Chapter 12

THE WALRASIAN AUCTION

CORINNE BRONFMAN, KEVIN MCCABE, DAVID PORTER, STEPHEN RASSENTI, VERNON SMITH

1. Introduction

Joyce (1984) reports the results of experiments with a Walrasian tatonnement auction showing that the mechanism is stable, exhibits strong convergence properties and generates efficiencies that average better than 97%.[1] He also found that when subjects could see part of the order flow (excess demand), prices tended to be lower. His experiments consisted of a stationary environment where subjects were provided single-unit supply and demand functions. We examine the robustness of his results in a more general multi-unit per subject setting and systematically investigate the effect of various order flow information and message restriction rules on the performance of the Walrasian auction.

2. Experimental Environments

2.1. Baseline

Consider the environment charted in Figure 1. Each buyer has a value for a discrete single unit. Each seller has the capacity to supply only one discrete unit to the market for a specified cost.

In this environment goods are to be allocated using the following Walrasian auction:

1. An initial price $P_0 > 0$ is selected by an auctioneer.
2. Each buyer and seller indicates to the auctioneer whether they wanted to buy or sell a single unit at the announced price.
3. If the number of buyers demanding a unit equals the number of sellers supplying a unit at that price, the process stops.
4. If there is an imbalance of supply and demand at that price, i.e. excess demand $E(P)$ is non-zero, the auctioneer updates the price using the following formula:

$$\Delta P = \begin{cases} \$0.05E(P) & \text{if } |E(P)| > 1, \\ \$ZE(P) & \text{if } |E(P)| = 1, \\ & \text{where } Z < \$0.05 \text{ is decided by auctioneer.} \end{cases}$$

[1] *Efficiency* is defined as the percent of the theoretical producer plus consumer surplus realized by a trading mechanism.

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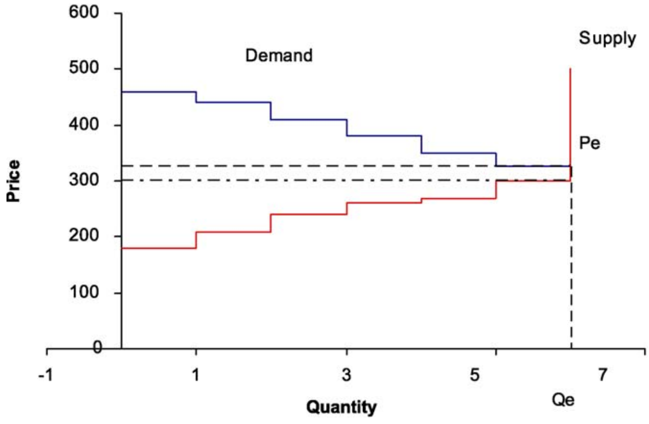

Figure 1. Simple single unit environment (E1). The figure shows an experimental environment with six buyers and six sellers. Each buyer has a resale value for one unit only and is represented by one of the steps on the demand function graphed in blue. Each seller has a cost for one unit only and is represented by one of the steps on the supply function graphed in red. The market equilibrium is given by the price tunnel P_e and quantity Q_e of six units.

When individuals have demands or supplies for one unit, non-revelation is a risky strategy since, should the market clear, the individual will fail to make a profitable transaction. In a single play of the game defined by this process, any pure strategy Nash equilibrium must be at (Q_e, P_e).

2.2. Multi-unit Non-stationary Supply and Demand Environment

When there are multiple units demanded/supplied by individual traders, the typical Nash equilibrium of the Walrasian auction results in underrevealation (see Hurwicz, 1972 and Otani and Sicilian, 1990). Consider the demand and supply configuration charted in Figure 2. The aggregate supply and demand arrays are step functions where each step identifies a particular buyer's or seller's value or cost. Only one trader is assigned to a step on these functions. In addition, each participant has multiple units to bid or to offer all on the same step. As shown in Figure 2, there are three buyers ($B1$, $B3$, $B5$) and three sellers ($S1$, $S2$, $S3$) with six units and two buyers ($B2$, $B4$) and two sellers ($S2$, $S3$) with three units. Thus, there are twenty-four buy and sell units in the market; of these, eighteen are potentially tradable in the equilibrium price tunnel (450, 470).

