Time Varying Weight Matrices and Autoregressive Parameters in Spatial Panel Models

Frank Davenport

February 15, 2010

1 Introduction

Panel data refers to repeated measurements on the same group of cross-sectional units. This type of data present the empirical analyst with several advantages and challenges. Advantages include the ability to account for unobserved heterogeneity in cross-sectional units,larger sample sizes (and thus more efficiency and degrees of freedom in estimation), and less risk of collinearity among exogenous regressors (provided they vary over time and or cross-sectional units) (Baltagi 2008). Disadvantages arise when panels are unbalanced due to attrition, self-selection, and or non-response. In addition the experimental design and data collection can be more costly and logistically difficult (Baltagi 2008). Finally inference and interpretation of regression estimates can be impacted if data exhibit either serial or cross-sectional correlation.

This paper discusses spatial panel models, a special case of panel data where correlation in cross-sectional units is some function of distance (geographic or otherwise). In cross-sectional spatial models, spatial correlation is dealt with by either an empirically derived or apriori specified measure of spatial dependence commonly referred to as the spatial weights (`W`) matrix.

The `W` matrix is an analyst specified measure of spatial correlation. Specifying the correct `W` for a dataset is a major challenge in spatial econometrics (Fingelton 2009; Getis and Aldstadt 2004). This is especially true when no satisfactory measure of distance between observations can be determined, as when the spatial support lies on irregular polygons (counties, regions, census tracts, et cetera). For point (discrete events in space or time) or raster (continuous spatial fields) data, weight matrices are often empirically derived as a direct function of spatial correlation provided through an empirical or theoretical variogram (Getis and Aldstadt 2004). However the majority of spatial economic data are either attributes of regions (ex. did county i implement policy y?) or micro-data aggregated up to the region level (average income in county i). In these cases the analyst is forced to pre-specify the W matrix based on theoretical and/or domain knowledge regarding the phenomena of interest.

Depending on the type of model and degree of misspecification, an incorrect W matrix can lead to biased estimates and or incorrect standard errors of regression coefficients and the spatial auto-regressive parameter ρ (a measure of the direction and magnitude of spatial correlation). This problem can be insidious in a panel setting where both W and ρ can theoretically change over time.

The W matrix and ρ can change over time if the nature of spatial dependence changes over time. For example, in their study of Indonesian rice farmers, Druska and Horace (2004), let ρ vary based on growing season. They assume that in 'wet' times, roads are less passable which induces greater isolation (and thus less spatial dependence) among farmers (Druska and Horrace 2004, p. 187). However a time-varying ρ only allows the global *magnitude* of spatial dependence to change but still assumes that the structure of that dependence (W) remains constant.

Trade in the greater European area provides an example of how the structure of spatial dependence might change over time. Prior to the fall of the Soviet Union, most eastern European countries were economically independent of those in the west. The creation of the European Union, and the inclusion of former Soviet Countries either in the EU or in EU trade agreements changed that relationship and thus the nature of spatial dependence between those countries. Similar examples could be drawn from international migration and domestic agriculture. Figure 1 provides a theoretical illustration of a time-varying W matrix.

An analyst faced with a time-varying W matrix must confront two problems: 1) The added theoretical burden (and danger of misspecification) of having to specify a different W matrix for each time period; and 2) Current spatial panel computation methods do not allow for a time varying W matrix. This paper focuses on the latter issue, by asking the following question: *How are estimates of the spatial auto-regressive parameter (ρ) impacted when the true spatial-temporal dgp includes a time varying W matrix*?

The remainder of the paper is as follows: The next section discusses different methods of constructing the W matrix. I then review some basic specifications of cross-sectional spatial models and show how a misspecified W impacts those models. These models are extended to a panel setting in the following discussion of spatial-temporal data generating processes. Three different techniques for estimating spatial-panels (one using fixed effects, the other two using random effects) are introduced. Finally these methods are tested in monte-carlo simulations where the spatial auto-regressive parameters (ρ) and the W matrix are allowed to vary through time. The last section provides some suggestions for further research.

2 Spatial Econometric Models and W matrices

2.1 The Spatial Weights W Matrix

The fundamental problem when conducting regression analysis on spatial data is that the observations are not independent. This can occur as a result of spill-over effects (housing price in location i effects housing price in location j),unobserved neighborhood or regional effects, or because spatial boundaries are often arbitrary administrative delineations (i.e. zip codes,census tracts, counties) irrespective of the phenomena being studied (Openshaw and Taylor 1979; Openshaw 1984; Anselin 2002).

Depending on the nature of the model, this can lead to incorrect standard errors or biased estimates. Thus analysts are typically forced to pre-specify some measure of spatial relationship in the form of the NxN `W` matrix. A `W` matrix is always square, rarely symmetric, and has zeroes on the diagonal. This last feature ensures that any single observation does not directly influence itself. Getis 2004, p. 91 provides a list of some standard `W` specifications (items in bold are independent of measures of precise distance and are the most frequently employed in spatial econometrics):

- **Spatially Contiguous Neighbors**
- Inverse distance raised to a power
- Ranked distances
- **Length of shared border divided by perimeter**
- N nearest neighbors
- All weighted centroids within distance d

Many other specifications exist, however most of these are modifications of the above list. In addition to specifying which observations influence each other, the analyst must also specify the actual weight of the influence. This is typically done by either row-standardizing (so that all weights for a given observation sum to one),standardizing across the whole matrix (so that the whole matrix sums to one), or some other theoretically specific weighting scheme. The vast majority of empirical studies use row-standardized weight matrices, largely for convenience of interpretation and computation, rather than theoretical validity (Kelejian and Prucha 2002; Kelejian et al. 2006). The use of row standardization is one of the reasons why `W` matrices are rarely symmetric.

Monte-carlo experiments have shown that misspecifying a `W` matrix can decrease the efficiency of estimated regression coefficients, as well as produce biased estimates of σ^2 and ρ (Florax and Rey 1995; Griffith 1996). However the specific impact of an incorrect `W` matrix is dependent on what components of the model are spatially dependent. In the next section I consider the implications of using an incorrect `W` matrix in two standard cross-sectional specifications: the Spatial Auto Regressive *lag* and *error* models.

2.2 Misspecifying Spatial Regression Models

The correct specification of a spatial econometric model depends on whether the correlation exists in the outcome variable (spatial lag), error term (spatial error), regressors (spatial Durbin), or some combination thereof (Anselin 1988; LeSage and Pace 2009). The two most common specifications, lag and error, are provided here as a baseline for discussing the role of the `W` matrix.

$$\begin{aligned} y &= \rho W y + X\beta + \epsilon \\ &= (I - \rho W)^{-1} X\beta + (I - \rho W)^{-1}\epsilon \end{aligned} \tag{2.1}$$

The spatial lag model (2.1) implies that the outcome variable is a function of the weighted (by `W`) average of it's neighbors (in addition to exogenous regressors and the error term).

