Evaluating Performance of Tarp Classifiers

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We use 10-fold cross-validation to evaluate the performance of 5 classifiers. A classifier will classify a pixel as belonging to a class in the set $\{Tarp, Vegetation\}$. A pixel is represented by a tuple of 3 integers. These integers represent intensities of color red, green, and blue and lie between 0 and 255 inclusive.

We load a data frame of classes and pixels based on an orthorectified image of Haiti at https://www.kaggle.com/datasets/billbasener/pixel-values-from-images-over-haiti?datasetId=1899167.

```
data_frame_of_classes_and_pixels <- read.csv(file = 'Data_Frame_Of_Classes_And_Pixels.csv')
head(x = data_frame_of_classes_and_pixels, n = 3)
```

```
#        Class Red Green Blue
# 1 Vegetation  64    67   50
# 2 Vegetation  64    67   50
# 3 Vegetation  64    66   49
```

TODO: Consider 5 number summaries and means and standard deviations of Red, Green, and Blue. and across classes.

According to https://stats.oarc.ucla.edu/r/dae/multinomial-logistic-regression/, "Below we use the `multinom` function from the `nnet` package to estimate a multinomial logistic regression model… we need to choose the level of our outcome that we wish to use as our baseline and specify this in the `relevel` function. Then, we run our model using `multinom`. The `multinom` package does not include `p`-value calculation for the regression coefficients, so we calculate `p`-values using Wald tests (here [two-tailed] z-tests)."

```
library(nnet)
factor_Class <- factor(x = data_frame_of_classes_and_pixels$Class)
data_frame_of_classes_and_pixels$Class <- relevel(
    x = factor_Class,
    ref = "Blue Tarp"
)
logistic_regression_model <- nnet::multinom(
    formula = Class ~ Red + Green + Blue,
    data = data_frame_of_classes_and_pixels
)
```

```
# # weights:  25 (16 variable)
# initial  value 101782.463020
# iter  10 value 77447.218043
# iter  20 value 29230.666252
# iter  30 value 24123.130528
# iter  40 value 23510.543863
```

```
# iter  50 value 23143.504515
# final  value 23072.994639
# converged
```

```
summary_of_logistic_regression_model <- summary(object = logistic_regression_model)
summary_of_logistic_regression_model
```

```
# Call:
# nnet::multinom(formula = Class ~ Red + Green + Blue, data = data_frame_of_classes_and_pixels)
#
# Coefficients:
#                   (Intercept)       Red      Green       Blue
# Rooftop             -3.001052 0.2394754 0.08565332 -0.3060410
# Soil               -11.994187 0.3168945 0.13645875 -0.4121712
# Various Non-Tarp    -2.917270 0.2516826 0.12434250 -0.3717489
# Vegetation          18.034812 0.1572812 0.38224636 -0.8164165
#
# Std. Errors:
#                   (Intercept)         Red      Green       Blue
# Rooftop           0.062157114 0.009033954 0.01213455 0.01209590
# Soil              0.086030376 0.008986432 0.01224512 0.01220922
# Various Non-Tarp 0.065428816 0.009095895 0.01226350 0.01221860
# Vegetation        0.003927454 0.011140036 0.01380552 0.01443139
#
# Residual Deviance: 46145.99
# AIC: 46177.99
```

```
coefficients <- summary_of_logistic_regression_model$coefficients
standard_errors <- summary_of_logistic_regression_model$standard.errors
z_scores <- coefficients / standard_errors
magnitudes_of_z_score <- abs(x = z_scores)
cumulative_density_function_values <- pnorm(q = magnitudes_of_z_score, mean = 0, sd = 1)
areas_in_one_tail <- 1 - cumulative_density_function_values
p_values <- areas_in_one_tail * 2
p_values
```

```
#                   (Intercept) Red        Green Blue
# Rooftop                     0   0 1.681544e-12    0
# Soil                        0   0 0.000000e+00    0
# Various Non-Tarp            0   0 0.000000e+00    0
# Vegetation                  0   0 0.000000e+00    0
```

The final negative log-likelihood l of our logistic regression model is $23,072.995$. The residual deviance $d = 2l = 46,145.990$.

TODO: Compare nested logistic regression models using residual deviance d.