During an experiment, buyers remained buyers and sellers remained sellers, period to period, although two important changes occurred each period:

1. The equilibrium prices were changed by parallel and equal shifts in the aggregate demand and supply arrays. In particular, from period to period, a random constant from the interval [100, 490] is added to (or subtracted from) each step on the aggregate demand and supply functions.

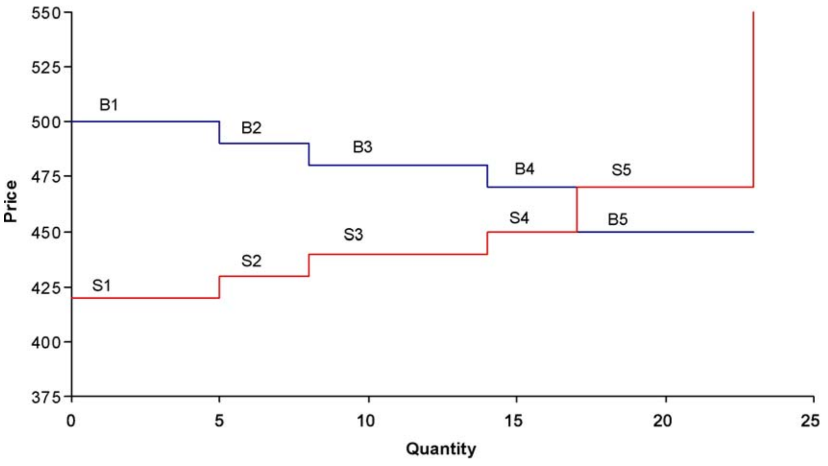

Figure 2. Multi-unit demand and supply environment (E2). This figure represents a multi-unit environment with five buyers and five sellers. The aggregate supply and demand arrays are step functions where each step identifies a particular buyer's or seller's value or cost. Only one subject is assigned to a step on these functions. In addition, each participant has multiple units to bid or to offer all on the same step. As shown in the figure, there are three buyers ($B1$, $B3$, $B5$) and three sellers ($S1$, $S2$, $S3$) with six units and two buyers ($B2$, $B4$) and two sellers ($S2$, $S3$) with three units. Thus, there are twenty-four buy and sell units in the market; of these, eighteen are potentially tradable in the equilibrium price tunnel (450, 470).

2. Within each period, buyers are assigned (by a random rotation procedure) to one of the demand steps ($B1$)–($B5$), and sellers are assigned to one of the supply steps ($S1$)–($S5$). Each period they are assigned to a new step. For example, buyer 1 with the right to resell up to 6 units at a price of 500, could be reassigned a step with 6 units, but with no tradable units within the equilibrium price tunnel, as is buyer 5. Alternatively, he or she could be on a step with only three units, as is buyer 4.

This experimental environment, which we will identify as E2, allows us to assess the performance of the Walrasian auction where participants have multiple units and where relative competitiveness is variable period to period. This environment has been used in previous experimental studies (see Campbell et al., 1991; McCabe, Rassenti, and Smith, 1992), and clearly stresses the price discovery process. The environment, from the participants' perspective, seems to be changing each period and thus relying on past market experience can hinder price discovery.

3. Walrasian Auction Design and Computerized Implementation

A Walrasian tatonnement must specify the following rules in order to implement the auction:

(i) The process must determine a starting or *initial price* P_0.

(ii) The *price adjustment function* we used in our experiments was the following piecewise rule:

$$P_t = P_{t-1} + \mathrm{rnd}\left(4\left[\frac{1}{2(1+\{t/4\})}\right]\left[D(P_{t-1}) - S(P_{t-1})\right]\right),$$

where $\{y\}$ denotes the greatest integer less than or equal to y, t is the current iteration in the period, and $\mathrm{rnd}(y)$ is the nearest integer to y. For example, in iteration 11 with an announced price of 200 and reported excess demand of 10, the price next period will be 207. Unlike the experiments conducted by Joyce, our experiments are computerized and thus there is no human auctioneer judging the "appropriate" price changes.

