Building A Model That Helps Locating Displaced People

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Introduction

In this project, we build a model that would help us locate people displaced by the earthquake in Haiti in 2010. More specifically, we build in a timely manner an accurate model that classifies pixels in geo-referenced aerial images of Haiti in 2010 as depicting blue tarps or depicting objects that are not blue tarps. People whose homes were destroyed by the earthquake often created temporary shelters using blue tarps. Blue tarps were good indicators of where displaced people lived.

Data

Our training and holdout data were collected likely by applying a Region Of Interest (ROI) Tool to high-resolution, orthorectified / geo-referenced images of Haiti in 2010. Our training image is presented below and is sourced from `HaitiOrthorectifiedImage.tif` at Pixel Values from Images over Haiti. One ROI tool is described at Region of Interest (ROI) Tool. Classes may be assigned to pixels by defining Regions Of Interest.

Figure 1: Orthorectified Image Of Haiti In 2010

According to What is orthorectified imagery?, an orthorectified image is an accurately georeferenced image that has been processed so that all pixels are in an accurate (x, y) position on the ground. Orthorectified images have been processed to apply corrections for optical distortions from the sensor system, and apparent changes in the position of ground objects caused by the perspective of the sensor view angle and ground terrain.

Our training data frame is sourced from a CSV file like `HaitiPixels.csv` at Pixel Values from Images over Haiti. Our training data frame consists of 63,241 observations. Each training observation consists of a class in the set $\{Vegetation, Soil, Rooftop, Various\ Non-Tarp, Blue\ Tarp\}$ and a pixel. A pixel is a colored dot. A pixel is represented by a tuple of intensities of color *Red*, *Green*, and *Blue* in the range 0 to 255. See below head and tail of our training data frame.

```
training_data_frame_of_classes_and_pixels <- read.csv(
    file = "Training_Data_Frame_Of_Classes_And_Pixels.csv"
)
head(x = training_data_frame_of_classes_and_pixels, n = 2)
```

```
#       Class Red Green Blue
# 1 Vegetation  64    67   50
# 2 Vegetation  64    67   50
```

```
tail(x = training_data_frame_of_classes_and_pixels, n = 2)
```

```
#           Class Red Green Blue
# 63240 Blue Tarp 132   139  149
# 63241 Blue Tarp 133   141  153
```

We conduct exploratory data analysis by considering the distributions of intensities of color *Red*, *Green*, and *Blue* in our training data frame of classes and pixels. The distribution of intensity of color Red is normal except for a relatively high proportion of high intensities. The distributions of intensity of colors Green and Blue each have two hills and are not normal.

Please explore TomLeversRPackage.

```
library(TomLeversRPackage)
plot_distribution(training_data_frame_of_classes_and_pixels, "Red", "red")
```

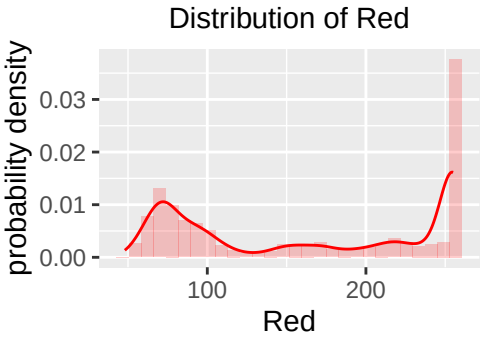


Figure 2: Distribution Of Intensities Of Color Red

```
plot_distribution(training_data_frame_of_classes_and_pixels, "Green", "green")
```

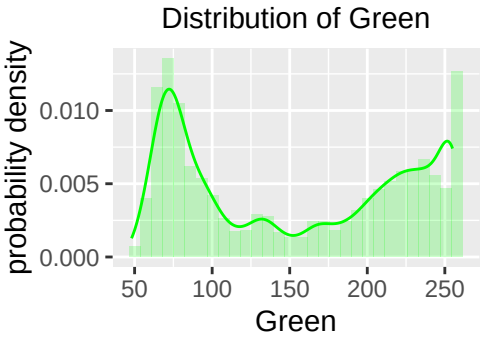


Figure 3: Distribution Of Intensities Of Color Green

```
plot_distribution(training_data_frame_of_classes_and_pixels, "Blue", "blue")
```

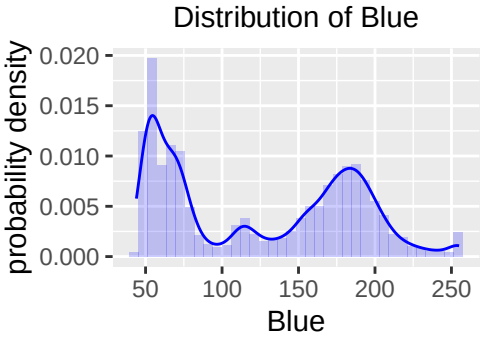


Figure 4: Distribution Of Intensities Of Color Blue

We examine the distribution of classes (such as *Blue Tarp*) in our training data frame in a space defined by intensities of color *Red*, *Green*, and *Blue*. The intensity space for pixels representing blue tarps is distinct from the intensity sapce for pixels representing objects that are not blue tarps.

```
distribution_of_classes_in_intensity_space <- plotly::plot_ly(
    data = training_data_frame_of_classes_and_pixels,
    x = ~Red,
    y = ~Green,
    z = ~Blue,
    color = ~Class,
    type = "scatter3d",
    mode = "markers",
    colors = c("blue", "black", "brown", "orange", "green")
)
htmlwidgets::saveWidget(
    widget = distribution_of_classes_in_intensity_space,
    file = "distribution_of_classes_in_intensity_space.html"
)
webshot2::webshot(
    url = "distribution_of_classes_in_intensity_space.html",
    file = "distribution_of_classes_in_intensity_space.png"
)
```

