DS-6030 Homework Module 6

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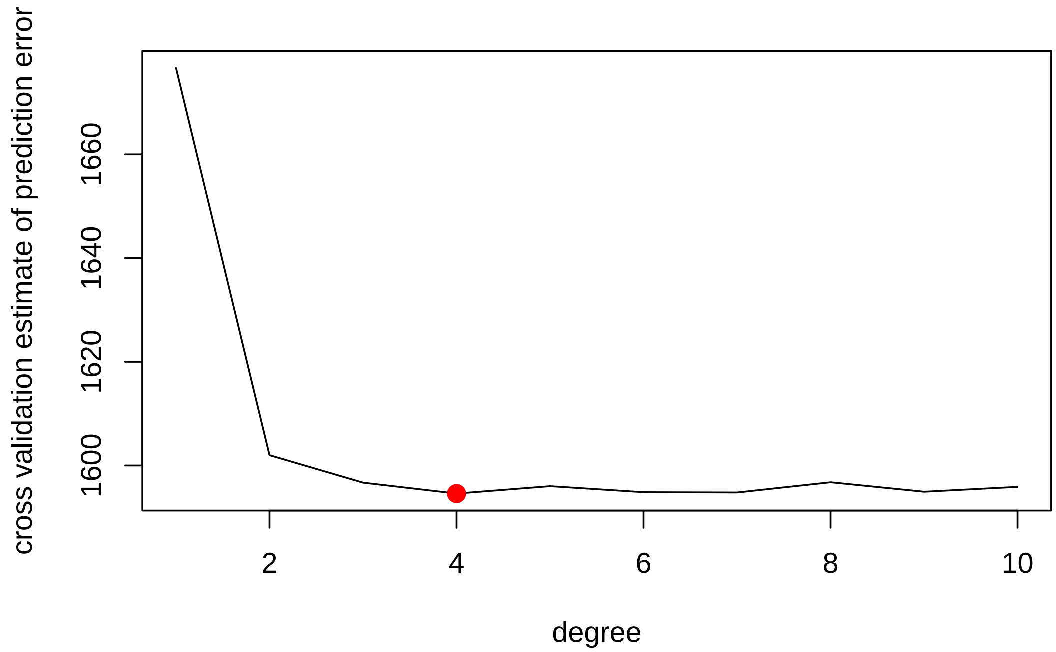
6. In this exercise, you will further analyze the `Wage` data set considered throughout this chapter.

   (a) Perform polynomial regression to predict wage using age. Use cross-validation to select the optimal degree d for the polynomial. What degree was chosen, and how does this compare to the results of hypothesis testing using ANOVA? Make a plot of the resulting polynomial fit to the data.

   The optimal degree for a polynomial regression to predict wage vs. age is $d = 4$. According to the documentation for `anova`, "When given a sequence of objects, `anova` tests the models against one another in the order specified... It produces a table which tests whether the model terms [for a given model] are significant [in the context of the previous model]." Examining the column of p values in below table, a term of degree 1 is significant in the context of an intercept-only model, a term of degree 2 is significant in the context of a polynomial of degree 2, a term of degree 3 is significant in the context of a polynomial of degree 2, a term of degree 4 is approximately significant in the context of a polynomial of degree 4, but a term of degree 5 is insignificant in the context of a polynomial of degree 4. This interpretation accords with using cross-validation to select the optimal degree for the polynomial.

```
library(ISLR2)
set.seed(4)
range_of_degrees <- 1:10
number_of_degrees <- length(range_of_degrees)
cross_validation_estimates_of_prediction_errors <- rep(NA, number_of_degrees)
for (degree in range_of_degrees) {
    the_glm <- glm(wage ~ poly(age, degree), data = Wage)
    cross_validation_estimates_of_prediction_errors[degree] <-
        boot::cv.glm(Wage, the_glm, K = 10)$delta[1]
}
plot(
    x = range_of_degrees,
    y = cross_validation_estimates_of_prediction_errors,
    xlab = "degree",
    ylab = "cross validation estimate of prediction error",
    type = "l"
)
optimal_degree <-
    which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_degree,
    y = cross_validation_estimates_of_prediction_errors[optimal_degree],
    col = "red",
    cex = 2,
```

