Lab6 - Parameterization

Geometry Processing (GPR)

1 Introduction

To parameterize a mesh with a single hole using Harmonic Maps, we need to:

1. Determine the border of the hole as an edge cycle. We will map this to the border of the parameter space, the 2D box $[0,1] \times [0,1]$.
2. Now, we solve for the 2D parameter coordinates, by constraining the points on the border of the hole to the limits of parameter space, and minimizing the Laplacian for the rest.

2 Algorithm

The algorithm step-by-step is:

1. Determine which half-edges are border half-edges, and so part of the hole. They are those for which their opposite half-edges do not exist. In other words, a half-edge H from a triangle T is a border half-edge, if triangle T does not have a neighbor through E.

```
set<int, int> halfEdges
For every triangle T
   For every half-edge H from T
      Look for half-edge opposite to H in halfEdges
      If found then
         Erase it from halfEdges
      else
         Add H to halfEdges
```

2. Build the edge cycle for the border of the hole, using the half edges that are still in halfEdges.

```
Choose first border edge from halfEdges
Repeat until the edge list closes
   Find next edge that connects
```

3. Assign coordinates to border edges.

```
Compute total length L of the border edge cycle
For each border vertex V
   Determine position P of vertex V inside the full length of the cycle
   Compute 2D parameter coordinates for that vertex
```

   Given the position P of a vertex V inside the full edge cycle of the hole, its 2D parameter space coordinates are computed as follows:

$$\lambda = P/L$$
$$\text{If } \lambda < 0.25 \rightarrow (u,v) = (4\lambda, 0)$$
$$\text{Else If } \lambda < 0.5 \rightarrow (u,v) = (1, 4(\lambda - 0.25))$$
$$\text{Else If } \lambda < 0.75 \rightarrow (u,v) = (1 - 4(\lambda - 0.5), 1)$$
$$\text{Else } \rightarrow (u,v) = (0, 1 - 4(\lambda - 0.75))$$

4. Build harmonic system, similar to the global laplacian, and solve it.

$$
\left[\begin{array}{c|c} \multicolumn{2}{c}{\mathbf{L_1}} \\ \hline \mathbf{0} & \mathbf{I} \end{array}\right] \cdot \begin{bmatrix} u'_1 & v'_1 \\ \vdots & \vdots \\ u'_n & v'_n \end{bmatrix} = \left[\begin{array}{cc} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \hline u_{m+1} & v_{m+1} \\ \vdots & \vdots \\ u_n & v_n \end{array}\right]
$$

where (u_i, v_i) are the fixed 2D parameter coordinates we have computed for the border vertices, and u'_i, v'_i are the final 2D harmonic coordinates we will assign to each vertex.