Required Problems

1. Let $A = \{1, 2, 3, 4\}$. Discribe a codomain B and a function $f : A \to B$ such that f is
   (a) onto B but not one-to one.
   (b) one-to-one but not onto B.
   (c) both one-to-one and onto B.
   (d) neither one-to-one nor onto B.

2. Consider the sequence $\{x_n\}_{n=1}^{\infty}$ such that

$$x_n = \frac{n+1}{n}$$

   To what does this sequence converge? Prove that this sequence converges to that limit.

3. Let S and T be convex sets. Prove that the intersection of S and T is also a convex set.

4. The set $S^{n-1} = \{\mathbf{x} | \sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, \ldots, n\}$ is called the $(n-1)$-dimensional unit simplex.
   (a) Prove that S^{n-1} is a convex set.
   (b) Prove that S^{n-1} is a compact set.

Practice Problems

5. Give a relation r from $A = \{5, 6, 7\}$ to $B = \{3, 4, 5\}$ such that
   (a) r is not a function
   (b) r is a function from A to B with the range $\mathcal{R}(r) = B$
   (c) r is a function from A to B with the range $\mathcal{R}(r) \neq B$

6. Identify the domain and range of each of the following mappings:
   (a) $\left\{(x, y) \in \mathbb{R}^2 \middle| y = \frac{1}{x+1}\right\}$
   (b) $\{(x, y) \in \mathbb{N} \times \mathbb{N} | y = x + 5\}$
   (c) $\left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} \middle| y = \frac{x^2-4}{x-2}\right\}$

7. Recall the definition of the inverse image associated with the function $f : X \to Y$, i.e.,

$$f^{-1}(B) = \{x \in X | f(x) \in B\}$$

   If $B \subset Y$ and $C \subset Y$, prove that $f^{-1}(B \cup C) = f^{-1}(B) \cup f^{-1}(C)$.

8. For each of the following sequences, list the first three terms:
   (a) $a_n = \frac{n+1}{2n+3}$
   (b) $b_n = \frac{1}{n!}$
   (c) $c_n = 1 - 2^{-n}$

9. Prove that if $x_n \to L$ and $y_n \to M$, then $x_n + y_n \to L + M$.

10. Prove that if $a_n \to a$ and $a_n \leq b$ for all n, then $a \leq b$.

11. Consider the following intervals in $\mathbb{R}$. For each, determine if it is closed. If so, give a proof:

    (a) $(-\infty, b]$
    (b) $(a, b]$
    (c) $[a, \infty)$
    (d) $[a, b)$

12. Consider the following sets. If the set is bounded, provide an M and a $\mathbf{x}$ such that $B_M(\mathbf{x})$ contains the set.

    (a) $A = \{x | x \in \mathbb{R} \wedge x^2 \leq 10\}$
    (b) $B = \left\{x | x \in \mathbb{R} \wedge x + \frac{1}{x} < 5\right\}$
    (c) $C = \{(x, y) | (x, y) \in \mathbb{R}_+^2 \wedge xy < 1\}$
    (d) $D = \{(x, y) | (x, y) \in \mathbb{R} \wedge |x| + |y| \leq 10\}$

13. Prove that the following functions are continuous using epsilon-delta proofs.

    (a) $f(x) = x + 3$
    (b) $g(x) = x^2$
    (c) $h(x) = |x|$