This is the softmax workbook for ECE C147/C247 Assignment #2

Please follow the notebook linearly to implement a softmax classifier.

Please print out the workbook entirely when completed.

We thank Serena Yeung & Justin Johnson for permission to use code written for the CS 231n class (cs231n.stanford.edu). These are the functions in the cs231n folders and code in the jupyer notebook to preprocess and show the images. The classifiers used are based off of code prepared for CS 231n as well.

The goal of this workbook is to give you experience with training a softmax classifier.

In [1]:
```
import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

%matplotlib inline
%load_ext autoreload
%autoreload 2
```

In [2]:
```
def get_CIFAR10_data(num_training=49000, num_validation=1000, num_test
=1000, num_dev=500):
    """
    Load the CIFAR-10 dataset from disk and perform preprocessing to p
repare
    it for the linear classifier. These are the same steps as we used
 for the
    SVM, but condensed to a single function.
    """
    # Load the raw CIFAR-10 data
    cifar10_dir = './cifar-10-batches-py' # You need to update this li
ne
    X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

    # subsample the data
    mask = list(range(num_training, num_training + num_validation))
    X_val = X_train[mask]
    y_val = y_train[mask]
    mask = list(range(num_training))
    X_train = X_train[mask]
    y_train = y_train[mask]
    mask = list(range(num_test))
    X_test = X_test[mask]
    y_test = y_test[mask]
    mask = np.random.choice(num_training, num_dev, replace=False)
    X_dev = X_train[mask]
    y_dev = y_train[mask]

    # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

    # Normalize the data: subtract the mean image
    mean_image = np.mean(X_train, axis = 0)
    X_train -= mean_image
    X_val -= mean_image
    X_test -= mean_image
    X_dev -= mean_image

    # add bias dimension and transform into columns
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

    return X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_de
v


# Invoke the above function to get our data.
X_train, y_train, X_val, y_val, X_test, y_test, X_dev, y_dev = get_CIF
AR10_data()
print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
```

```
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
print('dev data shape: ', X_dev.shape)
print('dev labels shape: ', y_dev.shape)
```

```
Train data shape:  (49000, 3073)
Train labels shape:  (49000,)
Validation data shape:  (1000, 3073)
Validation labels shape:  (1000,)
Test data shape:  (1000, 3073)
Test labels shape:  (1000,)
dev data shape:  (500, 3073)
dev labels shape:  (500,)
```

Training a softmax classifier.

The following cells will take you through building a softmax classifier. You will implement its loss function, then subsequently train it with gradient descent. Finally, you will choose the learning rate of gradient descent to optimize its classification performance.

In [3]:
```
from nndl import Softmax
```

In [4]:
```
# Declare an instance of the Softmax class.
# Weights are initialized to a random value.
# Note, to keep people's first solutions consistent, we are going to u
se a random seed.

np.random.seed(1)

num_classes = len(np.unique(y_train))
num_features = X_train.shape[1]

softmax = Softmax(dims=[num_classes, num_features])
```

Softmax loss

In [5]:
```
## Implement the loss function of the softmax using a for loop over
#  the number of examples

loss = softmax.loss(X_train, y_train)
```

In [6]:
```
print(loss)
```

```
2.3277607028048757
```

Question:

You'll notice the loss returned by the softmax is about 2.3 (if implemented correctly). Why does this make sense?

Answer:

The results tells us that on average, given our model, the $\log$ of the probability that an input example $x^{(i)}$ is classified to the class $y^{(i)}$, is approximately -2.32. Therefore, the probability that an input example $x^{(i)}$ is classified to the class $y^{(i)}$ is approximately $e^{-2.32} = 0.1$. This makes sense, because our model's parameters is randomly generated, so we expect that on average all classes are equally likely (we have 10 classes, each with probability 0.1 on average).

