

## MIXED STRATEGY NASH EQUILIBRIUM PREDICTIONS AS A MEANS OF ORGANIZING BEHAVIOR IN POSTED-OFFER MARKET EXPERIMENTS

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### 1. Introduction

In posted-offer markets the competitive outcome for a market, defined by the intersection of market supply and market demand curves, is frequently not a Nash equilibrium for the market viewed as a stage game. Rather, one or more sellers often have incentives to deviate unilaterally from the competitive outcome. A simple example illustrates. Consider a market with two sellers,  $S1$  and  $S2$ , and a single, fully revealing buyer. Sellers  $S1$  and  $S2$  can each offer two units for sale at a constant cost of  $c$ . The buyer will purchase up to three units at a maximum price of  $v_h$  per unit, and will pay up to  $v_l$  for a fourth unit, with  $v_h > v_l > c$ . In the competitive prediction for this market, four units trade at a price of  $v_l$ , and earnings for each seller are  $2(v_l - c)$ . But this competitive outcome is not a Nash equilibrium if  $v_h - c > 2(v_l - c)$ , since in this case either seller could increase earnings unilaterally by posting a price of  $v_h$  and selling a single unit.

Holt (1989) defines sellers as having market power in this instance when the competitive price is not a pure strategy Nash equilibrium. When market power arises, a Nash equilibrium in pure strategies typically does not exist. Notice above, for example, that the price  $v_h$  cannot be supported as a pure strategy Nash equilibrium, since a posting of  $v_h$ , by, say, seller  $S1$  will induce seller  $S2$  to post  $v_h - \varepsilon$ , thereby guaranteeing  $S2$  a sale of two units. Incentives to undercut remain for any price down to the point where earnings from selling two units as the low price seller are equal to earnings from selling a single unit at a price  $v_h$ . Defining this lowest price as  $p_{\min}$ , sellers will undercut each other until  $p_{\min} = (v_h + c)/2$ .<sup>1</sup> A common price of  $p_{\min}$  also cannot be supported as a pure strategy Nash equilibrium because only three units will trade at this price, and one of the sellers will fail to earn the security earnings available at the price  $v_h$ . Any static equilibrium for this market must involve mixing over the range  $p_{\min}$  to  $v_h$ .

Market power can arise inadvertently in posted-offer markets. One can easily imagine, for instance, that the competitive outcome in a market consisting of many units and

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<sup>1</sup> Price  $p_{\min}$  is found by solving the equation  $2(p_{\min} - c) = v_h - c$ .

many sellers might include the incentives illustrated above.<sup>2</sup> A wide variety of treatment changes within laboratory markets can also introduce or remove market power. Instances where treatments have intentionally affected market power include mergers (Davis and Holt, 1994), demand shifts (Wilson, 1998), changes in the buying queue (Kruse, 1993), and cost synergies (Davis and Wilson, 2000).

Market power has long been shown to affect prominently market performance (e.g., Dolbear et al., 1968; Ketcham, Smith and Williams, 1984; Holt, Langan and Villamil, 1986; Alger, 1987; Isaac and Reynolds, 2002; Davis and Williams, 1991; Kruse et al., 1994; Wellford, 1990). The purpose of this chapter is to describe generally how sellers behave in such contexts, and to evaluate the organizing power of mixed strategy Nash equilibrium predictions relative to alternative theories for organizing the data.

Prior to proceeding, we stress that the behavior of sellers in such markets should not be viewed as a direct test of Nash mixed strategy equilibrium predictions. The oligopoly issues that are typically of interest to researchers conducting posted-offer market experiments focus on the interactions of a fixed collection of sellers who engage in repeated interactions, often under conditions of less than complete information about the market. The consequent interdependencies in actions and informational imperfections violate the assumptions underlying the Nash equilibrium. Thus, the project is not to assess generally the behavioral relevance of static Nash mixing predictions, but rather to consider the organizing power of such predictions in a particular context somewhat outside the domain of the theory.

## 2. Equilibrium Mixed Strategy Pricing Distributions

Holt and Solis-Soberon (1992) develop a method for calculating mixing distributions for posted-offer markets when market power exists.<sup>3</sup> The procedure involves three steps. First, a range over which sellers will price is found. Second, a “security” point in that range, where seller earnings are assured is identified. Finally, the price distributions that each seller must use in order to make other sellers indifferent to every price over the range of randomization is calculated.

