**Discussion: Prisoners-Choice And Medical Testing Problems**

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*How is the Prisoners-Choice problem similar to and different from the Monty-Hall problem, and how do these differences and similarities affect the solution?*

Consider the Monty-Hall problem as presented by <https://www.cs.unm.edu/~forrest/classes/cs365/lectures/Bayes-1.pdf> . You are a contestant on a game show. There are 3 doors A, B, and C. There is a new car behind one door and goats behind the other two doors. Monty asks you to pick a door. You pick door A. Monty tells you that Monty will open another door that has a goat. Monty opens door B. Monty gives you a choice to stay with door A or switch to door C. Which option do you choose?

Consider the following three hypotheses / mutually exclusive parameters.

1. : Car is behind door A.
2. : Car is behind door B.
3. : Car is behind door C.

Consider the prior probabilities of these hypotheses.

Consider data : You choose door A. Monty opened door B and found a goat.

Consider the likelihoods of given each hypothesis.

1. : The likelihood that Monty will open door B given that the contestant chose door A and that the car is behind door A equals .
2. : The likelihood that Monty will open door B given that the contestant chose door A and that the car is behind door B equals .
3. : The likelihood that Monty will open door B given that the contestant chose door A and that the car is behind door C equals .

Consider the total probability of . By the Law Of Total Probability,

By the Bayes Theorem,

We choose to switch to door .

Consider the Prisoners-Choice problem. Prisoners A, B, and C are in jail. The jailer tells them that one will be executed and the other two set free. Prisoner A asks the jailer to tell Prisoner A the name of one of the prisoners to be set free. The jailer refuses. The jailer says that Prisoner A’s probability of execution would go from to . Is the jailer right?

Consider the following three hypotheses / mutually exclusive parameters.

1. : Prisoner A will be executed.
2. : Prisoner B will be executed.
3. : Prisoner C will be executed.

Consider the prior probabilities of these hypotheses.

Consider data : You are Prisoner A. The jailer tells Prisoner A that Prisoner B will be set free.

Consider the likelihoods of given each hypothesis.

1. : The likelihood that the jailer tells Prisoner A that Prisoner B will be set free given that Prisoner A will be executed equals .
2. : The likelihood that the jailer tells Prisoner A that Prisoner B will be set free given that Prisoner B will be executed equals .
3. : The likelihood that the jailer tells Prisoner A that Prisoner B will be set free given that Prisoner C will be executed equals .

Consider the total probability of . By the Law Of Total Probability,

By the Bayes Theorem,

The Monty-Hall problem and the Prisoners-Choice problem are similar in that:

* They deal with three entities labeled A, B, and C.
* They each involve two different outcomes. One outcome is more extreme than the other.
* In spirit, they each involve revealing data.
* The hypotheses, prior probabilities, data, first two likelihoods, calculations of total probability, and calculations of posterior probability are similar.

The Monty-Hall problem and the Prisoner-Choice problem are different in that:

* They deal with three doors and three prisoners.
* The extreme outcome in the Monty-Hall problem is an incentive. The extreme outcome in the Prisoner-Choice problem is a disincentive.
* Technically, in the Monty-Hall problem, data is revealed. In the Prisoner-Choice problem, data is not revealed. However, setting data in both problems allows us to see the difference between these two problems.
* Monty may not open the door of the contestant; the likelihood that Monty will open door B given that the contestant chose door A and that the car is behind door C equals . The jailer may tell Prisoner A that Prisoner A will be executed; the probability that the jailer tells Prisoner A that Prisoner B will be set free given that Prisoner C will be executed equals .
* For the Monty-Hall problem, the total probability . For the Prisoner-Choice problem, the total probability .
* The posterior probability that a car is behind door A given that Monty opened door B to reveal a goat is . The posterior probability that a car is behind door C given that Monty opened door C to reveal a goat is . The posterior probability that Prisoner A will be executed given that the jailer told Prisoner A the name of a prisoner who will be set free is . The posterior probability that Prisoner C will be executed given that the jailer told Prisoner A the name of a prisoner who will be set free is .
* Prisoner A cannot change Prisoner A’s identity, nor is given the opportunity.

*Suppose in the prisoner’s choice problem that the jailer tells us that prisoner B will be set free. Does that change the probability that prisoner C will be set free?*

Yes. As above, the posterior probability that Prisoner C will be executed given that the jailer told Prisoner A the name of a prisoner who will be set free is , which is different than the prior probability that Prisoner C will be executed.

*In the medical testing problem, compare the probability that the patient has ebola using frequentist and Bayesian probabilities.*

Consider the medical testing problem. Suppose an Ebola test is available with and . You have no reason to think you have Ebola, but you take the test and it comes back positive. If the prevalence of Ebola in the general population is 1/10000, what is the probability you have Ebola based on this test?

We assume the prevalence of Ebola in the general population is 1/10000. In the margin of the below confusion matrix, let’s assume that people have Ebola and 9,999,000 people do not have Ebola. The False Negative Rate . The number of False Negatives . The number of True Positives . . The number of False Positives . The number of True Negatives . The number of people testing negative . The number of people testing positive .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Has Ebola? | |  |
|  |  | N | Y |  |
| Tests positive? | N |  |  |  |
| Y |  |  |  |
|  |  |  |  |  |

The probability that you have Ebola based on testing positive

The probabilities that the patient has Ebola using frequentist and Bayesian probabilities are identical.