Chapter 45

EXPERIMENTAL BEAUTY CONTEST GAMES: LEVELS OF REASONING AND CONVERGENCE TO EQUILIBRIUM

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1. Introduction

John Maynard Keynes (1936, p. 155) suggested that the behavior of investors in financial markets could be likened to newspaper beauty contests in which readers were asked to choose the six prettiest faces from 100 photographs. The winner was the person whose preferences were closest to the average preferences of all participants. Keynes reasoned that contest participants, like financial investors, do not choose faces (investments) that they personally find the most attractive but are instead guided by their expectations of others' expectations. We present data from a series of experiments with human subjects to test how individuals form expectations of others' expectations. The experiment has much in common with Keynes' insight regarding the behavior of investors in financial markets, and therefore the game is called 'beauty contest' game in honor of Keynes.

In the basic beauty contest game, a group of subjects is asked to guess a number from 0 to 100. The winner is the person(s) whose guess is closest to p times the mean of the choices of all players, with, e.g., $p = 2/3$; p is known to all players and is positive. The winners split a prize of \$20. The same game is repeated several periods, and subjects are informed of the results in each period. In any period, for $p < 1$, in equilibrium all players should announce zero and thus everybody is a winner.[1] It is clear that deviating from zero would produce 0 payoffs for that player if all others choose zero. For $p > 1$, the upper bound of the interval is also an equilibrium.

The game is dominance solvable. Thus, as shown in Figure 1a, the process of iterated elimination of weakly dominated strategies, an important concept in game theory, with $p = 2/3$ and number chosen in the interval [0, 100] leads to the equilibrium choice of zero. A player might reason that the mean could be no higher than 100. Therefore, the winning number could be no greater than 2/3 of 100 or 66.667. Numbers above 66.667 are weakly dominated by 66.667 ($E(0)$ of Figure 1a, or 0-level of elimination of dominance). If our player believed that all others would choose 66.667, it would be rational for him to choose 44.444, approximately 2/3 of the new mean. Numbers between 66.667 and 44.444 are chosen by those players who eliminate dominated strategies, but

[1] If only integers are allowed, there is more than one equilibrium (see Lopez, 2002).

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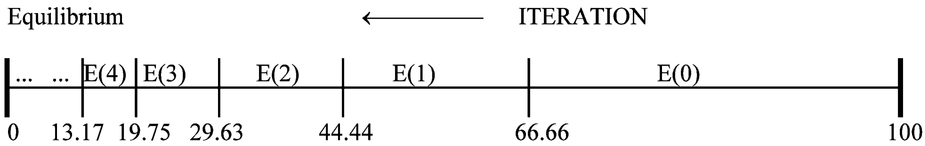

Figure 1a. Infinite process of iterated elimination of dominated strategies for $p = 2/3$-mean game (Infinite Threshold game). A choice in $E(0)$ means that a player is not rational (chooses a weakly dominated number), $E(1)$ is the interval of choices which indicates that the player is rational (does not choose dominated choices) but thinks that the other players are not rational and that they choose numbers which are in $E(0)$. $E(2)$ means that he believes that all others also think that everybody is rational, etc.

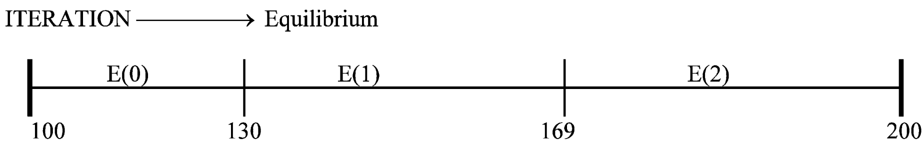

Figure 1b. Finite process of iterated elimination of dominated strategies for $p = 1.3$-mean game (Finite Threshold game). The iteration process starts from 100 upwards. Adjusted from Ho, Camerer, and Weigelt (1998).

believe that all others choose dominated strategies ($E(1)$, or 1-level of iterated dominance). This analysis can be continued. If a player eliminates dominated strategies and believes that all other eliminate dominated strategies and he also believes that all other players believe that nobody uses dominated strategies and so on, he arrives after infinitely many steps to the announcement zero.

A similar process is applied if $p > 1$ (see Figure 1b). The upper bound is the perfect equilibrium and the starting point of the elimination process is the lower (positive) bound of the interval. In this case only finitely many steps are needed to reach equilibrium.

If $p = 1$, the game is similar to a coordination game with any number, chosen by all subjects, forming an equilibrium (see survey of coordination games by Ochs, 1995).

A parameter p different from 1 means that a player wants to distinguish himself from the average, but by not too much. An interior equilibrium is reached if the objective is to be closest to p times the mean plus a constant [see for example 0.7 times (median + 18) in which case the equilibrium is 42]. The advantage is that a deviation from the equilibrium from above and below is possible (see Camerer and Ho, 1998). An economic interpretation could be that the average is exogenously increased by a strong or big outside player who has written down his bid already.

