DS-6030 Homework Module 6

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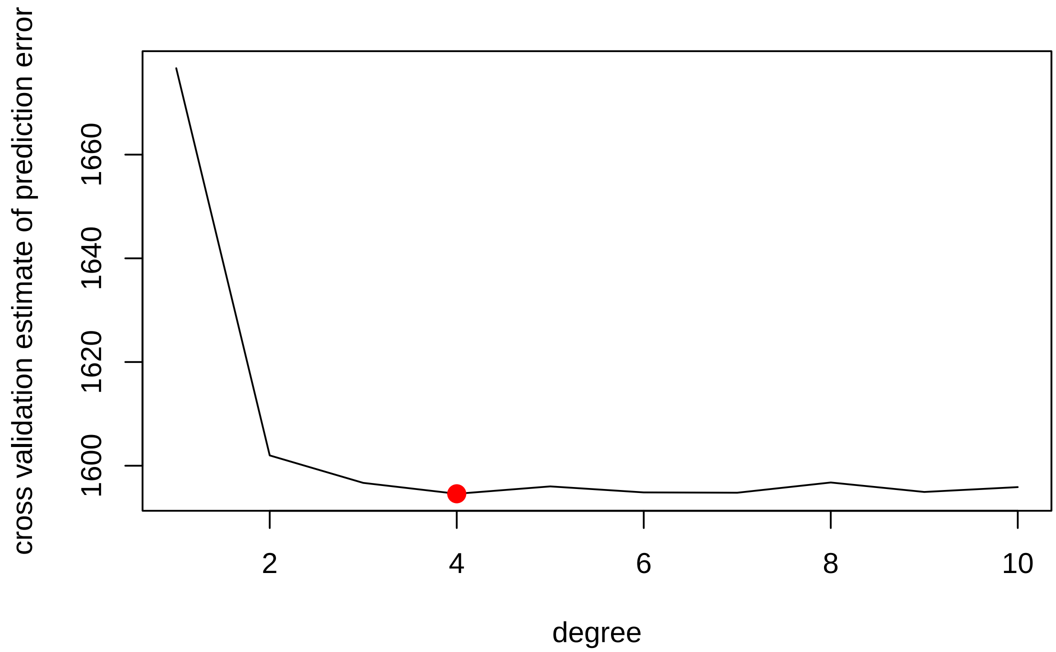
6. In this exercise, you will further analyze the `Wage` data set considered throughout this chapter.

   (a) Perform polynomial regression to predict wage using age. Use cross-validation to select the optimal degree d for the polynomial. What degree was chosen, and how does this compare to the results of hypothesis testing using ANOVA? Make a plot of the resulting polynomial fit to the data.

   The optimal degree for a polynomial regression to predict wage vs. age is $d = 4$. According to the documentation for `anova`, "When given a sequence of objects, `anova` tests the models against one another in the order specified… It produces a table which tests whether the model terms [for a given model] are significant [in the context of the previous model]." Examining the column of p values in below table, a term of degree 1 is significant in the context of an intercept-only model, a term of degree 2 is significant in the context of a polynomial of degree 2, a term of degree 3 is significant in the context of a polynomial of degree 2, a term of degree 4 is approximately significant in the context of a polynomial of degree 4, but a term of degree 5 is insignificant in the context of a polynomial of degree 4. This interpretation accords with using cross-validation to select the optimal degree for the polynomial.

```
library(ISLR2)
set.seed(4)
range_of_degrees <- 1:10
number_of_degrees <- length(range_of_degrees)
cross_validation_estimates_of_prediction_errors <- rep(NA, number_of_degrees)
for (degree in range_of_degrees) {
    the_glm <- glm(wage ~ poly(age, degree), data = Wage)
    cross_validation_estimates_of_prediction_errors[degree] <-
        boot::cv.glm(Wage, the_glm, K = 10)$delta[1]
}
plot(
    x = range_of_degrees,
    y = cross_validation_estimates_of_prediction_errors,
    xlab = "degree",
    ylab = "cross validation estimate of prediction error",
    type = "l"
)
optimal_degree <-
    which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_degree,
    y = cross_validation_estimates_of_prediction_errors[optimal_degree],
    col = "red",
    cex = 2,
```

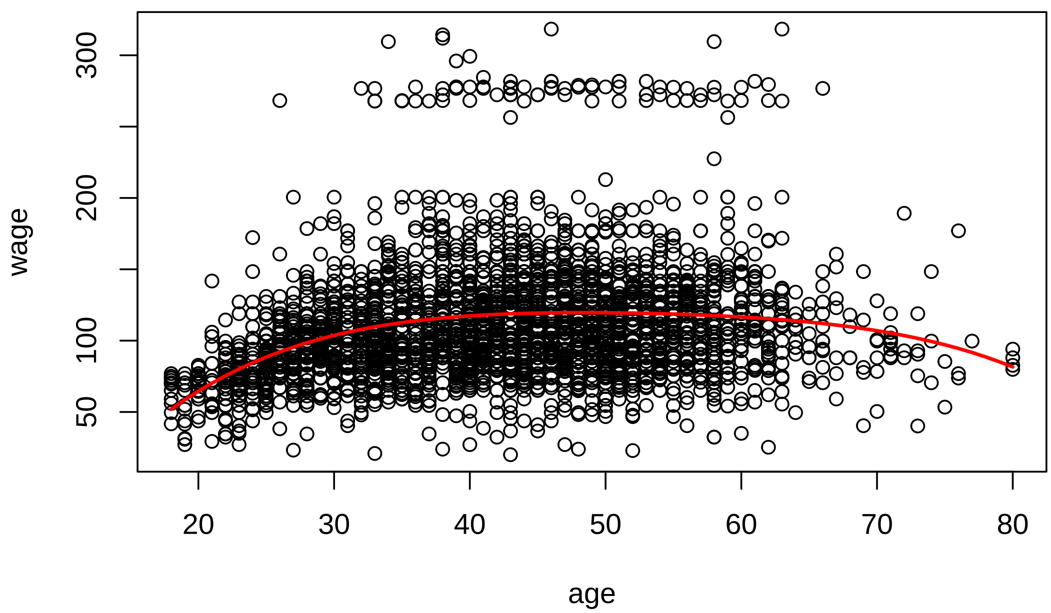
```
    pch = 20
)
```




```
lm_0 <- lm(wage ~ 1, data = Wage)
lm_1 <- lm(wage ~ age, data = Wage)
lm_2 <- lm(wage ~ poly(age, 2), data = Wage)
lm_3 <- lm(wage ~ poly(age, 3), data = Wage)
lm_4 <- lm(wage ~ poly(age, 4), data = Wage)
lm_5 <- lm(wage ~ poly(age, 5), data = Wage)
anova(lm_0, lm_1, lm_2, lm_3, lm_4, lm_5)
```

```
# Analysis of Variance Table
#
# Model 1: wage ~ 1
# Model 2: wage ~ age
# Model 3: wage ~ poly(age, 2)
# Model 4: wage ~ poly(age, 3)
# Model 5: wage ~ poly(age, 4)
# Model 6: wage ~ poly(age, 5)
#   Res.Df     RSS Df Sum of Sq        F    Pr(>F)
# 1   2999 5222086
# 2   2998 5022216  1    199870 125.4443 < 2.2e-16 ***
# 3   2997 4793430  1    228786 143.5931 < 2.2e-16 ***
# 4   2996 4777674  1     15756   9.8888  0.001679 **
# 5   2995 4771604  1      6070   3.8098  0.051046 .
# 6   2994 4770322  1      1283   0.8050  0.369682
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
plot(wage ~ age, data = Wage)
minimum_age <- min(Wage$age)
maximum_age <- max(Wage$age)
sequence_of_ages <- seq(from = minimum_age, to = maximum_age)
list_with_age <- list(age = sequence_of_ages)
vector_of_predicted_wages <- predict(object = lm_4, newdata = list_with_age)
lines(sequence_of_ages, vector_of_predicted_wages, col = "red", lwd = 2)
```