There are two interesting facets of the spatial lag model: 1) introducing spatial correlation in the outcome also implies correlation in the regressors and error term and 2)interpretation of marginal effects becomes more complicated. The first point is seen in the second line of (2.1). The second point is illustrated by considering the simultaneity implied by a lagged regressor. A one unit change in x_i will cause some change in y_i, which due to the model specification, causes a change in y_i's neighbors (y_j), which in-turn will feedback to influence y_i. This form of simultaneity can seem odd, unless one considers (2.1) to be the result of a long-run equilibrium process, as is often done in the regional convergence and growth literature (LeSage and Pace 2009; Rey and Montouri 1999). Thus the cross-sectional spatial lag model can viewed as the outcome of a spatial-temporal (panel) process (LeSage and Pace 2009). From this example, it becomes obvious that proper specification of `W` is essential to correctly interpret a spatial lag model.

The spatial error model is seen as a special case of correlated errors (non-zeros in the off-diagonal of the error covariance matrix) where correlation is a function of distance:

$$\begin{aligned} y &= X\beta + \epsilon \\ \epsilon &= \rho W \epsilon + u \\ E[u] &= \sigma^2 I \\ E[\epsilon\epsilon'] &= \sigma^2 (I - \rho W)^{-1} (I - \rho W)^{-1} \end{aligned} \tag{2.2}$$

The spatial error dgp is present if there is a spatially correlated omitted variable (i.e an unobserved region effect). If ignored this can lead to underestimation of standard errors and, possibly, over estimation of R^2. Worse, if a plausibly exogenous regressor is included that shares the same form of spatial correlation as the omitted variable (housing prices), this can lead to endogeneity and thus biased estimates.

The use of spatial heteroskedastic and autocorrelation standard errors (SHAC) methods (Conley 1999; Kelejian and Prucha 2007; Bester et al. 2009) makes proper specification a spatial error model less of an issue if the analyst is only concerned with hypothesis testing on non-spatially correlated exogenous regressors. However, if the goal of the analysis is to identify a latent spatial process, or simply to test whether the spatial error (versus lag or another specification) is the correct model to run, then the analyst will need an unbiased and efficient estimate of ρ which is dependent on proper specification of `W` matrix. The next section examines how these models have been extended to a panel setting

3 Spatial-Temporal Data Generating Processes and Estimation Procedures

3.1 Spatial Data Generating Processes

Several spatial-temporal dgp's have been discussed in the literature, though only the most basic extensions of the existing cross-sectional methods have been implemented in statistical software packages (Anselin et al. 2008; Elhorst 2009). Anselin (2008) provides a taxonomy of these dgp's based on the type of heterogeneity present in the process:

Temporal Heterogeneity

1. Fixed effects (time specific intercept) or random effects (time specific error tem) possibly augmented by a time specific spatial auto regressive parameter
2. Spatial **SUR** model that allows for serial correlation in the error term and a time specific autoregressive parameter

Spatial Heterogeneity

1. Fixed effects for spatial and/or temporal units. The specification of spatial correlation (both ρ and `W`) is assumed to be stationary through time.
2. Random effects for spatial and/or temporal units, where spatial correlation exists in both time-invariant (regional) and time-specific effects. Spatial correlation for each of these effects does not have to be the same.

Spatial-Temporal Heterogeneity

1. Pure space recursive models, where the outcome variable is dependent on a spatial lag from the previous time period ($\rho W_N y_{t-1}$)
2. Time-space recursive models, where a temporal lag term (y_{t-1}) is added to the to pure space recursive model
3. Time-space simultaneous model, features a temporal lag and a spatial lag from the same time period ($\rho W_N y_t$)
4. Time-space dynamic models, includes all of the components from the three previous specifications (time lag, space-time lag, and space lag)

5. Time-space error model, where the error covariance matrix is spatially correlated in each time period, and each of these covariance matrices is also serially correlated with the matrix from the previous term (the TxT serial correlation matrix is multiplied by the NxN spatial correlation matrix).

Anselin (2008) addresses the numerous estimation and identification issues that arise in the spatial-temporal models. Despite the complexity of these models, none of them explicitly address the possibility of `W` matrix changing over time.

3.2 Existing and Implemented Methods for Spatial Panel Estimation

I now focus on the three different spatially-heterogeneous panel models that have been implemented in various software programs (Matlab,R, and soon STATA). The processes are presented in more detail and the corresponding estimation techniques are discussed. With the exception of the fixed effects specification, these methods have only been implemented for estimating spatial panel error models (though a GM random effects lag model is presented in (Fingelton and Gallo 2008).

3.2.1 Fixed Effects

Elhorst (2003) specifies a one (individual) or two (time) way fixed effects model, estimated via a concentrated likelihood process. The basic fixed effects specification is as follows:

$$\begin{aligned} y &= (\iota_T \otimes \alpha) + X\beta + u \qquad (3.1) \\ u &= \lambda(\iota_T \otimes W_N)u + \epsilon \end{aligned}$$

Elhorst recommends first demeaning the model (subtracting within group means from both sides of the equation) to facilitate computation on large datasets, and then estimating via a least squares dummy variable approach. The residuals from this process are then fed into the log likelihood for the spatial autoregressive parameter (λ in this specification) which in turn is used in an FGLS to estimate (σ^2 is a function λ in the error specification) the regression coefficients and error covariance matrix. A new set of residuals is then obtained and used to re-estimate λ. The new λ is used again in a new FGLS and the process repeats until the estimates of the various coefficients converge (Elhorst 2003).

Anselin (2008, p. 640) correctly notes that this approach is inconsistent when spatial fixed effects are used. His argument is that the asymptotics of a spatial panel model are dependent on $N \rightarrow \infty$, and thus inclusion of individual fixed effects is subject to the incidental parameter problem (one new parameter for every observation).

3.2.2 Random Effects

While not subject to the inconsistency and loss of degrees of freedom of a fixed effects specification, random effects model have their own limitations. The principal assumption in random effects models is that the modeled unobserved effects in the error term are uncorrelated with the regressors. In a spatial panel context (where N is generally $<<$ than T) with time-random effects this is equivalent to arguing that unobserved 'temporal shocks' are independent of the predictors. If the panel includes spatial (individual) random effects, then the assumption is that unobserved characteristics inherent to that region are uncorrelated with observed attributes (regressors) of that region. The latter seems a stronger assumption than the former, but is obviously dependent on the nature of the problem.