2. Variations on the Beauty Contest Game

For the variations on the beauty contest game, see Table 1. This table states the authors who carried out beauty contest games (column 1), the number of replications of the

Table 1
Experimental designs and structure of the games by the different authors. Bold text indicates main focus of study

Authors	Subject pool	No. of sessions per treatment	No. of players in one group	Winning no. formular, (winner's choice closest to . . .)	Parameters p, c	Range of possible choices	Payoffs	No. of periods in a session	Information after each period	No. of iteration steps to reach perfect equilibrium
Nagel (1995)	Undergrads of various faculties Uni-Bonn, Germany	3–4	15–18 (total 166)	$p *$ mean	$\mathbf{p = 1/2}$, $\mathbf{2/3}$, $\mathbf{4/3}$	[0, 100], real numbers	20 DM = \$10 per period to the winners, 0 to losers, 5 DM show up fee	4	All choices (anonymous), mean $p *$ mean, winning number (s)	Infinite steps for $p < 1$ (equil. 0) n.a. for $p > 1$ (equil. 100)
Stahl (1996, 1998)	Data of Nagel (1995)									
Ho, Camerer, and Weigelt (1998)[a]	Undergrads of business quantitative methods class, Southeast Asia	6–7	**3 or 7** (total 277)	$p *$ mean	$p = 0.7, 0.9, 1.1, 1.3$	[0, 100] for $p < 1$, [100, 200] for $p > 1$ integers	expected \$.50 per person and period (\$3.50 for $n = 7$) (\$1.50 for $n = 3$) losers \$0	10	Mean, each subject was privately informed about his payoff (winner > \$0, loser \$0)	Infinite for $p < 1$ (equil. 100) 8 steps for $p = 1.1$; 3 steps for $p = 1.3$ (equil. 200)
Ho, Camerer (1999)	Data of Ho, Camerer, and Weigelt (1998)									

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Table 1
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Authors	Subject pool	No. of sessions per treatment	No. of players in one group	Winning no. formular, (winner's choice closest to . . .)	Parameters p, c	Range of possible choices	Payoffs	No. of periods in a session	Information after each period	No. of iteration steps to reach perfect equilibrium
Duffy and Nagel (1997)	Undergrads of various faculties Pittsburgh University	1 or 3	13–16 (total 175)	$p *$ **median** $p *$ **mean** $p *$ **max**	$p = 1/2$	[0, 100] real numbers	\$20 per period \$5 show-up fee	4, 10	All choices (anonymous), r, $p * r$ (r is mean, med. or max.) winning number (s)	infinite steps (equil. 0)
Camerer and Ho (1998)	Undergrads of UCLA and Penn (Philadelphia)	4, 6–7	7 (total 147)	$\mathbf{p} *$ (**median** $+$ **c**)	$p = 0.7, 0.8$ $c = 18$	[0, 100] real numbers	**\$5, \$7, or \$28** for winner, \$0 or -2 for others,	10	Median, each subject privately informed about his payoff	Infinite steps above below eq. ($p = .7$, eq. 42) ($p = .8$, eq. 72)
Nagel (1998)	Undergrads of various faculties Caltech, Pasadena	4 (additional sess. with fixed prize)	12–17 (total 59)	$p *$ mean	$p = 2/3$	[0, 100] real numbers	**winner gets \$$x$, x is his chosen number**, \$5 show-up fee	4	All choices (anonymous), mean $p *$ mean, winning number (s)	Infinite steps (equil. 0)

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Bosch et al. (2002)	**Game theorists, experi-menters, classroom**	2–3	32, 27	$p * \text{mean}$	$p = 2/3$	[0, 100] real numbers	Winner gets about $20	1	Winning number	Infinite steps (equil. 0)
	Newspaper data of Bosch and Nagel (1997b), Thaler (1997) and Selten and Nagel (1998)									
Bosch and Nagel (1997a)	**Readers of Expansion**	1 (3 weeks to decide)	3696	$p * \text{mean}$	$p = 2/3$	[1, 100] decimals	**Winner gets 100.000 Ptas = $800**	1	Rel. freq of choices, $p *$ mean, winner	12 steps (equil. 1)
Thaler (1997)	**Readers of Financial Times**	1 (1 week to decide)	1460	$p * \text{mean}$, best comment	$p = 2/3$	[0, 100] integers	**Winner gets two tickets to NY**	1	Abs. freq. of choices, $p *$ mean, winner	Infinite steps (equil. 0)
Selten and Nagel (1998)	**Readers of Spektrum der Wis-senschaft**	1 (2 weeks to decide)	2728	$p * \text{mean}$	$p = 2/3$	[0, 100] decimals	**Winner gets 1000DM = $800**	1	Rel. frey of choices, $p *$ mean, winner	Infinite steps (equil. 0)