(iii) We consider two alternative information structures:
   (a) *minimum information*: subjects are informed of the current trial price, the adjustment factor for the current trial, the number of seconds remaining for the current trial, and a full history of past trial prices and past order flow imbalances;
   (b) *complete order flow information*: In addition to the information in (a), subjects are provided on each trial with the real-time updated buy and sell orders as they arrived during the current price iteration, and what the next iteration price would be, based on the current imbalance information.

(iv) A *message restriction* specification limits the messages that can be sent. We used an improvement rule where a buyer who was willing to purchase m units at a price Y must be willing to purchase at least m units at prices lower than Y. Similarly, a seller who was willing to sell n units at price Z must be willing to sell at least n units at prices above Z.[2] The motivation for this rule was to restrict manipulation and obvious misrevelation (see McAfee, 1992). This rule restricts the potential buy and sell orders that can be placed during iteration t as a function of past responses.

(v) The stopping rule used in our experiments has two dimensions. First, during an iteration, the time remaining to submit an order is endogenous. A clock is set at 15 seconds when the iteration price is posted. Any new order quantity submitted at the price reinitializes the clock to 15 seconds. This rule provided an implementation of a "soft close" procedure. A soft close enforces a unanimity requirement in that no one can guarantee himself or herself the last say. The second dimension dealt with the exact close of the market period. We close the market period, at trial t^*, when $P_{t^*} = P_{t^*-1}$ or $E(P_{t^*}) = 0$. Notice that given

[2] In addition, in all replications we placed the following restrictions on the messages participants could send at each iteration:

1. Individuals could not sell short or buy on margin. Thus, individuals were not permitted to offer more units than their maximum declared capacity to buy or sell.
2. Once an order was sent to the market it could not be canceled.

Table 1
Experimental treatments* (number of experiments is listed in each cell)

Message restriction	Information	
	Minimal	Order flow
No	4	5
Yes	3	3

Notes. The experiments consist of two information treatments in which either *minimal information* is provided to subjects during an iteration,[3] i.e., subjects are informed of the current trial price, the adjustment factor for the current trial, the number of seconds remaining for the current iteration, and a full history of past trial prices and past order flow imbalances; or *complete order flow information* where subjects are provided, at each iteration, with real-time updated buy and sell orders as they arrived during the price iteration, and what the next iteration price would be, based on the current imbalance information. The other treatment uses a *message restriction* improvement rule placed on subject orders where a buyer who was willing to purchase m units at a price Y must be willing to purchase AT LEAST m units at prices lower than Y. Similarly, a seller who is willing to sell n units at price Z must be willing to sell AT LEAST n units at prices above Z. This rule restricts the potential buy and sell orders that can be placed during iteration t as a function of past responses.

our price adjustment rule, this stopping rule does not guarantee $E(P) = 0$. Thus, if at t^*, $E(P_{t^*}) = 0$, we ration by time priority.

Table 1 provides an overview of the experimental treatments and the number of experiments conducted per cell in our design. The design consists of two factors (improvement rule and order flow information) which are either present or not in each experiment.

4. Experimental Results

4.1. The E1 Environment Replication

Using E1 we replicated the Joyce experiments with a computer implementation of the Walrasian Tatonnement mechanism. Table 2 shows the mean efficiency and competitive price deviation in the later periods of the experiments. There is no significant difference in efficiency and price between the oral and computerized treatments.

4.2. Baseline and Treatment Effects

For the remainder of this paper, we will use the following abbreviations for the treatments in our design: FINI = full information with no bid-offer improvement; NINI =

[3] An iteration is the time between two successive price changes based on the buyer and seller responses on the amounts they are willing to purchase and sell at the stated price.

Table 2
Oral versus computerized treatment for E1

	Oral implementation	Computerized implementation
Mean efficiency	96.3	97.7
Mean price deviation	5.7	5.3

Notes. The statistics presented in the table are: Average efficiency, which is the percentage of the maximum of the producer and consumer surplus attained by the oral and computerized implementation of the Walrasian auction, and the average deviation of prices from the competitive equilibrium price, for the simple E1 environment in Figure 1, attained by each mechanism.

no information with no bid-offer improvement; FII = full information with bid-offer improvement; NII = no information with bid-offer improvement.