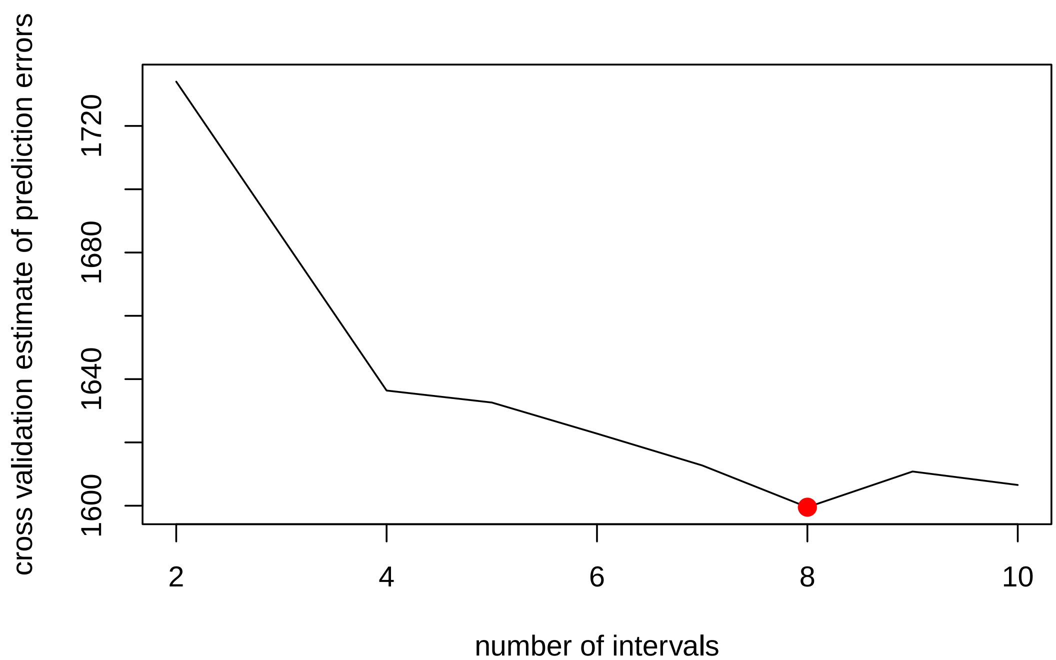


(b) Fit a step function to predict wage using age, and perform cross-validation to choose the optimal number of cuts. Make a plot of the fit obtained.

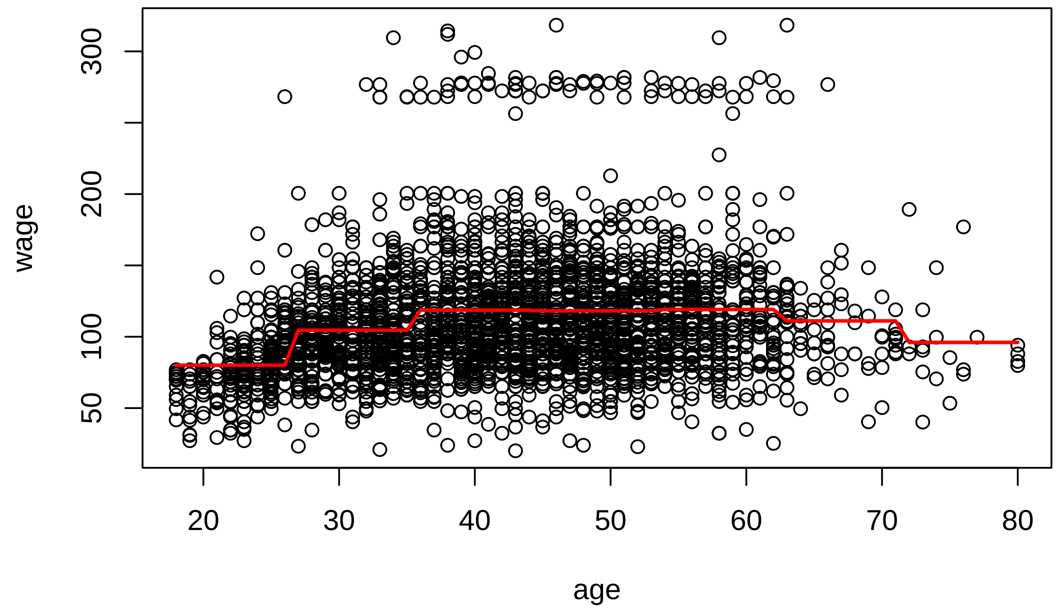
A cross-validated estimate of prediction error is minimum for 8 intervals.

```r
range_of_numbers_of_intervals <- 2:10
number_of_numbers_of_intervals <- length(range_of_numbers_of_intervals)
cross_validation_estimates_of_prediction_errors <-
    rep(NA, number_of_numbers_of_intervals)
for (number_of_intervals in range_of_numbers_of_intervals) {
    Wage$interval <- cut(Wage$age, number_of_intervals)
    the_glm <- glm(wage ~ interval, data = Wage)
    cross_validation_estimates_of_prediction_errors[number_of_intervals - 1] <-
       boot::cv.glm(Wage, the_glm, K = 10)$delta[1]
}
plot(
    x = range_of_numbers_of_intervals,
    y = cross_validation_estimates_of_prediction_errors,
    xlab = "number of intervals",
    ylab = "cross validation estimate of prediction errors",
    type = "l"
)
```

```r
optimal_number_of_intervals <-
    which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_number_of_intervals + 1,
    y = cross_validation_estimates_of_prediction_errors[
        optimal_number_of_intervals
    ],
    col = "red",
    cex = 2,
    pch = 20
)
```




```r
the_glm <- glm(wage ~ cut(age, optimal_number_of_intervals), data = Wage)
plot(wage ~ age, data = Wage)
vector_of_predicted_wages <- predict(object = the_glm, newdata = list_with_age)
lines(sequence_of_ages, vector_of_predicted_wages, col = "red", lwd = 2)
```



7. This question uses the variables `dis` (the weighted mean of distances to five Boston employment centers) and `nox` (nitrogen oxides concentration in parts per 10 million) from the Boston data. We will treat `dis` as the predictor and nox as the response.