Softmax gradient

In [7]:
```
## Calculate the gradient of the softmax loss in the Softmax class.
# For convenience, we'll write one function that computes the loss
#   and gradient together, softmax.loss_and_grad(X, y)
# You may copy and paste your loss code from softmax.loss() here, and
 then
#   use the appropriate intermediate values to calculate the gradient.

loss, grad = softmax.loss_and_grad(X_dev,y_dev)

# Compare your gradient to a gradient check we wrote.
# You should see relative gradient errors on the order of 1e-07 or les
s if you implemented the gradient correctly.
softmax.grad_check_sparse(X_dev, y_dev, grad)
```

```
numerical: -1.568360 analytic: -1.568360, relative error: 3.738697e-09
numerical: 0.260181 analytic: 0.260180, relative error: 1.180620e-07
numerical: 0.358200 analytic: 0.358200, relative error: 2.820070e-08
numerical: 1.874052 analytic: 1.874052, relative error: 9.416010e-09
numerical: -0.389514 analytic: -0.389514, relative error: 1.777949e-07
numerical: 1.839289 analytic: 1.839289, relative error: 3.048306e-08
numerical: -0.290178 analytic: -0.290178, relative error: 1.709662e-07
numerical: -2.698520 analytic: -2.698520, relative error: 1.978944e-09
numerical: 1.151592 analytic: 1.151592, relative error: 1.637834e-08
numerical: -3.445950 analytic: -3.445950, relative error: 1.882229e-08
```

A vectorized version of Softmax

To speed things up, we will vectorize the loss and gradient calculations. This will be helpful for stochastic gradient descent.

In [8]:
```
import time
```

In [9]:
```
## Implement softmax.fast_loss_and_grad which calculates the loss and
 gradient
#    WITHOUT using any for loops.

# Standard loss and gradient
tic = time.time()
loss, grad = softmax.loss_and_grad(X_dev, y_dev)
toc = time.time()
print('Normal loss / grad_norm: {} / {} computed in {}s'.format(loss,
np.linalg.norm(grad, 'fro'), toc - tic))

tic = time.time()
loss_vectorized, grad_vectorized = softmax.fast_loss_and_grad(X_dev, y
_dev)
toc = time.time()
print('Vectorized loss / grad: {} / {} computed in {}s'.format(loss_ve
ctorized, np.linalg.norm(grad_vectorized, 'fro'), toc - tic))

# The losses should match but your vectorized implementation should be
much faster.
print('difference in loss / grad: {} /{} '.format(loss - loss_vectoriz
ed, np.linalg.norm(grad - grad_vectorized)))

# You should notice a speedup with the same output.
```

```
Normal loss / grad_norm: 2.326425442427376 / 343.0326475036641 compute
d in 0.07816600799560547s
Vectorized loss / grad: 2.326425442427376 / 343.0326475036642 computed
in 0.05777096748352051s
difference in loss / grad: 0.0 /6.837463300096367e-14
```

Stochastic gradient descent

We now implement stochastic gradient descent. This uses the same principles of gradient descent we discussed in class, however, it calculates the gradient by only using examples from a subset of the training set (so each gradient calculation is faster).

Question:

How should the softmax gradient descent training step differ from the svm training step, if at all?

Answer:

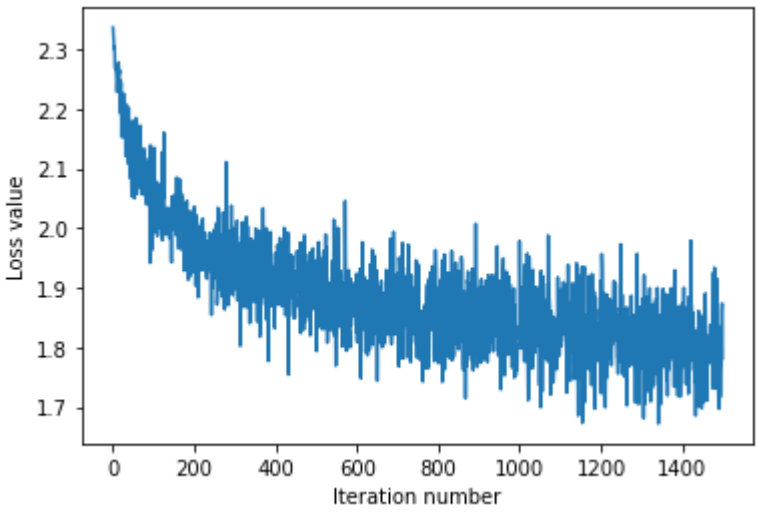
The softmax gradient descent training step is the same as the SVM training step.