Several authors argue that including a group mean (either $\bar{x}_t$ or $\bar{x}_n$, depending on the model) can control for potential endogenity in random effects (Skrondal and Rabe-Hesketh 2004; Bafumi and Gelman 2006; Bartels 2008). This is based on the assumption that the correlation between the random effect and the individual regressor is based on the presence of the group mean within that random effect (ie a special case of omitted variable bias) (see Skrondal and Rabe-Hesketh p. 52-53 for a full demonstration). Another solution (or additional step after adding the group mean) is to difference the group mean from the individual level regressor to prevent issues with collinearity. Again the assumption here is that the once the group mean has been removed from the random effect and included in the model, there is no other source of correlation between the individual regressor and whatever remains in the random effect. This may or may not apply depending on the nature of the problem.

The discussion of random effects is significant when considering the existing spatial panel estimation procedures, as both allow the analyst to specify time and/or spatial (individual) effects.

Random Effects via Maximum Likelihood (Baltagi et al. 2007)

The first approach (3.2) is presented in Baltagi (2007) and accounts for serial correlation (ρ) cross-sectional spatial correlation (λW) and unobserved random effects on the cross-section units(μ_N). The estimation procedure is similar to the concentrated likelihood used for the fixed effects estimation described above.

$$\begin{aligned} y_{it} &= X'_{it}\beta + u_{it}, i = 1, \ldots, N, t = 1, \ldots, T \\ u_t &= \mu_N + \epsilon_t \\ \epsilon_t &= \lambda W \epsilon_t + \nu_t \\ \nu_t &= \rho \nu_{t-1} + e_t \end{aligned} \tag{3.2}$$

Random Effects via Generalized Moments (Kapoor et al. 2007)

The second approach (3.3), proposed by Kapoor, Kelijean, and Pruha (2007- commonly referred to as KKP) uses a GM approach based on six moment conditions calculated using the sample moments of residuals, and spatial lags of the residuals. The first term ($\rho(I_T \otimes W)u_{it}$) in the second equation of (3.3) represents the cross-sectional spatial correlation ,while the third equation in (3.3) ϵ_N represents cross-sectional random effects and an idiosyncratic error term ν_N.

$$\begin{aligned} y_{it} &= X_{it}\beta + u_{it}, i = 1, \dots, N, t = 1, \dots, T \\ u_{it} &= \rho(I_T \otimes W)u_{it} + \epsilon_N \\ \epsilon_N &= (\iota_T \otimes I_N)\mu_N + \nu_N \end{aligned} \tag{3.3}$$

4 Monte-Carlo Simulations

4.1 Setup

As stated earlier, the above specifications and corresponding estimation methods rely on a pre-specified and time-constant `W` matrix. To test the robustness of these methods to a time-varying `W`, I conducted several monte-carlo simulations that compare the ability of the three estimation techniques to consistently estimate the ρ parameter, when the true data generating process includes ρ and or `W`'s that vary over time. All dgp's include a constant term ($\alpha = .5$), one regressor ($\beta = 1$), and a homoskedastic but spatially correlated error term:

$$\begin{aligned} y &= \alpha + X\beta + \epsilon \\ \epsilon &= \rho W \epsilon + u \\ E[u] &= \sigma^2 I \\ E[\epsilon\epsilon'] &= \sigma^2 (I - \rho W)^{-1} (I - \rho W)^{-1} \end{aligned} \tag{4.1}$$

The simulations differed in the specifications of ρ and `W`:

Simulation 1: $\rho = .5$; `W` = Constant[1]

Simulation 2: ρ = Varies (with $\bar{\rho}_T = .63$); `W` = Constant

Simulation 3: $\rho = .5$; `W` = Varies (grows over time as in figure 1)

Simulation 4: ρ = Varies; `W` = Varies

The `W` matrix has dimension $NTxNT$ with a block diagonal structure:

[1]Row standardized with queen contiguity

$$\mathtt{W} = \begin{pmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_T \end{pmatrix} \tag{4.2}$$

Each diagonal block of W (W_t) is an NxN row-standardized spatial weights matrix. In simulations 1 and 2, where W is constant, each diagonal block is equal ($W_1 = W_2 = \ldots W_T$). In contrast, the *varying* W simulations (3 and 4) follow the pattern of *growing* spatial correlation seen in figure 1. In this case, at t=1, the W matrix is quite sparse, as most observations are only correlated with one neighbor. However by t=10, the pattern of spatial correlation is equivalent to full queen contiguity, wherein all observations influence all their neighbors.

I ran 500 repetitions of each simulation. Each simulation had $N = 50$ observations generated on an irregular areal lattice (polygons) over T=10 time units. The continuous regressor and error term were re-created in each repetition while ρ and W remained fixed. Each method was run on the same dataset within each repetition. The results of individual simulations are presented in figures 2,3,4,and 5. Overall results are summarized below and in figures 6 and 7.[2]

4.2 Results

For the two methods that supply standard errors for ρ (FE,BalRe) the true value of ρ was always within two standard errors of the estimates (though the estimates were often still quite far). Those two methods falsely accepted the null hypothesis of $\rho = 0$, 2% of the time in simulations 3 and 4.

In the control simulation (1), the random effects methods performed the best while FE tended to underestimate the degree of spatial correlation (figure 2). This made the FE method perform better when ρ varied through time (figure 3), as both random effects methods overestimated the value of $\bar{\rho}_T$. This may be because the time periods with larger ρ tended to exert more influence on the estimation procedure.

When W varied through time all the methods underestimated ($\bar{\bar{\rho}}_{sim3} \approx .3$) and the true value of ρ was accuratley estimated on only a few repetitions. None of the methods estimated the value of ($\bar{\rho}_T$) when both ρ and W varied (figure 5).

Taken together the results of simulations 3 and 4, suggest that a misspecified W matrix exerts more influence than a misspecified ρ on estimating the ρ parameter. This is demonstrated in figure 6 where the distributions in the bottom two panes (simulations 3 and 4) are pulled more the to the

[2]For brevity I abbreviate the methods as follows:

- Fixed Effects (3.1)=FE
- Random Effects (Baltagi) (3.2)=BalRe
- Random Effects (KKP) (3.3)=KKPRe

.

left (lower estimates of ρ) those of the top two panes (simulations 1 and 2). Figure 7 demonstrates that, in general, each method performed similarly in each scenario.

5 Conclusions and Further Research

Time-varying `W` clearly exert influence on the estimation of the spatial auto-regressive parameter ρ. This may become irrelevant in spatial error models as SHAC standard error methods continue to develop. However as is clear from the dgp in section 3, `W` exerts considerably more influence on spatial lag models as it can effect estimation of the regression coefficients and interpretation of the model.

There are several avenues for extending the simulations in this paper:

- Performing similar tests with spatial-temporal lag processes
- Testing different types of time varying spatial dependence- for example where one spatial unit gradually influences more and more asymmetric influence on all other units (a 'hub and spoke') model of development
- Testing performance with more extreme and/or negative values of ρ

Regarding the role of time-varying W matrices, there are some existing approaches applied to both cross-sectional and panel data that might be useful. LeSage (2009) provides a Bayesian framework for estimating cross-sectional models with weight matrices and for 'averaging' models with different weight systems.Both Driscoll and Kraay (1998) and Chen and Conley (2001) introduce non-parametric methods for estimating spatial dependence in a panel setting. However, both of those methods require large T, and in the case of Driscoll and Kraay, T must be >> than N. Future work might look at expanding on these methods to the large N, 'moderate' T case, common in many spatial datasets.

