Chapter 64

LEARNING IN ENTRY LIMIT PRICING GAMES

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1. Introduction

Signaling games play a central role for many models in economic theory and information economics. Applications of signaling games include education (Spence, 1974; Riley, 1979), market entry games (Milgrom and Roberts, 1982; Cho, 1987; Bagwell and Ramey, 1988), and advertising (Milgrom and Roberts, 1982). For all of these applications, the conclusions depend on what outcomes are considered likely for signaling games.

When signaling games admit a large number of sequential equilibria, as is typical, game theory cannot provide a definitive answer as to which is most likely. Theorists have attempted to narrow down the set of possible equilibria via various processes of forward induction (see Fudenberg and Levine, 1998, pp. 446–460). This approach has not been entirely satisfactory; to reach fairly specific predictions, strong assumptions about the reasoning ability of players must be employed. Brandts and Holt (1992) demonstrate experimentally that violations can be generated of even the relatively weak Cho–Kreps intuitive criterion. As an alternative, attention has recently turned to models of learning in which boundedly rational players gradually learn to play a game through an adaptive process. While equilibrium may eventually emerge in these models, it reflects a steady-state of the dynamic process rather than the culmination of some reasoning process.

Cooper, Garvin, and Kagel (1997a, 1997b) examine whether refinements based on forward induction or simple adaptive learning models are better able to capture behavior in signaling game experiments. We find that observed behavior is inconsistent with either the equilibrium refinements literature or pure belief-based adaptive learning models. An augmented adaptive learning model in which some players recognize the existence of dominated strategies and their consequences successfully captures the major qualitative features of the data.

2. The Limit-pricing Game

Our research program has focused on Milgrom and Roberts' (1982) entry limit-pricing game which serves as a rich vehicle for investigating a number of properties of signaling games. In the Milgrom–Roberts model an incumbent monopolist (M) faces a potential entrant (E). There is asymmetric information as Es are uncertain whether they are facing a high (M_H) or low (M_L) cost M; ex ante, each type is equally likely. Entry is profitable against an M_H but not against an M_L. Before E decides to play IN or OUT it observes M's output which plays the role of a signal.

Handbook of Experimental Economics Results, Volume 1

DOI: 10.1016/S1574-0722(07)00064-9

Table 1
(Monopolist.) A player's payoffs as a function of B player's choice

Your Choice	A1 (High cost) X (IN)	A1 (High cost) Y (OUT)	A2 (Low cost) X (IN)	A2 (Low cost) Y (OUT)	Your choice
1	150	426	250	542	1
2	168	444	276	568	2
3	150	426	330	606	3
4	132	408	352	628	4
5	56	182	334	610	5
6	−188	−38	316	592	6
	38	*162*			
7	−292	−126	213	486	7
	20	*144*	**298**	**574**	

Note. Terms in ( ) not included in experiment. Payoffs are in "francs." Francs converted to dollars at 1 franc = \$0.001.

Table 2a
(Entrant.) B's payoffs

Your action choice	A1 (High cost)	A2 (Low cost)	Expected value[a]
A player's type			
X (IN)	300	74	187
Y (OUT)	250	250	250

Notes. Terms in ( ) and expected values not included in experiments. Payoffs are in "francs." Francs converted to dollars at 1 franc = \$0.001.
[a]Based on prior distribution (50% M_H, 50% M_L) of M types.

Table 1 shows the payoffs employed for Ms in our original sessions, referred to as SP (for standard payoff) sessions, in regular print. Modifications to these payoffs for a later set of sessions, referred to as ZA (for zero anticipation) sessions and MR (for mixed recognition), are shown in italics and boldface, respectively. Table 2a gives payoffs for "high cost" Es, and Table 2b gives payoffs for "low cost" Es.

Several features of the Ms' payoff tables are noteworthy. First, Ms are always better off, ceterus paribus, if the Es choose OUT. Second, M_Ls have more incentive to choose high output levels than M_Hs. In particular, suppose that Ms maximize their payoffs ignoring any effect their output has on Es' responses. The resulting "myopic" maxima are 2 for M_Hs and 4 for M_Ls. Finally, output levels 6 and 7 are strictly dominated strategies for M_Hs in all treatments.