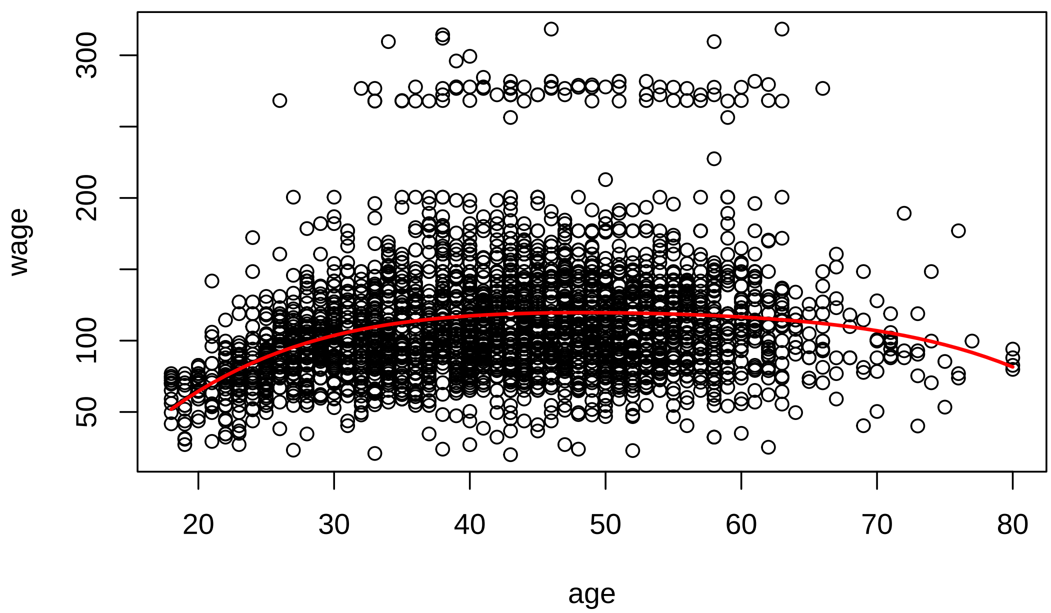
```r
    pch = 20
)
```




```r
lm_0 <- lm(wage ~ 1, data = Wage)
lm_1 <- lm(wage ~ age, data = Wage)
lm_2 <- lm(wage ~ poly(age, 2), data = Wage)
lm_3 <- lm(wage ~ poly(age, 3), data = Wage)
lm_4 <- lm(wage ~ poly(age, 4), data = Wage)
lm_5 <- lm(wage ~ poly(age, 5), data = Wage)
anova(lm_0, lm_1, lm_2, lm_3, lm_4, lm_5)
```

```
# Analysis of Variance Table
#
# Model 1: wage ~ 1
# Model 2: wage ~ age
# Model 3: wage ~ poly(age, 2)
# Model 4: wage ~ poly(age, 3)
# Model 5: wage ~ poly(age, 4)
# Model 6: wage ~ poly(age, 5)
#   Res.Df     RSS Df Sum of Sq        F    Pr(>F)
# 1   2999 5222086
# 2   2998 5022216  1    199870 125.4443 < 2.2e-16 ***
# 3   2997 4793430  1    228786 143.5931 < 2.2e-16 ***
# 4   2996 4777674  1     15756   9.8888  0.001679 **
# 5   2995 4771604  1      6070   3.8098  0.051046 .