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Fehr and Renninger (2000)	**Readers of DIE ZEIT**	1 (2 weeks to decide)		$p * \text{mean}$	$p = 2/3$	[0, 100] decimals	**Winner gets 1000DM = \$800**	1	**Open webpage to discuss with others before deciding**	Infinite steps (equil. 0)
Weber (2003)	Caltech undergrad-uates	3	8–10 (112 in total)	$p * \text{mean}$	$p = 2/3$	[0, 100]		10 periods	**4 treatments: full info**: all choices, $p * \text{mean}$ **no info**; **no info + explicit mentioning that average has been calculated**; **no info + asking for beliefs about other players choices**	Infinite steps (equil. 0)

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Grosskopf and Nagel (2001)	Under-graduates of Pompeu Fabra, (**Economists in various conferences and seminars**)	1 session (3 sessions with economists)	**2** (72 students in total + about 60 economists)	$p *$ mean	$p = 2/3$	[0, 100]	100 pesetas = 0.6 cents	**10 peri with 2 players then 3 periods with 18 players**(1 period for economist sessions)	**4 treatments: full info:** $p *$ mean, own payoff; **partial info:** own payoff; **no info + asking for beliefs about other player choices; no info**	**For 2 players: weakly dominant strategy 0 = 1** step
Costa-Gomes and Crawford (2006)	Under-graduates, graduates of UCSD		**2 person matching**, 13–21 in a session (125 students)	$p*$ **other choice**	$\mathbf{p}_i =$ **0.5, 0.7, 1.3, or 1.5; might be different for each player, $\mathbf{p}_i$ hidden by a "mouselab"**	**lower limit 100 or 300; upper limit 500 or 900; maybe be different for each player, limits hidden by a mouselab**	$3 for showing up on time, $8 for showing up 5 min. before, $0.04 for each of 0–300 possible points	**16 periods with a different game in each period, random matching**	No feedback	**2 to 51 steps; lower or upper limit** (**equilibrium**)

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Slonim (2005)	Ohio undergrads	3 sessions for SAME, 10 sessions for MIX	3 (27 for SAME, 70 for MIX in total)	$p *$ median	2/3	[0, 100]	$3 for each winner	**3 supergames with 3 peri: SAME = changing partner in each supergame MIX = 1 pl all periods (insider), 2 changing partners each supergame (outsiders)**	$p *$ mean, all choices	Infinite steps (equil. 0)

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Gueth, Kocher, and Sutter (2002)	Undergrads from first micro class in Humbolt University Berlin	5 sessions	4 (40 in total)	$p * (\text{mean} + c)$	$p = 1/2$, $c = 0$; $p = 1/2$, $c = 50$; (heterogen. sessions: 2 pla. $p = 2/3$, 2 players $p = 1/3$ and $c = 50$ or 0 for all players)	[0, 100]	**Payoff dependent on distance to winning number** (**average payoff about 80 cents per period**)	10 periods	All choices, $p*(\text{mean}+c)$	If $c = 0$, equil. 0, $p = 1/2$, $c = 50$ equil. 50; in heterog. session ($c = 50$): for $p = 1/3$ players: equil. $100/3$, for $p = 2/3$ players $200/3$
Kocher and Sutter (2005)	First year economic undergrads from Innsbruck	2–3	**Individ. treat:** 17 subjects; **Group treat:** 17 **groups of 3 pers.; within group communication**. **Heterog. treat: 2 individ. plus one group of 3 persons** (group = 1 pla.) (340 in total)	$p * \text{mean}$	$p = 2/3$	[0, 100]	\$6–10 for each winner	4 periods	All choices, $p * \text{mean}$	Infinite steps (equil. 0)

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Table 1
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Kaplan and Ruffle (2004)	**Economics students, psychology students of Ben Gurion University**	7 (3 with economic students, 4 with psych. students)	30	$p * \text{mean}$	$p = 2/3$	[0, 100]	About $100 for the winner plus **for each odd numbered subject: mean of all other 29 subjects/4 and for each even numbered subject 100-(mean of all other 29 subjects)/4**	1 period	All choices, $p * \text{mean}$	Infinite steps (equil. 0)

[a]Note that in Ho, Camerer, and Weigelt (1998) subjects played within a session and the same group a treatment $p > 1$ and then a treatment with $p < 1$ or vice versa.

same parameter set (called number of sessions per treatment, column 3) and the number of periods within a session (column 6), the number of players interacting (column 4), the parameters (p, c which are the deviation parameters from the average, column 5), information after each period (column 6) and the equilibrium predictions together with the number of reasoning steps (last column). Besides the test of level of reasoning (using different deviation parameters p from the average), the heterogeneity of the subject pool (see column 2, e.g., economists or subjects recruited through newspaper announcements vs students – see Bosch et al., 2002 – or differences in experience with the game – see Slonim, 2005) has become important issues in recent years. Gueth et al. (2002) and Costa-Gomes and Crawford (2006) introduce new asymmetric guessing games in which subjects of the same group may have different parameters p or intervals to choose from in order to separate leading decision rules. Furthermore, in Costa-Gomez and Crawford's experiment subjects need to actively search for information about own or other's parameters (via a so-called mouselab in which information is hidden in boxes). The search pattern is supposed to reveal the thinking process of a subject.