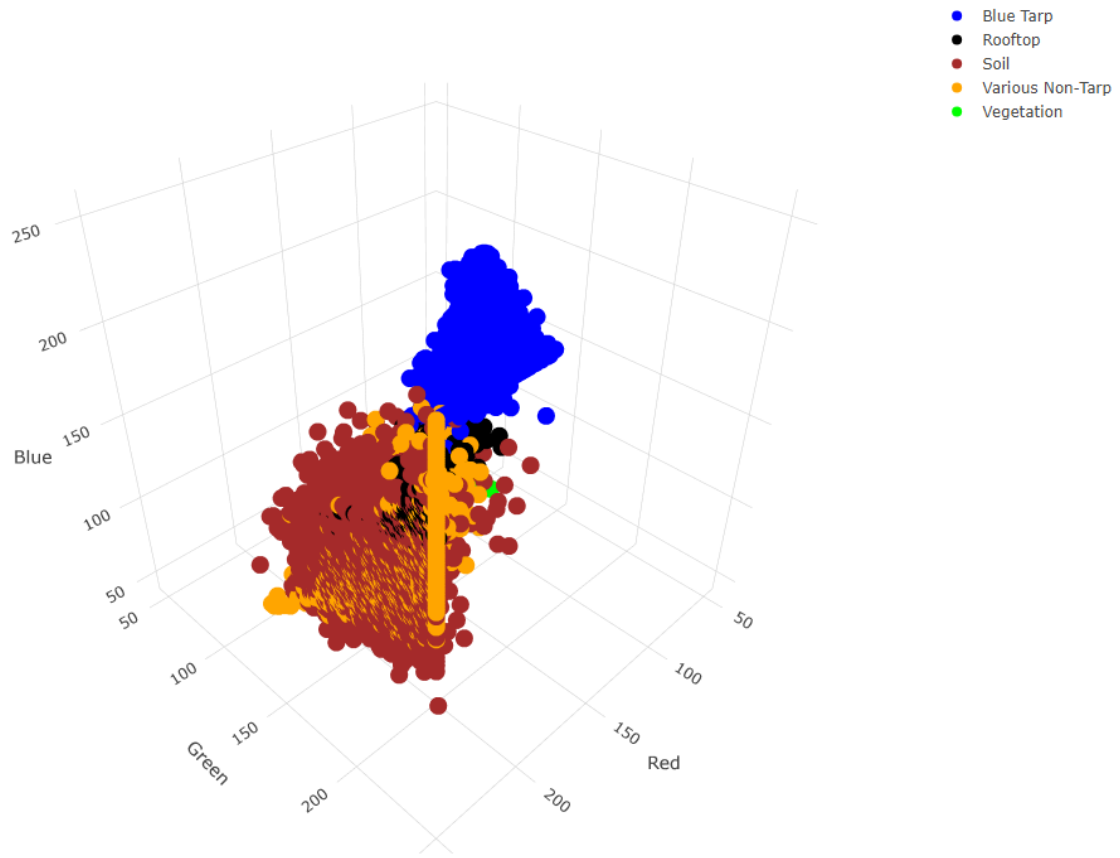


Figure 5: Distribution Of Classes In Intensity Space

Our holdout data frame is sourced from CSV files like `orthovnir057_ROI_NON_Blue_Tarps.txt`, `orthovnir067_ROI_Blue_Tarps.txt`, `orthovnir067_ROI_NOT_Blue_Tarps.txt`, `orthovnir069_ROI_Blue_Tarps.txt`, `orthovnir069_ROI_NOT_Blue_Tarps.txt`, and `orthovnir078_ROI_NON_Blue_Tarps.txt` at the above link and `orthovnir078_ROI_Blue_Tarps.txt` from Dr. Peter Gedeck. Each holdout observation consists of a class in the set $\{Not\ Blue\ Tarp,\ Blue\ Tarp\}$ and a pixel. See below head and tail of our holdout data frame.

```
holdout_data_frame_of_classes_and_pixels <- read.csv(
    file = "Holdout_Data_Frame_Of_Classes_And_Pixels.csv"
)
head(x = holdout_data_frame_of_classes_and_pixels, n = 2)
```

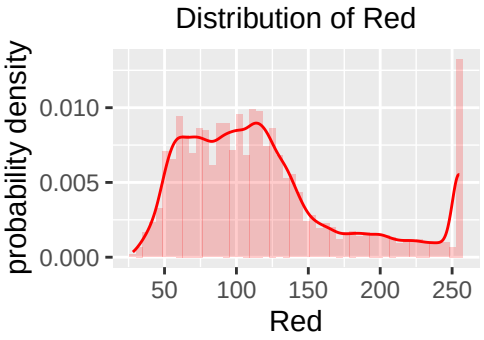
```
#           Class Red Green Blue
# 1 Not Blue Tarp 104    89   63
# 2 Not Blue Tarp 101    80   60
```

```
tail(x = holdout_data_frame_of_classes_and_pixels, n = 2)
```

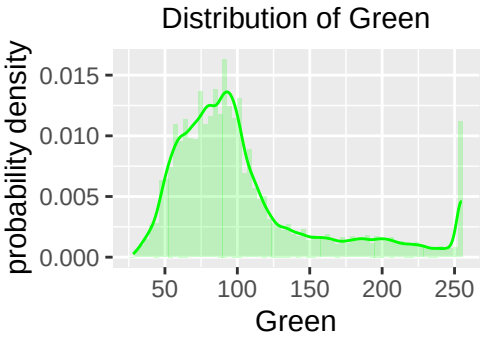
```
#                 Class Red Green Blue
# 2004176 Not Blue Tarp  88    75   73
# 2004177 Not Blue Tarp  72    65   62
```

We conduct exploratory data analysis by considering the distributions of intensities of color *Red*, *Green*, and *Blue* in our holdout data frame of classes and pixels. The distributions of intensity of colors *Red*, *Green* and *Blue* are roughly normal and broad except for long right tails and relatively high proportions of high intensities.

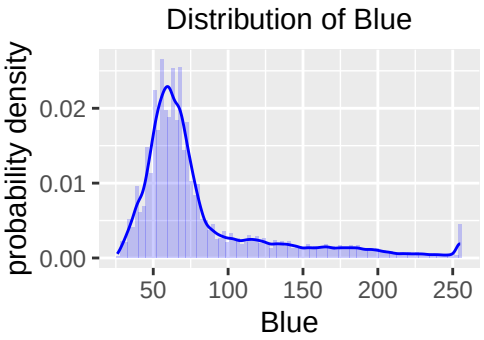
```
number_of_training_observations <- nrow(training_data_frame_of_classes_and_pixels)
number_of_holdout_observations <- nrow(holdout_data_frame_of_classes_and_pixels)
vector_of_random_indices <- sample(
    x = 1:number_of_holdout_observations,
    size = number_of_training_observations
)
slice_of_holdout_data_frame <- holdout_data_frame_of_classes_and_pixels[vector_of_random_indices, ]
plot_distribution(slice_of_holdout_data_frame, "Red", "red")
```




```
plot_distribution(slice_of_holdout_data_frame, "Green", "green")
```




```
plot_distribution(slice_of_holdout_data_frame, "Blue", "blue")
```



We examine the distribution of classes (such as *Blue Tarp*) in our holdout data frame in a space defined by intensities of color *Red*, *Green*, and *Blue*. The intensity space for pixels representing blue tarps is distinct from the intensity sapce for pixels representing objects that are not blue tarps.

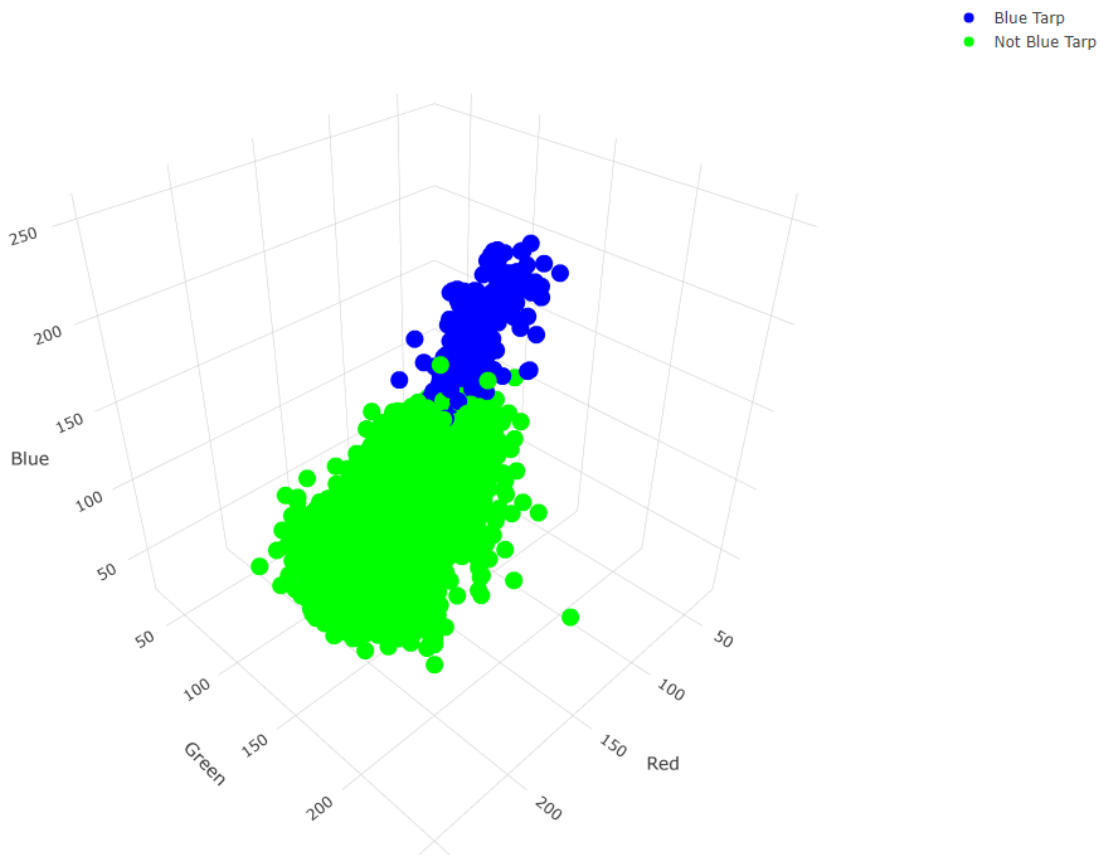


Figure 6: Distribution Of Classes In Intensity Space

The proportion of training observations that correspond to blue tarps is about 25 times the proportion of holdout observations that correspond to blue tarps.

```
number_of_training_observations_corresponding_to_blue_tarps <- log(nrow(
    training_data_frame_of_classes_and_pixels[
        training_data_frame_of_classes_and_pixels$Class == "Blue Tarp",
    ]
))
number_of_holdout_observations_corresponding_to_blue_tarps <- log(nrow(
    holdout_data_frame_of_classes_and_pixels[
        holdout_data_frame_of_classes_and_pixels$Class == "Blue Tarp",
    ]
))
proportion_of_training_observations_that_correspond_to_blue_tarps <-
    number_of_training_observations_corresponding_to_blue_tarps /
    number_of_training_observations
proportion_of_holdout_observations_that_correspond_to_blue_tarps <-
```

```
    number_of_holdout_observations_corresponding_to_blue_tarps /
    number_of_holdout_observations
proportion_of_training_observations_that_correspond_to_blue_tarps /
    proportion_of_holdout_observations_that_correspond_to_blue_tarps
```

```
# [1] 25.17897
```

Methods

Choosing A Binary Classifier

Since the intensity space for pixels representing blue tarps is distinct from the intensity space for pixels representing objects that are not blue tarps, we consider our optimal model to be a binary classifier that classifies pixels as depicting blue tarps or depicting objects that are not blue tarps. We may ignore non-binary classifiers that predict probabilities for all classes and may be used to locate pixels that more likely depict blue tarps than objects that are not blue tarps.

Choosing A Performance Metric

A threshold is a probability. A model classifies a pixel as representing a blue tarp if the probability that the pixel represents a blue tarp that the model predicts is greater than the threshold. False Positive Rate (FPR) is the ratio of false positives to actual negatives. Precision / Positive Predictive Value (PPV) is the ratio of true positives to predicted positives. Recall / True Positive Rate (TPR) is the ratio of true positives to actual positives. An F1 measure is the harmonic mean of PPV and TPR. A decimal of true positives is the ratio of number of true positives to total number of predictions. All of these quantities lie between 0 and 1.

Receiver Operator Characteristic (ROC) graphs are graphs of TPR Vs. FPR for different thresholds. According to ROC and AUC, Clearly Explained!, the Area Under The Curve (AUC) of an ROC graph "makes it easy to compare one ROC curve to another" and to compare one binary classifier to another. If the AUC for one ROC curve is greater than the AUC for another ROC curve, the binary classifier with the former ROC curve is better than the binary classifier with the latter ROC curve. A given model's threshold may be tuned so that its FPR is close to 0 and its TPR is close to 1. We may consider maximizing ROC AUC among binary classifiers and tuning a binary classifier's thresholds to minimize the distance between the corresponding point (FPR, TPR) and (0, 1).

"People often replace the False Positive Rate with Precision... If there were lots of samples that were not [Blue Tarps] relative to the number of [Blue Tarp] samples, then Precision might be more useful than the False Positive Rate. This is because Precision does not include the number of True Negatives in its calculation, and is not effected by the imbalance. In practice, this sort of imbalance occurs when studying a rare [occurrence]. In this case, the study will contain many more [pixels not corresponding to blue tarps than corresponding to blue tarps]. We may consider maximizing PR AUC among binary classifiers and tuning a binary classifier's thresholds to minimize the distance between the corresponding point (TPR, PR) and (1, 1).

According to Category graph: Precision-Recall vs. Threshold, "The ideal threshold setting is the highest possible recall and precision rate. This goal is not always achievable, because the higher the recall rate, the lower the precision rate, and vice versa. Setting the most appropriate threshold for a category is a trade-off between these two rates."

According to Optimal Thresholding of Classifiers to Maximize F1 Measure, "The [balanced] harmonic mean of precision and recall, the F1 measure is widely used to evaluate the success of a binary classifier when one class is rare." According to The truth of the F-measure, "the F-measure was first introduced to evaluation

tasks of information extraction technology at the Fourth Message Understanding Conference (MUC-4) in 1992."

The average of two ratios (e.g., precision and recall) is the balanced harmonic mean of those two ratios. According to Harmonic mean, "For instance, if a vehicle travels a certain distance d outbound at a speed s_1 (e.g., 60 km/h) and returns the distance at a speed s_2 (e.g., 20 km/h), then its average speed is the harmonic mean of s_1 and s_2 (30 km/h), not the arithmetic mean (40 km/h). The total travel time is the same as if it had traveled the whole distance at that average speed. This can be proven as follows:

$$Average\ speed\ for\ the\ entire\ journey = \frac{Total\ distance\ traveled}{Sum\ of\ time\ for\ each\ segment} = \frac{D}{T} = \frac{D}{t_1 + t_2} = \frac{2d}{\frac{d}{s_1} + \frac{d}{s_2}} = \frac{2}{\frac{1}{s_1} + \frac{1}{s_2}}$$

"[I]f each sub-trip covers the same distance, then the average speed is the [balanced] harmonic mean of [each] sub-trip speed." If one sub-trip covers a greater distance than the other sub-trip, "then a weighted harmonic mean is needed." To return to the F1 measure, if each of precision and recall has the same weight, then the F1 measure is the balanced harmonic mean of each of precision and recall. This balanced harmonic mean is

$$F1 = \frac{2}{\frac{1}{P} + \frac{1}{R}}$$

If recall has more weight than precision, then a weighted harmonic mean is needed. This weighted harmonic mean is

$$F_{w_P,\ w_R} = \frac{w_P + w_R}{\frac{w_P}{P} + \frac{w_R}{R}}$$

When searching for our optimal binary classifier, we choose as our optimal binary classifier the binary classifier with the highest F1 measure and $F_{w_P=1,\ w_R=1}$.

Regarding the weight of PPV and the weight of TPR, we prioritize TPR at least as much as PPV. We prioritize identifying as many pixels corresponding to blue tarps correctly as possible at least as much as having predictions that pixels correspond to blue tarps be correct. As we move toward providing resources for refugees, we may wish to prioritize identifying as many pixels corresponding to blue tarps as possible more than having predictions that pixels correspond to blue tarps be correct.