Table 2b
(Entrant.) B's payoffs

Your action choice	A1 (High cost)	A2 (Low cost)	Expected value[a]
A player's type			
X (IN)	500	200	350
Y (OUT)	250	250	250

Notes. Terms in ( ) and expected values not included in experiments. Payoffs are in "francs." Francs converted to dollars at 1 franc = \$0.001.
[a]Based on prior distribution (50% M_H, 50% M_L) of M types.

Turning to the Es' payoff tables, note that in either case an E should play IN if it knows it is facing an M_H and OUT if it knows it is facing an M_L. The two tables differ in the optimal response versus the prior distribution over types; the best response versus a 50–50 distribution over types is OUT with Table 2a and IN with Table 2b.

In analyzing the sequential equilibria of the limit-pricing game, we concentrate on the SP payoff table; the set of equilibria is unaffected by using the other payoff tables. With Es using Table 2a, there are pure strategy-pooling equilibria at output levels 1–5. For all of these equilibria, choice of the pooling outcome is followed by OUT; all other output levels induce choice of IN. There also exist two pure strategy separating equilibria. In both of these equilibria, M_Hs choose 2. M_Ls choose 6 in one separating equilibrium and 7 in the other. The M_Ls' equilibrium output generates a response of OUT in both equilibria, while all other output levels induce choice of IN.

With Es using Table 2b, no pure strategy equilibria exist. The same two pure strategy-separating equilibria exist as were described for Table 2a. This treatment also has several partial pooling (mixed strategy) equilibria. One of these is especially noteworthy, as it arises in a significant number of simulations. In this equilibrium, M_Ls always select 5 while M_Hs mix between 2 (probability .80) and 5 (probability .20). Es always enter on output levels other than 5, and enter on 5 with probability .11.

To reduce the set of equilibria, we follow the approach of the signaling game refinements literature by restricting the set of possible beliefs. One of the weakest equilibrium refinements is (non-iterated) elimination of dominated strategies. Applying a single round of deletion of dominated strategies to Table 1 removes play of 6 or 7 by M_Hs. This eliminates the separating equilibrium with M_Ls choosing 7, since this equilibrium depends on Es believing that M_Hs will play a strictly dominated strategy (choose 6 with sufficiently high probability to justify entry). The partial pooling (mixed strategy) equilibrium is likewise eliminated. A more demanding requirement is that beliefs are intuitive in the sense of Cho and Kreps (1987). The Cho and Kreps intuitive criterion eliminates the pooling equilibria at 1, 2, and 3 with Table 2a, as well as the inefficient separating equilibrium and the partial pooling equilibrium.

3. Experimental Procedures

Experimental sessions employed between 12 and 16 subjects who participated in 36 plays of the game (announced in advance). Ms' types were randomly determined in each play of the game. Subjects switched roles after every 6 games with Ms becoming Es and vice versa. Ms were randomly paired with a different E in each play of the game. Neutral terms were used throughout, with Ms called A players (type A1 for M_H, type A2 for M_L) and Es called B players. A players chose first, after which the B players decided between X (IN) and Y (OUT).

Following each play of the game subjects learned the outcome of their own choices and the type of M player they were paired with (but not the other player's identity). In addition, subjects' screens displayed the results for all pairings (Ms' choices, Es' responses, and Ms' type) with subjects' identification numbers suppressed and pairings presented in random order.

Sessions varied along several dimensions which are summarized in Table 3. While most of the treatment variables are self-explanatory, a few require additional explanation. Recall that the dominated strategies play a crucial role in picking out an equilibrium for the games with Table 2b. The treatments differed in how obvious it was to Es that M_Hs should never play these strategies. With the SP treatment, M_Hs had negative earnings from both of the dominated strategies, making it relatively obvious that these would not be used. In the ZA treatment, these strategies had positive earnings. While play of 6 or 7 was still dominated for M_Hs, this was less transparent in the ZA treatment than in the SP treatment. In some of the sessions, we restricted the choices available to M_Hs by not allowing them to choose either 6 and 7 or just 7. By forbidding use of 6 or 7 by M_Hs, we made it trivial for Es to realize that play of these strategies must come from an M_L. Combining the MR payoff table with forbidding use of 7 by M_Hs establishes perfect recognition that 7 is dominated for M_Hs and little recognition that 6 is, while increasing the returns to M_Ls choosing 7. This treatment was designed to induce the inefficient separating equilibrium.[1]

4. Adaptive Learning

We simulated an adaptive learning model to predict the outcomes for our experiments. In this model, players learn to play the game through a relatively simple process of trial and error loosely based on fictitious play (Brown, 1951). Players are rational in the limited sense that they maximize payoffs subject to their beliefs and update their beliefs in accordance with observed outcomes. However, they do not account for the changing distribution of opponents' strategies.