More detailed surveys on the guessing game experiments can be found in Nagel (1995, 1998). How to use the beauty contest game in the classroom is described in Nagel (2000).

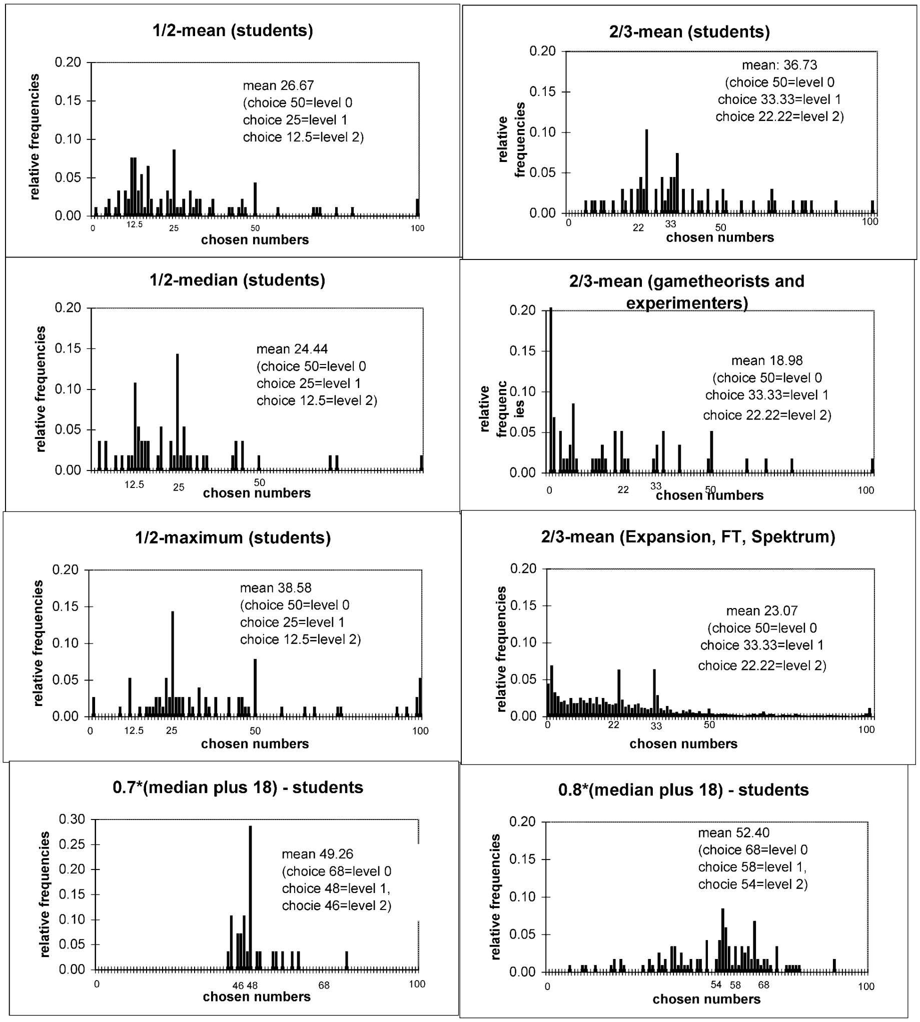
3. Bounded Rational Behavior

3.1. Iterated Best Reply Model

Figure 2 shows the relative frequencies of choices for the single treatments in the first period. For example, in the "1/2-median" game, the frequencies of choices around 25 or 12.5 stick out while no one chooses 0 (see Figure 2, 1/2-median game). Or in the 2/3-mean game choices near 33 and 22 are frequently observed (see 2/3-mean game in Figure 2). There are very few choices below numbers 12.5 or 22. In treatments with a small number of players, the midpoint of the interval seems most prominent (see 1.1-mean, 3 players).

How can this behavior be motivated with a common model? Ho, Camerer and Weigelt (1998), Nagel (1995) and Stahl (1996) apply a simple iterated best reply model:

1. Players choose uniformly randomly over the interval with an expected mean as the midpoint of the given interval, say 50, if choices are in interval [0, 100]; or they choose the midpoint as a focal point. This is the lowest level of reasoning (level 0-reasoning), not considering the task of being closest to the winning formula, for example, 1/2 times mean or 2/3 times mean in the particular treatment.
2. A player who thinks that the others just choose randomly might like to choose $1/2$ times $50 = 25$ or $2/3$ times $50 = 33$, respectively, where 50 is the expected mean of a uniform random distribution (level 1-reasoning).
3. A player might think that others choose 22 or 33, respectively, according to (2), and thus he likes to give best reply resulting in $50 * 1/2 * 1/2 = 12.5$ or $50 * 2/3 * 2/3 = 22$, respectively. This player behaves according to level 2, etc.