# 6   2994 4770322  1      1283   0.8050  0.369682
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

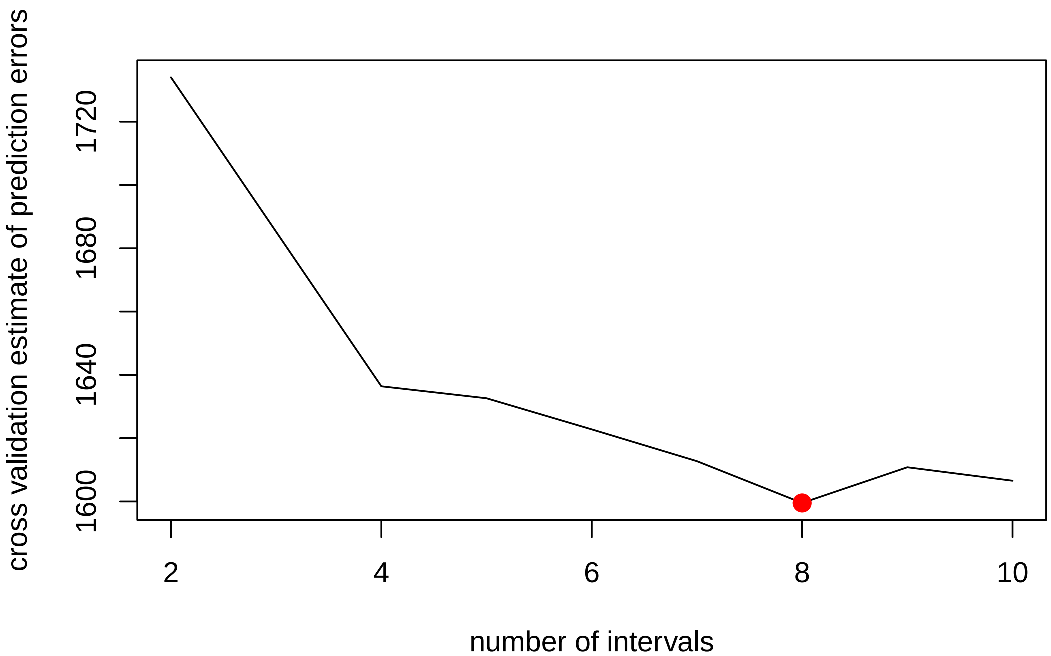
```r
plot(wage ~ age, data = Wage)
minimum_age <- min(Wage$age)
maximum_age <- max(Wage$age)
sequence_of_ages <- seq(from = minimum_age, to = maximum_age)
list_with_age <- list(age = sequence_of_ages)
vector_of_predicted_wages <- predict(object = lm_4, newdata = list_with_age)
lines(sequence_of_ages, vector_of_predicted_wages, col = "red", lwd = 2)
```



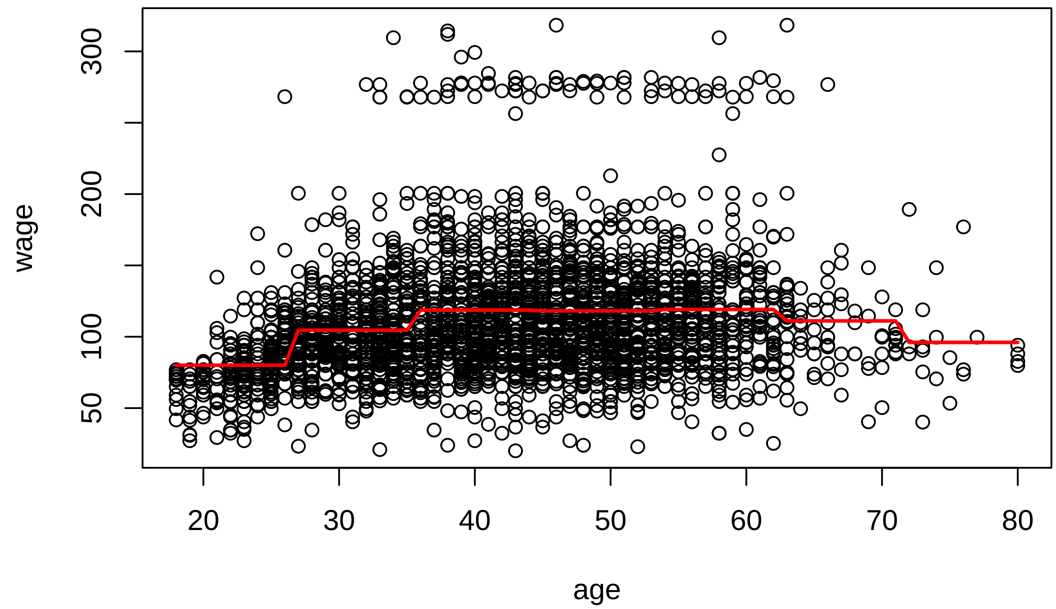
(b) Fit a step function to predict wage using age, and perform cross-validation to choose the optimal number of cuts. Make a plot of the fit obtained.
A cross-validated estimate of prediction error is minimum for 8 intervals.