In our model Ms begin with initial beliefs regarding the probability of Es choosing IN or OUT in response to each of their possible choices. Es start with initial beliefs

[1] Cooper, Garvin, and Kagel (1997a) also reports results from an earlier version of Treatment V.

Table 3
Experimental treatments

Treatment	Number of experimental sessions	Payoff tables	Strategies eliminated for $M_H s$	Subject experience	Cho–Kreps intuitive criterion prediction	Remarks
I	2 inexperienced 1 experienced	SP & 2A	None	None or same game	Pooling at 4 or 5 or separating with M_L at 6	Stronger equilibrium refinements predict pooling at 4
IB	2 experienced	SP & 2A	None	Prior experience with treatment II	Same as treatment I	Prior experience has no effect within equilibrium refinements approach
II	2 inexperienced 1 experienced	SP & 2B	None	None or same game	Separating with M_L at 6	Partial pooling equilibrium at 5 is a frequent outcome in simulations
III	2 inexperienced 1 experienced	ZA & 2B	None	None or same game	Same as treatment II	Positive payoffs on 6 and 7 for $M_H s$ to make recognition of dominated strategies more difficult
IV	2 inexperienced 1 experienced	SP & 2B	6 & 7	None or same game	Same as treatment II	$M_H s$ explicitly forbidden to use dominated strategies
V	2 inexperienced 1 experienced	MR & 2B	7	None or same game	Same as treatment II	Mixture of treatment III and IV with payoffs on 7 increased for $M_L s$

about the probability of an M_H or M_L choosing a particular output. The distributions of initial beliefs for Ms and Es in the simulations are fitted from first period data. After initial beliefs are randomly generated from these distributions, simulated Ms and Es are randomly matched to play sixty rounds of the entry limit pricing game. In each round, Ms' choices and Es' responses maximize their payoffs, conditional on beliefs. Following play of the game, beliefs are updated based on observed outcomes from own play and others' play in the previous period. The updating rule averages each players' initial beliefs and historical experience, where all prior experience receives equal weight (see Cooper, Garvin, and Kagel, 1997b for details). Augmented versions of the model permit some Es to recognize that M_Hs have dominated strategies. In this case these Es attach zero prior probability to M_H choosing 6 or 7 and never enter on 6 or 7.

Although this model does not converge to a steady state for all games, simulations consistently converge to a steady state for the limit-pricing game. In the limit-pricing game, observed steady state strategies must correspond to the outcomes induced by some sequential equilibrium. Figure 1 provides two typical examples of the simulation results. Play follows a distinctive pattern. Initially, choices by Ms are clustered around the myopic maxima, and Es do not distinguish strongly between output levels. Over time, entry rates rise on 2, the myopic maximum for M_Hs. In response, M_Hs move to pooling with M_Ls at 4. In simulations with high cost Es (Table 2a), play consistently converges to the pooling equilibrium at 4 with entry rates on 4 dropping to zero. With low cost Es (Table 2b), entry rates rise on 4, causing the M_Ls to separate to higher output levels. The results with low cost Es are sensitive to the percentage of Es who recognize the existence of dominated strategies. The larger this percentage becomes (i) the more rapidly M_Ls learn to limit price and (ii) the more likely play is to converge to the undominated separating equilibrium with M_Ls choosing 6 rather than the partial pooling equilibrium with M_Ls choosing 5. We hypothesize that increasing this proportion is equivalent to moving from Treatment III to Treatment II and then Treatment IV.