Source: Nagel (1995), Duffy and Nagel (1997), Thaler (1997), Bosch et al. (2002), Ho, Camerer, and Weigelt (1998), Selten and Nagel (1998). *Data*: Camerer and Ho (1999), Nagel (1998).

Figure 2. Relative frequencies of choices in the first period of the different treatments. The numbers on the x-axis indicate the choice related to the reasoning model $(50 + c)p^d$, where 50 is the expected value of an uniform distribution of numbers from [0, 100], p and c are given parameters and d is the depth of reasoning. For example 22 is approximately depth of reasoning 2 in the treatment with $p = 2/3$, $c = 0$ and hence $50 * 2/3^2 = 22.22$. Most choices are concentrated around levels 1–3 in the different graphs.

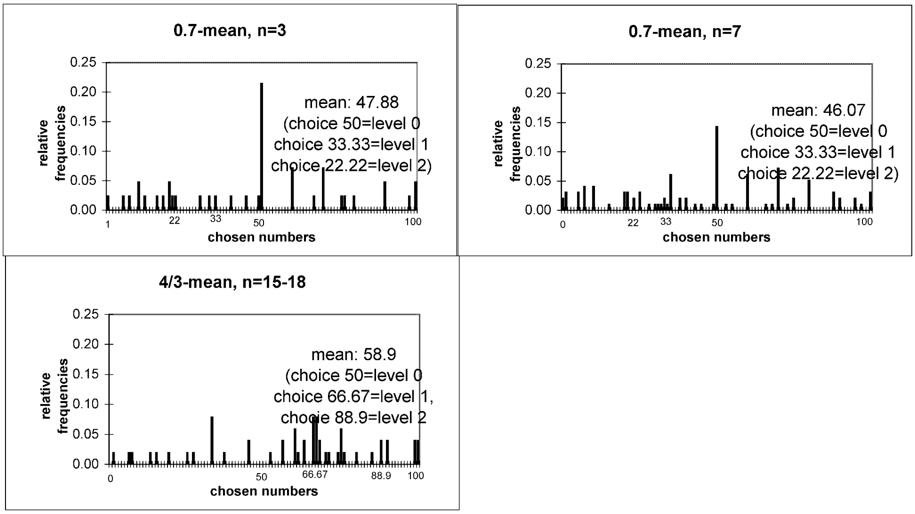
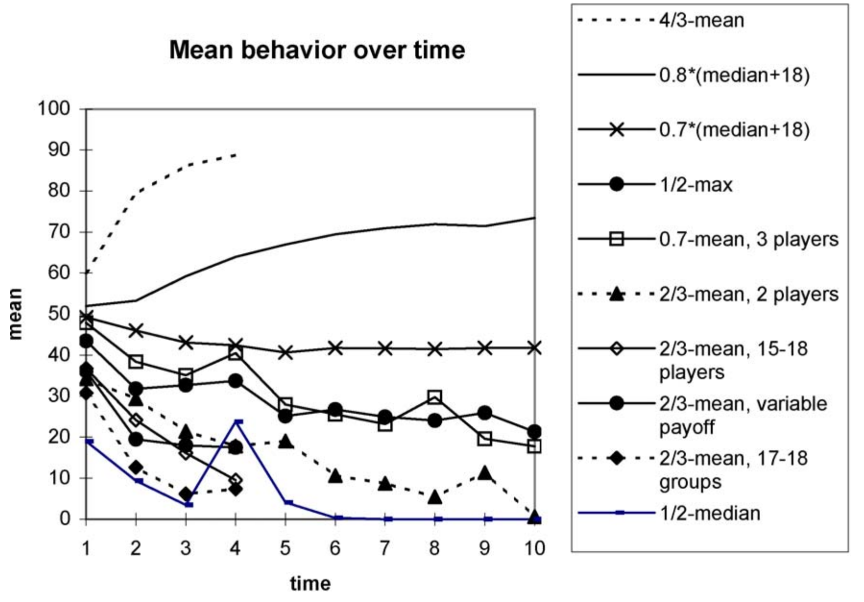


Figure 2. (*continued.*)

The process of iterative elimination of weakly dominated strategies mentioned above does not produce a common explanation for the behavior in the different treatments (see Nagel, 1995, 1998). Given the good fit of the iterated best reply model it is not surprising that less than 10% choose dominated strategies.