Figure 3 shows the efficiency distribution (boxplots) for each of the four treatments in our implementation of the Walrasian auction in the E2 environment. The boxplots show the median (the dot), interquartiles (the box), the 10th and 90th percentiles (bars below and above each box). In addition to the Walrasian treatments, we report the results of 6 baseline double auction (DA) experiments using the E2 environment.

The DA outperforms each of the Walrasian auction designs we tested. FINI performs best among the Walrasian auction treatments. The following efficiency rankings, for periods 7+, show that only the full information no improvement (FINI) treatment approaches the efficiency of the double auction: DA $\geqslant$ FINI $\geqslant$ NII = FII = NINI.

We summarize the following comparative static results.

(i) Conditional on having no bid-offer restriction rule, full information helps in obtaining more efficient allocations.

(ii) Conditional on only minimal information being provided, the improvement rule helps in obtaining more efficient allocations. However, the level of efficiency does not approach that of FINI or DA.

With respect to price formation, Figure 4 shows the price dispersion relative to the competitive equilibrium price tunnel. From the boxplots it is easy to see that each treatment results in prices that lie within the tunnel $(-10, +10)$. However, the low efficiencies reported in Table 2 show that the supply and demand match is not correct, and suggests the presence of significant underrevelation on both sides of the market: if either side underreveals to gain an advantage, the other side underreveals to neutralize that advantage.

4.3. Individual Behavior

Three types of individual behavior can be identified in our experiments:

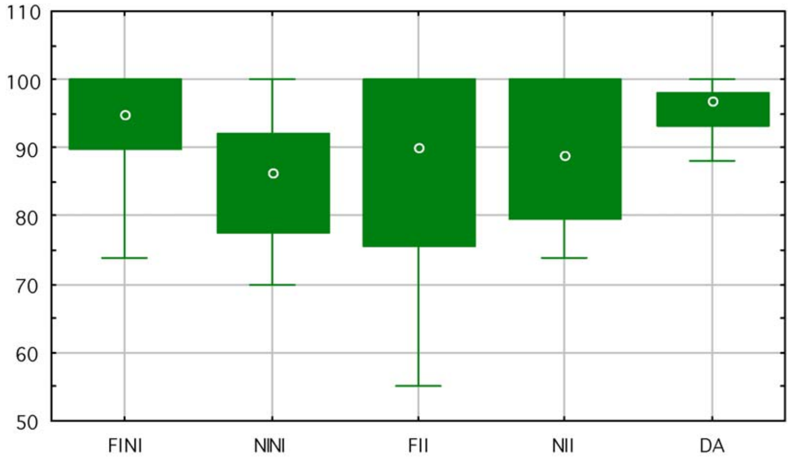

Figure 3. Distribution of efficiency across treatments. The figure shows the efficiency (the percentage of the maximum of the producer and consumer surplus attained) distribution for each of the four treatments in our implementation of the Walrasian auction in the E2 environment. The boxplots show the median (the dot), interquartiles (the box), the 5th and 95th percentiles (bars below and above each box). In addition to the Walrasian treatments, we report the results of 6 baseline double auction (DA) experiments using the E2 environment. The treatments are: FINI = full information with no bid-offer improvement; NINI = no information with no bid-offer improvement; FII = full information with bid-offer improvement; NII = no information with bid-offer improvement. *No information* means subjects are informed of the current trial price, the adjustment factor for the current trial, the number of seconds remaining for the current trial, and a full history of past trial prices and past order flow imbalances, while *full information* subjects were provided at each iteration with the real-time updated buy and sell orders as they arrived during the current price iteration, and what the next iteration price would be, based on the current imbalance information. The bid-offer improvement rule requires a buyer who was willing to purchase m units at a price Y must be willing to purchase AT LEAST m units at prices lower than Y. Similarly, a seller who was willing to sell n units at price Z must be willing to sell AT LEAST n units at prices above Z. This rule restricts the potential buy and sell orders that can be placed during iteration t as a function of past responses.