Simulations of our adaptive learning model with 0% of Es recognizing that 6 is dominated and 100% recognizing that 7 is, converge to the dominated separating equilibrium (M_Ls choose 7) 60% of the time. Further, permitting a few (15%) Ms to recognize that the 100% elimination of a dominated strategy by Es will result in zero entry when M_Ls choose 7 (these Ms attach zero prior probability to entry at 7), results in 95% of the simulations converging to the *dominated* separating equilibrium. We hypothesize that patterns of play in these simulations should be similar to behavior in Treatment V.

The adaptive learning model does not discard sequential equilibrium, but is an alternative way of characterizing how equilibria emerge; not by the careful reasoning through of the motives behind opponents strategies as in the refinements literature, but by a trial and error learning process in which subjects have some limited intelligence/perceptiveness.

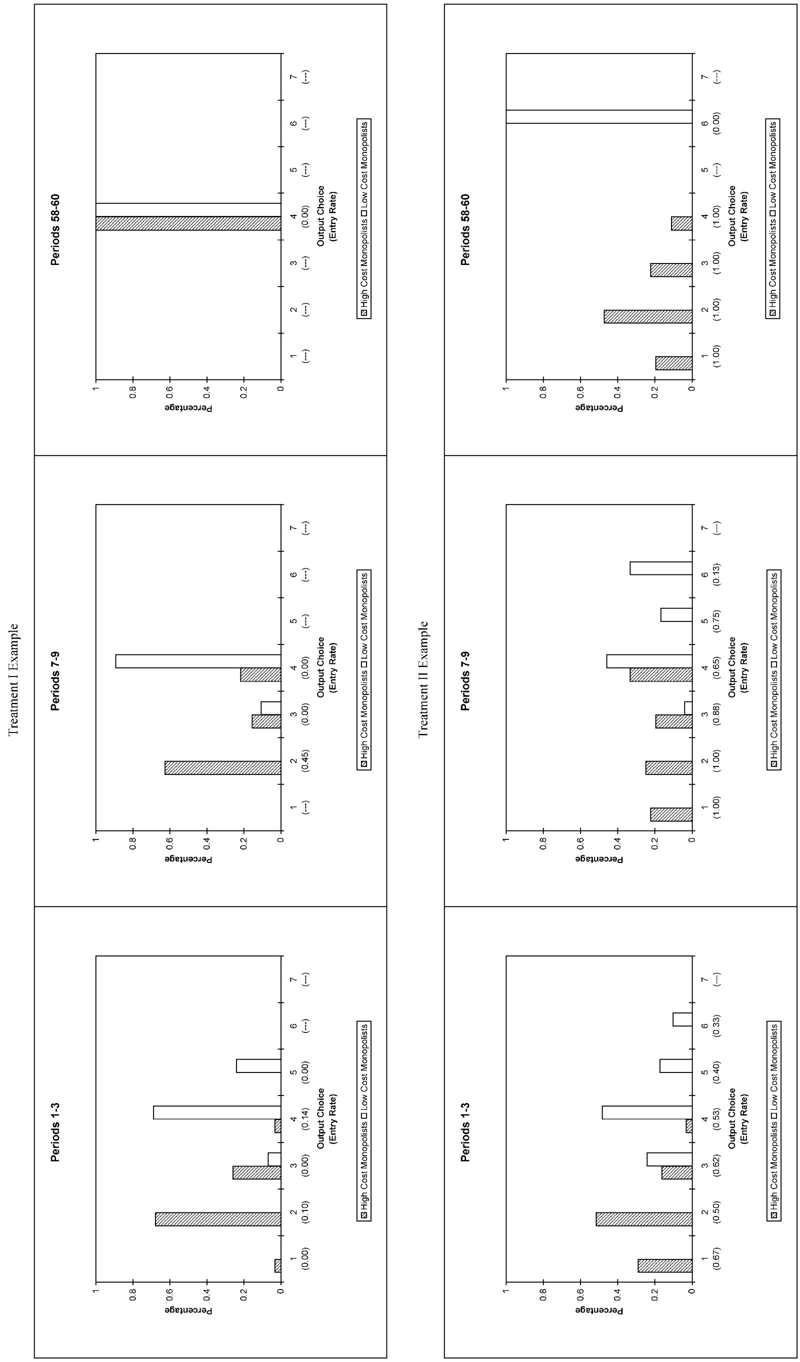


Figure 1. Sample simulation results.