In the interim, an analyst concerned with a time varying `W` could potentially generate a number of theoretically tractable `W` matrices for each time period, calculate estimates from each, and then compare the resulting estimates. However this process would still be limited by a)The analysts knowledge of spatial dependence within each time period and b)computational difficulties in large N and large T datasets.

To the Econ lab Group *This section is sparse as am I still trying to work out where I want to go with this. Below are some options and I would appreciate your feedback on these:*

- Performing similar tests with spatial-temporal lag processes. *This is where I want to go as this is the more interesting and relevant model specification, however the only available code for estimating this model is the Fixed effects, and the current version does not function very well. However there is published lit on other methods so I could try coding those and testing them*

- Testing different types of time varying spatial dependence- for example where one spatial unit gradually influences more and more asymmetric influence on all other units (a 'hub and spoke') model of development. *This is another angle I would like to pursue- in this case I would try and ground the varying W specifications on theoretical and/or empirical examples of changing spatial dependence over time- some examples are- trade growth in the EU, migration patterns in the late 20th century, regional agricultural linkages in/among developing countries*

- Testing performance with more extreme and/or negative values of ρ. *models tend to perform differently when rho is very high*

- *Another option with regards to rho would be to explore the some of the model specifications that allow rho to vary over time. Again this would entail coding them, but there are more existing examples/authors to draw from. Some issues brought up earlier that I am still exploring:* ***Why is the Fixed Effects performing so poorly*** *(though I think this an artifact of the code), and* ***Why is*** */rho*$_t$ ***upward biased****? (one option here is to simulate with different* ρ_t *vectors (ie one weighted more towards lower values of rho, another weighted towards higher values) and see how this effects the outcome).*

References

L. Anselin. *Spatial econometrics: methods and models*. Springer, 1988.

Luc Anselin. Under the hood : Issues in the specification and interpretation of spatial regression models. *Agricultural Economics*, 27(3):247–267, 2002.

Luc Anselin, Julie Le Gallo, and Hubert Jayet. Spatial panel econometrics. In *The Econometrics of Panel Data*, pages 625–660. 2008.

J. Bafumi and A. Gelman. Fitting multilevel models when predictors and group effects correlate. In *Annual Meeting of Midwest Political Science Association*, Chicago, IL, 2006.

B. H. Baltagi. *Econometric analysis of panel data*. 2008.

Badi H. Baltagi, Seuck Heun Song, Byoung Cheol Jung, and Won Koh. Testing for serial correlation, spatial autocorrelation and random effects using panel data. *Journal of Econometrics*, 140(1): 5–51, 2007.

B. Bartels. Beyond" fixed versus random effects": A framework for improving substantive and statistical analysis of panel, time-series cross-sectional, and multilevel data. *The Society for Political Methodology Working Paper*, 2008.

C. A. Bester, T. G. Conley, C. B. Hansen, and T. J. Vogelsang. Fixed-b asymptotics for spatially dependent robust nonparametric covariance matrix estimators. Technical report, 2009.

Xiaoheng Chen and Timothy G. Conley. A new semiparametric spatial model for panel time series. *Journal of Econometrics*, 105(1):59–83, 2001.

T. G. Conley. Gmm estimation with cross sectional dependence. *Journal of Econometrics*, 92(1): 1–45, 1999.

John C. Driscoll and Aart C. Kraay. Consistent covariance matrix estimation with spatially dependent panel data. *Review of Economics and Statistics*, 80(4):549–560, 1998.

Druska and Horrace. Generalized moments estimation for spatial panel data: Indonesian rice farming. *American Journal of Agricultural Economics*, 86(1):185–198, 2004. ISSN 1467-8276.

J. P. Elhorst. Spatial panel data models. In MM Fischer and A. Getis, editors, *Handbook of Applied Spatial Analysis*. Springer-Verlag, Berlin, 2009.

J. Paul Elhorst. Specification and estimation of spatial panel data models. *International Regional Science Review*, 26(3):244–268, July 1, 2003 2003.

Bernard Fingelton. Spatial autoregression. *Geographical Analysis*, 41(4):385–391, 2009. ISSN 1538-4632.

Bernard Fingelton and Julie Le Gallo. Estimating spatial models with endogenous variables, a spatial lag and spatially dependent disturbances: Finite sample properties*. *Papers in Regional Science*, 87(3):319–339, 2008. ISSN 1435-5957.

R. Florax and S. Rey. The impacts of misspecified spatial interaction in linear regression models. *New directions in spatial econometrics*, pages 111–135, 1995.

A. Getis and J. Aldstadt. Constructing the spatial weights matrix using a local statistic. *Geographical analysis*, 36(2):90–105, 2004.

D. A. Griffith. Some guidelines for specifying the geographics weight matrix contained in spatial statistical models. In *A Practical Handbook of Spatial Statistics*. CRC, 1996.

Mudit Kapoor, Harry H. Kelejian, and Ingmar R. Prucha. Panel data models with spatially correlated error components. *Journal of Econometrics*, 140(1):97–130, 2007.

H. H. Kelejian, I. R. Prucha, and Y. Yuzefovich. Estimation problems in models with spatial weighting matrices which have blocks of equal elements. *JOURNAL OF REGIONAL SCIENCE*, 46(3):507, 2006.

Harry H. Kelejian and Ingmar R. Prucha. 2sls and ols in a spatial autoregressive model with equal spatial weights. *Regional Science and Urban Economics*, 32(6):691–707, 2002.

Harry H. Kelejian and Ingmar R. Prucha. Hac estimation in a spatial framework. *Journal of Econometrics*, 140(1):131–154, 2007.

J. P. LeSage and R. K. Pace. *Introduction to spatial econometrics.* Chapman & Hall/CRC, 2009.

S. Openshaw. *The Modifiable Areal Unit Problem, Concepts and Techniques in Modern Geography.* Norwich: Geo Books. 1984.

S. Openshaw and P. J. Taylor. A million or so correlation coefficients: three experiments on the modifiable areal unit problem. *Statistical Applications in the Spatial Sciences*, 127:144, 1979.

Sergio J. Rey and Brett D. Montouri. Us regional income convergence: A spatial econometric perspective. *Regional Studies*, 33(2):143 – 156, 1999. ISSN 0034-3404

A. Skrondal and S. Rabe-Hesketh. *Generalized latent variable modeling: Multilevel, longitudinal, and structural equation models.* CRC Press, 2004.

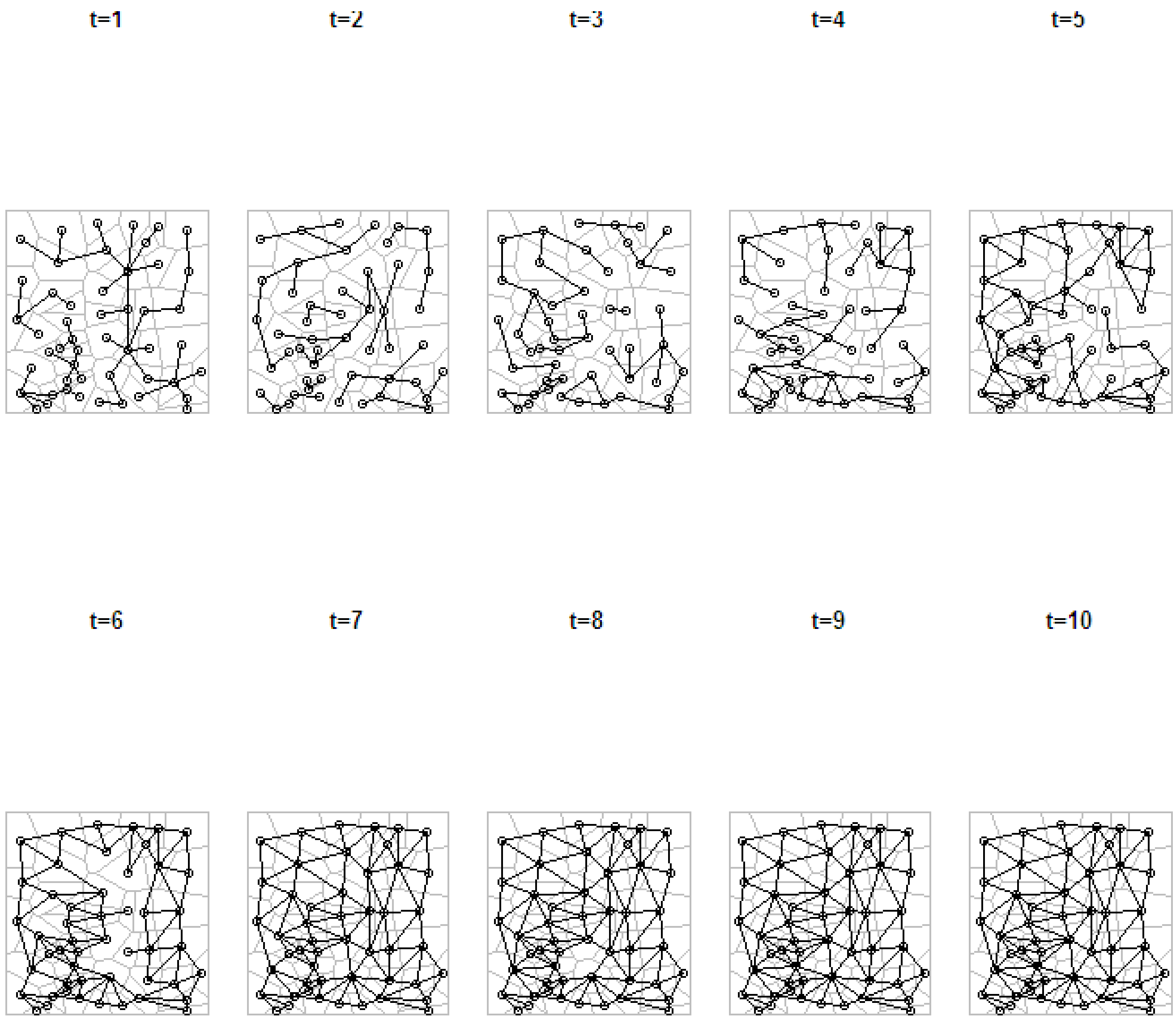

Figure 1: Time Varying W Matrix

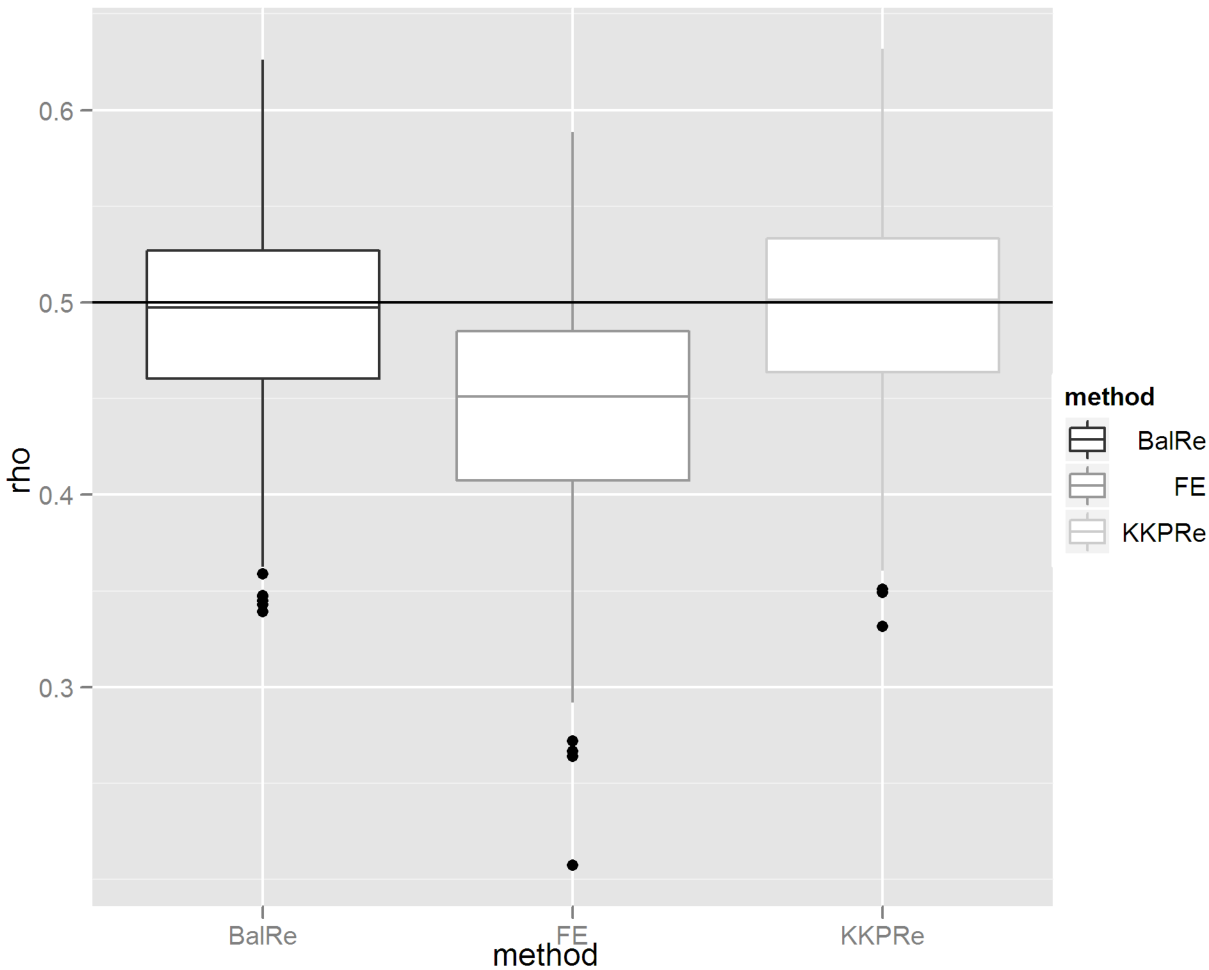

Figure 2: Sim 1: ρ and W constant (solid black line indicates true value of ρ)

