Chapter 10

STRATEGY-PROOF EQUILIBRIUM BEHAVIOR IN TWO-SIDED AUCTIONS

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Although we can think of an allocation mechanism as an institutional procedure that allows the preferences of individuals to be mapped into final allocations, this abstract formulation does not account for the fact that preferences are private and unobservable, and institutions have to rely upon the messages reported by agents, not their true preferences. Consequently, the standard theoretical proposition is that it is possible for an agent to affect prices and outcomes in a market by strategically misreporting his or her preferences. A buyer with a maximum willingness-to-pay of $10, $8 and $7, respectively, for three units of a good, who believes sellers are willing to sell for less, might strategically bid for all three units at $6 in an attempt to lower his or her purchase cost of the three units. Thus, allocation mechanisms are actually mappings from preferences, and each agent's information or beliefs about other agents, into allocations. This state of affairs has motivated the study of strategy-proof mechanisms designed to overcome the problem of strategic misrepresentation, but the results are not encouraging. Thus, stated informally, "an allocation mechanism is strategy-proof if every agent's utility-maximizing choice of what preferences to report depends only on his own preferences and not on his expectations concerning the preferences that other agents will report" (Satterthwaite, 1987, p. 519). This comes down to the strong requirement that each agent has a dominant strategy to report true preferences, and has led to impossibility theorems establishing the nonexistence of such a mechanism under certain minimal requirements (Gibbard, 1973; Satterthwaite, 1975).

Given these negative results, it is of particular interest to ask what people actually do in experimental environments in which the experimenter induces preferences on individual subjects so that the experimenter knows each agent's preferences but the subjects know only their own preferences. Although it is possible that an agent can obtain an advantage by strategically underrevealing his/her demand or supply, whether or not such action is successful depends upon the actions – possibly countervailing – of others. In particular, has society devised institutions in which forms of behavior arise that practically solve the problem of strategy-proofness in classical environments?

The best known example in which the answer to this question is "yes" is the continuous double oral (now widely computerized) auction used extensively in stock, bond, commodity, currency, futures and options markets. Since this institution is examined elsewhere in this volume, I will merely record here that in all but certain extreme classical environments (Smith, 1965; Holt, Langan, and Villamil, 1986; Davis and Williams, 1991) with a constant excess supply of only one or two units, convergence to competitive equilibria is rapid and leads to efficiencies of 100% or close thereto. Note that:

Handbook of Experimental Economics Results, Volume 1

DOI: 10.1016/S1574-0722(07)00010-8

(1) this is not a counter example to the impossibility results – the exceptions, although rare, confirm the generality claimed by the theorems; (2) the problem is never-the-less solved by the continuous double auction institution in an extremely wide class of environments in the practical workday world of organized exchange markets. Our theoretical understanding of why and how this is so is weak, and represents one of the outstanding unsolved problems in economic/game theory.

Are there other examples, offering a practical (if not perfect) solution to the problem of achieving strategy-proof equilibria? If so, what are the behavioral mechanisms that solve this problem? The answer is in the form of two versions of uniform price auctions: the uniform-price sealed bid-offer auction, and the uniform-price double auction (UPDA).

1. Strategy-Proof Equilibria in the Sealed Bid-Offer Auction

Figure 1 plots the normalized uniform price, period by period, for an 11 period sealed bid-offer auction with five buyers and five sellers. The exchange quantity, Q, and efficiency E, is indicated for each period at the bottom of the chart. Note the poor efficiency in the early periods and slow convergence of the blind bidding process even in a stationary supply and demand environment. See Smith et al. (1982); Friedman and Ostroy (1991); Van Boening (1991) for studies of the sealed bid-offer mechanism, sometimes called the Clearing House mechanism.

The bid-offer array cross for period 11, shown in Figure 2, reveals the strategies that each side evolves to insulate themselves from manipulation by the other side: on the buy side, bid units 9 through 15 in the ordered set of bids are only one cent or less above the clearing price ($7.14); offer units 9 through 14 are one cent below the clearing price. Each side gives the other only a cent or two of room for price manipulation. Since no buyer or seller has more than three intramarginal units, the seven units bid, and the six units offered that are within a cent of the clearing price prohibit any one (or two) buyers or sellers from manipulating the price. Thus, subjects grope around and latch on to a behavioral strategy-proof equilibrium.

