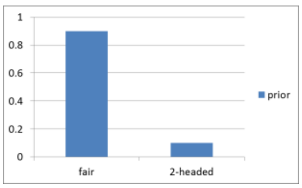
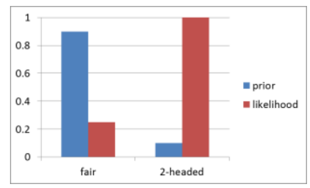
prior



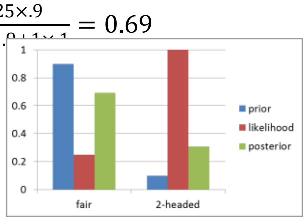
likelihood

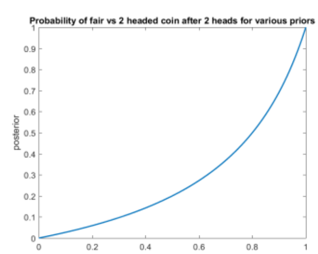
- $f(H=2|fair) = .5^2 = .25$
- $f(H=2|2\ headed) = 1^2 = 1$

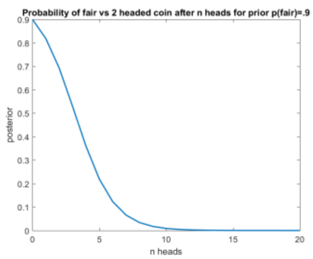


posterior

- $f(fair|H=2) = \frac{f(H=2|fair)\times f(fair)}{f(H=2|fair)\times f(fair)+f(H=2|2\ headed)\times f(2\ headed)}$
- $= \frac{.25\times.9}{.25\times.9+1\times.1} = 0.69$







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Normal mean with uniform prior

- Suppose $x_i \sim iidN(\mu, \sigma^2)$ with σ^2 known and I believe that θ is between $\pm c$, specifically, $\theta \sim U(-c, c)$.
- Likelihood: $\bar{x}|\theta \sim N\left(\theta, \frac{\sigma^2}{n}\right)$
- Prior: $f(\theta) = \begin{cases} \frac{1}{2c}, & -c < \theta < c \\ 0, & otherwise \end{cases}$
- Marginal likelihood:
- $f(\bar{x}) = \int_{-c}^{c} f(\bar{x}|\theta) \times \frac{1}{2c} d\theta = \frac{1}{2c}\left[\Phi\left(\frac{c-\theta}{\sqrt{\sigma^2/n}}\right) - \Phi\left(\frac{-c-\theta}{\sqrt{\sigma^2/n}}\right)\right]$
- Where $\int_{-\infty}^{b} \frac{1}{\sqrt{2\pi\sigma^2/n}} \exp\left(-\frac{1}{2\sigma^2/n}(x-\theta)^2\right) d\theta = \int_{-\infty}^{b} \frac{1}{\sqrt{2\pi\sigma^2/n}} \exp\left(-\frac{1}{2\sigma^2/n}(\theta-x)^2\right) d\theta$

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Normal mean with uniform prior

- $f(\theta|\bar{x}) = \begin{cases} \dfrac{\frac{1}{\sqrt{\sigma^2/n}}\phi\left((\theta-\bar{x})/\sqrt{\sigma^2/n}\right)\times\frac{1}{2c}}{\left[\Phi\left(\frac{c-\theta}{\sqrt{\sigma^2/n}}\right)-\Phi\left(\frac{-c-\theta}{\sqrt{\sigma^2/n}}\right)\right]\times\frac{1}{2c}}, & -c < \theta < c \\ 0, & otherwise \end{cases}$
-
- $\lim_{c\to\infty} f(\theta|\bar{x}) = \theta|\bar{x} \sim N\left(\bar{x}, \frac{\sigma^2}{n}\right)$

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