Data Frame For Modeling

In order to build binary classifiers, we create a composite data frame of our training data frame and our holdout data frame; substitute a column of classes for a column of indicators of whether of not a pixel depicts a blue tarp; add columns corresponding to normalized intensities and intensities normalized after transforming by the natural logarithm, the square root, the square, and interactions; and divide the composite data frame into training and holdout data frames again. We normalize as opposed to standardize intensities given that distributions of intensity are not normal. We shuffle the rows in our training and holdout data frames.

```
training_data_frame_of_indicators_and_pixels <- NULL
should_generate_training_and_holdout_data_frames_of_indicators_and_pixels <- FALSE
if (should_generate_training_and_holdout_data_frames_of_indicators_and_pixels) {
    composite_data_frame_of_classes_and_pixels <- rbind(
        training_data_frame_of_classes_and_pixels,
        holdout_data_frame_of_classes_and_pixels
    )
    number_of_training_and_holdout_observations <-
        nrow(composite_data_frame_of_classes_and_pixels)
    column_of_indicators <- rep(0, number_of_training_and_holdout_observations)
    condition <- composite_data_frame_of_classes_and_pixels$Class == "Blue Tarp"
    column_of_indicators[condition] <- 1
```

```
factor_of_indicators <- factor(column_of_indicators)
composite_data_frame_of_indicators_and_pixels <- data.frame(
    Indicator = factor_of_indicators,
    Normalized_Red = normalize_vector(composite_data_frame_of_classes_and_pixels[, "Red"]),
    Normalized_Green = normalize_vector(composite_data_frame_of_classes_and_pixels[, "Green"]),
    Normalized_Blue = normalize_vector(composite_data_frame_of_classes_and_pixels[, "Blue"]),
    Normalized_Natural_Logarithm_Of_Red =
        normalize_vector(log(composite_data_frame_of_classes_and_pixels[, "Red"])),
    Normalized_Natural_Logarithm_Of_Green =
        normalize_vector(log(composite_data_frame_of_classes_and_pixels[, "Green"])),
    Normalized_Natural_Logarithm_Of_Blue =
        normalize_vector(log(composite_data_frame_of_classes_and_pixels[, "Blue"])),
    Normalized_Square_Root_Of_Red =
        normalize_vector(sqrt(composite_data_frame_of_classes_and_pixels[, "Red"])),
    Normalized_Square_Root_Of_Green =
        normalize_vector(sqrt(composite_data_frame_of_classes_and_pixels[, "Green"])),
    Normalized_Square_Root_Of_Blue =
        normalize_vector(sqrt(composite_data_frame_of_classes_and_pixels[, "Blue"])),
    Normalized_Square_Of_Red =
        normalize_vector(I(composite_data_frame_of_classes_and_pixels[, "Red"]^2)),
    Normalized_Square_Of_Green =
        normalize_vector(I(composite_data_frame_of_classes_and_pixels[, "Green"]^2)),
    Normalized_Square_Of_Blue =
        normalize_vector(I(composite_data_frame_of_classes_and_pixels[, "Blue"]^2)),
    Normalized_Interaction_Of_Red_And_Green = normalize_vector(
        as.numeric(
            interaction(
                composite_data_frame_of_classes_and_pixels$Red,
                composite_data_frame_of_classes_and_pixels$Green
            )
        )
    ),
    Normalized_Interaction_Of_Red_And_Blue = normalize_vector(
        as.numeric(
            interaction(
                composite_data_frame_of_classes_and_pixels$Red,
                composite_data_frame_of_classes_and_pixels$Blue
            )
        )
    ),
    Normalized_Interaction_Of_Green_And_Blue = normalize_vector(
        as.numeric(
            interaction(
                composite_data_frame_of_classes_and_pixels$Green,
                composite_data_frame_of_classes_and_pixels$Blue
            )
        )
    )
)
training_data_frame_of_indicators_and_pixels <-
    composite_data_frame_of_indicators_and_pixels[1:number_of_training_observations, ]
holdout_data_frame_of_indicators_and_pixels <-
    composite_data_frame_of_indicators_and_pixels[
```

```
            (number_of_training_observations + 1):
            number_of_training_and_holdout_observations,
        ]
    set.seed(1)
    vector_of_random_indices <- sample(1:number_of_training_observations)
    training_data_frame_of_indicators_and_pixels <-
        training_data_frame_of_indicators_and_pixels[vector_of_random_indices, ]
    set.seed(1)
    vector_of_random_indices <- sample(1:number_of_holdout_observations)
    holdout_data_frame_of_indicators_and_pixels <-
        holdout_data_frame_of_indicators_and_pixels[vector_of_random_indices, ]
    write.csv(
        training_data_frame_of_indicators_and_pixels,
        "Training_Data_Frame_Of_Indicators_And_Pixels.csv",
        row.names = FALSE
    )
    write.csv(
        holdout_data_frame_of_indicators_and_pixels,
        "Holdout_Data_Frame_Of_Indicators_And_Pixels.csv",
        row.names = FALSE
    )
} else {
    training_data_frame_of_indicators_and_pixels <-
        read.csv("Training_Data_Frame_Of_Indicators_And_Pixels.csv", header = TRUE)
    training_data_frame_of_indicators_and_pixels$Indicator <-
        factor(training_data_frame_of_indicators_and_pixels$Indicator)
    holdout_data_frame_of_indicators_and_pixels <-
        read.csv("Holdout_Data_Frame_Of_Indicators_And_Pixels.csv", header = TRUE)
    holdout_data_frame_of_indicators_and_pixels$Indicator <-
        factor(holdout_data_frame_of_indicators_and_pixels$Indicator)
}
t(training_data_frame_of_indicators_and_pixels[1, ])
```

Grid Search

Dimensions

First Dimension: Type Of Classifier We conduct a grid search for an optimal binary classifier according to the $F1measure$. The primary dimension of our grid is type of classifier. We compare Logistic Regression (LR), Logistic Ridge Regression (LRR), Linear Discriminant Analysis (LDA), Quadratic Discriminant Analysis (QDA), K Nearest Neighbors (KNN), Random Forest (RF), and Support-Vector Machine (SVM) classifiers. Generally speaking, LR, LRR, LDA, and QDA classifiers are relatively inflexible with high bias and low variance; KNN, RF, and SVM classifiers are relatively flexible with low bias and high variance. KNN classifiers may be flexible when K is approximately 1 or inflexible as K approaches the number of observations in our training data set n.

Second Dimension: Set Of Predictive Terms The secondary dimension of our grid is the set of predictive terms. We consider classifiers with the terms in the above output other than *Indicator*. Per University Of Virginia courses Linear Models For Data Science and Introduction To Statistical Learning, these predictive terms involve common transformations of predictors.

Third Dimension: Value Of Primary Hyperparameter The third dimension of our grid is value of primary hyperparameter. For LRR classifiers, the primary hyperparameter is λ. LRR classifiers are penalized for inclusion of predictive terms proportionally to λ; setting λ to be greater than 0 may decrease the variance and increase the performance of LRR classifiers.

For KNN classifiers, the primary hyperparameter is K. The class of a test observation depends on determining the classes of the K nearest neighboring observations in the training data set. A KNN classifier with $K = 1$ is relatively flexible with low bias and high variance; such a model may overfit the training data and identify patterns with performance that is less than ideal. A KNN classifier with $K = n$ predicts that all observations have one class and has high bias and low variance. A KNN classifier with $1 < K < n$ may perform best.

For RF classifiers, the primary hyperparameter is *mtry*, the number of variables randomly sampled as candidates at each split / fraction of features available at each split. According to Dr. Bill Basener, "by selecting only some of the features for each split, we make our trees to be different from each other. We're creating more variety among our trees and are decorrelating the trees... [mtry] is probably the most important parameter... for a random forest. And the way this works: A lower fraction creates more decorrelation among the trees. So the trees are more different. But it also creates lower accuracy in the individual trees because they don't have as many parameters to select from for their classification. And so there's a bit of a tradeoff there."