The true induced supply (S) and demand (D) overlaying the bid-offer cross illustrates the extent of under-revelation. All buyer units are under-revealed; all but the first three seller units are under-revealed. Three seller units are over-revealed – essentially offered safely "at market," as all other units on the sell side protect against manipulation by buyers. By offering these units "at market" these sellers free ride on the under-revealing messages of other sellers who erect a barrier to strategic manipulation by the buyers. Of particular interest is the empirical observation that this free riding is rare. As long as someone on your side of the market incurs the risk of offering their intramarginal units near the clearing price, your incentive is to guarantee to trade the maximum number of profitable units by revealing your demand or supply.

These experimental results make it plain that the theoretical condition for a strategy-proof equilibrium – that each agent have a dominate strategy to reveal true willingness-

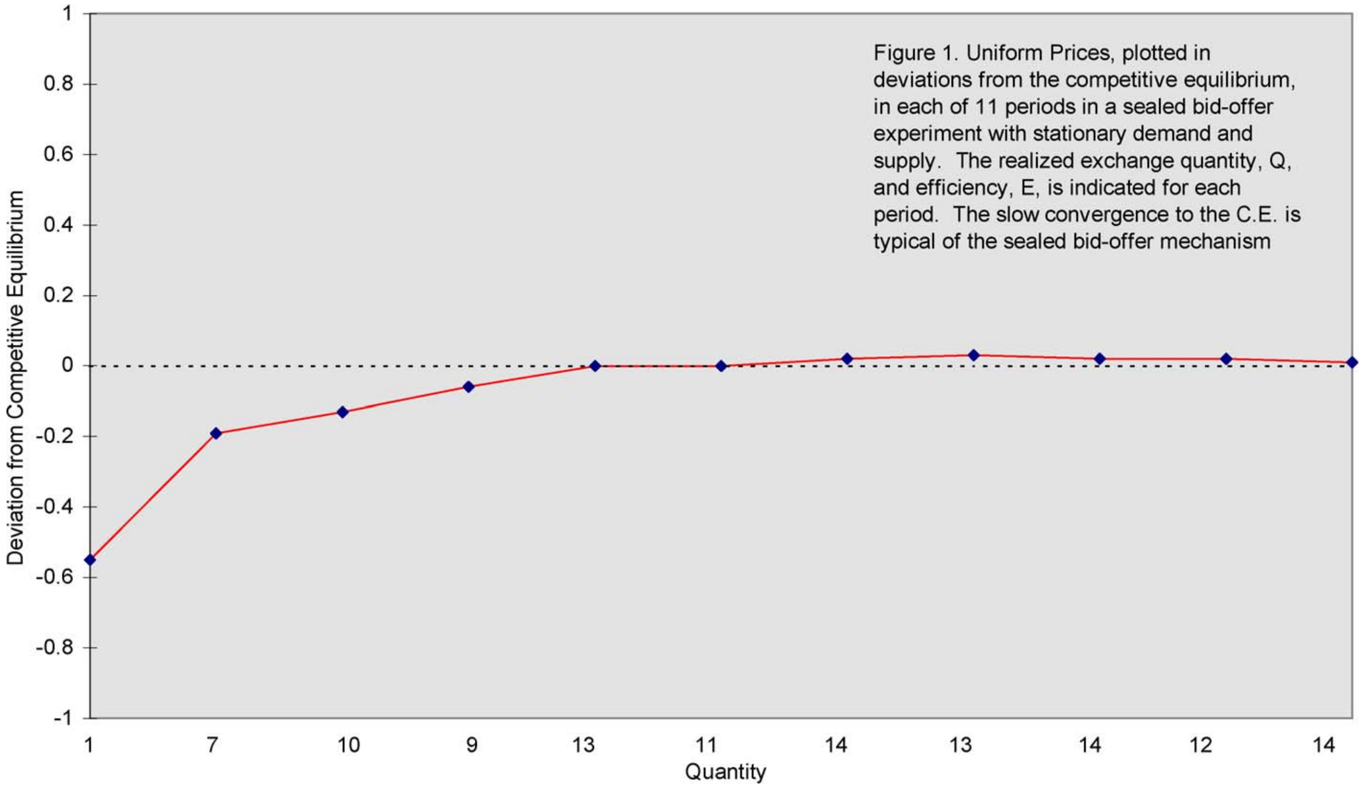

Figure 1. Uniform prices, plotted in deviations from competitive equilibrium, in each of 11 periods in a sealed bid-offer experiment with stationary demand and supply. The realized exchange quantity, Q, and efficiency, E, is indicated for each period. The slow convergence to the C.E. is typical of the sealed bid-offer mechanism.