5. Experimental Results

Figure 2 summarizes data from typical inexperienced subject sessions for Treatment I and Treatment II, respectively.[2] Figure 3 reproduces the corresponding experienced subject sessions. For all treatments, play starts with Ms clustered around the myopic maxima. As entry on 2 increases, M_Hs attempt to pool with M_Ls. In Treatment I sessions, the entry rate falls on 4, leading to pooling at 4. In Treatment II sessions, the entry rate rise on 4. This is followed by the M_Ls separating to 6. A clear separating equilibrium only emerges for experienced subjects. Although sequential equilibria which fulfill the intuitive criterion emerge in both treatments, the pattern of play in the experiments bears a striking resemblance to that predicted by the simulations of the adaptive learning model.

This conclusion is reinforced by considering the results of Treatment Ib (reported in Figure 4). Here, rather than converging to the pooling equilibrium at 4, play flirts with pooling at 4 and then returns to the separating equilibrium. The refinements literature has nothing to say about this result, but the adaptive learning model is easily able to capture this result as an effect of accumulated beliefs.

Figure 5 compares results from Treatments II, III, and IV. Limit pricing develops progressively faster as we move from Treatment III to Treatment II and then Treatment IV. Comparing Treatment II with Treatment III, play tends more toward the partial pooling equilibrium rather than the undominated separating equilibrium. In Treatment IV, entry on 6 and 7 is close to zero and play converges much more rapidly to the undominated separating equilibrium than in Treatment II. These results are consistent with simulation results for the adaptive learning model.

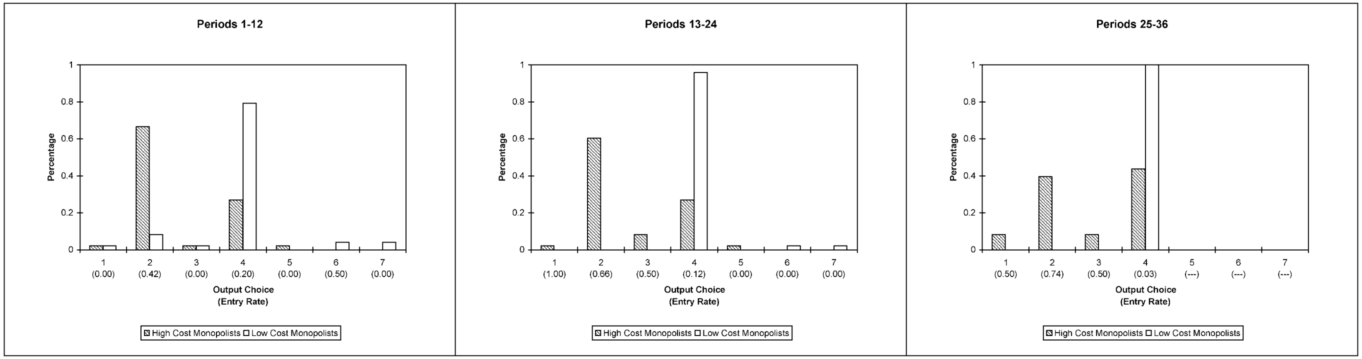
Like Treatment IV, Treatment V leads to rapid emergence of a separating equilibrium. However, virtually all limit pricing takes place at 7 rather than 6. By the end of the experienced subject session, extremely strong convergence to the dominated separating equilibrium is observed. Output level 6 is chosen only 3 times in the inexperienced subject sessions and never in the experienced subject session. This outcome is a strong violation of all existing refinements to sequential equilibrium, but is consistent with simulations of the adaptive learning model.

6. Conclusions

Equilibrium refinements do quite poorly in capturing our results. The refinements say nothing about the observed dynamics of play and cannot predict observed differences between Treatments I and IB and between Treatments II, III, and IV. The results of Treatment V are completely inconsistent with any refinement to sequential equilibrium. All of these results can be characterized by a simple belief-based learning model augmented to allow for some limited reasoning ability by players.

[2] See Cooper, Garvin, and Kagel (1997b) for a full accounting of all sessions run. For replication of these results, see Cooper and Kagel (2002).