   (a) Use the `poly()` function to fit a cubic polynomial regression to predict `nox` using `dis`. Report the regression output, and plot the resulting data and polynomial fits.
   Per the regression output, all coefficients are significant in the context of the multiple linear model.

```
library(ggplot2)
library(MASS)
```

```
#
# Attaching package: 'MASS'

# The following object is masked from 'package:ISLR2':
#
#     Boston
```

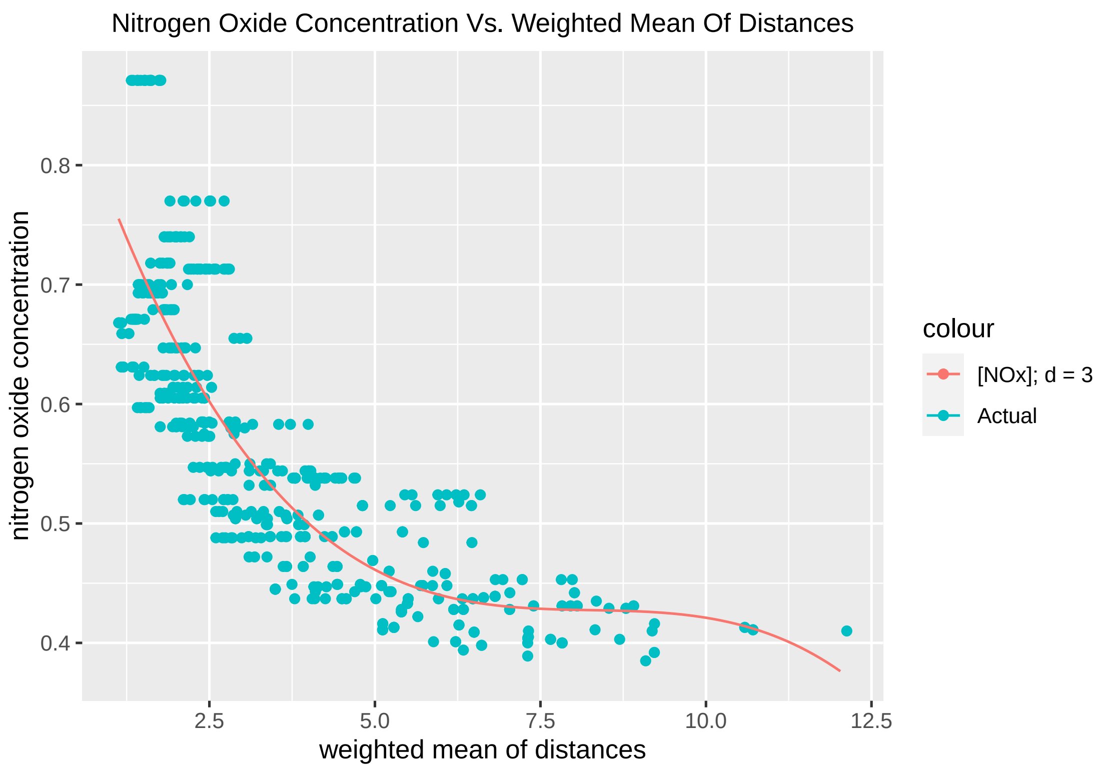
```
set.seed(1)
lm_3 <- lm(nox ~ poly(dis, 3), data = Boston)
summary(lm_3)
```

```
#
# Call:
# lm(formula = nox ~ poly(dis, 3), data = Boston)
#
# Residuals:
#       Min        1Q    Median        3Q       Max
# -0.121130 -0.040619 -0.009738  0.023385  0.194904
#
```

```
# Coefficients:
#                Estimate Std. Error t value Pr(>|t|)
# (Intercept)    0.554695   0.002759 201.021  < 2e-16 ***
# poly(dis, 3)1 -2.003096   0.062071 -32.271  < 2e-16 ***
# poly(dis, 3)2  0.856330   0.062071  13.796  < 2e-16 ***
# poly(dis, 3)3 -0.318049   0.062071  -5.124 4.27e-07 ***
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
# Residual standard error: 0.06207 on 502 degrees of freedom
# Multiple R-squared:  0.7148,  Adjusted R-squared:  0.7131
# F-statistic: 419.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

```r
minimum_weighted_mean_of_distances <- min(Boston$dis)
maximum_weighted_mean_of_distances <- max(Boston$dis)
sequence_of_weighted_means_of_distances <- seq(
    from = minimum_weighted_mean_of_distances,
    to = maximum_weighted_mean_of_distances,
    by = 0.1
)
list_with_dis <- list(dis = sequence_of_weighted_means_of_distances)
data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations <-
    data.frame(
        weighted_mean_of_distances =
            sequence_of_weighted_means_of_distances
    )
for (degree in range_of_degrees) {
    the_lm <- lm(nox ~ poly(dis, degree), data = Boston)
    vector_of_predicted_nitrogen_oxide_concentrations <- predict(
        object = the_lm,
        list_with_dis
    )
    column_label <- paste("NOx_concentration_predicted_by_polynomial_of_degree_", degree, sep =
    data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations[
        ,
        column_label
    ] <- vector_of_predicted_nitrogen_oxide_concentrations
}
ggplot() +
 geom_point(
     data = Boston,
     mapping = aes(
         x = dis,
         y = nox,
         color = "Actual"
     )
 ) +
 geom_line(
     data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
     mapping = aes(
         x = weighted_mean_of_distances,
         y = NOx_concentration_predicted_by_polynomial_of_degree_3,
         color = "[NOx]; d = 3"
     )
```

```r
) +
labs(
    x = "weighted mean of distances",
    y = "nitrogen oxide concentration",
    title = "Nitrogen Oxide Concentration Vs. Weighted Mean Of Distances"
) +
theme(
 plot.title = element_text(hjust = 0.5, size = 11),
)
```

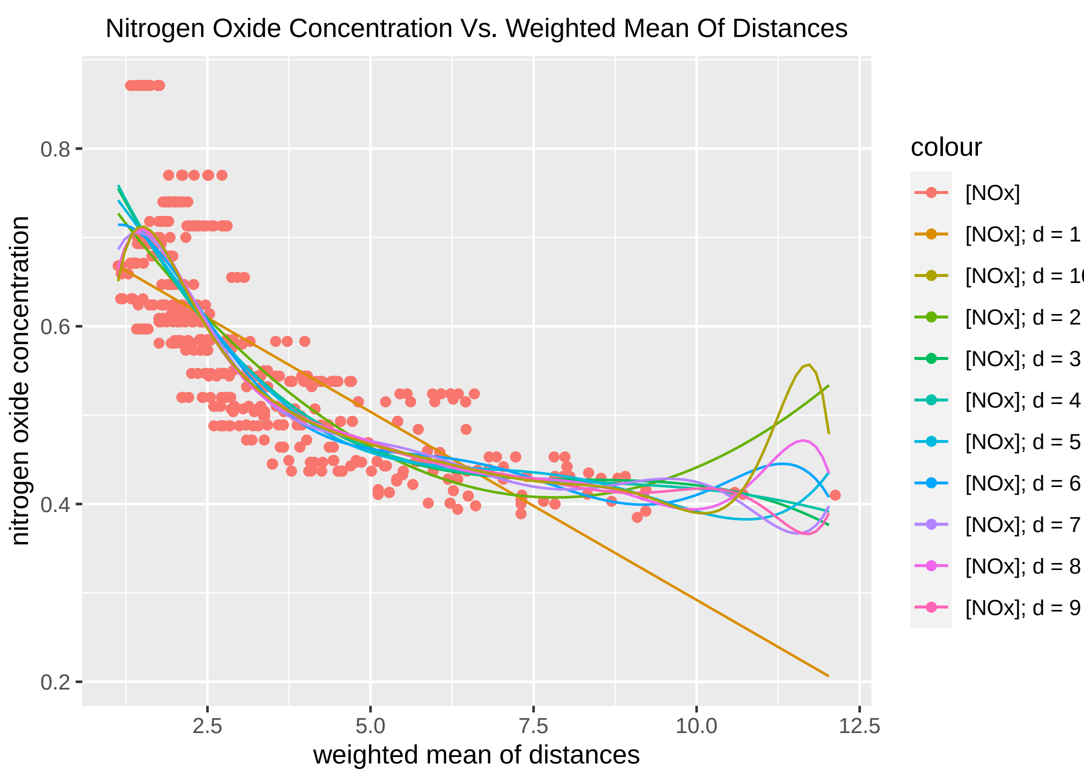


(b) Plot the polynomial fits for a range of different polynomial degrees (say, from 1 to 10), and report the associated residual sum of squares.

