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This folder contains the R code and the datasets used for the glacier application appearing in Section 5 of the paper. If you wish to reproduce the results, we recommend that you run the different code steps in the order they appear. However, you should be warned that the entire process might require over a day, therefore we suggest that you save the different folders in the proposed file architecture, and run only parts 3 and 7 (maximin geodesic distance and IMSPE-optimal designs, respectively). Hereby is a short description of the different steps:

- **1 - Glacier_Scaling.R** : reads the original locations on the glacier from the file Glacier_Interior_and_Boundary.txt (in the /Auxiliary_Files directory, first 247 rows define the glacier's boundary), scales and centers them so that they are contained in the unit cube and writes the new locations into the Glacier_Interior_and_Boundary_Scaled.txt file in the same directory.

- **2 - Pre_Optimization.R** : sources the functions file Pre_Optimization_Functions.R, performs Isomap on the glacier data (using a 2×10^{-5} ball about each location to define its neighbors) and writes the following files into the /Auxiliary_Files directory:

  - Glacier_Embeddings_4D.txt : the original locations in their representation in the new coordinate system in the embedding space after MDS.
  - Glacier_Geodesic_Distances.txt : The approximate geodesic distance matrix for the candidates.

- **3 - Glacier_Maximin_Design.R** : reads the set of scaled locations Glacier_Interior_and_Boundary_Scaled.txt and its respective list of embedded locations Glacier_Embeddings_4D.txt and chooses an optimal subset of size design.size using an exchange algorithm, based on the Maximin geodesic distance criterion. The results are written into the /Glacier_Maximin_Designs directory, in the form of the following files:
  - Glacier_Size_(design.size)_Maximin_Design.txt : the resultant design in the original coordinates.
  - Glacier_Size_(design.size)_Embedded_Maximin_Design.txt : the resultant design in the new coordinate system in the embedding space.

- 4 - **Embedding_Integration_Pts.R** : reads a set of 5922 locations on the glacier from the file Glacier_Integration_Points.txtGlacier_Integration_Points.txt in the /Auxiliary_Files directory and

uses the Nystrom approximation to embed those in the higher dimensional space, writing the output into Embedded_Integration_Pts_Glacier_Scaled_all.txt in the same directory.

- **5 - Glacier_Uniform_Integration_Pts.R** : reads Embedded_Integration_Pts_Glacier_Scaled_all.txt (written in the previous step) and selects a subset of uniformly spaced location for integration over the embedded region, using the maximin geodesic distance (more precisely, its approximation) criterion. The selected subset is written into the file Glacier_Uniform_Integration_Pts.txt in the /Auxiliary_Files directory.

- **6 - Embedding_Glacier_Data_2009.R** : reads the 2009 glacier ablation data set (consisting of 14 observations) from the Glacier_Data_2009.txt file in the /Glacier_2009_Measurements directory, scales it so that it is contained in the unit cube (in a way that is consistent with the scaling of the candidates points), uses Nystrom's method to embed it in the 4-dimensional region and writes the embedded data set into the Embedded_2009_Data_Glacier_Scaled.txt file in the same directory.

- 7 - **Glacier_IMSPE_Optimal_Design.R** : Sources the functions file IMSPE_Optimal_Design_Functions.R and generates an IMSPE-optimal design of any size on the glacier, based on the geodesic distance. Uses the maximin geodesic distance design of the same size, generated in step 3 and the maximum likelihood estimators of the Gaussian process parameters, estimated from the 2009 data. The results are written into Glacier_Size_(design.size)_Embedded_IMSPE_Design_2009.txt and Glacier_Size_(design.size)_IMSPE_Design_2009.txt in the /Glacier_IMSPE_Optimal_Designs/2009_Designs directory.