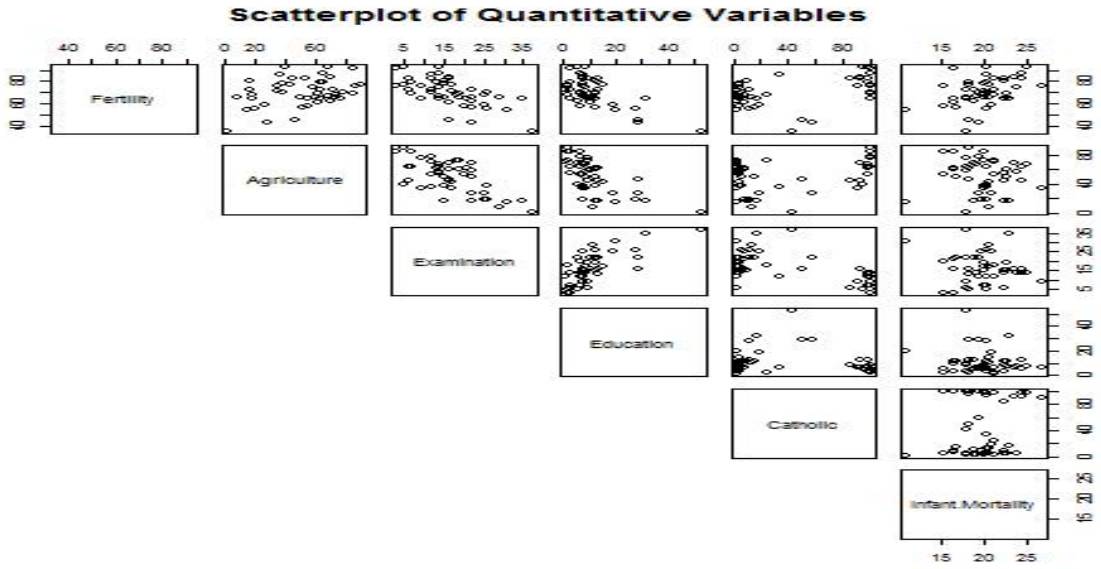
Stat 6021: Homework Set 6 Solutions

1. (a) The scatterplot matrix is displayed below:



The correlation matrix is shown below:

	Fertility	Agriculture	Examination	Education	Catholic
Fertility	1.000	0.353	-0.646	-0.664	0.464
Agriculture	0.353	1.000	-0.687	-0.640	0.401
Examination	-0.646	-0.687	1.000	0.698	-0.573
Education	-0.664	-0.640	0.698	1.000	-0.154
Catholic	0.464	0.401	-0.573	-0.154	1.000
Infant.Mortality	0.417	-0.061	-0.114	-0.099	0.175

	Infant.Mortality
Fertility	0.417
Agriculture	-0.061
Examination	-0.114
Education	-0.099
Catholic	0.175
Infant.Mortality	1.000

i. The predictors Examination and Education have the highest (negative) correlations with the fertility measure.

ii. A number of predictors are correlated with one another. For example, the percent of males involved in agriculture is correlated Examination and Education.

(b) The R output from the MLR:

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Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      66.91518   10.70604   6.250 1.91e-07 ***
Agriculture      -0.17211    0.07030  -2.448  0.01873 *
Examination      -0.25801    0.25388  -1.016  0.31546
Education        -0.87094    0.18303  -4.758 2.43e-05 ***
Catholic          0.10412    0.03526   2.953  0.00519 **
Infant.Mortality  1.07705    0.38172   2.822  0.00734 **