```r
ggplot() +
 geom_point(
     data = Boston,
     mapping = aes(
         x = dis,
         y = nox,
         color = "[NOx]"
     )
 ) +
 geom_line(
     data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
     mapping = aes(
         x = weighted_mean_of_distances,
         y = NOx_concentration_predicted_by_polynomial_of_degree_1,
         color = "[NOx]; d = 1"
     )
```

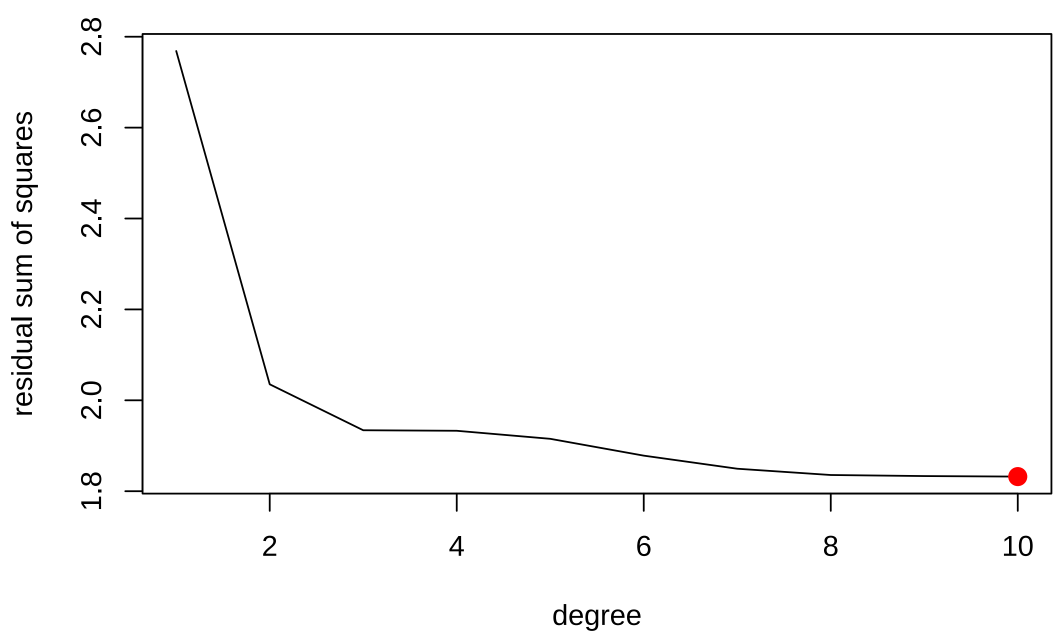
```
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_2,
        color = "[NOx]; d = 2"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_3,
        color = "[NOx]; d = 3"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_4,
        color = "[NOx]; d = 4"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_5,
        color = "[NOx]; d = 5"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_6,
        color = "[NOx]; d = 6"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_7,
        color = "[NOx]; d = 7"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
```

```
        y = NOx_concentration_predicted_by_polynomial_of_degree_8,
        color = "[NOx]; d = 8"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_9,
        color = "[NOx]; d = 9"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_polynomial_of_degree_10,
        color = "[NOx]; d = 10"
    )
) +
labs(
    x = "weighted mean of distances",
    y = "nitrogen oxide concentration",
    title = "Nitrogen Oxide Concentration Vs. Weighted Mean Of Distances"
) +
theme(
 plot.title = element_text(hjust = 0.5, size = 11),
)
```



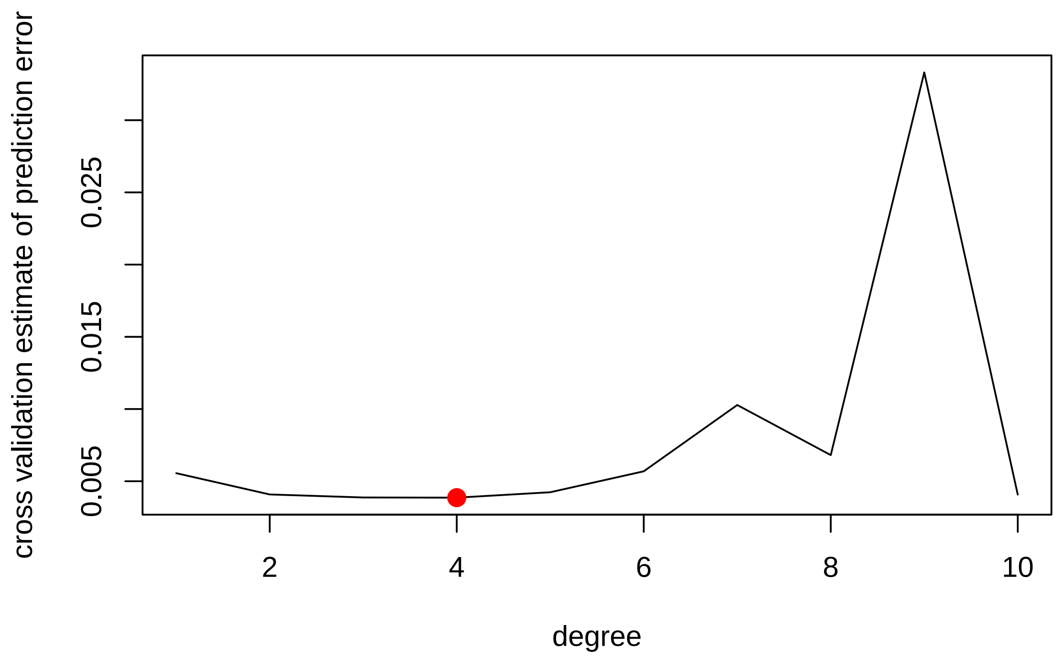
Per a plot of residual sum of squares vs. degree of polynomial, residual sum of squares decreases with degree of polynomial to a minimum for degree 10.

```
range_of_degrees <- 1:10
number_of_degrees <- length(range_of_degrees)
vector_of_residual_sums_of_squares <- rep(NA, number_of_degrees)
for (degree in range_of_degrees) {
    the_lm <- lm(nox ~ poly(dis, degree), data = Boston)
    vector_of_residual_sums_of_squares[degree] <- sum(the_lm$residuals^2)
}
plot(
    x = range_of_degrees,
    y = vector_of_residual_sums_of_squares,
    xlab = "degree",
    ylab = "residual sum of squares",
    type = "l"
)
optimal_degree <-
    which.min(vector_of_residual_sums_of_squares)
points(
    x = optimal_degree,
    y = vector_of_residual_sums_of_squares[optimal_degree],
    col = "red",
    cex = 2,
    pch = 20
)
```



(c) Perform cross-validation or another approach to select the optimal degree for the polynomial, and explain your results.
The minimum cross-validation estimate of prediction error occurs for degree 4.