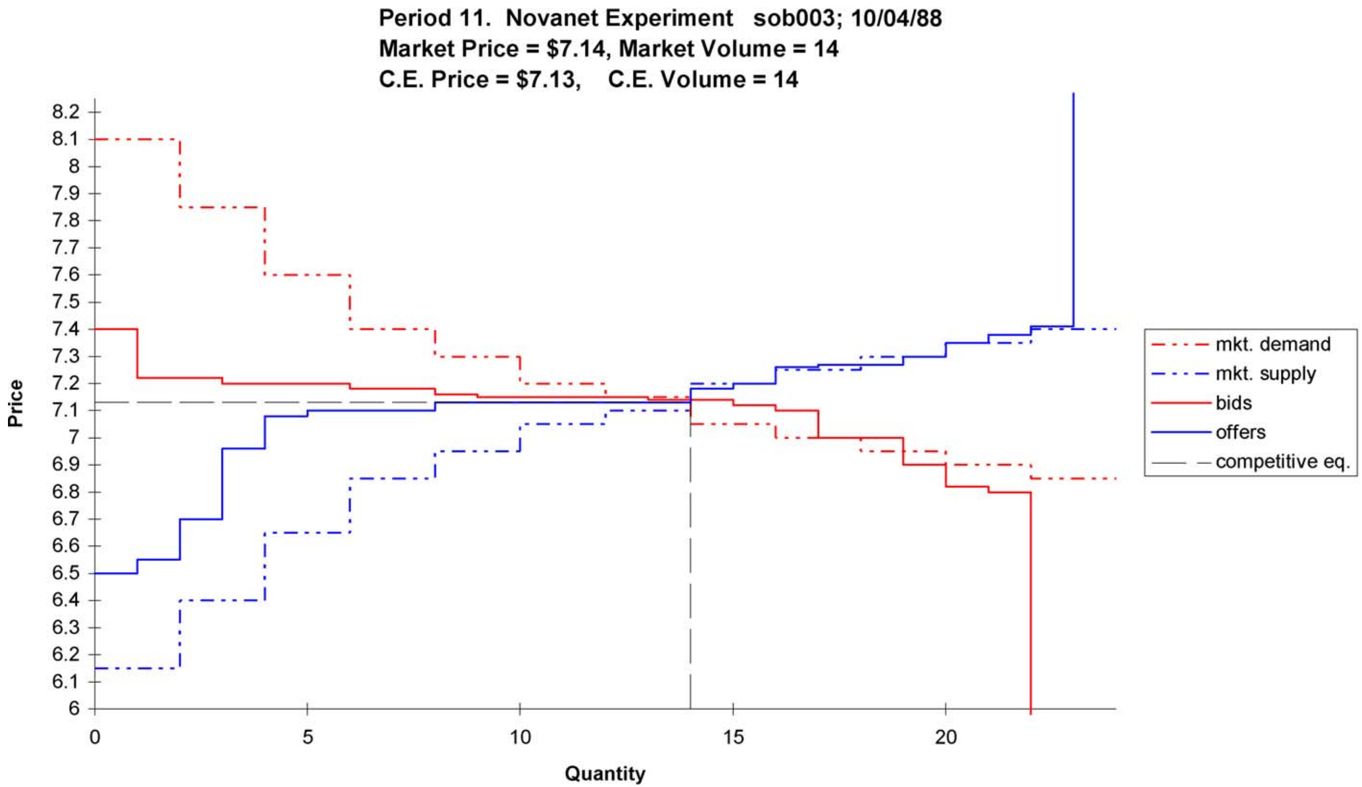

Figure 2. Chart shows the true supply (S) and demand (D) overlaid by the reported array of bids and offers for period 11 of the experiment shown in Figure 1. Bid units 9 to 15 are within one cent of the clearing price ($7.14) and offer units 9 to 14 are one cent below this price. Since no buyer or seller has more than 3 intramarginal units, each side of the market prevents the other side from moving the price by more than one cent.

to-pay or willingness-to-accept for all units, and not just units near the margin – is much too strong. The above results from blind two-sided auctions, however, also show that there is a cost to the achievement of a strategy-proof equilibrium: blind two-sided auctions converge more slowly to the competitive equilibrium than continuous double auctions, and upon converging, are somewhat less efficient.