To illustrate, consider the example developed in the introduction. The first two steps have been completed already. Sellers  $S_1$  and  $S_2$  will each mix over the range  $p_{\min}$  to  $v_h$ , and the limit price  $v_h$  ensures each seller of security earnings of  $v_h - c$ . For the third step, define  $F(p)$  as the cumulative pricing distribution that, say, seller  $S_1$  must follow in order to make seller  $S_2$  indifferent between the security price and any other price in the mixing range. For any price  $p$ ,  $p_{\min} \leq p \leq v_h$ , expected earnings must just exactly equal those available at  $v_h$ , or  $v_h - c = [1 - F(p)](2p - 2c) + [F(p)](p - c)$ . Solving,

<sup>2</sup> Indeed the authors know from their own research experience that market power is almost insidious, and can arise when completely unintended.

<sup>3</sup> Kruse et al. (1994) also develop an alternative method for calculating mixed strategy equilibria in posted-offer markets with many small buyers, and nearly smooth demand curves.

$F(p) = (2p - v_h - c)/(p - c)$ . In a symmetric equilibrium, seller  $S2$  must also price according to  $F(p)$ , in order to make seller  $S1$  indifferent to mixing over the support  $p_{\min} \leq p \leq v_h$ .

In this case, calculating the equilibrium mixing distributions is straightforward. However, even relatively minor cost asymmetries, and/or additional demand steps complicate quickly these calculations. For designs using the pronounced cost and value asymmetries typical of posted-offer market experiments with “normal” looking supply and demand arrays, a closed form solution for a well-defined cumulative distribution function often does not exist, making infeasible the calculation of the equilibrium mixing distributions. Thus, although the Nash equilibrium differs from the competitive outcome in many markets, only a carefully designed subset of those instances permits direct evaluation of static Nash predictions.

### 3. Pricing Performance with Market Power

When significant power exists, and when sellers realize their power, pricing patterns typically involve a series of price cycles that deteriorate as the session progresses. The upper left panel of [Figure 1](#) illustrates the sequence of price postings for a representative market. The market, taken from [Davis and Holt \(1994\)](#), involves a design where two sellers have power, and one or more sellers have no power. Market power exists for two sellers because each can sell profitably a single unit at a limit price  $p_{\lim}$ , independent of the actions of the other sellers. Power sellers can sell up to four units, and theoretically these sellers will mix over the range  $p_{\lim}$  to a lower bound  $p_{\min}$ , where earnings from selling four units as the low price seller equal earnings from selling a single unit at the limit price. Capacities for the sellers without power range between one and three units, but these sellers sell nothing with a unilateral deviation above the prices of the other sellers. In this experiment, demand was simulated, sellers had complete information about both demand and costs, and the market lasted for 60 periods. Solid dots (seller  $S1$ ) and hollow blue dots (seller  $S2$ ) illustrate the price choices for sellers with power. Price choices for sellers without power are illustrated by red, green and black dots. As is evident from the chart only a single no-power seller was present in the first 30 periods, while three sellers without power participated in the second 30 periods. While it is not obvious from the figure, the change in the number of sellers without power is theoretically irrelevant because it does not affect the mixing predictions for either the sellers with power or those without power.<sup>4</sup> Calculation of the theoretical mixing distribution is rather involved (see [Davis and Holt, 1994](#) for details), and sellers with and without power have different distributions. Power sellers  $S1$  and  $S2$  mix over the range  $p_{\min}$  to  $p_{\lim}$ . In equilibrium, no-power sellers  $S3$ – $S5$  mix over a range from  $p_{\min}$  to slightly above  $p_{\min}$ .

<sup>4</sup> But behaviorally, reducing the number of sellers without power did result in a small but significant mean price increase. See [Davis and Holt \(1994\)](#).

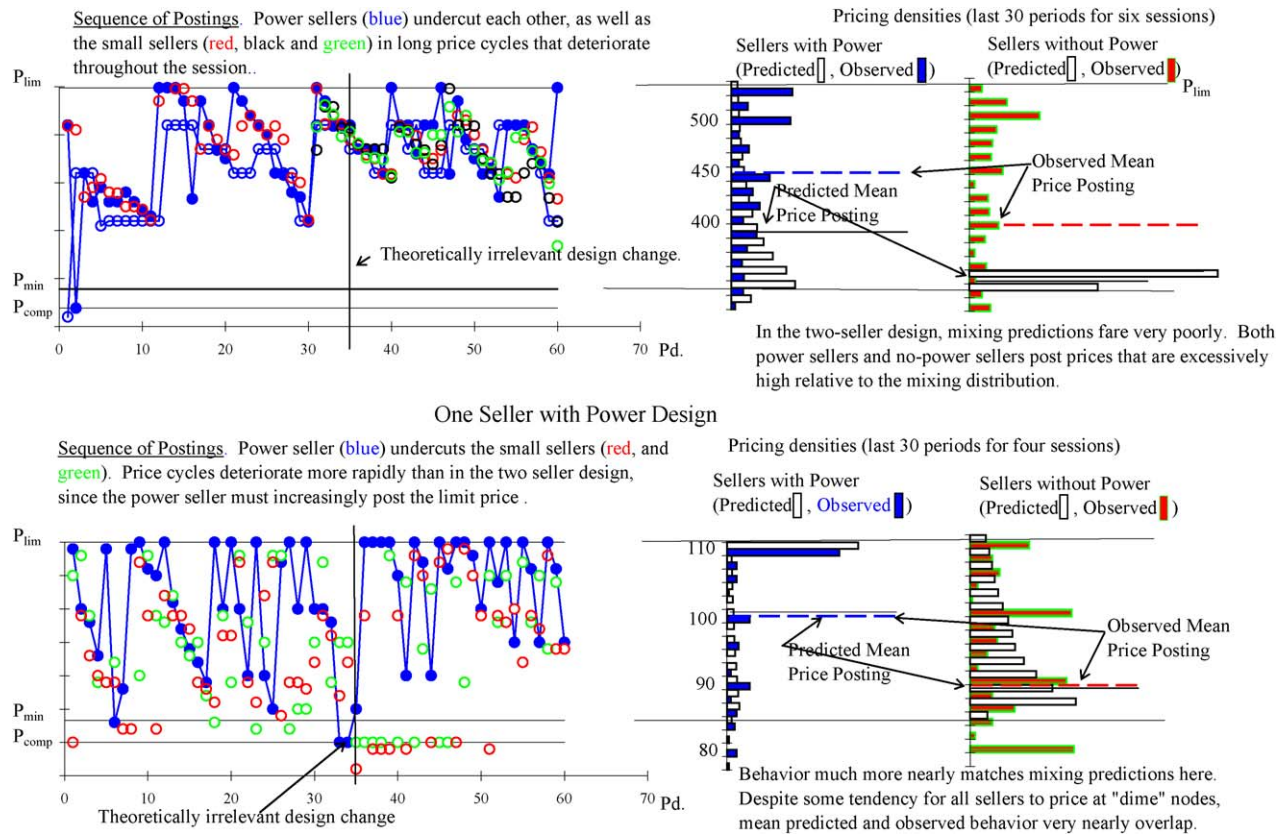


Figure 1. Two sellers with power design.

In the first twelve periods, power seller  $S1$  and no-power seller  $S3$  gradually undercut each other while power seller  $S2$ , prices generally below them. (Incidentally, seller  $S3$ 's aggressive behavior was rather costly for this seller. Fairly aggressive behavior by large no-power sellers was not uncommon.) In period 13 seller  $S1$  raises his price to  $p_{lim}$  and maintains that posting for three periods, again drawing up seller  $S3$ . Seller  $S1$  then undercuts  $S3$ , starting another cycle that persists until period 20. Seller  $S1$  again posts the limit price in period 21, starting another cycle that lasts until the treatment change in period 30. A fourth cycle occurs between periods 30 and 40. After period 40, however, the cycles deteriorate somewhat, and are more difficult to characterize.

"Edgeworth" price cycles of this type are typical of such markets, and have been commonly observed by researchers who have investigated market power in the laboratory (see, e.g., Kruse, 1993; Kruse et al., 1994; Davis and Holt, 1994; Wilson, 1998; Davis and Wilson, 2000). But both the amplitude and the regularity of such cycles vary considerably from experiment to experiment. Although the sources of cycle variability have not been studied formally, cycles are undoubtedly affected by design and procedural factors such as subject experience, session length, the amount of information given to sellers, and the obviousness of the endowed power in the supply and demand schedules.<sup>5</sup>