```r
cross_validation_estimates_of_prediction_errors <- rep(NA, number_of_degrees)
for (degree in range_of_degrees) {
    the_glm <- glm(nox ~ poly(dis, degree), data = Boston)
    cross_validation_estimates_of_prediction_errors[degree] <-
        boot::cv.glm(Boston, the_glm, K = 10)$delta[1]
}
plot(
    range_of_degrees,
    cross_validation_estimates_of_prediction_errors,
    xlab = "degree",
    ylab = "cross validation estimate of prediction error",
    type = "l"
)
optimal_degree <- which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_degree,
    y = cross_validation_estimates_of_prediction_errors[optimal_degree],
    col = "red",
    cex = 2,
    pch = 20
)
```



(d) Use the bs() function to fit a regression spline to predict nox using dis. Report the output for the fit using four degrees of freedom. How did you choose the knots? Plot the resulting fit.

The R interpreter determines knots automatically.

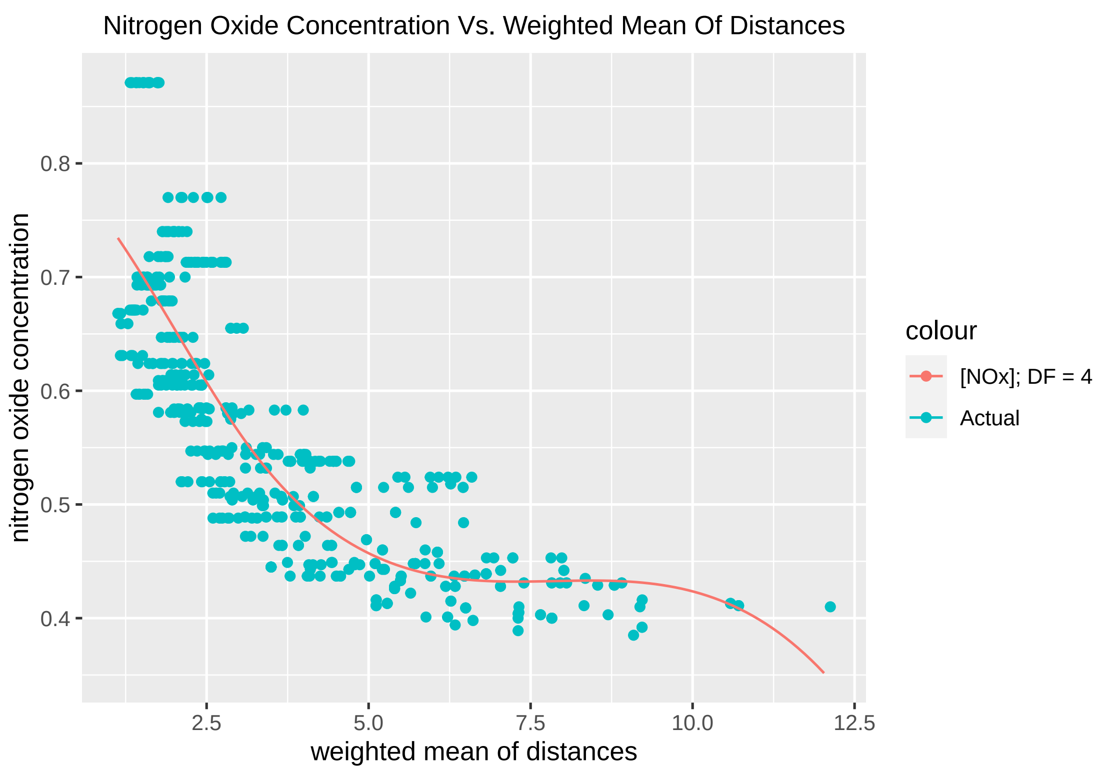
```r
library(ggplot2)
library(MASS)
```

```r
lm_4 <- lm(nox ~ splines::bs(dis, df = 4), data = Boston)
summary(lm_4)
```

```
#
# Call:
# lm(formula = nox ~ splines::bs(dis, df = 4), data = Boston)
#
# Residuals:
#       Min        1Q    Median        3Q       Max
# -0.124622 -0.039259 -0.008514  0.020850  0.193891
#
# Coefficients:
#                            Estimate Std. Error t value Pr(>|t|)
# (Intercept)                 0.73447    0.01460  50.306  < 2e-16 ***
# splines::bs(dis, df = 4)1 -0.05810    0.02186  -2.658  0.00812 **
# splines::bs(dis, df = 4)2 -0.46356    0.02366 -19.596  < 2e-16 ***
# splines::bs(dis, df = 4)3 -0.19979    0.04311  -4.634 4.58e-06 ***
# splines::bs(dis, df = 4)4 -0.38881    0.04551  -8.544  < 2e-16 ***
# ---
# Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#
# Residual standard error: 0.06195 on 501 degrees of freedom
# Multiple R-squared:  0.7164,  Adjusted R-squared:  0.7142
# F-statistic: 316.5 on 4 and 501 DF,  p-value: < 2.2e-16
```

```r
data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations <-
    data.frame(
        weighted_mean_of_distances =
            sequence_of_weighted_means_of_distances
    )
numbers_of_degrees_of_freedom <- 3:10
for (number_of_degrees_of_freedom in numbers_of_degrees_of_freedom) {
    the_lm <- lm(nox ~ splines::bs(dis, df = number_of_degrees_of_freedom), data = Boston)
    vector_of_predicted_nitrogen_oxide_concentrations <- predict(
        object = the_lm,
        list_with_dis
    )
    column_label <- paste(
        "NOx_concentration_predicted_by_B_spline_with_DF_",
        number_of_degrees_of_freedom,
        sep = ""
    )
    data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations[
        ,
        column_label
    ] <- vector_of_predicted_nitrogen_oxide_concentrations
}
ggplot() +
 geom_point(
     data = Boston,
     mapping = aes(
         x = dis,
         y = nox,
         color = "Actual"
```

```
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_4,
        color = "[NOx]; DF = 4"
    )
) +
labs(
    x = "weighted mean of distances",
    y = "nitrogen oxide concentration",
    title = "Nitrogen Oxide Concentration Vs. Weighted Mean Of Distances"
) +
theme(
 plot.title = element_text(hjust = 0.5, size = 11),
)
```

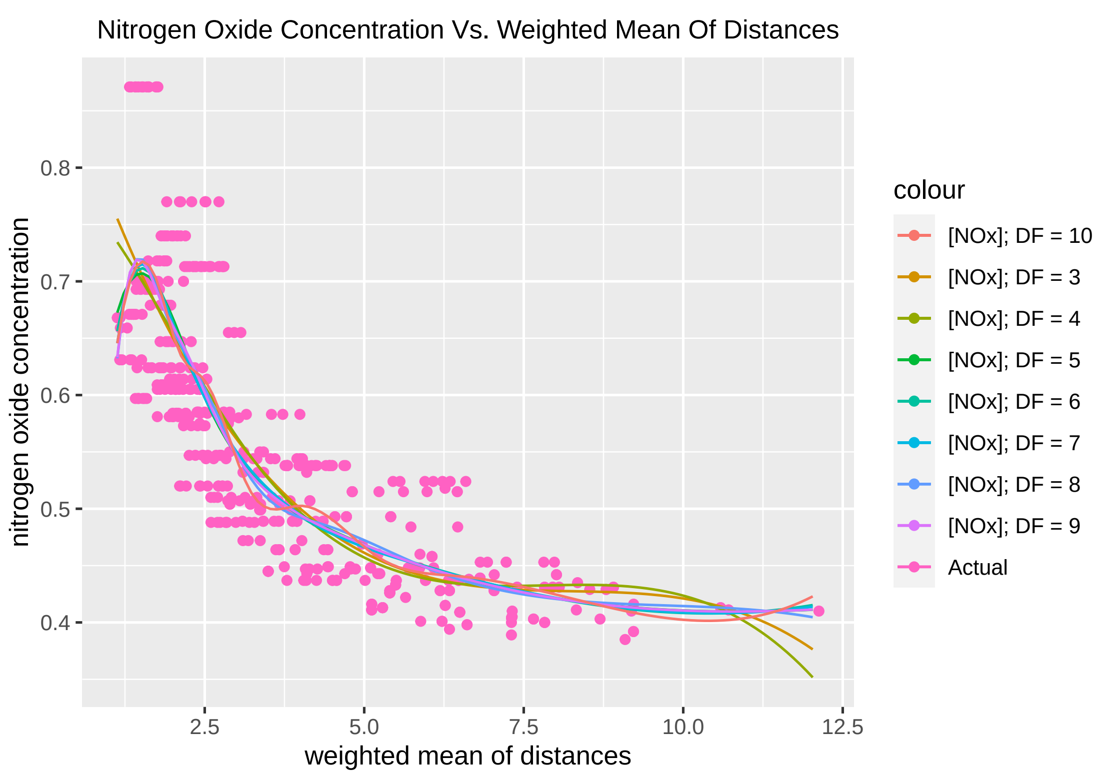


(e) Now fit a regression spline for a range of degrees of freedom, and plot the resulting fits and report the resulting RSS. Describe the results obtained.