For SVM classifiers, the primary hyperparameter is cost C. According to *An Introduction to Statistical Learning$ (James et al. 2023), "Cost C is a nonnegative tuning parameter... C determines the number of severity of the violations to the margin (and to the hyperplane) that we will tolerate. We can think of C as a budget for the amount that the margin can be violated by the n observations. If $C = 0$ then there is no budget for violations to the margin... For $C > 0$ no more than C observations can be on the wrong side of the hyperplane... As the budget C increases, we become more tolerant of violations to the margin, and so the margin will widen. Conversely, as C decreases, we become less tolerant of violations to the margin and so the margin narrows...

C controls the bias-variance trade-off of a support-vector [machine]. When C is small, we seek narrow margins that are rarely violated; this amounts to a classifier that is highly fit to the data, which may have low bias but high variance. On the other hand, when C is larger, the margin is wide and we allow more violations to it; this amounts to fitting the data less hard and obtaining a classifier that is potentially more biased but may have lower variance...

When the tuning parameter C is large, then the margin is wide, many observations violate the margin, and so there are many support vectors. In this case, many observations are involved in determining the hyperplane. This classifier has low variance (since many observations are support vectors) but potentially high bias. In contrast, if C is small, then there will be fewer support vectors and hence the resulting classifier will have low bias but high variance...

When C is large, then there is a high tolerance for observations being on the wrong side of the margin, and so the margin will be large. As C decreases, the tolerance for observations being on the wrong side of the margin decreases, and the margin narrows."

Fourth Dimension: Value Of Secondary Hyperparameter The fourth dimension of our grid is value of secondary hyperparameter. For RF classifiers, the secondary hyperparameter is *ntree*, the number of trees that we want to create. According to Dr. Basener, "And what is the effect of this parameter? So the more trees [you] have the more accuracy you get, but up to a point. Typically, you'll get a lot of increase when you go from 1 to 2 to 10 or 20 trees, and then you'll increase more up to 100. But at some point, increasing doesn't give you any more accuracy. So that may be at 500 that you don't want to create any more trees, or 2000 you have enough trees. It depends a lot of how complicated of a decision you need to make, how complicated your space is, how many features you have, how much data you have." According to Dr. Peter Gedeck, "a good rule of thumb is that usually from 100 trees onward, not much happens."

For SVM classifiers, the secondary hyperparameter is degree d for classifiers with polynomial kernels and γ for classifiers with radial kernels. According to *An Introduction to Statistical Learning*, "It can be shown

that... The linear support vector classifier can be represented as

$$f(x) = \beta_0 + \sum_{i=1}^{n} [\alpha_i \langle x, x_i \rangle]$$

where there are n parameters α_i, one per training observation."

According to Dr. Basener, "we can take this hold framework and instead of using the inner product there, we can replace that by what's called a kernel. And so it's a way of measuring distance between two points: between x, a new observation, and each x_i in our training set. And so we can use what's called the linear kernel

$$K(x_i, x_{i'}) = \sum_{j=1}^{p} [x_{ij} x_{i'j}]$$

, which is [the inner product in] the linear support vector classifier. We can use a polynomial kernel of degree [d]

$$K(x_i, x_{i'}) = \left(1 + \sum_{j=1}^{p} [x_{ij} x_{i'j}]\right)^d$$

So all we're doing is replacing th[e inner product] with one of these functions here. So [there's] a polynomial kernel of degree d or what's called a radial kernel

$$K(x_i, x_{i'}) = \exp\left\{-\gamma \sum_{j=1}^{p} \left[(x_{ij} - x_{i'j})^2\right]\right\}$$

And this is a common kernel to use. It's also called a radial basis function... So this is an exponential. Let's think about this function. There's an exponential of [a scaled] distance between the two points squared. So this is going to give some sort of bell-shaped curve and that γ tells us how quickly that bell-shaped curve drops off."

Fifth Dimension: Value Of Threshold The fifth dimension of our grid is value of threshold. Varying the threshold according to which a classifier classifies an observations as corresponding to a blue tarp or not will result in different numbers of false negatives, false positives, true negatives, and true positives and different F1 measures. We seek to maximize F1 measure.

Cross Validation

According to *An Introduction to Statistical Learning (Second Edition)* by James et al., for models with categorical responses such as binary classifiers, "k-fold cross validation involves randomly dividing a set of observations into k groups, or folds, of approximately equal size. The first fold is treated as a validation set, and the method is fit on the remaining $k - 1$ folds." A performance metric like F1 measure is "computed on the observations in the held-out fold. This procedure is repeated k times; each time, a different group of observations is treated as a validation set. This process results in k estimates of the" performance metric, $F_1, F_2, ..., F_k$. "The k-fold cross validation estimate is computed by averaging these values".

"[W]e are interested only in the location of the [maximum] point in the estimated test [F1 measure] curve. This is because we might be performing cross-validation on a number of statistical learning methods, or on a single method using different [hyperparameters], in order to identify the method that results in the [highest F1 measure. D]espite the fact that they sometimes [misestimate] the true test [F1 measure, most] of the CV curves [of F1 measure vs. threshold] come close to identifying the correct [threshold] corresponding to the [highest F1 measure]."

"[T]here is some variability in the CV estimates as a result of the variability in how the observations are divided into ten folds. But this variability is typically much lower than the variability in the test [F1 measures] that result from the validation set approach."

As described below, we apply cross validation to averaging performance metrics and choosing λ for LRR classifiers, K for KNN classifiers, and C and γ for SVM classifiers. We apply cross validation to averaging performance metrics using `rsample::vfold_cv`. We use `vfold_cv` by passing `vfold_cv` our full training data frame of indicators and pixels, and specifications of numbers of partitions of the data set and times to repeat v-fold partitioning. We apply cross validation to choosing λ, K, and C and γ using `caret::train` by passing `train` a `caret::trainControl` constructed with `method = "cv"`.

Bidirectional Selection

To search for an optimal LR classifier, we perform manual bidirectional selection. For each formula, we perform 10-fold cross validation. For each fold, we record 100 thresholds and corresponding performance metrics including F1 measure. For each formula, we record 100 thresholds and corresponding average performance metrics. For each formula, we find the maximum average F1 measure. For each formula, we choose the threshold that yields the highest maximum average F1 measure. We choose the LR classifier with the formula and threshold with the highest maximum average F1 measure. For our optimal LR classifier with optimal formula and optimal threshold, we provide a plot of Average Performance Metrics Vs. Threshold, a Precision-Recall Curve, an ROC curve, and a data frame of optimal performance metrics.

To describe bidirectional selection in detail, we determine the maximum average F1 measure for LR classifiers with formula $Indicator \sim normalize(Red)$. Similarly, we calculate the maximum average F1 measure for LR classifiers with formula $Indicator \sim normalize(Green), Indicator \sim normalize(Blue), ..., Indicator \sim normalize\,(Green : Blue)$. The classifiers with formula $Indicator \sim normalize(Blue^2)$ have the highest maximum average F1 measure of 0.446. We consider the maximum average F1 measures for LR classifiers with formulas $Indicator \sim normalize(Blue^2) + normalize(Red), ..., Indicator \sim normalize(Blue^2) + normalize(Green : Blue)$. The classifiers with formula $Indicator \sim normalize(Blue^2) + normalize(Red^2)$ have the highest maximum average F1 measure of 0.912. Since we have already considered the maximum average F1 measures for classifiers with formulas $Indicator \sim normalize(Red^2)$ and $Indicator \sim normalize(Blue^2)$, we do not drop the other predictor and do not consider the F1 measures of these classifiers. We consider the maximum average F1 measures for classifiers with formulas $Indicator \sim normalize(Blue^2) + normalize(Red^2) + normalize(Red), ..., Indicator \sim normalize(Blue^2) + normalize(Red^2) + normalize(Green : Blue)$. The classifiers with formula $Indicator \sim normalize(Blue^2) + normalize(Red^2) + normalize\left(\sqrt{Blue}\right)$ have the highest maximum average F1 measure of 0.937. We drop predictor $normalize(Blue^2)$ and consider the maximum average F1 measure for classifiers with formula $Indicator \sim normalize(Red^2) + normalize\left(\sqrt{Blue}\right)$. These classifiers have a maximum average F1 measure of 0.903. We choose a LR classifier with formula $Indicator \sim normalize(Blue^2) + normalize(Red^2) + normalize\left(\sqrt{Blue}\right)$.