Figures 2 also distinguish relative frequencies of choices of the first period of different subject pools. Subjects with training in game theory show much higher level of reasoning and therefore their numbers tend to be closer to equilibrium (see Figure 2, 2/3-mean, game theorists and experimenters) than those choices of students (see Figure 2, 2/3 mean, students). The beauty contest game with $p = 2/3$ has recently been sponsored by three daily business newspapers (*Financial Times* (FT) in Great Britain; Thaler, 1997; *Expansion* in Spain; Bosch and Nagel, 1997a and by *Spektrum der Wissenschaft*; Selten and Nagel, 1998). Prizes were as high as \$1000. A rich set of comments from these participants has been gathered (see, for example, the comment in Appendix B). The main results of previous laboratory experiments were confirmed by the newspaper experiments (see Figure 2, 2/3-mean, FT, Expansion and Spektrum). The main difference was the high number of frequencies at or near the equilibrium. The choices closest to the winning number were typically those which came from subjects who did their own pre-experiments with students, friends or even with a newsgroup experiment.

The idea that subjects apply a low degree of iterated dominance has been shown in many other experimental studies, mostly in mixed motive games (see, for example, van Huyck, Wildenthal, and Battalio, 2002; Stahl and Wilson, 1994; McKelvey and Palfrey, 1992; and Nagel and Tang, 1998). The advantage of the beauty contest experiment over these mixed motive games is that the beauty contest game is a constant game, so that non-game-theory features like altruism or reciprocity should not matter. Thus one can



Data source: Nagel (1995), Duffy and Nagel (1997), Camerer and Ho (1999), Ho, Camerer, and Weigelt (1998), Nagel (1998), Grosskopf and Nagel (2001), Kocher and Sutter (2005).

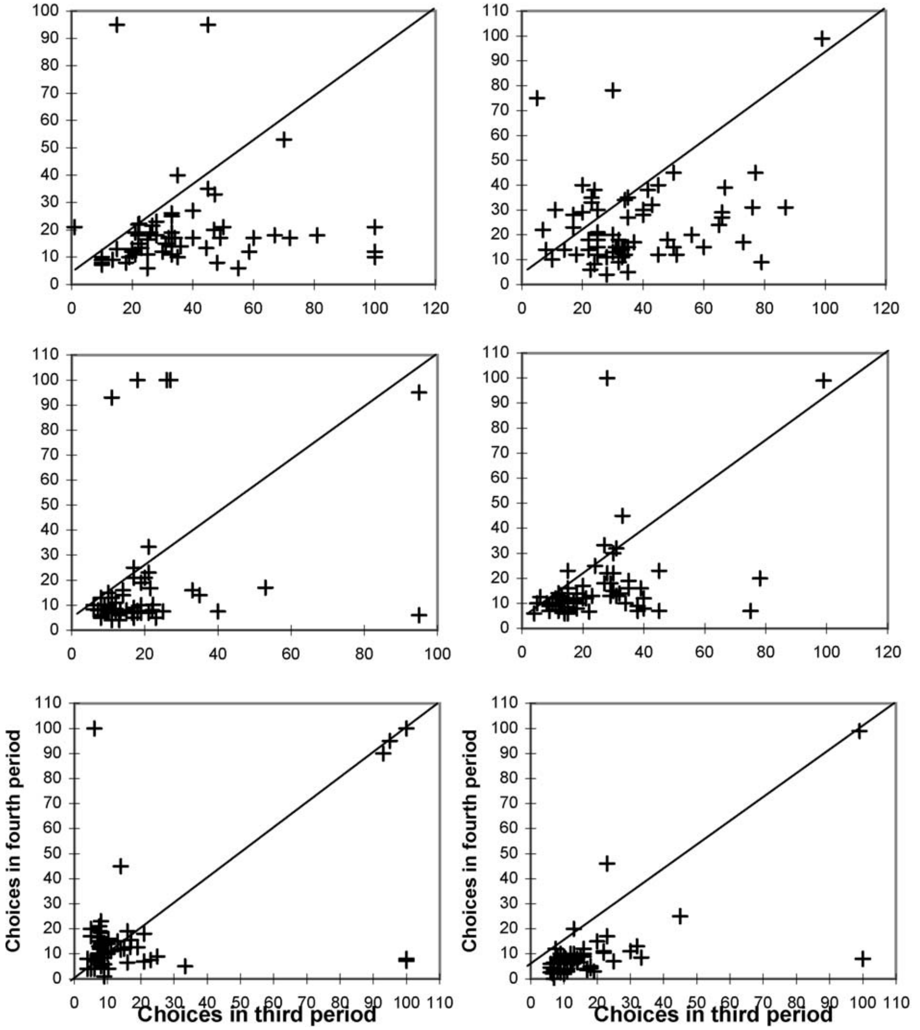
Figure 3. Mean behavior over time from various treatments. In all treatments behavior converges to equilibrium, albeit at different speed.

concentrate on strategic reasoning. Furthermore, one can construct games with a few number of iteration steps to an infinite number of iteration steps to reach an equilibrium (compare Figures 1a and 1b) without complicating the instructions.

