(z) \implies y = z\big)$$

   Describe in plain english the qualifer $\exists!$.

10. Let x and y be integers. Prove the following propositions:

    (a) If x and y are even, then xy is even.

    (b) If x and y are odd, then $x + y$ is even.

    (c) If X is even and y is odd, then $x + y$ is odd.

11. Let a and b be real numbers. Prove that $|a + b| \leq |a| + |b|$.

12. Let x be an integer. Write a proof by contraposition to show that if x is even, then $x + 1$ is odd.

13. Suppose a and b are positive integers. Write a proof by contradiction to show that if ab is odd, the both a and b are odd.

14. Prove that if $x \notin B$ and $A \subset B$, then $x \notin A$.

15. Let $A = \{1, 3, 5, 7, 9\}$, $B = \{0, 2, 4, 6, 8\}$, $C = \{1, 2, 4, 5, 7, 8\}$, and $D = \{1, 2, 3, 5, 6, 7, 8, 9, 10\}$. Find the following:

    (a) $A \cup B$

    (b) $A - B$

    (c) $(A \cap C) \cap D$

    (d) $A \cup (C \cap D)$

16. Let A, B, C, and D be sets. Prove that if $C \subset A$ and $D \subset B$ and A and B are disjoint, then C and D are disjoint.