```r
range_of_numbers_of_intervals <- 2:10
number_of_numbers_of_intervals <- length(range_of_numbers_of_intervals)
cross_validation_estimates_of_prediction_errors <-
    rep(NA, number_of_numbers_of_intervals)
for (number_of_intervals in range_of_numbers_of_intervals) {
    Wage$interval <- cut(Wage$age, number_of_intervals)
    the_glm <- glm(wage ~ interval, data = Wage)
    cross_validation_estimates_of_prediction_errors[number_of_intervals - 1] <-
       boot::cv.glm(Wage, the_glm, K = 10)$delta[1]
}
plot(
    x = range_of_numbers_of_intervals,
    y = cross_validation_estimates_of_prediction_errors,
    xlab = "number of intervals",
    ylab = "cross validation estimate of prediction errors",
    type = "l"
)
```

```r
optimal_number_of_intervals <-
    which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_number_of_intervals + 1,
    y = cross_validation_estimates_of_prediction_errors[
        optimal_number_of_intervals
    ],
    col = "red",
    cex = 2,
    pch = 20
)
```




```r
the_glm <- glm(wage ~ cut(age, optimal_number_of_intervals), data = Wage)
plot(wage ~ age, data = Wage)
vector_of_predicted_wages <- predict(object = the_glm, newdata = list_with_age)
lines(sequence_of_ages, vector_of_predicted_wages, col = "red", lwd = 2)
```



7. This question uses the variables `dis` (the weighted mean of distances to five Boston employment centers) and `nox` (nitrogen oxides concentration in parts per 10 million) from the Boston data. We will treat `dis` as the predictor and nox as the response.

   (a) Use the `poly()` function to fit a cubic polynomial regression to predict `nox` using `dis`. Report the regression output, and plot the resulting data and polynomial fits.

```r
library(MASS)
```

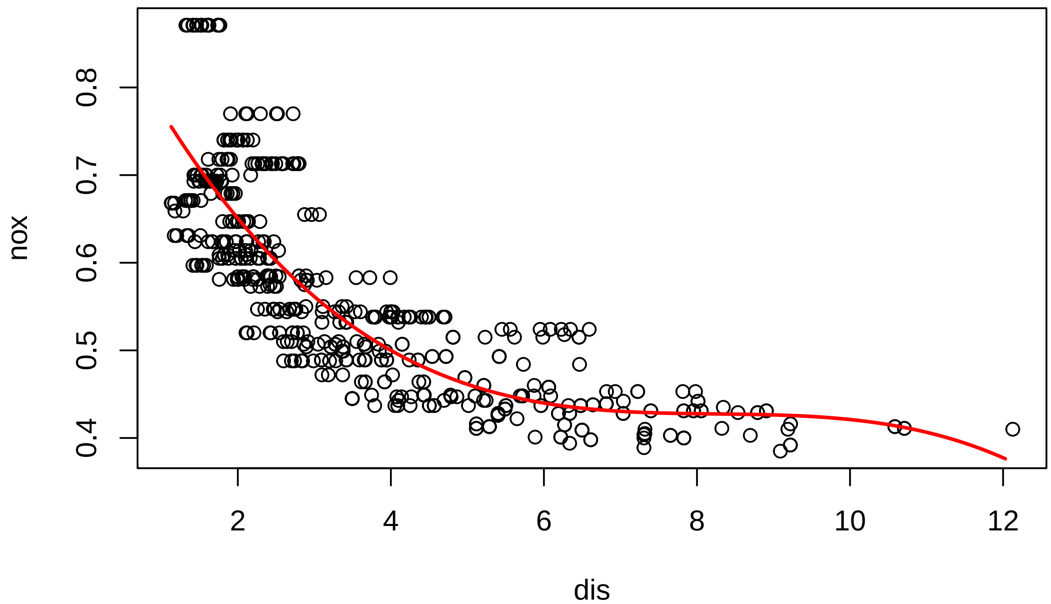
```
#
# Attaching package: 'MASS'

# The following object is masked from 'package:ISLR2':
#
#     Boston
```

```r
set.seed(1)
the_lm <- lm(nox ~ poly(dis, 3), data = Boston)
summary(the_lm)
```

```
#
# Call:
# lm(formula = nox ~ poly(dis, 3), data = Boston)
#
# Residuals:
#       Min        1Q    Median        3Q       Max
# -0.121130 -0.040619 -0.009738  0.023385  0.194904
#
# Coefficients:
#                 Estimate Std. Error t value Pr(>|t|)
```

```
# (Intercept)    0.554695   0.002759 201.021  < 2e-16 ***
# poly(dis, 3)1 -2.003096   0.062071 -32.271  < 2e-16 ***
# poly(dis, 3)2  0.856330   0.062071  13.796  < 2e-16 ***
# poly(dis, 3)3 -0.318049   0.062071  -5.124 4.27e-07 ***
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
# Residual standard error: 0.06207 on 502 degrees of freedom
# Multiple R-squared:  0.7148,  Adjusted R-squared:  0.7131
# F-statistic: 419.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

```r
minimum_weighted_mean_of_distances <- min(Boston$dis)
maximum_weighted_mean_of_distances <- max(Boston$dis)
sequence_of_weighted_means_of_distances <- seq(
    from = minimum_weighted_mean_of_distances,
    to = maximum_weighted_mean_of_distances,
    by = 0.1
)
list_with_dis <- list(dis = sequence_of_weighted_means_of_distances)
vector_of_predicted_nitrogen_oxide_concentrations <- predict(
    object = the_lm,
    list_with_dis
)
plot(nox ~ dis, data = Boston)
lines(
    sequence_of_weighted_means_of_distances,
    vector_of_predicted_nitrogen_oxide_concentrations,
    col = "red",
    lwd = 2
)
```



(b) Plot the polynomial fits for a range of different polynomial degrees (say, from 1 to 10), and report the associated residual sum of squares.

(c) Perform cross-validation or another approach to select the optimal degree for the polynomial, and explain your results.

(d) Use the `bs()` function to fit a regression spline to predict `nox` using `dis`. Report the output for the fit using four degrees of freedom. How did you choose the knots? Plot the resulting fit.

(e) Now fit a regression spline for a range of degrees of freedom, and plot the resulting fits and report the resulting RSS. Describe the results obtained.

(f) Perform cross-validation or another approach in order to select the best degrees of freedom for a regression spline on this data. Describe your results.

10. This question relates to the College data set.

(a) Split the data into a training set and a test set. Using out-of-state tuition as the response and the other variables as the predictors, perform forward stepwise selection on the training set in order to identify a satisfactory model that uses just a subset of the predictors.

(b) Fit a GAM on the training data, using out-of-state tuition as the response and the features selected in the previous step as the predictors. Plot the results, and explain your findings.

(c) Evaluate the model obtained on the test set, and explain the results obtained.

(d) For which variables, if any, is there evidence of a non-linear relationship with the response?