2. Strategy-Proof Equilibria in the Uniform Price Double Auction (UPDA)

UPDA is a real-time, continuous feedback mechanism clearing all trades at a single price in each trading period. It comes in several forms depending upon whether there is a fixed time, or endogenous, close rule (the market closes if there is no new trade after a prespecified period), an open or closed book (the list of all bids and offers is displayed or not), and whether accepted bids enjoy a conditional time priority (a better bid or offer cannot displace an accepted one unless it meets the offer or bid on the other side). See McCabe, Rassenti and Smith (1991, pp. 311–316) for a report of 49 UPDA experiments comparing these different versions with double auction; also Friedman (1991), Cason and Friedman (1996). All of these versions yield even more under-revelation of demand and supply than the blind two-sided auction discussed above, but efficiency tends to be higher and, in one form (endogenous close, open book, the "other side" rule with conditional time priority), exceeds that of the continuous double auction.

Table 1 lists a period-by-period summary of the results from a typical experiment (up 43 with 5 buyers and 5 sellers). The market used a fixed close time, open book and the conditional time priority ("other side") rule. The environment is particularly demanding in that a random constant is added to all buyer values and seller costs, and the individual value/cost assignments are re-randomized, in each of 15 trading periods. Column 2 in Table 1 lists the fluctuating competitive equilibrium prices, P_e, induced by these random shifts (the competitive equilibrium quantity remains constant at $Q_e = 18$). Note that this equilibrium shifts randomly in a range of realizations from 260 to 610 across the 15 trading periods and thereby exposes the subjects to extreme exogenous uncertainty. Column 3 shows the realized clearing price, P_r, and quantity, Q_r, based on the reported bids and offers from all subjects. Finally, column 4 contains the market efficiency, Eff_e, achieved in each period, and the percentage of the true surplus that is revealed in the reported bids and offers, Eff_r. Efficiencies of 100% are achieved three times with less than 10% of the surplus revealed – periods 2, 5, and 14; in period 14 only 5% is revealed. Efficiency averages 95% across all 15 periods, while the average percentage of the surplus revealed is only 27%.

Figure 3 plots the true demand and supply (shown dashed) and the realized bid (B) and offer (O) arrays (shown solid) for period 14. Note that the true demand and supply has the following property: if all agents reveal their true demand or supply with the exception of one intramarginal buyer or seller, then that agent can manipulate the price to his or her advantage.

Table 1
Summary of results: up 43; 5,5

	P_e	Q_e	P_r	Q_r	Eff_e	Eff_r
1	295	18	300	16	91%	22%
2	405	18	400	18	100%	7%
3	545	18	540	18	100%	14%
4	460	18	448	18	92%	14%
5	360	18	350	18	100%	9%
6	500	18	500	18	98%	12%
7	260	18	250	17	96%	26%
8	565	18	553	15	92%	28%
9	300	18	300	18	100%	28%
10	610	18	610	18	100%	33%
11	365	18	350	15	85%	88%
12	550	18	558	15	88%	55%
13	450	18	450	18	100%	31%
14	410	18	410	18	100%	5%
15	485	18	484	19	89%	39%
				$\mu = 17.3$	95%	27%
				$\sigma = 1.3$	5	21

Notes. The trading period 1–15 is shown in column 1. The randomly shifting midpoint of the set of competitive equilibrium prices is listed under P_e, Q_e. The reported UPDA outcomes are listed under P_r, Q_r. Market efficiency is shown under Eff_e, while Eff_r is the percentage of the available surplus that is revealed by the reported bid and offer arrays. μ and σ are the means and standard deviations of the column data.

Thus, if the agent is a buyer, he or she has only to reveal all units except the last and bid that unit at \$4. But this truth is irrelevant. The relevant question is what behavior is manifest when all agents have the potential for manipulating the price. In Figure 3 we observe that bid and offer units 6 through 18 are all tied at the competitive equilibrium clearing price at \$4.10, and no agent has nearly enough capacity to alter this price. In effect each side erects a solid barrier against manipulation by the other side. In this example only 5% of the true surplus is revealed, yet the participants capture 100% of the possible gains from exchange.

3. Summary

The experimental evidence from three prominent auction trading institutions: (1) the continuous double auction, (2) the uniform-price sealed bid-offer auction, and (3) the uniform-price double auction (with continuous feedback of real time information on the acceptance status of bids and offers), shows that subjects are able to work out a behavioral equilibrium which is strategy-proof. In institutions (2) and (3) this is achieved by a groping process which (a) approximates the competitive equilibrium, and (b) produces