Treatment I Example



Treatment II Example

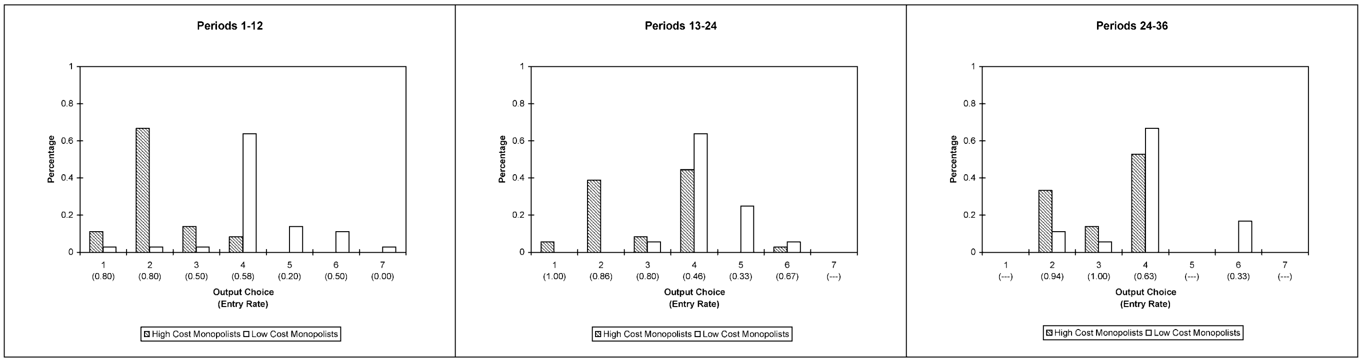


Figure 2. Sample inexperienced session results from Cooper, Garvin, and Kagel (1997b).