```
ggplot() +
 geom_point(
    data = Boston,
    mapping = aes(
        x = dis,
        y = nox,
        color = "Actual"
```

```
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_3,
        color = "[NOx]; DF = 3"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_4,
        color = "[NOx]; DF = 4"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_5,
        color = "[NOx]; DF = 5"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_6,
        color = "[NOx]; DF = 6"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_7,
        color = "[NOx]; DF = 7"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_8,
        color = "[NOx]; DF = 8"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
```

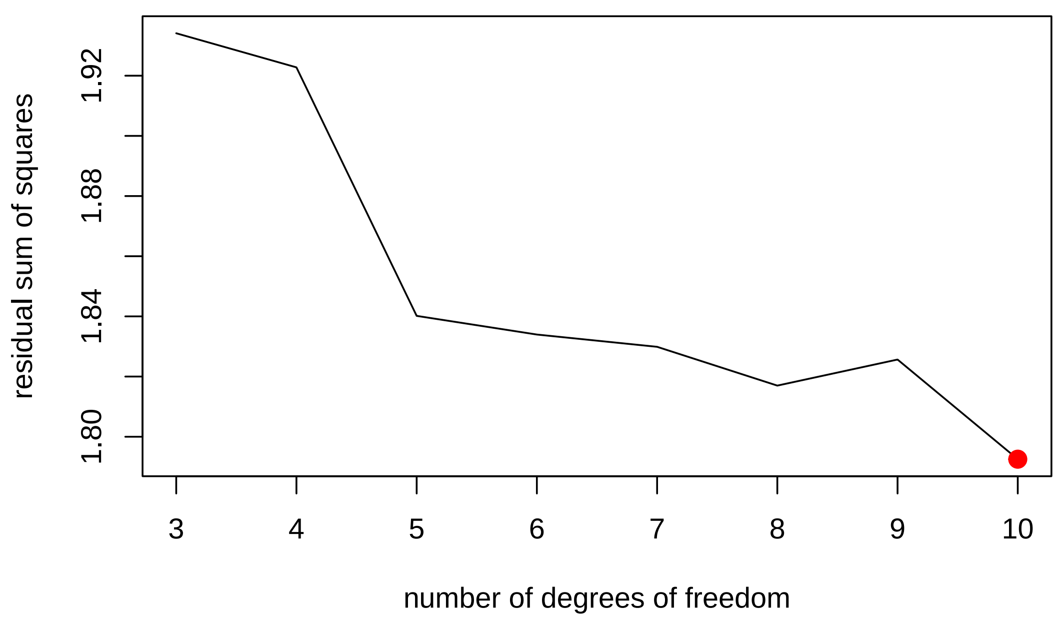
```
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_9,
        color = "[NOx]; DF = 9"
    )
) +
geom_line(
    data = data_frame_of_weighted_means_of_distances_and_predicted_NOx_concentrations,
    mapping = aes(
        x = weighted_mean_of_distances,
        y = NOx_concentration_predicted_by_B_spline_with_DF_10,
        color = "[NOx]; DF = 10"
    )
) +
labs(
    x = "weighted mean of distances",
    y = "nitrogen oxide concentration",
    title = "Nitrogen Oxide Concentration Vs. Weighted Mean Of Distances"
) +
theme(
 plot.title = element_text(hjust = 0.5, size = 11),
)
```



Per a plot of residual sum of squares vs. number of degrees of freedom of B-spline, residual sum of squares decreases with number of degrees of freedom of B-spline to a minimum for number of degrees of freedom 10.

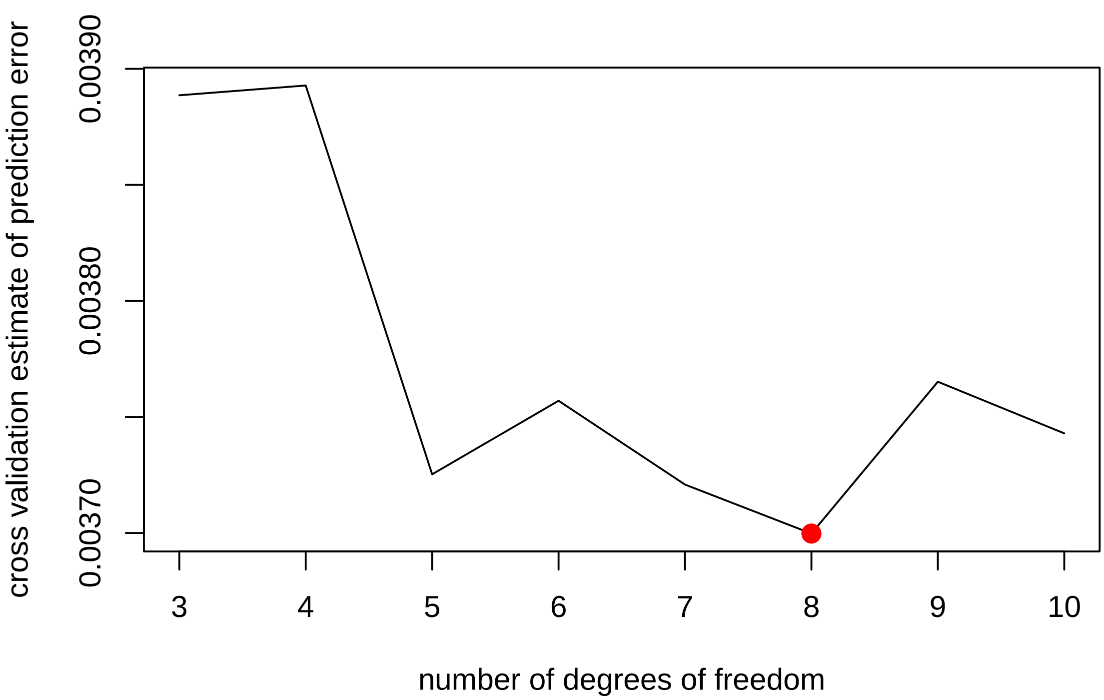
```
numbers_of_degrees_of_freedom <- 3:10
number_of_numbers_of_degrees_of_freedom <- length(numbers_of_degrees_of_freedom)
vector_of_residual_sums_of_squares <-
```

```r
    rep(NA, number_of_numbers_of_degrees_of_freedom)
for (number_of_degrees_of_freedom in numbers_of_degrees_of_freedom) {
    the_lm <- lm(
        nox ~ splines::bs(dis, df = number_of_degrees_of_freedom),
        data = Boston
    )
    vector_of_residual_sums_of_squares[number_of_degrees_of_freedom - 2] <-
        sum(the_lm$residuals^2)
}
plot(
    x = numbers_of_degrees_of_freedom,
    y = vector_of_residual_sums_of_squares,
    xlab = "number of degrees of freedom",
    ylab = "residual sum of squares",
    type = "l"
)
optimal_number_of_degrees_of_freedom <-
    which.min(vector_of_residual_sums_of_squares)
points(
    x = optimal_number_of_degrees_of_freedom + 2,
    y = vector_of_residual_sums_of_squares[
        optimal_number_of_degrees_of_freedom
    ],
    col = "red",
    cex = 2,
    pch = 20
)
```



(f) Perform cross-validation or another approach in order to select the best degrees of freedom for a regression spline on this data. Describe your results.