Our optimal LR classifier has formula $Indicator \sim normalize(Blue^2) + normalize(Red^2) + normalize\left(\sqrt{Blue}\right)$. The maximum average F1 measure is 0.937; the corresponding threshold is 0.25.

Average accuracy is nearly 1 for about 95 percent of thresholds. Average F1 measure is nearly 0.937 for thresholds between about 0.1 and about 0.4. Average PPV increases logarithmically. Average TPR increases linearly from a threshold of about 0.95 to 0. A Precision-Recall curve has an AUC of 0.984. An ROC curve has an AUC of 0.99920.

```
# "glm.fit: fitted probabilities numerically 0 or 1" occurs due to predicted
# probabilities of the Logistic Regression binary classifier being close to 0 and 1.
# "collapsing to unique 'x' values" is an artifact of MESS::auc.
set.seed(1)
```

```
summary_of_performance <- summarize_performance_of_cross_validated_classifiers(
    type_of_model = "Logistic Regression",
    formula = Indicator ~ Normalized_Square_Of_Blue + Normalized_Square_Of_Red + Normalized_Square_Root_
    data_frame = training_data_frame_of_indicators_and_pixels
)
```

```
summary_of_performance$plot_of_performance_metrics_vs_threshold
```

```
summary_of_performance$PR_curve
summary_of_performance$ROC_curve
t(summary_of_performance$data_frame_of_optimal_performance_metrics)
data_frame_of_fold_ids_and_maximum_F1_measures_for_Logistic_Regression <-
    summary_of_performance$data_frame_of_fold_ids_and_maximum_F1_measures
```

To search for an optimal LRR classifier, we perform manual bidirectional selection as above. Unfortunately, bidirectional selection from predictor $normalize(Blue^2)$ results ultimately classifiers with formula $Indicator\ normalize(Blue^2) + normalize(Green : Blue) + normalize(\ln(Blue))$ and maximum average F1 measure 0.447. We conduct an exhaustive search for optimal classifiers with a formula with two predictors; the optimal classifiers have formula $Indicator\ normalize(\ln(Blue)) + normalize(\sqrt{(Red)})$. We perform manual bidirectional selection using this formula as a baseline.

For a given formula, before cross validation, we search for one value of hyperparameter λ. We use `caret::train` to choose $\lambda = 0.000123$ for all of our LRR classifiers. According to the documentation for `caret::train`, `caret::train` "sets up a grid of tuning parameters for a number of classification and regression routines, fits each model, and calculates a resampling based performance measure". Our inputs include our formula specifying response and predictive terms, our full data frame of indicators and pixels, the method "glmnet", the metric "F1_measure", a `trainControl` object, and a data frame containing one row for each combination of $\alpha = 0$ and value of λ to be evaluated. We construct the `trainControl` object with method "cv" denoting resampling method Cross Validation and summary function `calculate_F1_measure` to compute F1 measures across resamples. $\alpha = 0$ indicates a combination of ridge regression and lasso regression that is effectively ridge regression.

λ is a constant that determines the importance of squares of coefficients to the quantity to be minimized when conducting LRR. We may use `glmnet::glmnet` to choose a sequence of values of *lambda* to be evaluated. According to the documentation for `glmnet::glmnet`, `glmnet` fits "a generalized linear model via penalized maximum likelihood." Our inputs include a matrix of the values of the predictive terms in our data frame of indicators and pixels, a vector of response values, an indication that the generalized linear model is a LR model, and $alpha = 0$.

Unfortunately, `glmnet` chooses a sequence of values of *lambda* with a minimum of 0.0035. `caret::train` considers the optimal value of λ to be this minimum. We assume that the true optimal value of *lambda* is less than 0.0035. We use `glmnet::cv.glmnet` to graph $Misclassification\ Error Vs. ln(\lambda)$ for $-9 < ln(-\lambda) < -8$. Misclassification error is an approximation of F1 measure. Our software fails to provide graphs of Performance Metrics Vs. Threshold and PR and ROC curves for some values of $ln(\lambda)$ less than -9. We assume that the optimal value of λ is less than $exp(-9) = 0.000123$.

Our optimal LRR classifier has formula $Indicator \sim normalize(\ln(Blue)) + normalize(\sqrt{Red}) + normalize(\ln(Green)) + normalize(\ln(Red))$. Our optimal value for λ is 0.000123; the maximum average F1 measure is 0.938; the corresponding threshold is 0.18.

Average accuracy is nearly 1 for about 87.5 percent of thresholds but in general is not quite as good as for our optimal LR classifier. Average F1 measure is nearly 0.938 for maybe 11 percent of thresholds; the maximum average F1 measure is slightly higher than that for our optimal LR classifier but about the maximum for far fewer thresholds. Average PPV increases logarithmically. From about a threshold of 0.14 PPV is greater than that for our optimal LR classifier. Average TPR increases in the manner of $y = sqrt(x)$ from a threshold

