DS-6030 Homework Module 7

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8. In the lab, a classification tree was applied to the Carseats data set after converting Sales into a qualitative response variable.

   Now we will seek to predict Sales using regression trees and related approaches, treating the response as a quantitative variable.

   (a) Split the data set into a training set and a test set.

```
set.seed(1)
library(ISLR2)
library(TomLeversRPackage)
training_and_testing_data <- split_data_set_into_training_and_testing_data(
    Carseats,
    proportion_of_training_data = 0.9
)
training_data <- training_and_testing_data$training_data
testing_data <- training_and_testing_data$testing_data
head(training_data, n = 2)
```

```
#     Sales CompPrice Income Advertising Population Price ShelveLoc Age Education
# 324 10.36       107    105          18        428   103    Medium  34        12
# 167  6.71       119     67          17        151   137    Medium  55        11
#     Urban  US
# 324   Yes Yes
# 167   Yes Yes
```

```
head(testing_data, n = 2)
```

```
#     Sales CompPrice Income Advertising Population Price ShelveLoc Age Education
# 8   11.85       136     81          15        425   120      Good  67        10
# 123  6.88       119    100           5         45   108    Medium  75        10
#     Urban  US
# 8     Yes Yes
# 123   Yes Yes
```

   (b) Fit a regression tree to the training set. Plot the tree, and interpret the results. What test MSE do you obtain?

```
library(tree)
full_tree <- tree(Sales ~ ., data = training_data)
summary(full_tree)
```

```
#
# Regression tree:
# tree(formula = Sales ~ ., data = training_data)
# Variables actually used in tree construction:
# [1] "ShelveLoc"   "Price"        "Age"            "Income"       "CompPrice"
# [6] "Advertising"
# Number of terminal nodes:  17
# Residual mean deviance:  2.653 = 910.1 / 343
# Distribution of residuals:
#     Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
# -5.18600 -1.09000  0.05305  0.00000  1.08300  4.63100
```

```
plot(full_tree)
text(full_tree, pretty = 0)
```
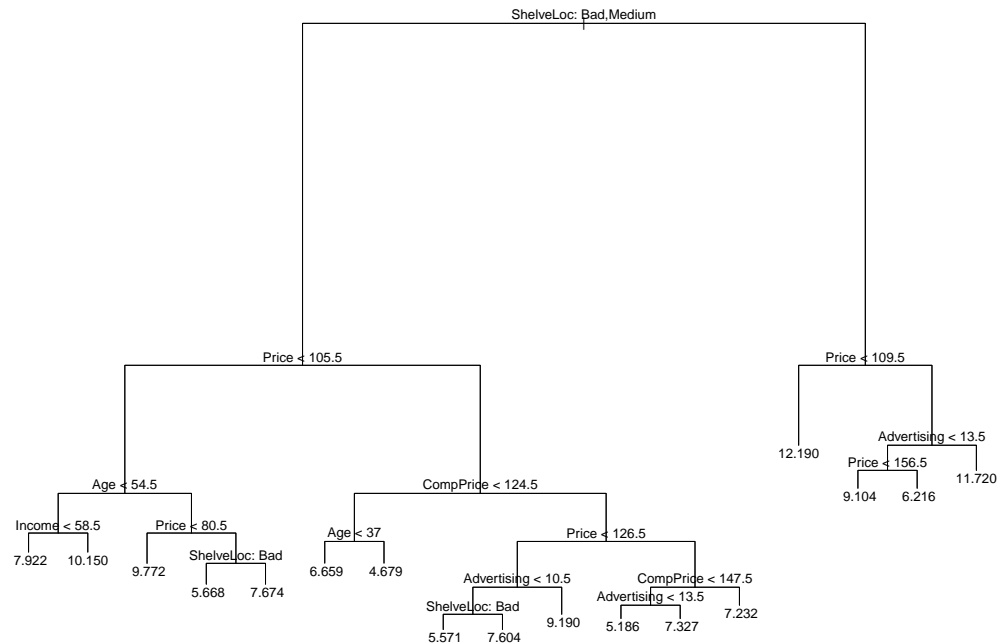


```
vector_of_predicted_sales <- predict(full_tree, newdata = testing_data)
vector_of_actual_sales <- testing_data$Sales
calculate_mean_squared_error(vector_of_predicted_sales, vector_of_actual_sales)
```

```
# [1] 4.896065
```

When shelf location is good and price is less than 109.5 monetary units, our tree predicts that 12.190 thousand child car seats will be sold at each location in each time period.

The test Mean Squared Error of our tree when predicting sales is 4.896 $thousand^2$.

(c) Use cross-validation in order to determine the optimal level of tree complexity. Does pruning the tree improve the test MSE?

```
object_of_types_prune_and_tree_sequence <- cv.tree(full_tree)
vector_of_numbers_of_leaves <- object_of_types_prune_and_tree_sequence$size
vector_of_deviances <- object_of_types_prune_and_tree_sequence$dev
plot(vector_of_numbers_of_leaves, vector_of_deviances, type = "l")
index_of_minimum_deviance <- which.min(vector_of_deviances)
optimal_number_of_leaves <-
    vector_of_numbers_of_leaves[index_of_minimum_deviance]
minimum_deviance <- min(vector_of_deviances)
points(
    optimal_number_of_leaves,
    minimum_deviance,
    col = "red",
    cex = 2,
    pch = 20
)
```
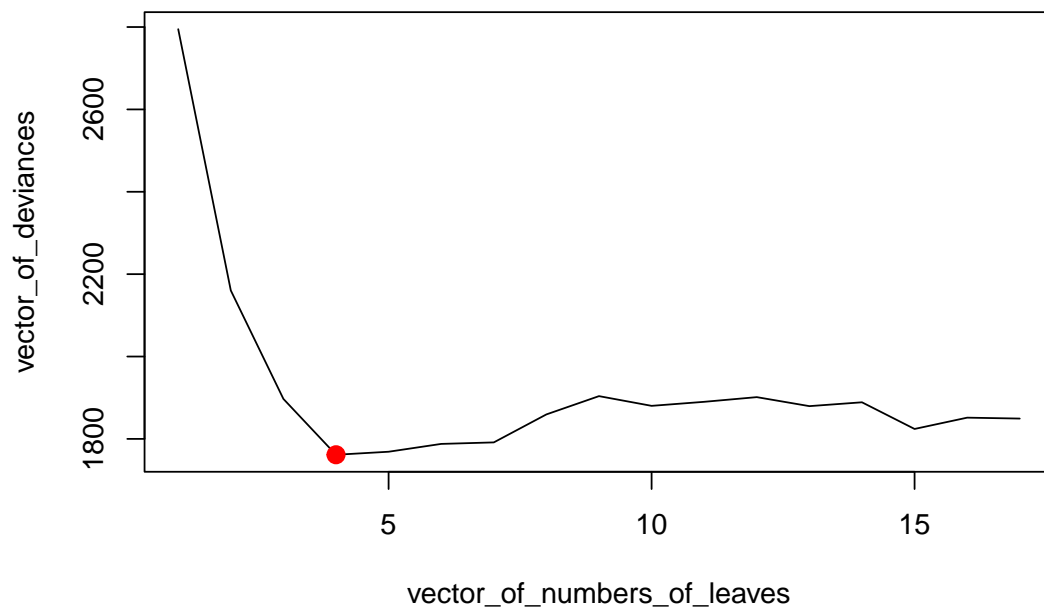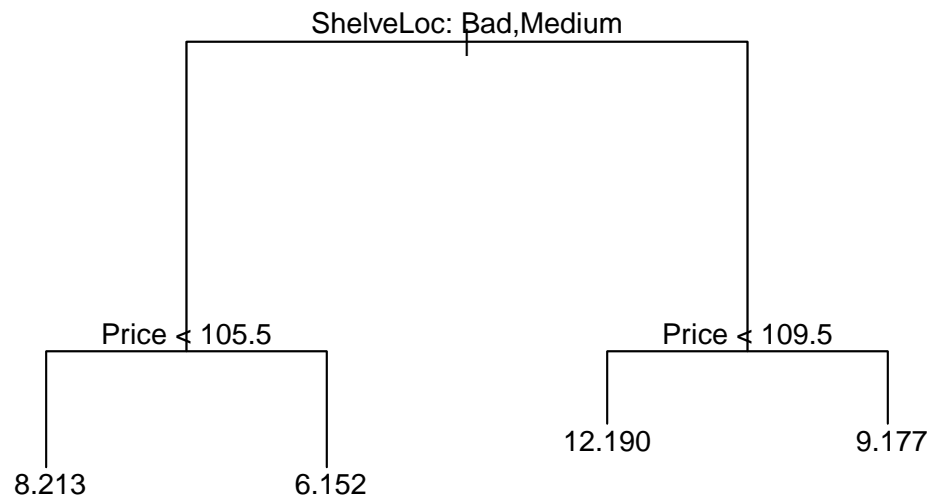


```
pruned_tree <- prune.tree(full_tree, best = optimal_number_of_leaves)
plot(pruned_tree)
text(pruned_tree, pretty = 0)
```

ShelveLoc: Bad,Medium

Price < 105.5   Price < 109.5

                                    12.190   9.177

8.213   6.152

```
vector_of_predicted_sales <- predict(pruned_tree, newdata = testing_data)
calculate_mean_squared_error(vector_of_predicted_sales, vector_of_actual_sales)
```

```
# [1] 6.785063
```

The test Mean Squared Error for the pruned tree is greater and less desirable than the Mean Squared Error for the full tree.

(d) Use the bagging approach in order to analyze this data. What test MSE do you obtain? Use the `importance()` function to determine which variables are most important.

Per *An Introduction to Statistical Learning* (Second Edition), bagging "is simply a special case of a random forest with [the number of variables randomly sampled as candidates at each split] $m = p$ the number of predictors."

```
library(randomForest)
```

```
# randomForest 4.7-1.1
```

```
# Type rfNews() to see new features/changes/bug fixes.
```

```
index_of_column_Sales <-
    get_index_of_column_of_data_frame(training_data, "Sales")
data_frame_of_predictors <- training_data[, -index_of_column_Sales]
data_frame_of_sales <- training_data[, index_of_column_Sales]
number_of_predictors <- ncol(data_frame_of_predictors)
get_test_MSE_and_vector_of_ordered_percents_increase_in_MSE_for_random_forest <-
    function(mtry) {
    the_randomForest <- randomForest(
        formula = Sales ~ .,
        data = training_data,
```

```
        mtry = mtry,
        importance = TRUE
    )
    vector_of_predicted_sales <-
        predict(the_randomForest, newdata = testing_data)
    test_MSE <- calculate_mean_squared_error(
        vector_of_predicted_sales,
        vector_of_actual_sales
    )
    matrix_of_importance_metrics <- importance(the_randomForest)
    vector_of_percents_increase_in_MSE <-
       matrix_of_importance_metrics[, "%IncMSE"]
    vector_of_indices_of_ordered_percents_increase_in_MSE <-
        order(vector_of_percents_increase_in_MSE, decreasing = TRUE)
    vector_of_ordered_percents_increase_in_MSE <-
        vector_of_percents_increase_in_MSE[
            vector_of_indices_of_ordered_percents_increase_in_MSE
        ]
    list_of_test_MSE_and_vector_of_ordered_percents_increase_in_MSE_for_random_forest <-
        list(
            test_MSE = test_MSE,
            vector_of_ordered_percents_increase_in_MSE =
                vector_of_ordered_percents_increase_in_MSE
        )
    return(
        list_of_test_MSE_and_vector_of_ordered_percents_increase_in_MSE_for_random_forest
    )
}
get_test_MSE_and_vector_of_ordered_percents_increase_in_MSE_for_random_forest(
    mtry = number_of_predictors
)
```

```
# $test_MSE
# [1] 2.912954
#
# $vector_of_ordered_percents_increase_in_MSE
#   ShelveLoc       Price  CompPrice Advertising         Age      Income
#   81.267672   79.323197  38.593171   25.822124   25.716498   14.313945
#   Education          US      Urban  Population
#    2.885808    2.270115  -1.691179   -2.211183
```

The test Mean Squared Error for our bootstrap aggregation (BAg) is 2.974, which is 0.607 of the MSE for our full tree and 0.438 of the MSE for our pruned tree.

According to In a random forest, is larger %IncMSE better or worse?, "%IncMSE is the most robust and informative measure. IT is the increase in mse of predictions(estimated with out-of-bag-CV) as a result of variable j being permuted(values randomly shuffled)... the higher the number, the more important."

$\%IncMSE$ is highest for *ShelveLoc* followed by *Price*; *ShelveLoc* and *Price* are the two most important variables.

(e) Use random forests to analyze this data. What test MSE do you obtain? Use the `importance()` function to determine which variables are most important. Describe the effect of m, the number of variables considered at each split, on the error rate obtained.

```r
data_frame_of_values_of_mtry_and_test_MSEs <- data.frame(
    matrix(NA, nrow = number_of_predictors, ncol = 2)
)
colnames(data_frame_of_values_of_mtry_and_test_MSEs) <- c("mtry", "test_MSE")
for (mtry in 1:number_of_predictors) {
    print(paste("mtry: ", mtry, sep = ""))
    data_frame_of_values_of_mtry_and_test_MSEs[mtry, "mtry"] <- mtry
    test_MSE_and_vector_of_ordered_percents_increase_in_MSE <-
        get_test_MSE_and_vector_of_ordered_percents_increase_in_MSE_for_random_forest(
            mtry = mtry
        )
    test_MSE <- test_MSE_and_vector_of_ordered_percents_increase_in_MSE$test_MSE
    vector_of_ordered_percents_increase_in_MSE <-
        test_MSE_and_vector_of_ordered_percents_increase_in_MSE$
            vector_of_ordered_percents_increase_in_MSE
    print(vector_of_ordered_percents_increase_in_MSE)
    data_frame_of_values_of_mtry_and_test_MSEs[mtry, "test_MSE"] <- test_MSE
}
```

```
# [1] "mtry: 1"
#    ShelveLoc        Price          Age  Advertising    CompPrice           US
#   27.4145011   22.6376857   12.1022390   11.8362136    9.1428466    6.3777495
#       Income    Education        Urban   Population
#    5.7156306    2.2635817   -0.2072851   -0.9091637
# [1] "mtry: 2"
#    ShelveLoc        Price  Advertising          Age    CompPrice       Income
#    44.880078    37.860838    17.212010    16.137100    14.665327     6.413410
#          US    Education        Urban   Population
#     6.210358     1.257871    -1.022202    -1.798896
# [1] "mtry: 3"
#    ShelveLoc        Price  Advertising          Age    CompPrice       Income
#    57.917795    48.708554    19.677702    19.528156    18.673141     7.957699
#          US    Education   Population        Urban
#     5.868734     3.956120    -2.433051    -2.642151
# [1] "mtry: 4"
#    ShelveLoc        Price    CompPrice          Age  Advertising       Income
#    60.737660    56.391682    23.604401    21.854985    18.236436     8.220434
#          US    Education        Urban   Population
#     6.106552     1.320007    -1.574042    -1.839350
# [1] "mtry: 5"
#    ShelveLoc        Price    CompPrice          Age  Advertising       Income
#    68.562969    61.860220    25.200712    23.425230    20.589795    10.621514
#          US    Education   Population        Urban
#     5.444433     3.662041    -1.106284    -2.907471
# [1] "mtry: 6"
#    ShelveLoc        Price    CompPrice          Age  Advertising       Income
#    76.681110    67.104963    29.750847    24.706524    21.958387    11.035165
#          US    Education        Urban   Population
#     3.985181     2.732473    -1.483772    -2.223225
# [1] "mtry: 7"
#    ShelveLoc        Price    CompPrice          Age  Advertising       Income
#   79.7220186   72.1876642   34.2439225   24.6451061   23.4062353   11.4780558
#          US    Education   Population        Urban
#    4.3765943    2.7168918   -0.4954921   -1.9197438
```

6

```
# [1] "mtry: 8"
#    ShelveLoc        Price    CompPrice          Age Advertising        Income
#  79.5222605   74.7577736   33.7322095   24.6121401   24.4295814   14.3477921
#          US    Education   Population        Urban
#   4.9868483    2.4070209   -0.2734359   -1.8060475
# [1] "mtry: 9"
#    ShelveLoc        Price    CompPrice          Age Advertising        Income
#   83.989493    78.130492    38.157897    26.980376    23.366049    13.876704
#    Education          US   Population        Urban
#    3.103712     2.183155    -1.673313    -2.381346
# [1] "mtry: 10"
#    ShelveLoc        Price    CompPrice  Advertising          Age        Income
#   81.363209    77.647089    38.547174    27.190982    22.778387    14.981041
#          US    Education        Urban   Population
#    4.123539     3.332881    -1.602569    -2.302125
```

```r
print(data_frame_of_values_of_mtry_and_test_MSEs)
```

```
#     mtry test_MSE
# 1      1 6.661409
# 2      2 4.600594
# 3      3 3.789455
# 4      4 3.396240
# 5      5 3.224155
# 6      6 3.121334
# 7      7 2.950383
# 8      8 2.876444
# 9      9 2.927813
# 10    10 2.981307
```

```r
vector_of_values_of_mtry <- data_frame_of_values_of_mtry_and_test_MSEs$mtry
vector_of_test_MSEs <- data_frame_of_values_of_mtry_and_test_MSEs$test_MSE
plot(
    x = vector_of_values_of_mtry,
    y = vector_of_test_MSEs,
    type = "l"
)
index_of_minimum_test_MSE <- which.min(vector_of_test_MSEs)
optimal_value_of_mtry <-
    vector_of_values_of_mtry[index_of_minimum_test_MSE]
minimum_test_MSE <- min(vector_of_test_MSEs)
points(
    optimal_value_of_mtry,
    minimum_test_MSE,
    col = "red",
    cex = 2,
    pch = 20
)
```
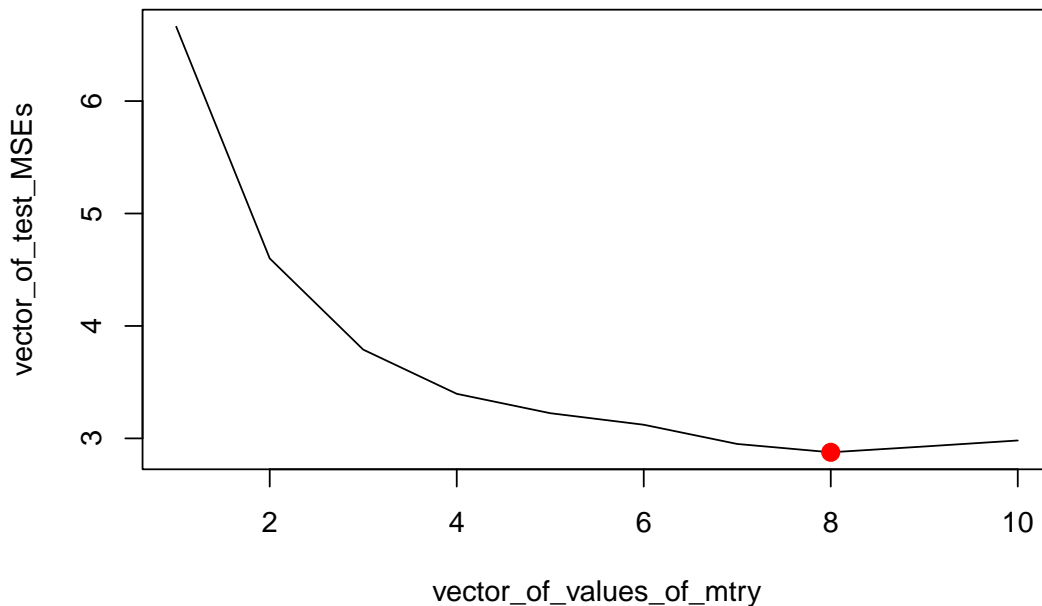
See above plot for test Mean Squared Errors for different values of the number of variables randomly sampled as candidates at each split m. Test MSE decreases parabolically with number of variables to a minimum for $m = 8$. In all cases *ShelveLoc* and *Price* are the most important predictors.

(f) Now analyze the data using BART, and report your results. (skip this exercise)

9. This problem involves the OJ data set which is part of the ISLR package.

   (a) Create a training set containing a random sample of 800 observations, and a test set containing the remaining observations.

```
training_and_testing_data <- split_data_set_into_training_and_testing_data(
    OJ,
    number_of_training_data = 800
)
training_data <- training_and_testing_data$training_data
testing_data <- training_and_testing_data$testing_data
```

   (b) Fit a tree to the training data, with Purchase as the response and the other variables as predictors. Use the summary() function to produce summary statistics about the tree, and describe the results obtained. What is the training error rate? How many terminal nodes does the tree have?

```
full_tree <- tree(Purchase ~ ., data = training_data)
summary(full_tree)

#
# Classification tree:
# tree(formula = Purchase ~ ., data = training_data)
# Variables actually used in tree construction:
# [1] "LoyalCH"    "SalePriceMM" "SpecialCH"   "PriceDiff"   "STORE"
```

```
# Number of terminal nodes:  9
# Residual mean deviance:  0.7139 = 564.7 / 791
# Misclassification error rate: 0.1575 = 126 / 800
```

Our full tree is a classification tree that predicts whether a customer will purchase Citrus Hill or Minute Maid orange juice. A tree is grown by binary recursive partitioning using the response in the specified formula, $Purchase$, and choosing splits from the terms of the right-hand-side, which in our case are all terms besides $Purchase$. The predictors actually used in tree construction are $LoyalCH$, $SalePriceMM$, $SpecialCH$, $PriceDiff$, and $STORE$. $Purchase$ is a factor with levels CH and MM indicating whether the customer purchased Citrus Hill or Minute Maid Orange Juice. $LoyalCH$ seems to be a rate of customer brand loyalty for CH between 0 and 1. $SalePriceMM$ seems to be the net sale price of Minute Maid orange juice in dollars. $SpecialCH$ seems to be an indicator of whether or not there is a special on Citrus Hill orange juice. $PriceDiff$ seems to be net sale price of Minute Maid orange juice less net sale price of Citrus Hill orange juice in dollars. $STORE$ seems to be a categorical variable indicating at which of 5 possible stores the purchase occurred. In our full tree there are 9 terminal nodes / leaves. The deviance of our full tree is 564.7. A small deviance indicates a tree that provides a good fit to the training data. The residual mean deviance for our full tree is $564.7/(800 - 9)$. The training error rate / misclassification error rate for our full tree is $126/800$.

(c) Type in the name of the tree object in order to get a detailed text output. Pick one of the terminal nodes, and interpret the information displayed.

```
full_tree
```

```
# node), split, n, deviance, yval, (yprob)
#       * denotes terminal node
#
#  1) root 800 1077.00 CH ( 0.60000 0.40000 )
#     2) LoyalCH < 0.5036 341  371.50 MM ( 0.23460 0.76540 )
#       4) LoyalCH < 0.282272 167  114.20 MM ( 0.10778 0.89222 ) *
#       5) LoyalCH > 0.282272 174  226.60 MM ( 0.35632 0.64368 )
#        10) SalePriceMM < 2.04 97  101.40 MM ( 0.21649 0.78351 )
#          20) SpecialCH < 0.5 75   58.90 MM ( 0.13333 0.86667 ) *
#          21) SpecialCH > 0.5 22   30.50 CH ( 0.50000 0.50000 ) *
#        11) SalePriceMM > 2.04 77  106.40 CH ( 0.53247 0.46753 ) *
#     3) LoyalCH > 0.5036 459  352.10 CH ( 0.87146 0.12854 )
#       6) PriceDiff < -0.39 22   27.52 MM ( 0.31818 0.68182 ) *
#       7) PriceDiff > -0.39 437  285.40 CH ( 0.89931 0.10069 )
#        14) LoyalCH < 0.764572 175  172.30 CH ( 0.80571 0.19429 )
#          28) SalePriceMM < 2.125 106  126.30 CH ( 0.71698 0.28302 ) *
#          29) SalePriceMM > 2.125 69   30.55 CH ( 0.94203 0.05797 ) *
#        15) LoyalCH > 0.764572 262   84.93 CH ( 0.96183 0.03817 )
#          30) STORE < 1.5 133    0.00 CH ( 1.00000 0.00000 ) *
#          31) STORE > 1.5 129   70.35 CH ( 0.92248 0.07752 ) *
```

A terminal node / leaf / branch to leaf is indicated by an asterisk. Because `full_tree` outputs information for each of two branches for each internal node / node other than the root node / trunk and the leaves, we speak in terms of branches. Let us consider the terminal node / leaf / branch to leaf labeled 4. The prediction of the full tree associated with this branch to leaf is MM. We arrive at this branch when the split criterion $LoyalCH$ is less than 0.504 and less than 0.282. The split criterion associated with this branch is $LoyalCH < 0.282$. The number of observations / purchases in our training data set is 800. Of those purchases, 341 purchases are by customers with loyalty to Citrus Hill less than 0.504. Of those purchases, 167 purchases are by customers with loyalty to Citrus Hill less than 0.282. The number of purchases associated with our branch is 167 with a deviance of 114.2. 0.108 of purchases associated with our branch were of Citrus Hill orange juice. 0.892 of purchases associated with our branch were of Minute Maid orange juice.