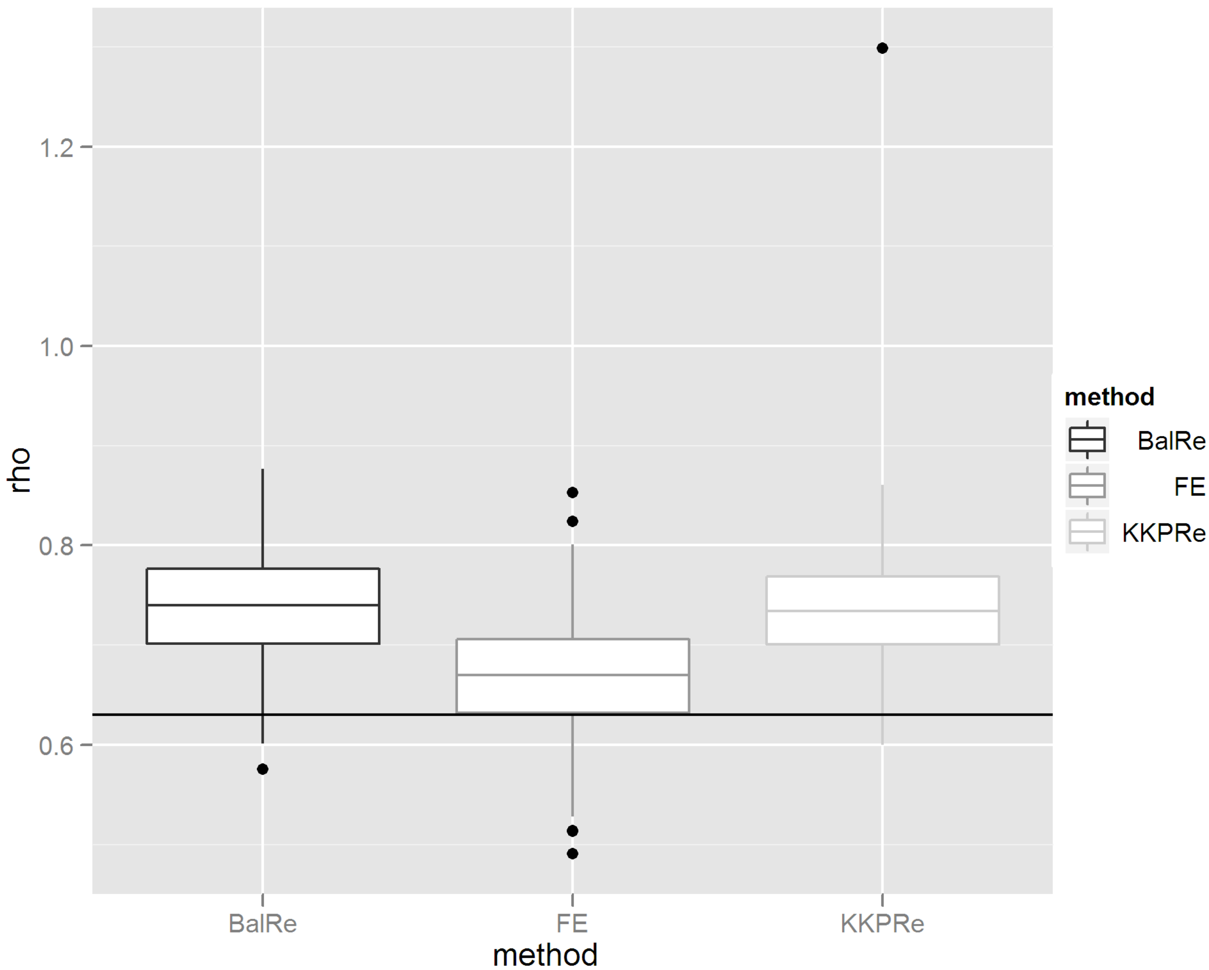

Figure 3: Sim 2: ρ varies, W constant (solid black line indicates true value of ρ)