The sequence of postings shown in the lower left portion of Figure 1 illustrates some of the variability in pricing patterns possible across experiments. This sequence, taken from Davis and Wilson (2000), involves a single seller with power,  $S1$ , represented by the solid blue dots, and two sellers without power,  $S2$ , and  $S3$ , shown as hollow green and hollow red dots, respectively. Seller  $S1$  can with certainty sell a single unit at  $p_{lim}$ , independent of the other sellers' choices. Seller  $S1$  has a capacity of three units, and will find undercutting  $S2$  and  $S3$  profitable down to the price  $p_{min}$ . The market consists of 60 trading periods, although a theoretically irrelevant design change was imposed after period 30 (in this case the cost fell on  $S1$ 's inframarginal unit). Sellers were given full information about demand, but only private cost information.<sup>6</sup>

Notice that the cycles are much less regular in this sequence than in the first sequence presented. After two more or less regular cycles in the first twenty periods, pricing becomes quite erratic, as seller  $S1$  alternates between the limit price and a series of lower

<sup>5</sup> Full information probably facilitates exercise of market power. For example, although power was clearly exercised when present in each of 12 full-information sessions reported by Davis and Holt (1994), and in each of six full-information sessions conducted in a variant of the Davis and Holt design reported by Wilson (1998), Wilson reports that competitive performance was observed in four of 12 power sequences conducted under conditions of private information. This result complements the findings of researchers dating to Fouraker and Siegel (1963) who have observed that incomplete information tends to facilitate competitive outcomes. Notably, competitive outcomes have also been generated in some other incomplete information markets where some power existed (e.g., Ketcham, Smith and Williams, 1984, design 1 and Davis and Holt, 1998, baseline sessions).

<sup>6</sup> It is doubtful that sellers had less useful information here than in Davis and Holt (1994). The cost structure was simple, and sales quantities were revealed publicly *ex post* each period. After several periods, sellers undoubtedly knew the consequences of price changes on their and their rivals' sales quantities.

prices, in an effort to lure sellers  $S2$  and  $S3$  into higher postings. At the same time sellers  $S2$  and  $S3$  become very leery being of caught pricing above  $S1$ , suffering exclusion from the market. Observe in particular that repeated postings at the competitive price by  $S1$  in periods 33 and 34 prominently affect the pricing decisions of  $S2$  and  $S3$  for the next 10 periods. Following  $S1$ 's competitive choices, neither seller  $S2$  nor  $S3$  regularly post prices above the competitive threshold again until period 48, this despite  $S1$ 's persistent postings at  $p_{\text{lim}}$ .

#### 4. Pricing Densities Relative to Static Nash Equilibrium Predictions

The price autocorrelations evident in both of the illustrated price sequences obviously invalidate any strict interpretation of Nash mixing predictions. However, the mixing distributions may still possibly organize the distribution of observed prices fairly well. A reasonable behavioral conjecture in this context is that sellers try to anticipate the actions of the others in making pricing decisions each period. This process becomes more difficult over time, as revealed by the decay of the price cycles. Eventually such a process could drive sellers to the predicted equilibrium mixing distributions.

In several instances, some rough conformity between predicted and observed distributions has been found. Pricing densities for the final 30 periods of the four sessions conducted in the one-seller-with-power design are displayed in the bottom right quadrant of Figure 1. Postings by the power sellers, illustrated by the blue bars, reveal an apparent tendency for pricing at the dime nodes of 80, 90 and 100 cents. These deviations are enough to drive a rejection of a distributional test.<sup>7</sup> However, a strong mode at the limit price is observed, as predicted, and only a relatively small portion of the pricing density occurs below the lower limit  $p_{\text{min}}$ . Moreover, the observed mean of the pricing distribution for power sellers shown as a dashed blue line (101.54 cents), is only slightly below the predicted mean, shown as a thin line (103.14 cents). Similarly, the observed mean for sellers without power, shown as a dashed red line (93.22 cents) deviates only slightly from the predicted mean, shown as a thin line (92.47 cents), despite some tendency for the sellers without power to post prices below  $p_{\text{min}}$  and to post at even-dime nodes. Similar results have been observed in very different designs (e.g., Kruse et al., 1994; Kruse, 1993) and have given rise to the conjecture that perhaps at least the central moments of predicted mixing distributions may characterize behavior reasonably well.