```
cross_validation_estimates_of_prediction_errors <-
    rep(NA, number_of_numbers_of_degrees_of_freedom)
for (number_of_degrees_of_freedom in numbers_of_degrees_of_freedom) {
    the_glm <- glm(
        nox ~ splines::bs(dis, df = number_of_degrees_of_freedom),
        data = Boston
    )
    cross_validation_estimates_of_prediction_errors[number_of_degrees_of_freedom - 2] <-
        boot::cv.glm(Boston, the_glm, K = 10)$delta[1]
}
plot(
    numbers_of_degrees_of_freedom,
    cross_validation_estimates_of_prediction_errors,
    xlab = "number of degrees of freedom",
    ylab = "cross validation estimate of prediction error",
    type = "l"
)
optimal_degree <- which.min(cross_validation_estimates_of_prediction_errors)
points(
    x = optimal_degree + 2,
    y = cross_validation_estimates_of_prediction_errors[optimal_degree],
    col = "red",
    cex = 2,
    pch = 20
)
```

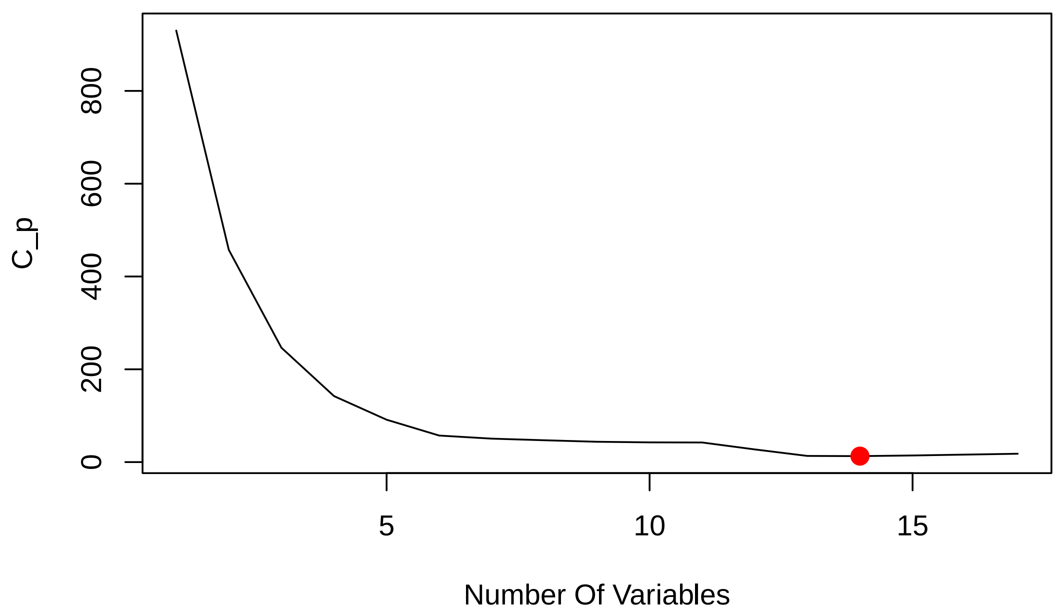


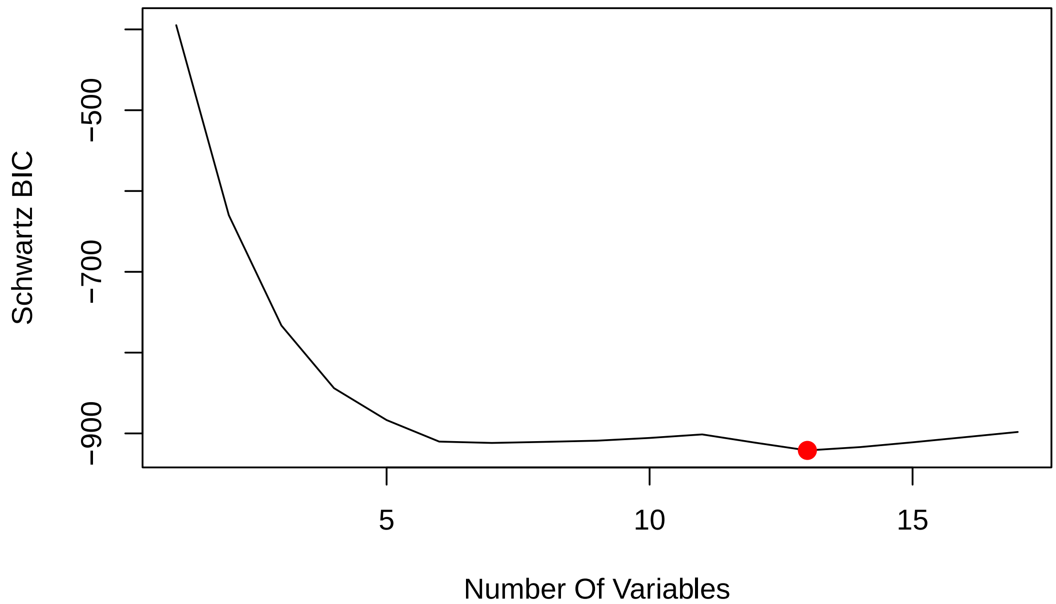
10. This question relates to the College data set.

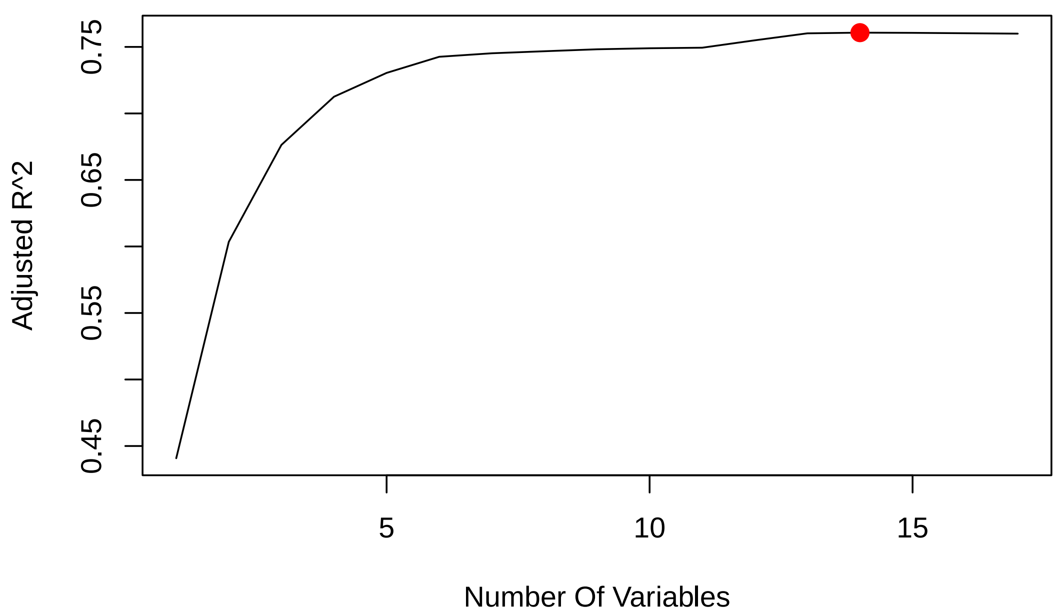
    (a) Split the data into a training set and a test set. Using out-of-state tuition as the response and the other variables as the predictors, perform forward stepwise selection on the training set in order to identify a satisfactory model that uses just a subset of the predictors.

        According to Mallow's C_p, adjusted R^2, and Schwartz Bayesian Information Criterion approximately, the best model by forward selection on the training set is a model that uses 14 predictors.