In [10]:
```
# Implement softmax.train() by filling in the code to extract a batch
 of data
# and perform the gradient step.
import time


tic = time.time()
loss_hist = softmax.train(X_train, y_train, learning_rate=1e-7,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took {}s'.format(toc - tic))

plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```

```
iteration 0 / 1500: loss 2.3365926606637544
iteration 100 / 1500: loss 2.0557222613850827
iteration 200 / 1500: loss 2.035774512066282
iteration 300 / 1500: loss 1.9813348165609888
iteration 400 / 1500: loss 1.9583142443981612
iteration 500 / 1500: loss 1.8622653073541355
iteration 600 / 1500: loss 1.8532611454359385
iteration 700 / 1500: loss 1.8353062223725827
iteration 800 / 1500: loss 1.829389246882764
iteration 900 / 1500: loss 1.8992158530357484
iteration 1000 / 1500: loss 1.9783503540252303
iteration 1100 / 1500: loss 1.8470797913532633
iteration 1200 / 1500: loss 1.8411450268664085
iteration 1300 / 1500: loss 1.7910402495792102
iteration 1400 / 1500: loss 1.870580302938226
That took 37.36585307121277s
```



Evaluate the performance of the trained softmax classifier on the validation data.

In [11]:
```
## Implement softmax.predict() and use it to compute the training and
 testing error.

y_train_pred = softmax.predict(X_train)
print('training accuracy: {}'.format(np.mean(np.equal(y_train,y_train_
pred), )))
y_val_pred = softmax.predict(X_val)
print('validation accuracy: {}'.format(np.mean(np.equal(y_val, y_val_p
red)), ))
```

```
training accuracy: 0.3811428571428571
validation accuracy: 0.398
```

Optimize the softmax classifier

You may copy and paste your optimization code from the SVM here.

In [12]:
```
np.finfo(float).eps
```

Out[12]:
```
2.220446049250313e-16
```

In [14]:
```
# ================================================================ #
# YOUR CODE HERE:
#   Train the Softmax classifier with different learning rates and
#     evaluate on the validation data.
#   Report:
#     - The best learning rate of the ones you tested.
#     - The best validation accuracy corresponding to the best validat
ion error.
#
#   Select the SVM that achieved the best validation error and report
#     its error rate on the test set.
# ================================================================ #
best_learning_rate = 0
best_val_accur = 0
best_test_accur = 0
for i in range(1, 10000, 200):
    learning_rate = i*1e-5
    softmax.train(X_train, y_train, learning_rate=learning_rate,
                      num_iters=500, verbose=False)
    y_train_pred = softmax.predict(X_train)
    train_accur = np.mean(np.equal(y_train,y_train_pred))
    y_val_pred = softmax.predict(X_val)
    val_accur = np.mean(np.equal(y_val, y_val_pred))

    if val_accur > best_val_accur:
        best_val_accur = val_accur
        best_learning_rate = learning_rate

softmax.train(X_train, y_train, learning_rate=best_learning_rate,
                      num_iters=500, verbose=False)

y_test_pred = softmax.predict(X_test)
test_error = 1 - np.mean(np.equal(y_test, y_test_pred))

print('best validation accuracy: {}'.format(best_val_accur))
print('best learning rate: {}'.format(best_learning_rate))
print('test set error using the best learning rate: {}'.format(test_er
ror))
# ================================================================ #
# END YOUR CODE HERE
# ================================================================ #
```

```
best validation accuracy: 0.339
best learning rate: 0.04401
test set error using the best learning rate: 0.763
```