Econ 241A Prof. Startz

Midterm Fall 2016

Please answer all questions. Show your work.
The exam is open book/open note; closed any devices that can communicate. (No laptops, cell phones, Morse code keys, signal fires, etc.)

1 Consider a sample of $x_1, ..., x_n$ where $x_i \sim N(\mu, \sigma^2)$
Your TA asks you to consider an estimator given by $\tilde{x} = -\frac{1}{n_e} \sum_{i \text{ is even}} x_i + 2\frac{1}{n_o} \sum_{j \text{ is odd}} x_j$

a) Is $\tilde{x}$ an unbiased estimator of μ?
b) Find $Var(\tilde{x})$. How does it compare to $Var(\bar{x})$?

2 Consider two independent variables X and Y with marginal densities given by:
$f_X(x) = \frac{x}{2}$ if $x \in [0, 2]$ and $f_Y(y) = \frac{3}{2} - y$ if $y \in [0, 2]$

a) Find $f_{X,Y}(1, 2)$ and $f_{X,Y}(2, 0)$
b) Find $P(X - Y > 0)$

3 The ranking General in the Prussian army just watched the movie Concussion and is concerned about the prevalence of CTE in his soldiers from being kicked in the head by bunnies. He asks you to model the distribution of the number of kicks that a soldier receives (per month) as poisson with parameter λ.

a) In terms of λ, what percent of soldiers have been kicked more than twice?

The General supplies you with the following data on kicking incidents:

# Kicks	# of Soldiers
0	50
1	10
2	5
3	5
4	3
10	1

b) Estimate and interpret $\hat{\lambda}$.
c) What is your estimate of Part a?
d) Suppose 50% of the soldiers are cavalry and are therefore not at risk of being kicked by bunnies. What other distribution would be more appropriate to model this data? Provide estimates for the parameters of this model (they don't have to be exact, just approximate).

4 Consider three variables that are distributed jointly normal:

$$\begin{pmatrix} \theta \\ \nu \\ \epsilon \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \right)$$

Define $X = \theta + \nu$, $Y = \theta + \epsilon$, and $Z = \theta | X$.

a) Find $\mathbb{E}[Z]$.
b) Find $\mathbb{E}[Z|Y]$. (You can assume $Cov(Z, Y) = a$)

5 $X \sim U(-2, 4)$
$g(x) = abs(x)$
$Y = g(X)$
Find $F_Y(y)$ and plot it.