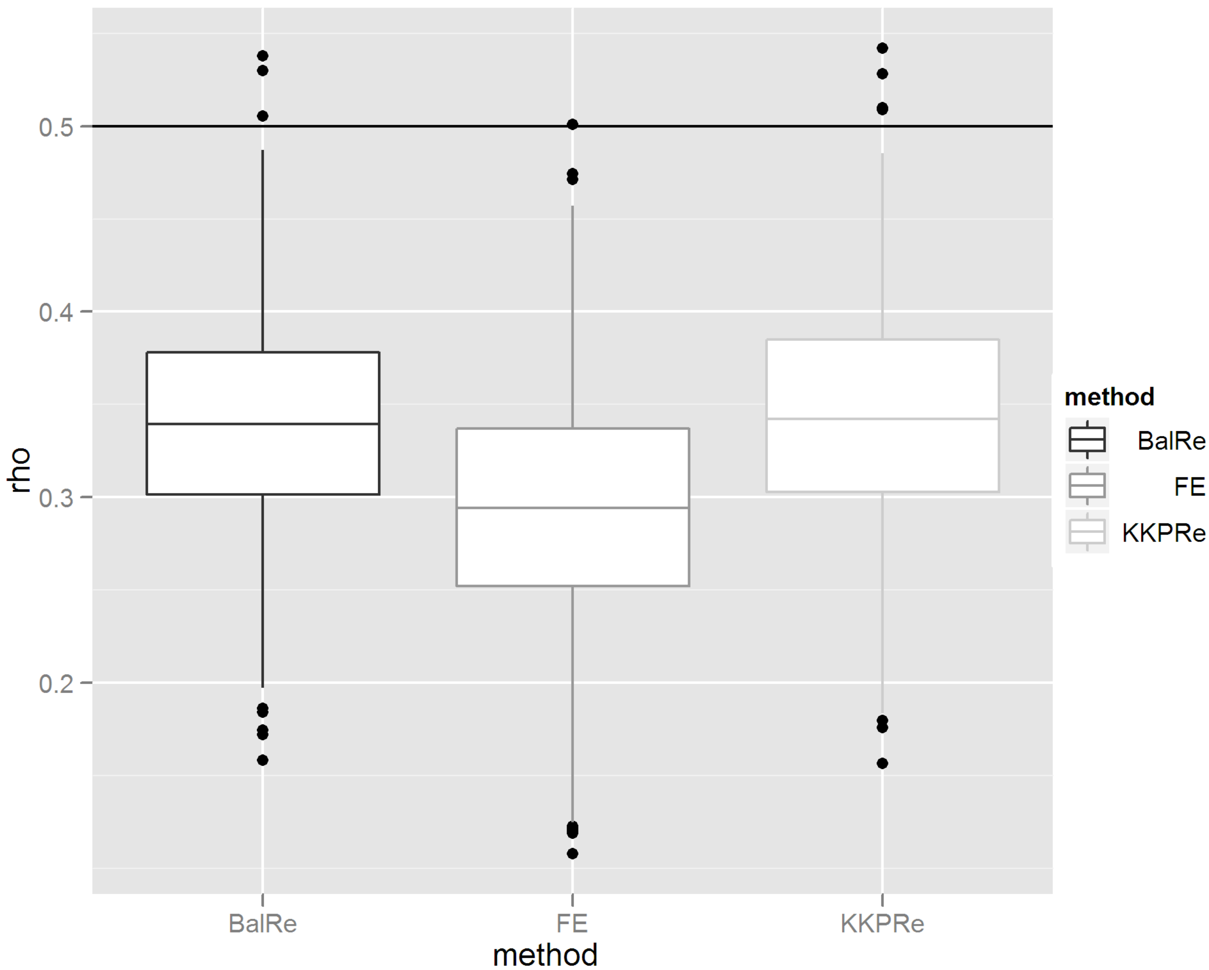

Figure 4: Sim 3: ρ constant, W varies (solid black line indicates true value of ρ)

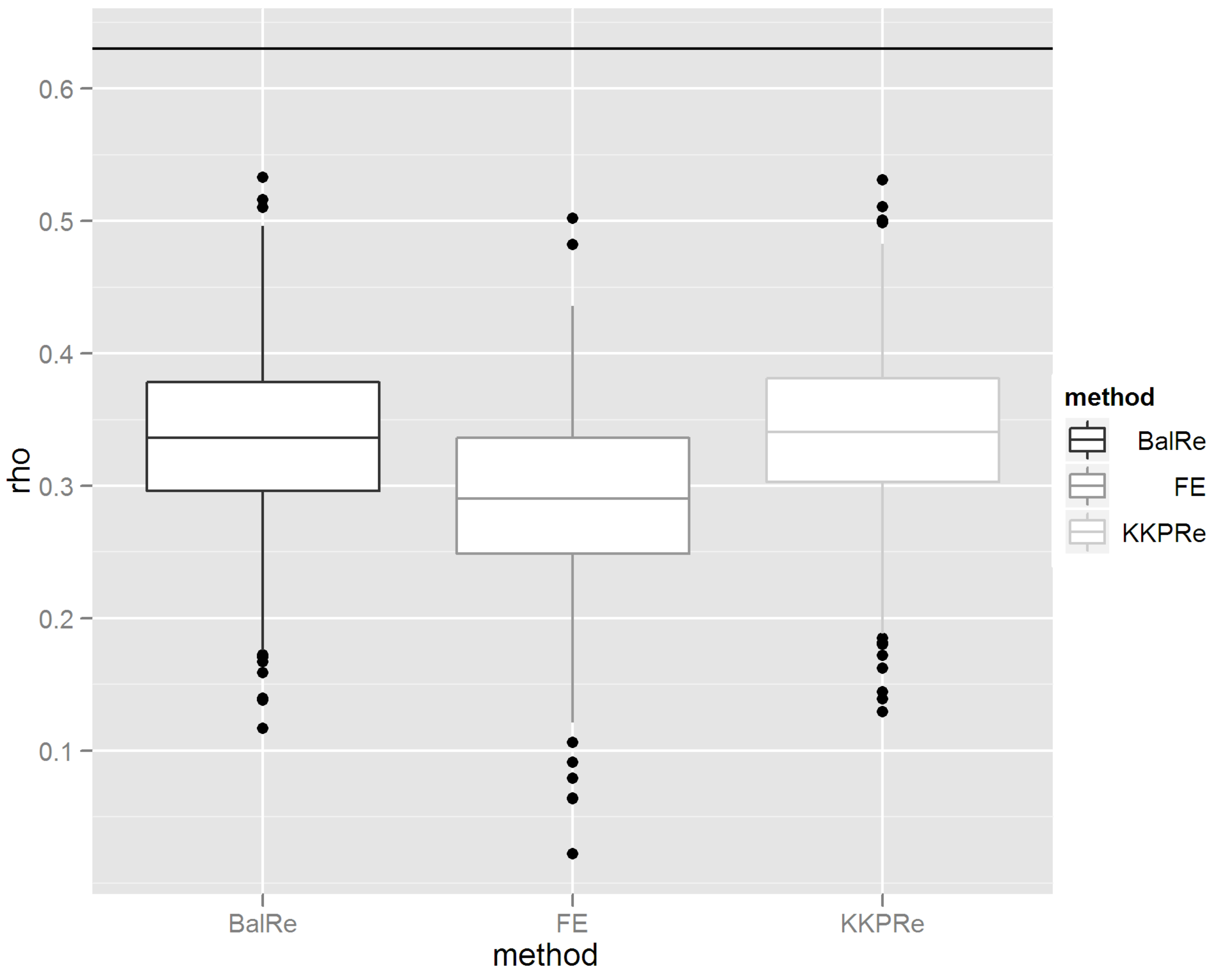

Figure 5: Sim 4: ρ and W vary (solid black line indicates true value of ρ)(solid black line indicates true value of ρ)