3.2. Learning

Figure 3 shows average behavior of some treatments over time. In most of them behavior converges to equilibrium. When p is smaller, the convergence is faster (compare 1/2-median game with 2/3-mean game in Figure 3). If the number of subjects is small, convergence to equilibrium is much slower than with a high number of subjects (see Figure 3, 2/3-mean with 15–18 subjects vs 0.7-mean with 3 subjects, or 2/3 with 2 subjects where a player even has a weakly dominant strategy). If a player's decision is first discussed in a group of three, first period behavior is not different, but convergence over time is faster than if only individuals interact (see Figure 3, 2/3 mean with 15–18 subjects vs 2/3 mean with 17–18 groups, where each group consisted of 3 members).

Note when the size of the prize to the winner is determined by his chosen number (see Nagel, 1998) – i.e., if the winner chooses 5 he gets \$5, instead of a fixed prize – the convergence towards equilibrium is slowed down in comparison to the game with fixed



Data source: Nagel (1995) and Nagel (1998).

Figure 4. Transition behavior from period t to period $t+1$ in $2/3$-mean variable payoff treatment on the left side and $2/3$-mean fixed payoff treatment on the right side. Each cross presents the behavior of one subject The transition behavior in the rounds 1 to 3 are very similar in the two treatments, however, in the forth period subjects continue to decrease their choices in the right graph, but increase their choices again in the left graph, since otherwise potential gains become too small.

prize (compare 2/3-mean and 2/3-mean variable-payoff in Figure 3). See Figure 4, which shows the transition from period 1 to period 2, period 2 to 3 and period 3 to 4 of these two treatments, respectively. A player's choices in period t and $t+1$are indicated

Table 2

Classification of choices according to depth of reasoning in 10-round sessions (underlined numbers are modal frequencies); $d < 0$ in $\frac{1}{2}$-median means choices below the median of the previous period, $d = 0$: choices around the median of the previous period, $d = 1$ choices around $\frac{1}{2}^{d=1}$$*$-median of previous period, etc. $d = 2$ choices around $\left(\frac{1}{2}\right)^{d=2}$$*$median of previous period, etc. In mean games the reference point is the mean and in maximum game it is the maximum. Only in the $\frac{1}{2}$-median game there is a clear increase of levels of reasoning after 7 periods with more than 50% choosing levels higher than 2. *Source*: Duffy and Nagel (1997)

Period	1	2	3	4	5	6	7	8	9	10
(a) $1/2$-*median session*										
$d > 3$	0	0	0	0.15	0.08	0	0.15	0.23	<u>0.62</u>	<u>1.00</u>
$d = 3$	0.08	0.08	0	0	0.08	0.08	0.31	0.31	0.23	0
$d = 2$	0.38	0.31	<u>0.62</u>	<u>0.46</u>	<u>0.31</u>	<u>0.54</u>	<u>0.54</u>	<u>0.46</u>	0.15	0
$d = 1$	<u>0.54</u>	<u>0.38</u>	0.31	0.15	0.08	0.15	0	0	0	0
$d = 0$	0	0.23	0.08	0	0.08	0.08	0	0	0	0
$d < 0$	0	0	0	0.23	0.38	0.15	0	0	0	0
(b) $1/2$-*mean session*										
$d > 3$	0	0	0	0	0.07	0	0.14	0.07	0.07	0
$d = 3$	0	0.14	0	0	0.07	0.14	0.29	0.14	0.14	0
$d = 2$	0.07	0.14	0.36	0.29	0.14	<u>0.36</u>	<u>0.57</u>	<u>0.29</u>	<u>0.29</u>	<u>0.50</u>
$d = 1$	<u>0.71</u>	<u>0.57</u>	<u>0.43</u>	<u>0.57</u>	<u>0.71</u>	0.29	0	<u>0.29</u>	<u>0.29</u>	0.29
$d = 0$	0.14	0	0.14	0	0	0.14	0	0.14	0	0
$d < 0$	0.07	0.14	0.07	0.14	0	0.07	0	0.07	0.21	0.21
(c) $1/2$-*maximum session*										
$d > 3$	0	0	0	0	0	0	0	0	0	0.07
$d = 3$	0.07	0.13	0	0	0	0	0	0	0	0.07
$d = 2$	<u>0.40</u>	<u>0.53</u>	<u>0.40</u>	0.13	0.07	0.27	0	<u>0.73</u>	0.07	<u>0.80</u>
$d = 1$	0.33	0.33	<u>0.40</u>	<u>0.87</u>	<u>0.67</u>	<u>0.73</u>	<u>0.80</u>	0.27	<u>0.67</u>	0.07
$d = 0$	0.20	0	0.07	0	0.20	0	0.13	0	0.07	0
$d < 0$	0	0	0.13	0	0.07	0	0.07	0	0.20	0

by a cross. In the variable treatment the behavior from period 3 to 4 might resemble the beginning of a bubble (see Sunder, 1995). About 50% of the players increase their choices whereas in the fixed-payoff treatments most subjects choose a number smaller than in period 3 (choices below the diagonal-line). The variable-payoff treatment is a mixed-motive game, in structure similar to a finitely repeated prisoner's dilemma game or the centipede game in which the equilibrium is not Pareto-optimal.