```
library(leaps)
colleges <- College
list_of_training_and_testing_data <-
    TomLeversRPackage::split_data_set_into_training_and_testing_data(
        data_frame = colleges,
        proportion_of_training_data = 0.9
    )
training_data <- list_of_training_and_testing_data$training_data
testing_data <- list_of_training_and_testing_data$testing_data
subset_selection_object <- regsubsets(
    Outstate ~ .,
    data = training_data,
    nvmax = 17,
    method = "forward"
)
TomLeversRPackage::analyze_subset_selection_object(subset_selection_object)
```








```
# $coefficients_by_Mallows_Cp
#   (Intercept)    PrivateYes          Apps        Accept        Enroll
# -1.675751e+03  2.190844e+03 -2.922056e-01  7.816260e-01 -5.591633e-01
#     Top10perc   F.Undergrad    Room.Board      Personal           PhD
```

```
#  2.544466e+01 -8.696432e-02  8.412901e-01 -2.799464e-01  1.473434e+01
#      Terminal     S.F.Ratio   perc.alumni        Expend     Grad.Rate
#  2.135001e+01 -5.027035e+01  4.148921e+01  1.894437e-01  2.599650e+01
#
# $coefficients_by_Schwartz_BIC
#   (Intercept)    PrivateYes          Apps        Accept     Top10perc
# -1760.7876572  2203.6868443    -0.2745499     0.6941289    24.1023505
#    F.Undergrad    Room.Board      Personal           PhD      Terminal
#    -0.1629121     0.8580574    -0.2835728    14.7886440    22.1407883
#     S.F.Ratio   perc.alumni        Expend     Grad.Rate
#   -50.0931302    40.3891913     0.1877100    25.7705077
#
# $coefficients_by_adjusted_R2
#   (Intercept)    PrivateYes          Apps        Accept        Enroll
# -1.675751e+03  2.190844e+03 -2.922056e-01  7.816260e-01 -5.591633e-01
#     Top10perc   F.Undergrad    Room.Board      Personal           PhD
#  2.544466e+01 -8.696432e-02  8.412901e-01 -2.799464e-01  1.473434e+01
#      Terminal     S.F.Ratio   perc.alumni        Expend     Grad.Rate
#  2.135001e+01 -5.027035e+01  4.148921e+01  1.894437e-01  2.599650e+01
```

The names of the 14 predictors of the best 14-predictor model according to forward selection are *Private*, *Apps*, *Accept*, *Enroll*, *Top10perc*, *F.Undergrad*, *Room.Board*, *Personal*, *PhD*, *Terminal*, *S.F.Ratio*, *perc.alumni*, *Expend*, and *Grad.Rate*.

(b) Fit a GAM on the training data, using out-of-state tuition as the response and the features selected in the previous step as the predictors. Plot the results, and explain your findings.

According to the documentation for `gam::s`, `s` is a "symbolic wrapper to indicate a smooth term in a formala argument to gam."

Below is a plot of a Generalized Additive Model (GAM) with subplots of `s(x = x, df = 4)`, where x represents one of our 14 predictors. In these plots, the function of *Room.Board* looks relatively linear. We can perform a series of ANOVA tests in order to determine which of these three models is best: a GAM that excludes *Room.Board* (M1), a GAM that uses a linear function of *Room.Board* (M2), or a GAM that uses a spline function of *Room.Board* (M3).

```r
library(gam)
```

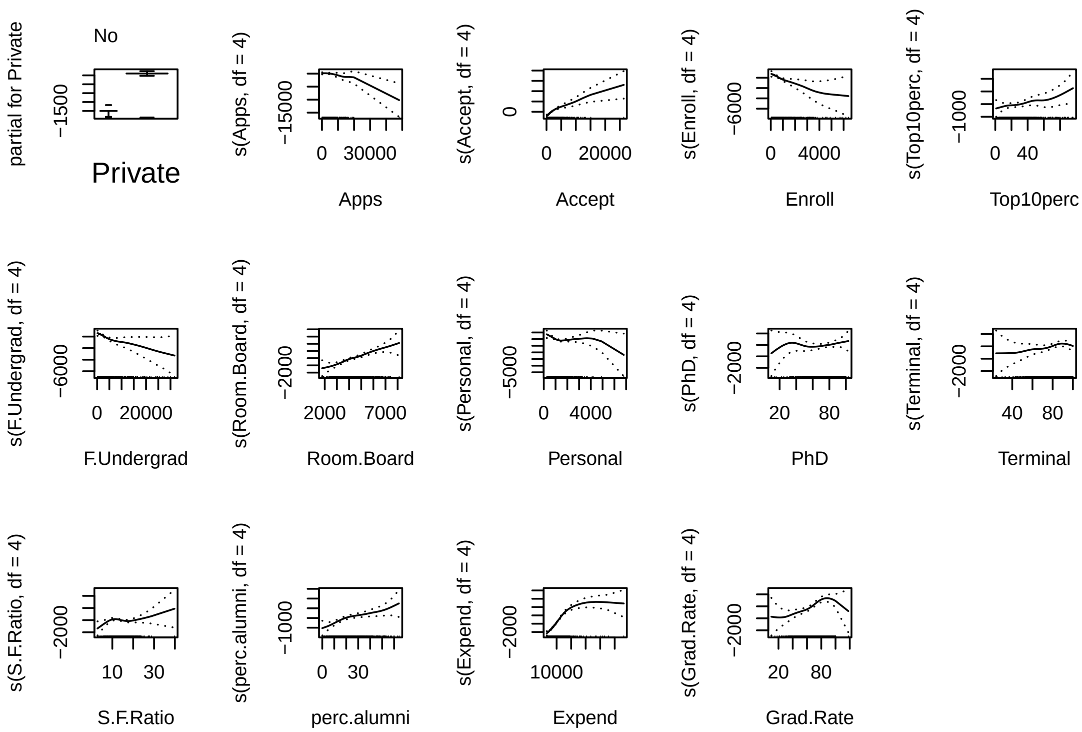
```
# Loading required package: splines

# Loading required package: foreach

# Loaded gam 1.22-2
```

```r
the_gam <- gam(
    Outstate ~
        Private +
        s(Apps, df = 4) +
        s(Accept, df = 4) +
        s(Enroll, df = 4) +
        s(Top10perc, df = 4) +
        s(F.Undergrad, df = 4) +
        s(Room.Board, df = 4) +
        s(Personal, df = 4) +
        s(PhD, df = 4) +
        s(Terminal, df = 4) +
        s(S.F.Ratio, df = 4) +
        s(perc.alumni, df = 4) +
```

```r
        s(Expend, df = 4) +
        s(Grad.Rate, df = 4),
    data = training_data
)
par(mfrow = c(3, 5))
plot(the_gam, se = TRUE)
```



(c) Evaluate the model obtained on the test set, and explain the results obtained.

(d) For which variables, if any, is there evidence of a non-linear relationship with the response?