The glaring discrepancies between predicted and observed densities for the last 30 periods of six sessions conducted in two-sellers-with-power design, which are shown in the upper right quadrant of Figure 1, suggest that such conjectures are preliminary. Consider first the power sellers, illustrated by the blue bars. Not only do they exhibit a tendency to price at prominent 50 cent nodes (e.g., in this case 400, 450 and 500 cents),

<sup>7</sup> For example, the Kolmogorov–Smirnov Test statistic for the large sellers 0.158, which exceeds the  $p = 0.05$  critical value (240 d.f.).

but hardly any prices were observed at the predicted price mode, and far too much density was observed at the upper end of the distribution. The observed mean, shown as a dashed blue line (455 cents) far exceeds the predicted mean, shown as a thin line (388 cents). Similarly, the no-power sellers priced well above the predicted distribution, with the observed mean, which is displayed as a red dashed line (407 cents), far exceeding the thin-line predicted mean (335 cents). [Wilson \(1998\)](#) observes comparable results in a variation of the [Davis and Holt \(1994\)](#) design.

The wide discrepancies between predicted and observed performance in the [Davis and Holt \(1994\)](#) design merit some reflection. To the best of our knowledge, such wide discrepancies between observed and predicted performance in posted-offer markets are unique to this design.<sup>8</sup> It is possible that this particular type of design interacts with the sellers' decision-making process in a way that generates persistent deviations from static Nash-predicted outcomes. Sellers, for example, may generally make decisions with errors, but the error-process may generate persistent deviations from Nash-predicted outcomes only when the asymmetric sellers are sufficiently balanced. The central moments of static Nash mixing distributions may summarize pricing choices quite well either when sellers are symmetric (as in by [Kruse, 1993](#); and [Kruse et al., 1994](#)) or when the asymmetries are so pronounced that the decision process is again simple (as in [Davis and Wilson, 2000](#)). Gaining some understanding of this apparent anomaly remains an important unanswered question.<sup>9</sup>

## 5. Performance of Alternative Theories

The somewhat erratic performance of Nash mixing predictions should be considered in light of the available alternative theories. Perhaps most obviously, competitive predictions do not characterize outcomes well in this context. Market power raises prices above the competitive prediction, and in the direction of the Nash equilibrium.<sup>10</sup> It is also difficult to ascribe the observed performance to some type of collusion. As illustrated in the sequences illustrated on the left side of [Figure 1](#), prices rise above the competitive prediction, but prices tend neither to the joint profit maximizing price ( $p_{lim}$

<sup>8</sup> [Davis and Reilly \(1998\)](#), however, do observe rather wide discrepancies between bids and static mixing predictions in asymmetric variants of an all-pay auction. [Anderson, Goeree and Holt \(1998\)](#) argue that the deviations observed in the Davis and Reilly auction can be explained in a logit-equilibrium with decision errors.

<sup>9</sup> The capacity of each no-power seller relative to total market capacity may also affect performance, as was observed above in the discussion of the sequence of postings for the two-sellers with power design, shown in [Figure 1](#).

<sup>10</sup> There are, of course exceptions. In markets, of relatively short duration, where market power is slight, and where participants have only limited information about market costs and market demand, market power does not always generate deviations from competitive prediction. See, for example, the references listed at the end of note 5.



in each case), nor to any other stable outcome. Notice further that the kind of price-cutting behavior observed does little to suggest that sellers mete out punishments for defections with their cuts, as would be a part of any repeated game collusive strategy. [In a clever duopoly experiment, Kruse (1993) demonstrates that static predictions outperform dynamic predictions in repeated games.] A final possible means of organizing such data suggested by Figure 1, would be some variant of a standard theory of price cycles (Edgeworth, 1925). This final alternative is more promising, as was observed by Kruse et al. (1994). Although changes in the frequency and amplitude of the cycles invalidate any mechanical application of a static theory of price cycles, a modified version of the Edgeworth cycle theory may provide a good model for disequilibrium price formation. Notably, a modified Edgeworth cycle theory is not necessarily inconsistent with a distribution of prices that eventually approximates the static Nash mixing distributions.

## 6. Summary

Market power arises in many posted-offer markets and drives a distinction between the competitive prediction and the Nash equilibrium for the market viewed as a stage game. Pricing patterns in such markets tend to be characterized by “Edgeworth cycles” that deteriorate as the sessions progress. The amplitude and frequency of the cycles are sensitive to design and procedural details, and vary considerably from experiment to experiment. Although persistent serial correlation in pricing proscribe any direct test of static Nash mixing predictions, rough correspondence between the central moments of predicted and observed densities has been observed in a variety of different instances. However, the persistent and very prominent deviations observed in an asymmetric design by Davis and Holt (1994) suggest that Nash mixing predictions do not uniformly organize behavior well. The circumstances under which mixing predictions may organize outcomes well merits further investigation.

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