(1) *Overrevelation*: A buy or sell response that can result in a marginal loss in profit if the process stops, i.e., for each iteration t and participant i at the price P_t [4]

$$D_t^i(P_t) > d_t^i(P_t), \qquad S_t^j(P_t) > s_t^j(P_t).$$

(2) *Underrevelation*: A buy or sell response that is less than the number of units that are profitable at the current price

$$D_t^i(P_t) < d_t^i(P_t), \qquad S_t^i(P_t) < s_t^j(P_t).$$

[4] We define $D_t = (D_O(P_O), D_1(P_1), \ldots, D_t(P_t))$, $S_t = (S_O(P_O), S_1(P_1), \ldots, S_t(P_t))$ as the aggregate supply and demand responses for each price iteration up to P_t and D_t^i, S_t^j the individual supply and demand responses for each price iteration up to P_t. We will represent the true demands and supplies with the lower case letters, d_t, s_t, d_t^i, s_t^j.

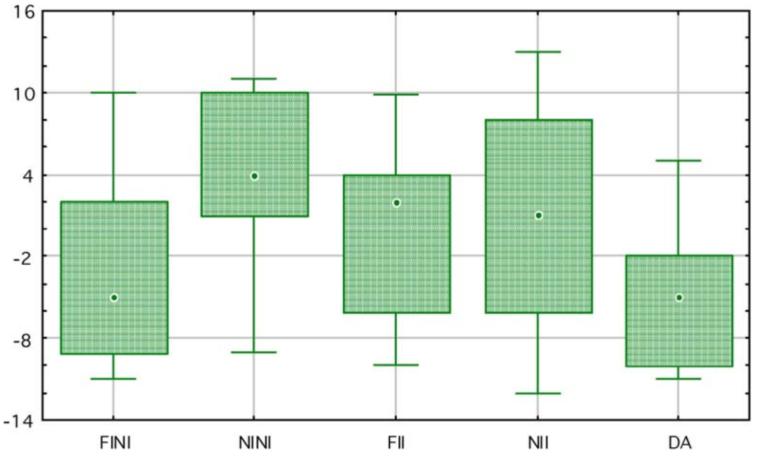

Figure 4. Distribution of prices relative to competitive equilibrium. The figure shows the distribution of the deviation of prices, in cents, from the competitive equilibrium price for each of the four treatments in our implementation of the Walrasian auction in the E2 environment. The boxplots show the median (the dot), interquartiles (the box), the 10th and 90th percentiles (bars below and above each box). In addition to the Walrasian treatments, we report the results of 6 baseline double auction (DA) experiments using the E2 environment. The treatments are: FINI = full information with no bid-offer improvement; NINI = no information with no bid-offer improvement; FII = full information with bid-offer improvement; NII = no information with bid-offer improvement. *No information* means subjects are informed of the current trial price, the adjustment factor for the current trial, the number of seconds remaining for the current trial, and a full history of past trial prices and past order flow imbalances, while *full information* subjects were provided at each iteration with the real-time updated buy and sell orders as they arrived during the current price iteration, and what the next iteration price would be, based on the current imbalance information. The bid-offer improvement rule requires a buyer who was willing to purchase m units at a price Y must be willing to purchase AT LEAST m units at prices lower than Y. Similarly, a seller who was willing to sell n units at price Z must be willing to sell AT LEAST n units at prices above Z. This rule restricts the potential buy and sell orders that can be placed during iteration t as a function of past responses.

(3) *Revelation*: A buy or sell response that contains all profitable units and no unprofitable units at the current price

$$D_t^i(P_t) = d_t^i(P_t), \qquad S_t^i(P_t) = s_t^j(P_t).$$

Overrevelation rarely occurs (3% of responses), however, both buyers and sellers underreveal nearly one-third of the time. In FII, over 65% of the buyer responses are consistent with underrevelation.

Notice that under our improvement rule, in later periods, once a buyer (seller) has revealed a willingness to purchase (sell) x units at a particular price, he or she is required to purchase (sell) that many units at a lower (higher) price; therefore, underrevealing at the beginning of a period is the only way to obtain strategic bargaining room later in a period. Consequently, the improvement rule fosters underrevelation by motivating people to begin their bargaining from a more "advantageous" position.

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