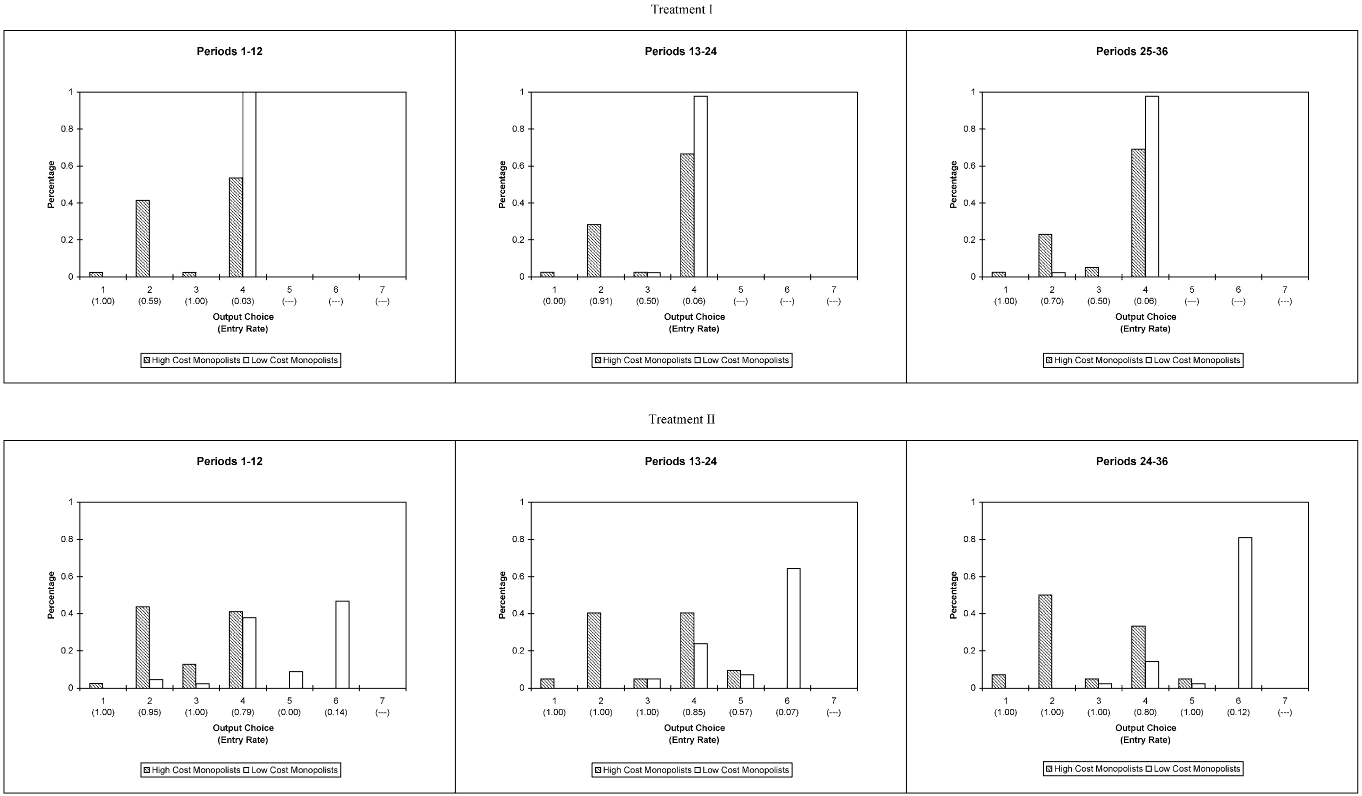


Figure 3. Experienced session results from Cooper, Garvin, and Kagel (1997b).

