Econ241a: PS 1

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Consider

...the problem of a farmer who plants seeds using labor L and grows a crop of size X. Suppose the production function is

$$X = \alpha L$$

and that the farmer's utility is

$$U(X, L) = X^{\beta} + (\bar{L} - L)$$

where $\bar{L} - L$ is leisure and $\beta < 1$. How many hours should the farmer put in and how does the marginal productivity of labor effect the decision?

- Marginal productivity of labor is α, since one additional unit of labor L will produce α additional units of crop.
- To optimize the labor, substitute the production function into the utility function and differentiate with respect to L, and set equal to zero:

$$U(X, L) = X^{\beta} + (\bar{L} - L) = (\alpha L)^{\beta} + (\bar{L} - L)$$

$$\frac{dU}{dL} = \beta \alpha^{\beta} L^{\beta - 1} - 1 = 0$$

$$L^* = \left(\frac{1}{\alpha^{\beta} \beta}\right)^{\beta - 1}$$

Since $\beta - 1$ is negative, turn it into $1 - \beta$ and flip the fraction:

$$L^* = (\alpha^{\beta} \beta)^{1 - \beta}$$

Write a Matlab function

...that plots the optimal labor supply against the marginal product of labor.

```
function [] = plotLstar(a_vec, b_vec)
%% Econ 241A lecture 1 assignment:
% This function plots the optimal labor against a grid of alpha and beta
% values provided by the user.
%
% The marginal product of labor is dX/dL = alpha.  Each additional unit
%   of labor increases output by alpha units.
%
% The optimal labor occurs when dU/dL = 0.
%   U(L) = (alpha*L)^beta + (L_bar - L) = 0
```

```
%   L* = [1/(a^b * b)]^(1 / (b - 1)) = (a^b * b) ^ (1 / (1 - b))
% ------------
% Input: a_vec and b_vec are vectors of possible values for parameters
%   alpha > 0 and 0 < beta < 1.  These vectors will be used to create
%   a grid upon which to calculate L*.
% ------------
% Output: none; simply plots the function.

%% use meshgrid to create a matrix of alpha and beta from vectors
%    provided by the user.
    [a, b] = meshgrid(a_vec, b_vec)

%% Calculate L_star; note matrix operators
    L_star = ((a.^b) .* b) .^ (1 ./ (1 - b))

%% Plot using surfc
    surfc(a, b, L_star)

end
```

Calling this function with `a_vec = [0:.2:2]` and `b_vec = [0:.1:.9]` results in:

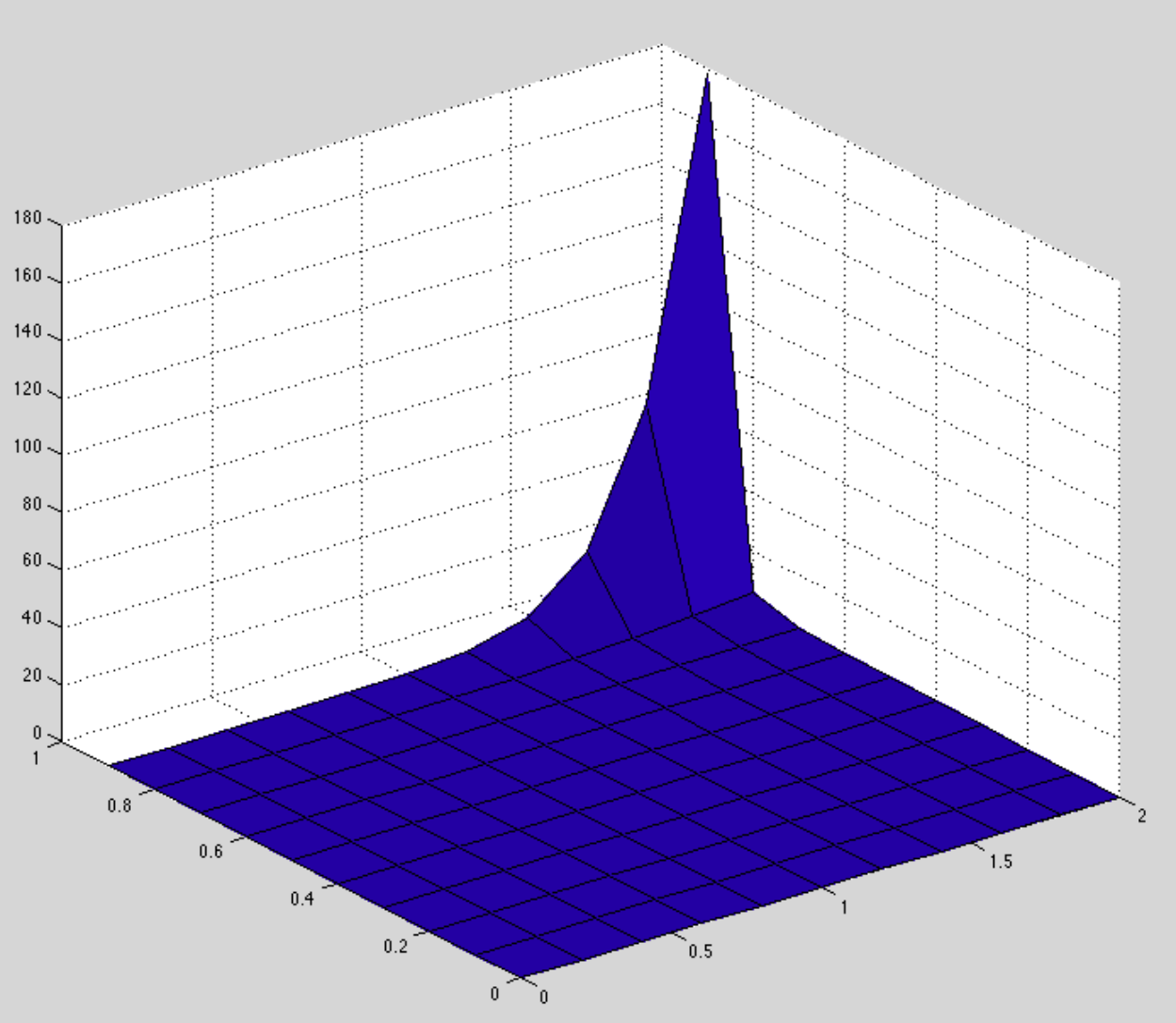

Figure 1: