

Univariate Probability 1/19

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Random Variables

Discrete Randon Variables

Continuous Random Variables

Probability Review 3 - Univariate Random Variables and Distributions

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Random Variables

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Random Variables

Variables
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- Random variable function from sample space to \Re or a subset of \Re
- Discrete random variable at most a countable number of values
- Continuous random variable uncountable set of possible values



Discrete Random Variables

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Probability mass function: p(x) = P(A = x)

Cumulative distribution function: $F(a) = \sum_{x_i < a} p(x_i)$



Bernoulli and Binomial Distributions

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Bernoulli:

$$X = \begin{cases} 1 & \text{with probability p} \\ 0 & \text{with probability 1-p} \end{cases}$$

 Binomial: Suppose independent Bernoulli trials with probability p, then the probability of x successes in N trials is

$$p(x) = \binom{N}{x} p^x (1-p)^{N-x}$$



Binomial Distribution

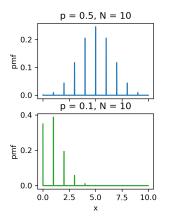
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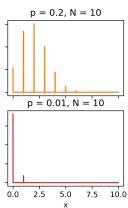
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Poisson Distribution

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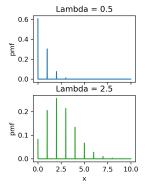
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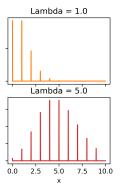
variables

Discrete Random

Continuous Randon Variables Poisson: For $\lambda > 0$ and some x = 0, 1, ...,

$$p(k) = e^{-\lambda} \frac{\lambda^x}{x!}$$







Continuous Random Variables

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Probability density function : For $A \subset \Re$ with f(x) such that $P(X \in A) = \int_A f(x) dx$

Cumulative distribution function : $F(a) = \int_{-\infty}^{a} f(x)dx$

Example : Suppose $f(x) = x, x \in (0, \sqrt{2})$ and f(x) = 0 otherwise.

Graph f(x) and F(x)



Example

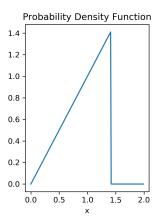
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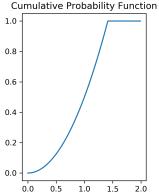
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Uniform Distribution

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Continuous Random Variables Uniform:

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$



Uniform Distribution

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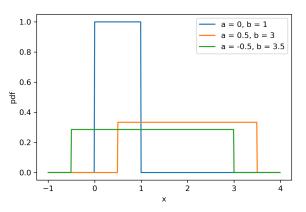
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Beta Distribution

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• Beta: For a > 0, b > 0 and $x \in [0, 1]$

$$f(x) = \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1}$$

where

$$B(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx$$



Beta Distribution

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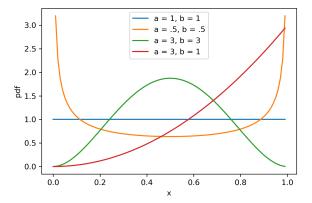
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Gamma Distribution

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• Gamma: For $\lambda > 0, \alpha > 0$ and $x \ge 0$

$$f(x) = \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha - 1}}{\Gamma(\alpha)}$$

where

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha - 1} dx$$

for $\alpha \in \mathbb{Z}^+$ (i.e., a positive integer)

$$\Gamma(\alpha) = (\alpha - 1)!$$

Also

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$



Gamma Distribution

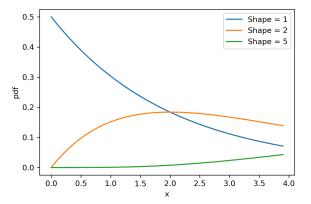
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Gaussian Distribution

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• Gaussian: For $\mu \in \Re, \sigma^2 > 0$ and $x \in \Re$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

- Suppose $X \sim N(\mu, \sigma^2)$ what is the distribution of Y = aX + b?
- $Y \sim N(a\mu + b, a^2\sigma^2)$



Gaussian Distribution

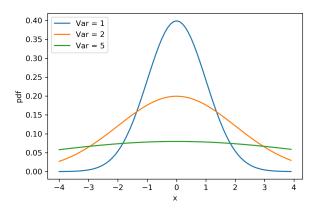
Univariate Probability 17/ 19

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t Distribution

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Continuous Random Variables • t Distribution: For $\mu \in \Re$, $\hat{\sigma}$, $\nu > 0$ and $x \in \Re$

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{2\pi}\hat{\sigma}} \left(1 + \frac{1}{\nu} \frac{(x-\mu)^2}{\hat{\sigma}^2}\right)^{-\frac{\nu+1}{2}}$$

- μ is location, ν is the degrees of freedom, and $\hat{\sigma}$ is the scale
- As $\nu \to \infty$ the distribution converges to the Gaussian
- For $\nu = 1$ known as the Cauchy distribution



t Distribution

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