of about 1 to 0. Average TPR is generally less than that for our optimal LR classifier. We prioritize TPR over precision but under F1 measure. A Precision-Recall curve has an AUC of 0.982. An ROC curve has an AUC of 0.99923. The PR AUC is less than that for our optimal LR classifier; the ROC AUC is greater than that for our optimal LR classifier.

```
formula <-
    Indicator ~
        Normalized_Natural_Logarithm_Of_Blue +
        Normalized_Square_Root_Of_Red +
        Normalized_Natural_Logarithm_Of_Green +
        Normalized_Natural_Logarithm_Of_Red
full_model_matrix <-
    model.matrix(object = formula, data = training_data_frame_of_indicators_and_pixels)[, -1]
sequence_of_lambda_values <- exp(seq(-9, -8, length = 100))
set.seed(1)
the_cv.glmnet <- glmnet::cv.glmnet(
    x = full_model_matrix,
    y = training_data_frame_of_indicators_and_pixels$Indicator,
    family = "binomial",
    type.measure = "class",
    alpha = 0,
    lambda = sequence_of_lambda_values
)
data_frame <- data.frame(
    lambda = the_cv.glmnet$lambda,
    misclassification_error_rate = the_cv.glmnet$cvm,
    maximum_misclassification_error_rate = the_cv.glmnet$cvup,
    minimum_misclassification_error_rate = the_cv.glmnet$cvlo
)
library(ggplot2)
the_ggplot <- ggplot(
    data = data_frame,
    mapping = aes(
        x = lambda,
        y = misclassification_error_rate,
        ymin = minimum_misclassification_error_rate,
        ymax = maximum_misclassification_error_rate
    )
) +
    geom_point() +
    scale_x_log10() +
    geom_errorbar() +
   labs(
    x = "lambda",
    y = "Misclassification Error Rate",
    title = "Misclassification Error Rate Vs. lambda"
   ) +
   theme(
    plot.title = element_text(hjust = 0.5, size = 11),
   )
print(the_ggplot)
optimal_lambda <- the_cv.glmnet$lambda.min
optimal_lambda
summary_of_performance <- summarize_performance_of_cross_validated_classifiers(
```

```
    type_of_model = "Logistic Ridge Regression",
    formula = formula,
    data_frame = training_data_frame_of_indicators_and_pixels,
    optimal_lambda = optimal_lambda
)
```

```
summary_of_performance$plot_of_performance_metrics_vs_threshold
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$PR_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$ROC_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
t(summary_of_performance$data_frame_of_optimal_performance_metrics)
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

To search for an optimal LDA binary classifier, we perform manual bidirectional selection as above. Our optimal LDA binary classifier has formula $Indicator \sim normalize(Blue) + normalize(Red^2) + normalize\left(Green^2\right)$. The maximum average F1 measure is 0.891; the corresponding threshold is 0.69.

Average accuracy is nearly 1 for about 95 percent of thresholds but in general is not quite as good as that for our optimal Logistic Regression binary classifier or that for our optimal Logistic Ridge Regression binary classifier. Average F1 measure is nearly 0.891 for maybe 6 percent of thresholds; the maximum average F1 measure is lower than that for our optimal Logistic Regression binary classifier and that for our optimal Logistic Ridge Regression binary classifier. The maximum average F1 measure is about 0.891 for about the same percent of thresholds as for our optimal Logistic Ridge Regression binary classifier. Average PPV increases logarithmically from a threshold of about 0 to our optimal threshold of 0.69, at which point average PPV jumps up to 1. From about a threshold of 0.69 PPV is greater than that for our optimal Logistic Regression binary classifier and equal to that for our optimal Logistic Ridge Regression binary classifier. For thresholds lower than our optimal threshold of 0.69 PPV is less than those for our optimal Logistic Regression and Logistic Ridge Regression binary classifiers. Average TPR generally has a similar trend and is less than that for our optimal Logistic Regression binary classifier. Average TPR is generally less than that for our optimal Logistic Ridge Regression binary classifier for thresholds less than 0.38 and greater than that for our optimal Logistic Ridge Regression binary classifier for thresholds greater than 0.38. We prioritize TPR over precision but under F1 measure. A Precision-Recall curve has an AUC of 0.903. An ROC curve has an AUC of 0.949. These AUC's are less than those for our optimal Logistic Regression and Logistic Ridge Regression binary classifiers.

```
summary_of_performance <- summarize_performance_of_cross_validated_classifiers(
    type_of_model = "LDA",
    formula = Indicator ~ Normalized_Blue + Normalized_Square_Of_Red + Normalized_Square_Of_Green,
    data_frame = data_frame_of_indicators_and_pixels
)
```

```
summary_of_performance$plot_of_performance_metrics_vs_threshold
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$PR_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$ROC_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
t(summary_of_performance$data_frame_of_optimal_performance_metrics)
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

To search for an optimal QDA binary classifier, we perform manual bidirectional selection as above. Our optimal QDA binary classifier has formula $Indicator \sim normalize\left(Blue^2\right) + normalize\left(Red^2\right) + normalize\left(Red\right)$. The maximum average F1 measure is 0.939; the corresponding threshold is 0.960.

Average accuracy is nearly 1 for about 95 percent of thresholds but in general is not quite as good as that for our optimal Logistic Regression binary classifier or that for our optimal Logistic Ridge Regression binary classifier. In generally average accuracy is better than for our LDA binary classifier. Average F1 measure is nearly 0.939 for few thresholds; the maximum average F1 measure is lower than that for our optimal Logistic Regression binary classifier and that for our optimal Logistic Ridge Regression binary classifier but higher than that for our LDA binary classifier. Average PPV increases in the manner of $y = tan(x)$ from a threshold of about 0 to 1. In general PPV is less than that for one of the preceding models. Average TPR is nearly 1 for thresholds less than 0.75. We prioritize TPR over precision but under F1 measure. A Precision-Recall curve has an AUC of 0.966. An ROC curve has an AUC of 0.998. These AUC's are less than those for our optimal Logistic Regression and Logistic Ridge Regression binary classifiers but greater than that for our LDA binary classifier.

```
summary_of_performance <- summarize_performance_of_cross_validated_classifiers(
    type_of_model = "QDA",
    formula = Indicator ~ Normalized_Square_Of_Blue + Normalized_Square_Of_Red + Normalized_Red + Normal
    data_frame = data_frame_of_indicators_and_pixels
)
```

```
summary_of_performance$plot_of_performance_metrics_vs_threshold
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$PR_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$ROC_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
t(summary_of_performance$data_frame_of_optimal_performance_metrics)
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

To search for an optimal K Nearest Neighbors binary classifier, we perform manual bidirectional selection as above.

For a given formula, before cross validation, we search for one value of hyperparameter K. We use `caret::train` to choose $K = 1$ for all of our Logistic Ridge Regression models. According to the documentation for `caret::train`, `caret::train` "sets up a grid of tuning parameters for a number of classification and regression routines, fits each model, and calculates a resampling based performance measure". Our inputs include our formula specifying response and predictive terms, our full data frame of indicators and pixels, the method "knn", the metric "F1_measure", a `trainControl` object, and a data frame containing one row for each K to be evaluated. Dr. Peter Gedeck recommended evaluating $K \in [1, 25]$. We construct the `trainControl` object with method "cv" denoting resampling method cross validation and summary function `calculate_F1_measure` to compute F1 measures across resamples. K is a constant that determines the number of neighboring training observations used to determine the probability that a testing observation corresponds to a blue tarp.

See below graph of average F1 measure vs. K for 10 cross-validated KNN binary classifiers trained on the full data frame of indicators and pixels. `caret::train` chose the value of K for all KNN binary classifiers corresponding to the maximum average F1 measure.

Our optimal KNN binary classifier has formula $Indicator \sim normalize(Red : Blue) + normalize(Green : Blue) + normalize\left(\sqrt{Blue}\right)$. The maximum average F1 measure is 0.946; the corresponding threshold is 0.080.

Average accuracy is nearly 1 for about 100 percent of thresholds and is better than that for any of the previous binary classifiers. Average F1 measure is nearly 0.946 for 90 percent of thresholds; the maximum average F1 measure is higher than that for any previous binary classifier. Average F1 measure, PPV, and TPR all jump to about 0.942 for a threshold of about 0.1. A Precision-Recall curve has an AUC of 0.948. An ROC curve has an AUC of 0.997. These AUC's are less than those for our optimal Logistic Regression, Logistic Ridge Regression, and QDA binary classifiers but greater than that for our LDA binary classifier.

```
summary_of_performance <- summarize_performance_of_cross_validated_classifiers(
    type_of_model = "KNN",
    formula = Indicator ~ Normalized_Interaction_Of_Red_And_Blue + Normalized_Interaction_Of_Green_And_
    data_frame = data_frame_of_indicators_and_pixels
)
```

```
summary_of_performance$plot_of_performance_metrics_vs_threshold
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$PR_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
summary_of_performance$ROC_curve
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
t(summary_of_performance$data_frame_of_optimal_performance_metrics)
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

```
data_frame_of_fold_ids_and_maximum_F1_measures_for_KNN <-
    summary_of_performance$data_frame_of_fold_ids_and_maximum_F1_measures
```

```
# Error in eval(expr, envir, enclos): object 'summary_of_performance' not found
```

Performance Table

Per our Performance Table, our KNN binary classifier with $K = 1$ and threshold $t = 0.080$ has the best optimal F1 measure and corresponding accuracy. Our KNN binary classifier has the second-best corresponding TPR, FPR, and PPV. However, our KNN binary classifier has the second-worst Area Under The Precision-Recall Curve and Area Under The ROC Curve. Our Logistic Regression binary classifier has the best Area Under The Precision-Recall Curve, Area Under The ROC Curve, and corresponding TPR. Our Logistic Regression binary classifier has the second-best corresponding optimal measure and corresponding accuracy. However, our Logistic Regression binary classifier has the worst corresponding FPR and PPV. Beautifully, our QDA binary classifier is completely middle of the road. Our Logistic Ridge Regression binary classifier is second-best in Area Under The Precision-Recall Curve and Area Under The ROC curve and second-worst in everything else. Our LDA binary classifier has the best corresponding FPR and PPV and is worst in everything else.