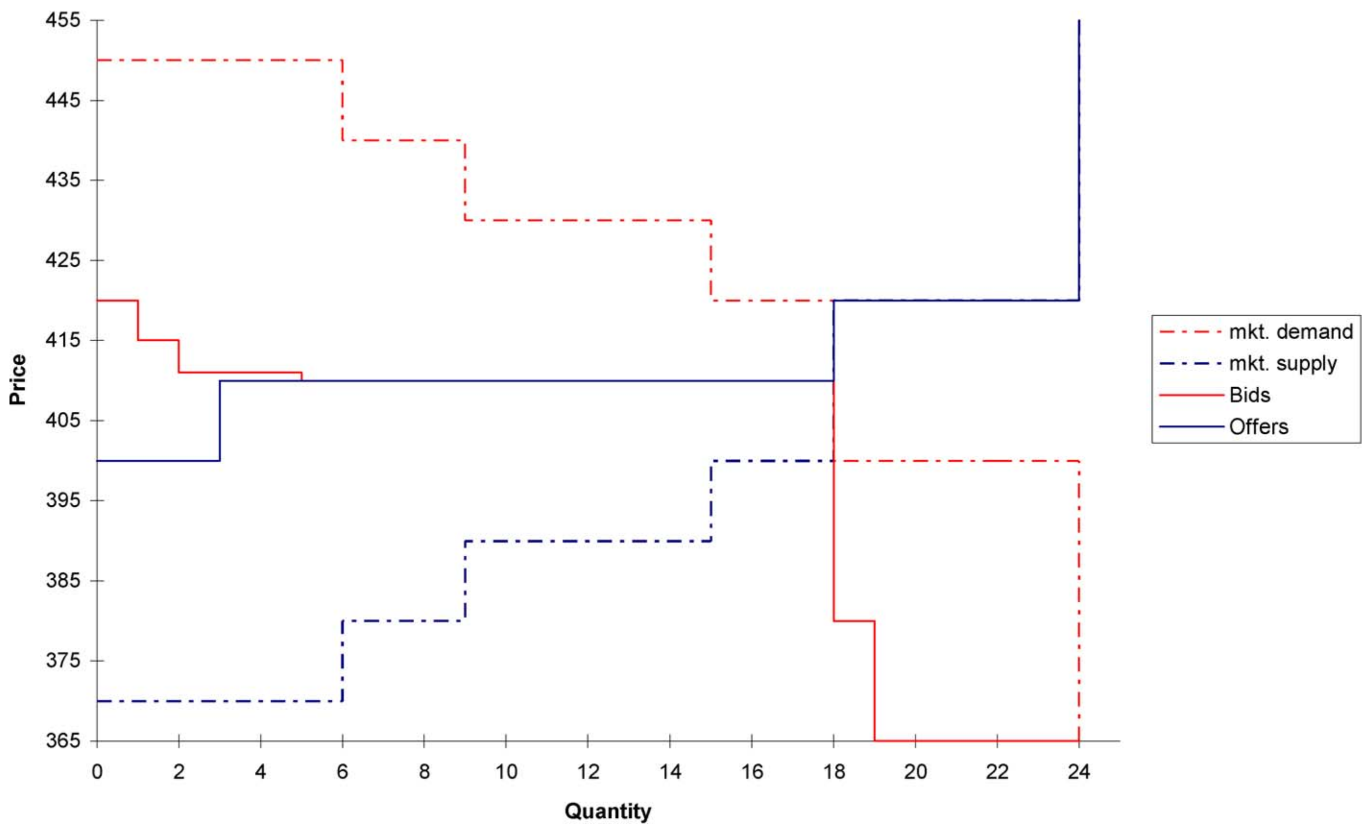

Figure 3. Chart shows the true supply (S) and demand (D) and the reported array of bids (B) and offers (O) for period 14 of UPDA experiment 43 with 5 buyers and 5 sellers (Table 1). S and D shift randomly each period, each buyer (seller) has all the units on a step, with the steps reassigned to buyers (sellers) each period.

a large number of bids and offers that are tied, or nearly tied. In (3) there are typically a great many tied bids and offers at the clearing price. The number of such ties often greatly exceeds the capacity of any one – typically several – agents. Consequently, no individual can take action that can strategically move the price to his or her advantage. This grid-lock, non-cooperative equilibrium is a form of unconscious order, in the sense that subjects are not cognizant of having achieved a strategy-proof equilibrium, and would not in fact know the meaning of such a statement.

Impossibility theorems provide important insight as to why the above two-sided mechanisms have difficulty achieving perfection, but they do not address the question as to why such field mechanisms, when studied in the laboratory, perform so well – in fact surprisingly well given the theorems and the fact that we observe massive under-revelation in the experiments. Fresh new theory, addressing the regularities in behavior that produce such good empirical outcomes, is much needed.

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