Table 2 shows the relative frequencies of choices classified to the different levels of reasoning, separately for each period and different order statistics with $p = 1/2$. In period 1 the starting point for the level of reasoning is 50 and in the subsequent rounds it is the mean, median or maximum of the previous period, depending on the treatment. Apparently players continue to use low levels of reasoning. Only in the 1/2-median game do more than 50% of the players choose a level of reasoning greater than

2 ($d > 2$) after the 7th period. In all other cases of all treatments the majority plays around level 1 or 2 (the underlined numbers in Table 2 present modal choices).

Various learning models have been applied or developed to study this behavior over time. Nagel (1995) and Duffy and Nagel (1997) apply a learning direction theory á la Selten and Stoecker (1986). Stahl (1996, 1998) developed a rule-learning model which incorporates the level-model described above and elements of learning direction theory and reinforcement learning. Camerer and Ho (1999) apply an experienced-based attraction model for various normal form games such as the beauty contest game. Their model is a generalization of reinforcement (see, for example, Roth and Erev, 1995) and fictitious play models. An iterated best reply model is applied by Ho et al. (1998). These models show that both the order-statistic (e.g., mean or median or maximum choice of all chosen numbers) and the parameter p, which determine the winning number, matter for how quickly players move toward the equilibrium (see Figure 3). The question how the number of subjects influences the convergence over time remains unexplained by the descriptive learning models. In addition, the sudden increase of choices in the variable-payoff-treatment remains unexplained.

In summary, experiments on the beauty contest are easy to perform and are ideal for studying iterated dominance and numbers of levels of reasoning applied by real subjects. In the beginning, behavior is far away from equilibrium but converges to equilibrium over time. A simple bounded rational reasoning model describes the behavior in a consistent way across several treatments, especially in the first period. Convergence over time can be explained by various adaptive learning models and continuous low level of rationality.

Appendix A: Instructions (from Duffy and Nagel, 1997)

A.1. General

You are taking part in an experiment in the economics of decision making. If you have any questions, please feel free to ask. You should have four response cards in front of you, each with a title: first, second, third, or fourth round. You should also have an "explanation sheet."

This experiment will consist of four rounds. The rules, described below, are valid for all four rounds and for all participants.

A.2. The Rules

In each round you will be asked to choose a number between 0 and 100. Write the number you choose on the card of the corresponding round. At the end of each round all cards for that round will be collected, the numbers chosen will be written on the board, and the *median* will be determined. This will be done without identifying any participant. The winner of each round is the person who is closest to $1/2$ *times the median* of all chosen numbers for that round.

A.3. What is the Median

The median is found by ranking all chosen numbers from lowest to highest (or from highest to lowest) and picking out the middle number. For example if we have five numbers represented by the letters a, b, c, d, and e, and it is the case that $a < b < c < d < e$, then the middle number, c, is the median. Thus, the median is the number such that half of all numbers lie below it and half of all numbers lie above it.

Remember, however, that you want to choose a number that you believe will be closest to $1/2$ *times the median*.

A.4. Payoffs

l participant will receive a $5 payment provided that they complete all four rounds of this experimental session. In addition, the winner of each round will get $20. If, in any round, there are several participants at an equal distance to $1/2$ *of the median*, the $20 prize will be divided among them. All payments will be made at the end of the session.

A.5. Explanation Sheet

Briefly describe your decision for each round on the explanation sheet.

ARE THERE ANY QUESTIONS BEFORE WE BEGIN?

Appendix B

Comment by a school-class (see Bosch et al., 2002), translation from German to English: I would like to submit the proposal of a class grade 8e of the Felix-Klein-Gymnasium Goettingen for your game: 0.228623. How did this value come up? Johanna ... asked in the math-class whether we should not participate in this contest. The idea was accepted with great enthusiasm and lot of suggestions were made immediately. About half of the class wanted to submit their favorite numbers. To send one number for all, maybe one could take the average of all these numbers. A first concern came from Ulfert, who stated that numbers greater than 66 2/3 had no chance to win. Sonja suggested to take 2/3 of the average. At that point it got too complicated to some students and the finding of the decision was postponed. In the next class Helena proposed to multiply 33 1/3 with 2/3 and again with 2/3. However, Ulfert disagreed, because starting like that one could multiply it again with 2/3. Others agreed with him that this process then could be continued. They tried and realized that the numbers became smaller and smaller. A lot of students gave up at that point, thinking that this way a solution could not be found. Other believed to have found the path of the solution: one just has to submit a very small number.

However, one could not agree how many of the people who participate realized this process. Johanna supposed that the people who read this newspaper are quite sophisticated. At the end of the class 7 to 8 students heatedly continued to discuss this problem.

The next day the math teacher received the following message: ... We think it best to submit number 0.228623.

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