We calculate binary-classifier scores for each type of binary classifier. Each binary-classifier score is a weighted sum of the first 7 colored performance metrics above it in the Performance Table.

Quantity	Method Of Determination	Logistic Regression	Logistic Ridge Regression	LDA	QDA	KNN
corresponding threshold	value corresponding to optimal F1 measure	0.1800	0.1800	0.6900	0.9600	0.0800
alpha	minimum	-	0.0000	-	-	-
optimal lambda	lambda in sequence provided by glmnet::cv.glmnet	-	0.0001	-	-	-
optimal K	K in sequence provided by Peter Gedeck corresponding to maximum F1 measure	-	-	-	-	1
optimal Area Under The Precision-Recall Curve	average	0.9844	0.9811	0.9037	0.9656	0.9486
optimal Area Under The ROC Curve	average	0.9993	0.9992	0.9494	0.9983	0.9772
corresponding accuracy	value corresponding to optimal F1 measure	0.9962	0.9957	0.9937	0.9961	0.9965
corresponding True Positive Rate	value corresponding to optimal F1 measure	0.9667	0.9444	0.8097	0.9369	0.9457
corresponding False Positive Rate	value corresponding to optimal F1 measure	0.0029	0.0026	0.0002	0.0019	0.0018
corresponding precision	value corresponding to optimal F1 measure	0.9179	0.9232	0.9916	0.9412	0.9461
optimal F1 measure	maximum average	0.9416	0.9335	0.8912	0.9389	0.9458
binary-classifier score	weighted sum	25	18	15	21	26

Figure 7: Performance Table

Conclusions

Our KNN binary classifier has the highest binary classifier score, the highest optimal F1 measure, and the highest corresponding accuracy among our optimal binary classifiers. On these bases, we recommend our KNN binary classifier.

That being said, we determined the optimal binary classifier for each type of binary classifier based on an industry standard; i.e., the use of F1 measure. Leaders responsible for deploying resources to refugees may wish to replicate this research by choosing optimal binary classifiers for each type of binary classifier based on an F measure where PPV and TPR have different weights. Leaders may wish to choose thresholds for a binary classifier that results in greater TPR than the TPR corresponding to the maximum average F1 measure for that binary classifier. In such a case our Logistic Regression binary classifier may outperform our KNN binary classifier.

Results may improve by hybridizing our Logistic Ridge Regression binary classifier with a Logistic Lasso Regression binary classifier by specifying *alpha* to be greater than 0 and less than or equal to 1. Results may improve by developing software that can find the minimum in a relatively continuous plot of F1 measure Vs. $log(\lambda)$. Results may improve by considering values of K between 1 and the number of observations in our full data frame of indicators and pixels n.

According to Data Scientist Stephen Allwright, "A good F1 score is dependent on the data you are working with and the use case. For example, a model predicting the occurrence of a disease would have a very different expectation than a custometer churn model. However, there is a general rule of thumb when it comes to F1 scores, which is as follows:"

- An F1 score greater than 0.9 is very good.
- An F1 score of 0.8 to 0.9 is good.
- An F1 score of 0.5 to 0.8 is okay.
- An F1 score less than 0.5 is not good.

All of our binary classifiers are adequately performing. Our Logistic Regression and KNN binary classifiers have a statistically insignificant difference in average maximum F1 measures. Following Text-Mining Evaluation: Statistical Tests, we train and test our Logistic Regression and KNN binary classifiers using 10-fold cross validation. We use the same folds for both systems. We calculate maximum F1 measures for each held-out fold. We construct a data frame of fold ID's and maximum F1 measure for each binary classifier and fold. We calculate the average maximum F1 measures across held-out folds. We consider the difference between the average maximum F1 measures to be a test statistic. We consider an alternate hypothesis to be that our Logistic Regression binary classifier has a higher average maximum F1 measure than our KNN binary classifier for all possible test sets. We consider a null hypothesis to be that our Logistic Regression binary classifier has an equal average maximum F1 measure to that of our KNN binary classifier. We consider a p value to be the probability of observing that the difference in average maximum F1 measures for a random sample / test set is greater than our test statistic. We consider our confidence level to be $CL = 0.95$ and our significance level to be $\alpha = 1 - CL = 0.05$. If the p value is greater than or equal to the significance level α, we fail to reject the null hypothesis that our Logistic Regression binary classifier has an average maximum F1 measure equal to that of our KNN binary classifier. If the p value is less than or equal to the significance level α, we reject the null hypothesis and have sufficient evidence to support the alternate hypothesis that our Logistic Regression binary classifier has an average maximum F1 measure greater than that of our KNN binary classifier.

To calculate our p value, we conduct Fisher's Randomization Test. We calculate our p value as the ratio of a count C and a number of iterations I. Our count C is the number of iterations $i \leq I$ for which a modified difference between an average maximum F1 measure for Logistic Regression and an average maximum F1 for KNN is greater than our test statistic. For each iteration, an average maximum F1 measure for Logistic Regression is computed effectively by tossing a fair coin for each row in our data frame of fold ID's and maximum F1 measures, if heads swapping the maximum F1 measures for the present fold, and calculating the mean of the maximum F1 measures in the column corresponding to Logistic Regression. An average maximum F1 measure for KNN is computed similarly. A modified difference between the average maximum F1 measure for Logistic Regression and the average maximum F1 measure for KNN is compared to our test statistic. If the modified difference is greater than our test statistic, count C is incremented. Our p value is the ratio of count C and number of iterations I.

The p value for our Logistic Regression and KNN binary classifiers is 0.4. The probability of observing a random test statistic greater than our test statistic is 0.4. Because our p value is greater than our significance level, we cannot confidently say that our test statistic is not due to random chance. Our difference between the average maximum F1 measures of our Logistic Regression and KNN models is statistically insignificant.

```
calculate_probability_for_randomization_test(
    data_frame_of_fold_ids_and_maximum_F1_measures_for_Logistic_Regression,
    data_frame_of_fold_ids_and_maximum_F1_measures_for_KNN
)
```

We are able to use predictive modeling tools as our data set of classes and pixels may be represented as a complete data frame of natural numbers. We are able to use classifiers as the data in column *Class* is nominal. We are able to use binary classifiers as pixels representing blue tarps exist in a distinct subspace defined by intensities of color *Red*, *Green*, and *Blue*. We are able to normalize intensities of color *Red*, *Green*, and *Blue*.

Our data set seems particularly well-suited to the class of prediction methods Binary Classifier as our data set is complete, observations may be represented as natural numbers, class of pixel is nominal, and pixels representing blue tarps exist in a distinct subspace defined by intensities of color *Red*, *Green*, and *Blue*.

Given that according to Stephen Allwright the maximum average F1 measures of our binary classifiers are very good, our research may be very effective in terms of locating people displaced by the earthquake in Haiti in 2010 and helping to save human life.