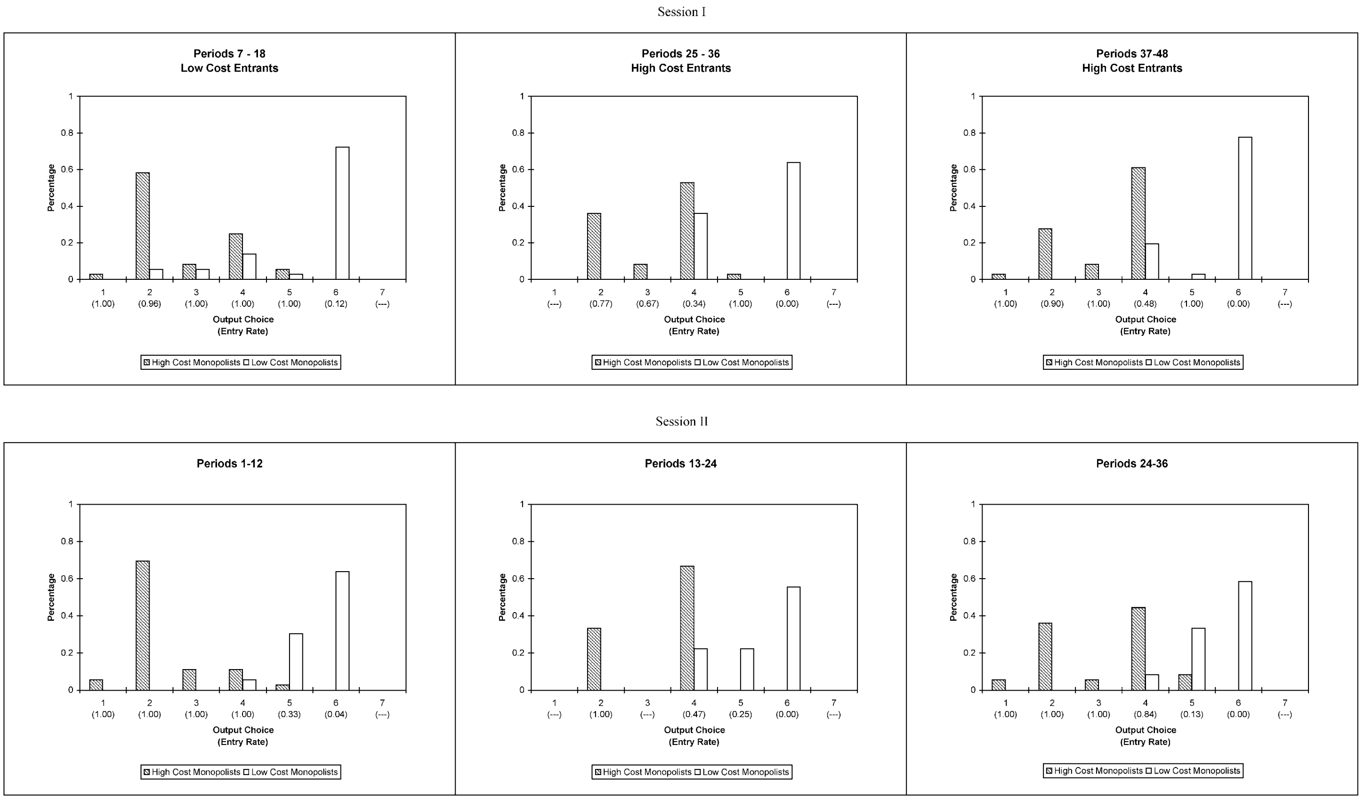


Figure 4. Treatment IB, crossover from low cost entrants to high cost entrants (Cooper, Garvin, and Kagel, 1997b).

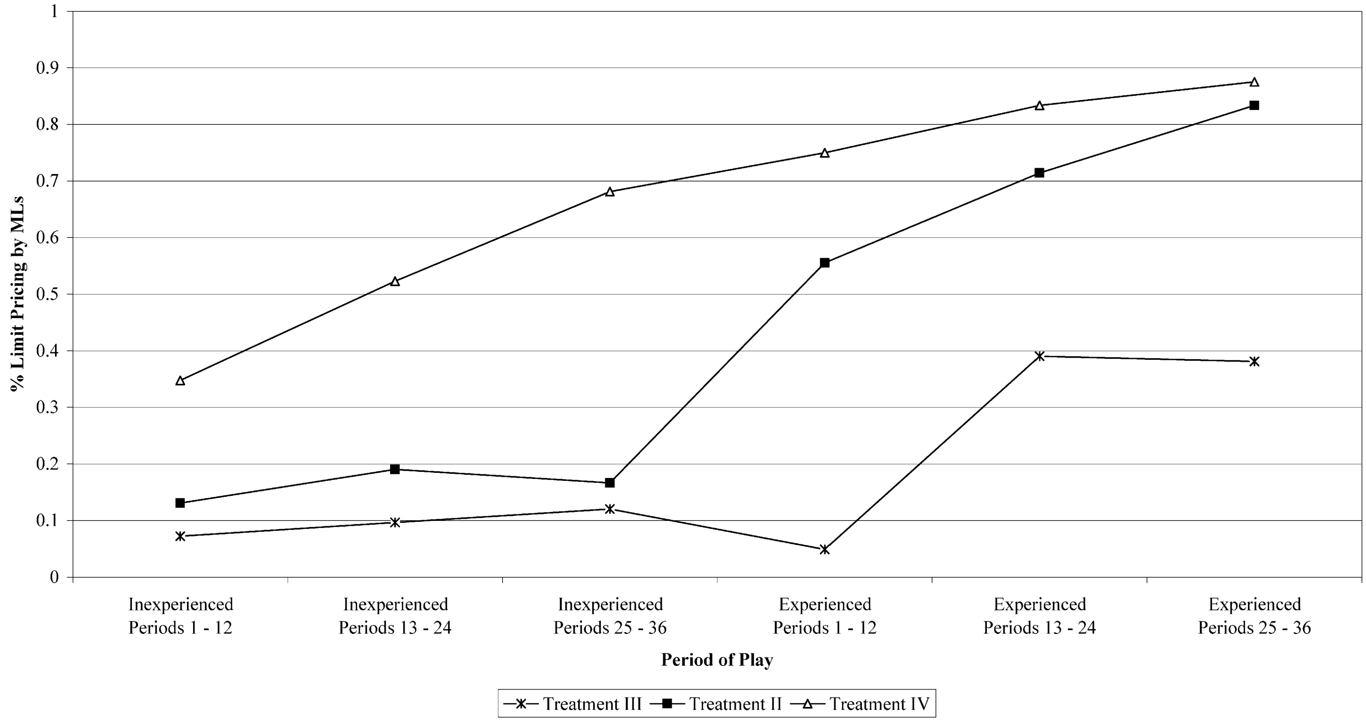


Figure 5. Comparison of limit pricing by $M_L s$ across Treatments II, III, and IV from Cooper, Garvin, and Kagel (1997a).

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