Economics 241B
Exercise 6

1. Asymptotic Behavior of Hypothesis Tests

Your fieldwork requires that you test $H_0 : \beta_k = \bar{\beta}_k$ vs. $H_1 : \beta_k \neq \bar{\beta}_k$.

a. You know that under H_0,

$$\sqrt{n}\left(B_k - \bar{\beta}_k\right) \xrightarrow{d} N\left(0, Avar\left(B_k\right)\right) \tag{1}$$
$$\widehat{Avar\left(B_k\right)} \xrightarrow{p} Avar\left(B_k\right),$$

where $\widehat{Avar\left(B_k\right)} = S_{xx}^{-1}\hat{S}S_{xx}^{-1}$. To perform your test, you wish to use a t-statistic and you want to make sure that the limit distribution is standard normal. Show that (1) implies that under H_0,

$$\frac{B_k - \bar{\beta}_k}{SE^*\left(B_k\right)} \xrightarrow{d} N\left(0, 1\right), \tag{2}$$

where $SE^*\left(B_k\right) = \sqrt{\frac{1}{n}\widehat{Avar\left(B_k\right)}}$.

b. When performing the test, under what circumstances would you select the critical values from a normal distribution versus a t distribution?

c. Show what $SE^*\left(B_k\right)$ converges in probability to.

d. With your answer from part c, intuitively explain the convergence in (2).

2. Consistency and Conditional Mean Independence

You want to estimate the following scalar equation (in deviation-from-means form):

$$y_t = \beta x_t + u_t.$$

There are two potentially important assumptions you can make:

(A1): $\mathbb{E}\left(x_t u_t\right) = 0$ for all t, and $\mathbb{E}\left(x_t^2\right) = \sigma_{xx} < \infty$

(A2): $\mathbb{E}\left(u_t | X\right) = 0$ for all t, and $\mathbb{E}\left(x_t^2\right) = \sigma_{xx} < \infty$.

a. Show that, under (A1), the OLS estimator is a consistent estimator for β.

b. Your coauthor suggests you run a first-difference model in which your outcome of interest is no longer the level of y in period t, but rather the change in y from time period $t-1$ to t.

Denote $\Delta y_t = y_t - y_{t-1}$ and $\Delta x_t = x_t - x_{t-1}$. Will an OLS regression of Δy_t on Δx_t yield a consistent estimator of β under (A1)? Will it yield a consistent estimator under (A2)?

c. Imagine you and your coauthor are working on a project in which you observe individuals (indexed by i) in two time-periods, $t = 1$ and $t = 2$. You observe outcomes ($y_{i,t}$) and a mean-zero scalar regressor ($x_{i,t}$) for each individual in both time periods. There also exists a characteristic specific to each individual that does not change over time (α_i) and that you cannot observe, yet it influences the outcome of interest. You and your coauthor want to estimate the following scalar equation

$$y_{i,t} = \alpha_i + \beta x_{i,t} + u_{i,t}.$$

The assumptions that correspond to A1 and A2 are, for each value of i:

(A1′): $\mathbb{E}\left(x_{i,t}u_{i,t}\right) = 0$ for all t, and $\mathbb{E}\left(x_{i,t}^2\right) = \sigma_{xx} < \infty$

(A2′): $\mathbb{E}\left(u_{i,t}|X\right) = 0$ for all t, and $\mathbb{E}\left(x_{i,t}^2\right) = \sigma_{xx} < \infty$.

If α_i is uncorrelated with $x_{i,t}$ and $x_{i,t-1}$, will the OLS estimator be a consistent estimator for β under (A1′)?

d. Suppose you have reason to believe that the individual characteristic may in fact be correlated with the regressor of interest. In light of your answer in part b, can you construct a consistent estimator for β under (A1′)? Can you construct a consistent estimator for β under (A2′)?