Econ 241A Probability, Statistics and Econometrics
Fall 2017

Problem Set 8

1. Let $X_1, X_2, ..., X_n$ be iid with pdf $f(x|\theta) = \frac{1}{\theta}$.

   (a) What is the MLE of θ?

   (b) What is the Method of Moments estimator of θ?

   (c) Denote the MLE of θ, $\hat{\theta}$. Also, denote the Method of Moments estimator of θ as $\tilde{\theta}$. Finally, define $\hat{\hat{\theta}} = \frac{n+1}{n}\hat{\theta}$. Fill the following table:

Estimator	Expected Value	Variance	Mean Squared Error
$\tilde{\theta}$			
$\hat{\theta}$			
$\hat{\hat{\theta}}$			

   Hint: For the expected value of $\hat{\theta}$, derive first the exact cdf, $F_{\hat{\theta_n}}(x|n)$. Then, calculate $f_{\hat{\theta}_n}(x|n)$.

   (d) What is the Cramer-Rao lower bound for the variance of θ?

   (e) Does any estimate among $\tilde{\theta}$, $\hat{\theta}$, and $\hat{\hat{\theta}}$ 'attain' the Cramer-Rao lower bound for unbiased estimates of θ (note that not all of them are unbiased)? Discuss (Hint: notice that the uniform distribution does not belong to the exponential family).

In addition, solve the following problems from Casella and Berger: 7.19, 7.20, 7.21, 7.48, 8.2 and 8.18.