The summary for our logistic regression model includes a data frame of coefficients. Each row of coefficients corresponds to a model equation. Interpreting the rows,

$$\begin{aligned} ln\left[\frac{P(Class=Rooftop)}{P(Class=Blue\ Tarp)}\right] &= \beta_{Rooftop,\ Intercept} + \beta_{Rooftop,\ Red}\ Red + \beta_{Rooftop,\ Green}\ Green + \beta_{Rooftop,\ Blue}\ Blue \\ &= -3.001 + 0.239\ Red + 0.0857\ Green - 0.306\ Blue \end{aligned}$$

$$
\begin{aligned}
ln\left[\frac{P(Class=Soil)}{P(Class=Blue\ Tarp)}\right] &= \beta_{Soil,\ Intercept} + \beta_{Soil,\ Red}\ Red + \beta_{Soil,\ Green}\ Green + \beta_{Soil,\ Blue}\ Blue \\
&= -11.994 + 0.317\ Red + 0.136\ Green - 0.412\ Blue
\end{aligned}
$$

Odds and relative risk are synonymous. An increase of 1 unit in predictor *Red* is associated with a change of 0.239 in the log odds of a pixel depicting a rooftop versus a pixel depicting a blue tarp. An increase of 1 unit in predictor *Blue* is associated with a change of -0.412 in the log odds of a pixel depicting soil versus a pixel depicting a blue tarp. Each predictor coefficient represents the log odds for a change of 1 unit in the predictor.

```
exp(x = coefficients)
```

```
#                    (Intercept)      Red    Green      Blue
# Rooftop           4.973474e-02 1.270582 1.089429 0.7363565
# Soil              6.180031e-06 1.372858 1.146208 0.6622109
# Various Non-Tarp  5.408115e-02 1.286188 1.132404 0.6895274
# Vegetation        6.798597e+07 1.170325 1.465573 0.4420128
```

The odds of a pixel depicting various non-tarp objects versus a pixel depicting a blue tarp is 1.132 for an increase of 1 unit in predictor *Green*. The odds of a pixel depicting vegetation versus a pixel depicting a blue tarp is 0.442 for an increase of 1 unit in predictor *Blue*.

The predicted probabilities for our first 3 observations and each class are presented below.

```
predicted_probabilities <- fitted(object = logistic_regression_model)
head(x = predicted_probabilities, n = 3)
```

```
#      Blue Tarp      Rooftop         Soil Various Non-Tarp Vegetation
# 1 2.521696e-06 3.992604e-05 1.049958e-07     4.741077e-05  0.9999100
# 2 2.521696e-06 3.992604e-05 1.049958e-07     4.741077e-05  0.9999100
# 3 1.633587e-06 3.224178e-05 8.961114e-08     3.933451e-05  0.9999267
```

The below data frame allows us to consider the changes in predicted probability associated with holding the intensity of color *Red* equal to the mean intensity of color *Red*, holding the intensity of color *Green* equal to the mean intensity of color *Green*, and increasing the intensity of color *Blue* from 0 to 255 inclusive linearly across 10 intensities. As the intensity of color *Blue* increases from 0 to 255, the predicted probability of a pixel depicting a blue tarp increases from 0 to 1 and the predicted probability of a pixel depicting vegetation decreases from 1 to 0.

```
library(pracma)
mean_intensity_of_color_Red <- mean(data_frame_of_classes_and_pixels$Red)
mean_intensity_of_color_Green <- mean(data_frame_of_classes_and_pixels$Green)
linearly_spaced_intensities <- pracma::linspace(x1 = 0, x2 = 255, n = 10)
data_frame <- data.frame(
    Red = mean_intensity_of_color_Red,
    Green = mean_intensity_of_color_Green,
    Blue = linearly_spaced_intensities
)
predicted_probabilities <- predict(
    object = logistic_regression_model,
    newdata = data_frame,
    type = "probs"
)
predicted_probabilities
```

```
#       Blue Tarp      Rooftop         Soil Various Non-Tarp   Vegetation
# 1  3.355845e-45 7.744803e-24 7.139919e-19     2.351567e-20 1.000000e+00
# 2  3.730912e-35 1.476340e-17 6.728758e-14     6.966213e-15 1.000000e+00
# 3  4.147898e-25 2.814247e-11 6.341275e-09     2.063650e-09 1.000000e+00
# 4  4.605674e-15 5.357846e-05 5.968569e-04     6.105587e-04 9.987390e-01
# 5  1.504899e-07 3.001701e-01 1.653153e-01     5.315791e-01 2.935305e-03
# 6  2.239992e-03 7.660713e-01 2.085838e-02     2.108303e-01 3.929878e-09
# 7  9.423066e-01 5.525580e-02 7.437979e-05     2.363225e-03 1.487003e-16
# 8  9.999899e-01 1.005412e-05 6.690936e-10     6.682435e-08 1.419391e-26
# 9  1.000000e+00 1.723897e-09 5.671786e-15     1.780596e-12 1.276713e-36
# 10 1.000000e+00 2.955794e-13 4.807822e-20     4.744513e-17 1.148366e-46
```