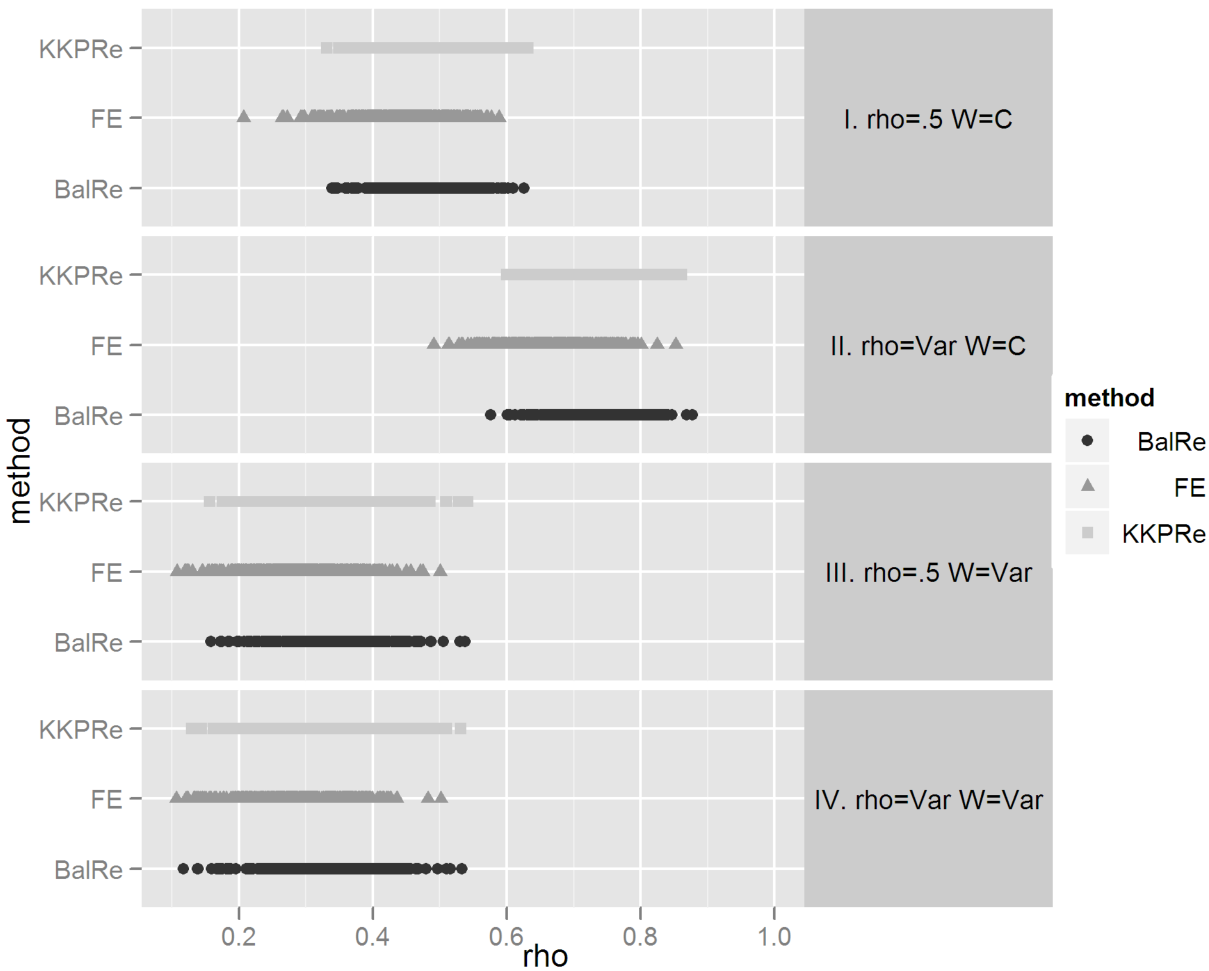

Figure 6: Summary of All Simulations: Comparing Performance of Different Methods within Scenarios

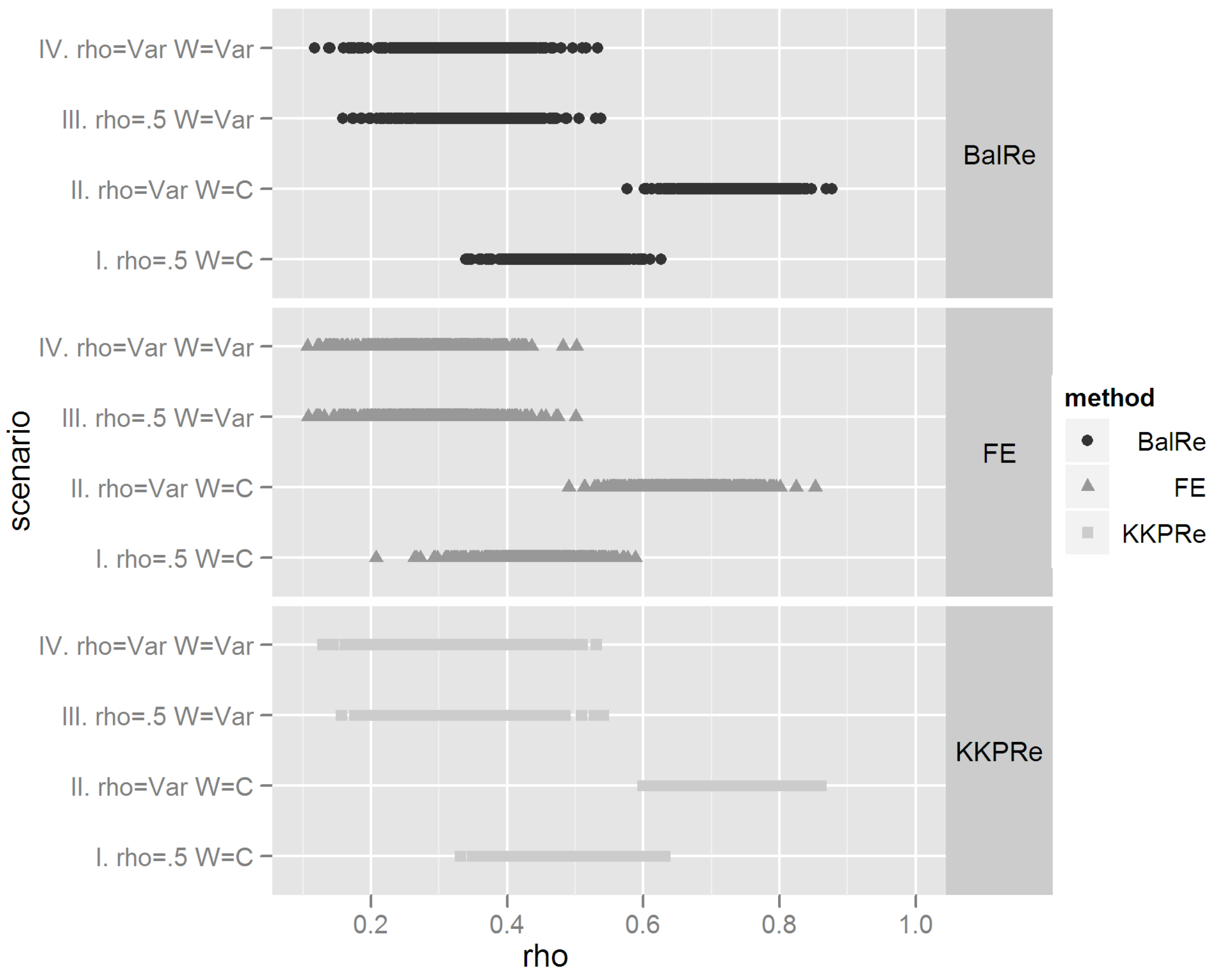

Figure 7: Summary of All Simulations: Comparing Performance of the same Method across Scenarios