Residual standard error: 7.165 on 41 degrees of freedom
Multiple R-squared:  0.7067,    Adjusted R-squared:  0.671
F-statistic: 19.76 on 5 and 41 DF,  p-value: 5.594e-10
```

i. The ANOVA F statistic is testing whether our MLR model is useful in predicting the response variable. With a small p-value, our data supports the claim that our model is useful in predicting the fertility measure.

ii. A couple of "contradictions":

- We noted in the previous part that the response variable was most correlated with the predictors Examination and Education. However, in the MLR, the variable Examination is insignificant. This informs us that we could drop Examination from the model. This apparent contradiction is due to the fact that Examination is highly correlated with a number of other predictors, so when those predictors are present, we do not need to add Examination to the model.
- Also note that the relationship between the percent of males in agriculture and the fertility measure is positive, but in the MLR, the slope is negative. This apparent contradiction is due to the scatterplot and correlation not taking the effect of the predictors into account, whereas the MLR model is taking the other predictors into account.

2. (a) The estimated coefficient of *Stay* is 0.237209. The estimated *InfctRsk* increases by 0.237209 percent, for every one unit increase in the average length of stay in the hospital, when the other predictors are held constant.

(b) The hypothesis statements are
$H_0 : \beta_2 = 0$.
$H_0 : \beta_2 \neq 0$.

The t statistic is $\frac{-0.014071}{0.022708} = -0.6196$. The pvalue is $2 \times pt(-0.6196, 108)$ which is 0.5368.

Since p-value is greater than 0.05, we fail to reject the null hypothesis. Our data suggests that *Age* is not useful in predicting the response *InfctRsk*, when the other predictors are already in the model.

(c) I disagree. The t test in a multiple linear regression does not test if a predictor is linearly related to the response, on its own.

(d) The confidence interval using the Bonferroni method is

$$\hat{\beta}_j \pm t_{(1-\frac{\alpha}{2\times g}, n-p)} se(\hat{\beta}_j), \tag{1}$$

where g denotes the number of intervals created. Therefore, we have

- For β_1: $0.237209 \pm t_{1-\frac{0.05}{2\times 3}, 118-5} \times 0.060957$ which gives (0.0890, 0.3854).
- For β_2: $-0.014071 \pm t_{1-\frac{0.05}{2\times 3}, 118-5} \times 0.022708$ which gives (-0.0693, 0.0412).
- For β_3: $0.020383 \pm t_{1-\frac{0.05}{2\times 3}, 118-5} \times 0.005524$ which gives (0.0069, 0.0338).

$t_{1-\frac{0.05}{2\times 3}, 113-5}$ is found using `qt(1-0.05/(2*3), 108)` in R, which gives 2.431841.

(e)
- Residual standard error $= s = 1.04$, so MSE $= s^2 = 1.04^2$, and SSE $= 108 \times 1.04^2 = 116.8128$.
- $F = \dfrac{\text{MSR}}{\text{MSE}} \implies$ MSR $= F \times$ MSE $= 19.56 \times 1.04^2 = 21.1561$. So SSR $= 21.1561 \times 4 = 84.6244$.
- SST $=$ SSR $+$ SSE $= 84.6244 + 116.8128 = 201.4372$.
- DF for regression $= p - 1 = 5 - 1 = 4$.
- DF for error $= n - p = 113 - 5 = 108$.
- DF for total $= n - 1 = 113 - 1 = 112$.

Source of Variation	SS	df	MS
Regression	84.6244	4	21.1561
Error	116.8128	108	1.0816
Total	201.4372	112	*********

(f) $R^2 = \frac{SS_R}{SS_T} = \frac{84.6244}{201.4372} = 0.4201$.

About 42% of the variation in the response variable *InfctRsk* is explained by our model.

(g) $R^2_{adj} = 1 - \frac{MS_{res}}{MS_T} = 1 - \frac{1.0816}{201.4372/112} = 0.3986$.

3. The classmate's concern is not warranted. For the ANOVA F test, the hypothesis statements are
$H_0 : \beta_1 = \beta_2 = 0$.
H_a : not all β_1, β_2 equal zero.

Since the p-value is less than 0.05, we reject the null hypothesis. Our data suggests that our model is useful in predicting the response *LeftArm*.

Based on the t tests, we see that both of them do not have statistically significant results. We make the following conclusions:
LeftFoot can be removed from the model, when *RtFoot* is already in the model.
Likewise, *RtFoot* can be removed from the model, when *LeftFoot* is already in the model.

The conclusion for each t-test is that the variable is not a useful predictor given the presence of the other predictor in the model. Combined with the result of the F test, this means that we should only need one predictor, not both, to adequately predict left forearm length.

4.

$$\begin{aligned} \boldsymbol{HH} &= \left(\boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'}\right)\left(\boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'}\right) \\ &= \boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'}\boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'} \\ &= \boldsymbol{X}\boldsymbol{I}(\boldsymbol{X'X})^{-1}\boldsymbol{X'} \\ &= \boldsymbol{X}(\boldsymbol{X'X})^{-1}\boldsymbol{X'} \\ &= \boldsymbol{H} \end{aligned}$$