(d) Create a plot of the tree, and interpret the results.

```
plot(full_tree)
text(full_tree, pretty = 0)
```
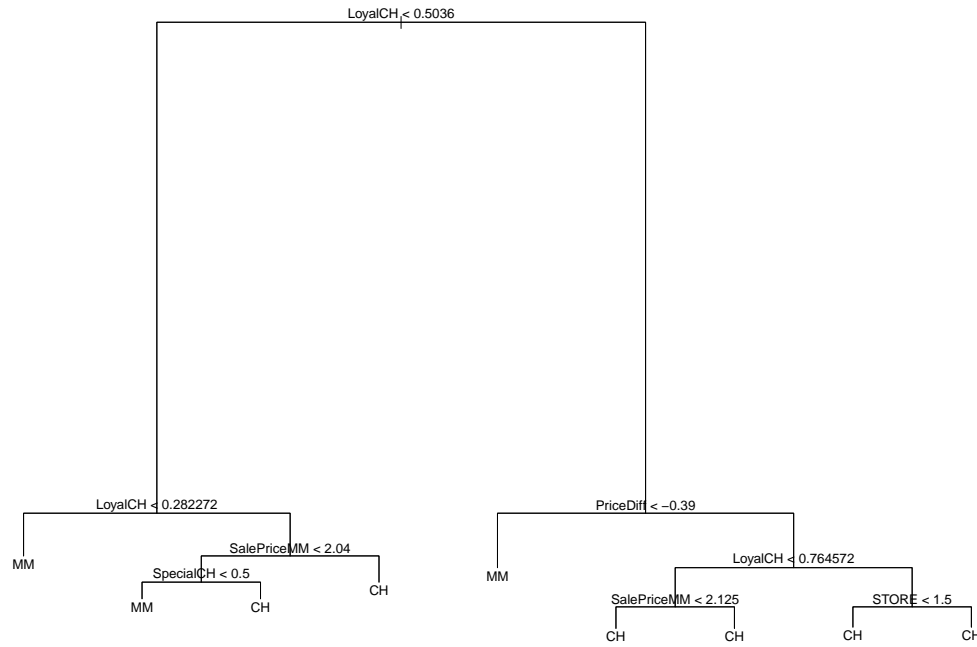

Per our full tree, the most important predictor of whether a customer will purchase Citrus Hill or Minute Maid orange juice is loyalty to Citrus Hill. The split criterion for the first non-root / internal node / the first pair of branches is *LoyalCH*. The split criteria for the second internal node in the second echelon is also *LoyalCH*.

(e) Predict the response on the test data, and produce a confusion matrix comparing the test labels to the predicted test labels. What is the test error rate?

```
vector_of_predicted_purchases <- predict(full_tree, testing_data, type = "class")
confusion_matrix <- table(vector_of_predicted_purchases, testing_data$Purchase)
confusion_matrix

#
# vector_of_predicted_purchases  CH  MM
#                             CH 144  27
#                             MM  29  70

number_of_purchases_of_Citrus_Hill_orange_juice_predicted_correctly <- 144
number_of_purchases_of_Minute_Maid_orange_juice_predicted_correctly <- 70
number_of_purchases_predicted_correctly <-
    number_of_purchases_of_Citrus_Hill_orange_juice_predicted_correctly +
    number_of_purchases_of_Minute_Maid_orange_juice_predicted_correctly
number_of_purchases <- nrow(testing_data)
test_accuracy <- number_of_purchases_predicted_correctly / number_of_purchases
```

10

```
test_error_rate <- 1 - test_accuracy
test_error_rate
```

[1] 0.2074074

The test error rate is about 0.207.

(f) Apply the `cv.tree()` function to the training set in order to determine the optimal tree size.

(g) Produce a plot with tree size on the x-axis and cross-validated classification error rate on the y-axis.

(h) Which tree size corresponds to the lowest cross-validated classification error rate?

(i) Produce a pruned tree corresponding to the optimal tree size obtained using cross-validation. If cross-validation does not lead to selection of a pruned tree, then create a pruned tree with five terminal nodes.

(j) Compare the training error rates between the pruned and unpruned trees. Which is higher?

(k) Compare the test error rates between the pruned and unpruned trees. Which is higher?