Econ 241A Prof. Startz

Midterm Fall 2015

Please answer all questions. Show your work.

The exam is open book/open note; closed any devices that can communicate. (No laptops, cell phones, Morse code keys.)

1. Suppose $x \sim U(0,1)$. Find the covariance between x and x^2.

2. There were two kinds of Sneetches in the world, the Star-Belly Sneetches had bellies with stars. The Plain-Belly Sneetches had none upon thars. 90 percent of Sneetches had a star. The Star-Belly Sneetches believed that when they saw a bad thing, two-thirds of the time it was due to a Plain-Belly Sneetch. In other words, people believed that 2/3rd of bad Sneetches were Plain-Bellies. (The Star-Belly Sneetches were wrong about this, but for the purpose of the problem pretend they were right.) Among all Sneetches, only 1 percent were really bad.

   If a Star-Belly comes upon a Plain-Belly, what is the probability that the Plain-Belly is bad?

3. The probability of dying is distributed exponentially with expected number of years $1/\lambda$. Consider an annuity that pays out continuously a rate p per year and stops payment at death. If the continuously compounded interest rate is r, (so a dollar at time t is worth e^{-rt} dollars now, then the present value of payments through year τ is
$$\frac{p}{r}[1 - e^{-r\tau}]$$
   What is the expected net present value of the annuity?

4. Suppose that $x, \varepsilon, \text{and } v$ are jointly normally distributed
$$\begin{bmatrix} x \\ \varepsilon \\ v \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{bmatrix}\right)$$
   Further, $y = \beta x + \varepsilon$ and $z = x + v$.
   Find
$$\frac{\operatorname{cov}(y, z)}{\operatorname{var}(z)}$$

5. Consider a simulation that produces a yes/no answer where the probability of "yes" is p. The total number of independent Monte Carlo trials is n. If we observe k yeses, we estimate

$$\hat{p} = \frac{k}{n}$$

(a) Find mean, μ, and the variance, V, of $\hat{p}$ in terms of $p, k, \text{and } n$.

(b) In a large number of trials, $\hat{p}$ is approximately normally distributed. Taking $\hat{p} \sim N(\mu, V)$, then it can be shown $P\left(|\hat{p} - \mu| > 1.96\sqrt{V}\right) = .05$. If we think $p = .1$, how many observations do we need do that the probability $\hat{p}$ is off by 0.01 is five percent?