Lab 3 - Reconstruction

Geometry Processing (GPR)

1 Simple reconstruction

Approach due to Hoppe et al [1]. Their implicit function is really a distance function that takes a point $\mathbf{p}$ as an argument and is defined as:

$i \leftarrow$ Index of $\mathbf{p_i}$, closest point to $\mathbf{p}$
{ Compute $\mathbf{z}$ as the projection of $\mathbf{p}$ onto the tangent plane at $\mathbf{p_i}$ }
$\mathbf{z} \leftarrow \mathbf{p_i} - ((\mathbf{p} - \mathbf{p_i}) \cdot \mathbf{n_i}) \cdot \mathbf{n_i}$
if $distance(\mathbf{z}, \mathbf{p_i}) \leq \rho + \delta$ **then**
    $f(\mathbf{p}) \leftarrow (\mathbf{p} - \mathbf{p_i}) \cdot \mathbf{n_i}$
else
    $f(\mathbf{p}) \leftarrow undefined$
end if

2 Radial Basis Functions (RBF)

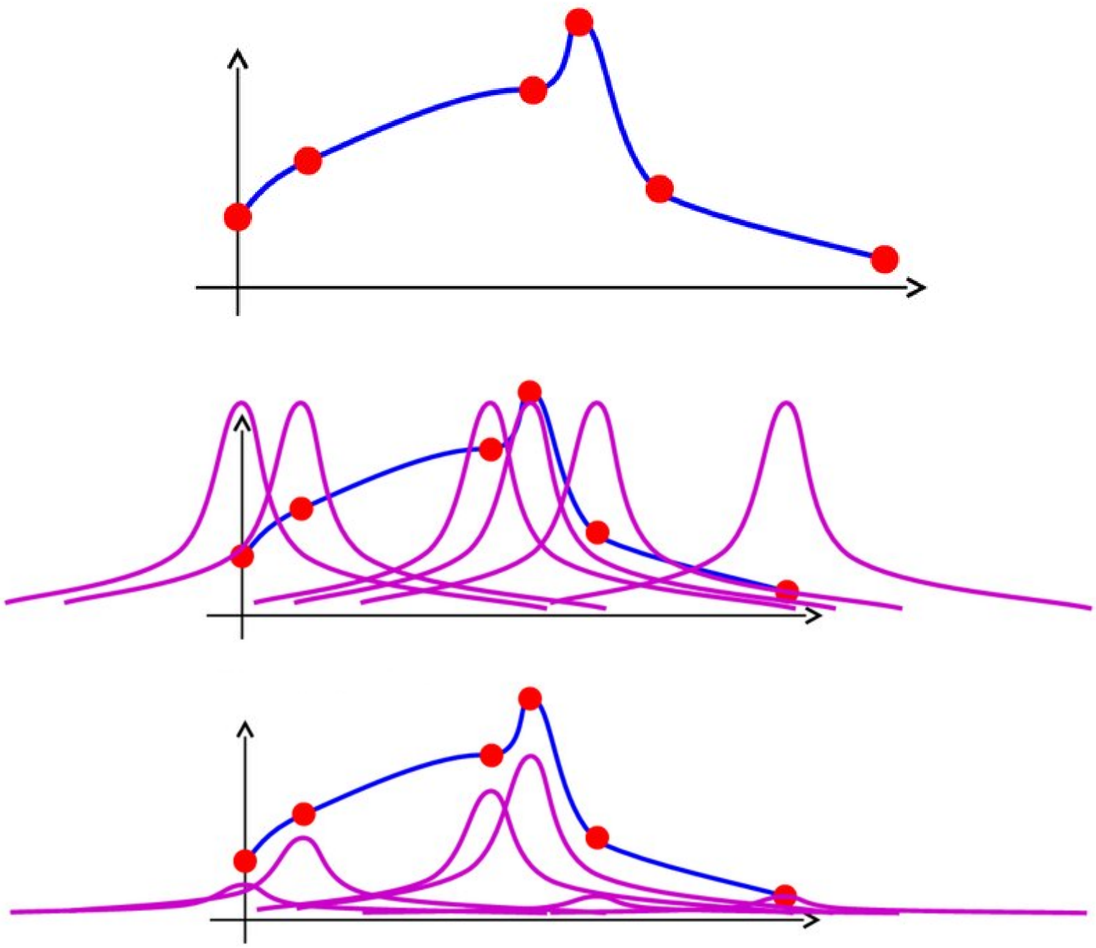

Figure 1: Interpolating with RBFs.

Input: Set of pairs $(p_i, v_i), p_i \in \mathbb{R}^3, v_i \in \mathbb{R}$

Output: Smooth interpolating function f.

$$f(p) = \sum_{i=1}^{m} f_i(p) \qquad f_i(p) = \phi(\|p - p_i\|) \cdot c_i$$

Gaussian RBF:

$$\phi(r) = exp(-r^2/2c^2)$$

Interpolating conditions:

$$f(p_i) = v_i \implies f(p_i) = \sum_{j=1}^{m} f_j(p_i) = v_i \implies \sum_{j=1}^{m} \phi(\|p_i - p_j\|) \cdot c_j = v_i \implies \mathbf{A} \cdot \mathbf{c} = \mathbf{v}$$

$$\mathbf{A} = \begin{pmatrix} \phi(\|p_1 - p_1\|) & \phi(\|p_1 - p_2\|) & \cdots & \phi(\|p_1 - p_m\|) \\ \phi(\|p_2 - p_1\|) & \phi(\|p_2 - p_2\|) & \cdots & \phi(\|p_2 - p_m\|) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(\|p_m - p_1\|) & \phi(\|p_m - p_2\|) & \cdots & \phi(\|p_m - p_m\|) \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} c_1 & c_2 & \cdots & c_m \end{pmatrix}^T \qquad \mathbf{v} = \begin{pmatrix} v_1 & v_2 & \cdots & v_m \end{pmatrix}^T$$

Problems:

- We want to interpolate, so $f(p_i) = 0$ for all input points. Which means $\mathbf{A} \cdot \mathbf{c} = \mathbf{0}$. The trivial solution is $\mathbf{c} = \mathbf{0}$.
  - Create artificial points. For each p_i, create $p_i^+ = p_i + d \cdot n_i$ and $p_i^- = p_i - d \cdot n_i$, such that $f(p_i^+) = d$ and $f(p_i^-) = -d$.
- As m increases matrix $\mathbf{A}$ is going to become ill-conditioned.
  - Apply a regularization by $\mathbf{A}' = \mathbf{A} + \lambda\mathbf{I}$.
- Matrix $\mathbf{A}$ is dense and can be very large ($m \times m$).
  - Use gaussian RBFs with compact support for a sparse matrix.
  - Loses its generalization power (holes are undefined).
  - Solution: Give increasing support to a decreasing number of points.

$$\phi(r) = \begin{cases} exp(-r^2/2c^2) & \text{if } r < 3c \\ 0 & \text{otherwise} \end{cases}$$

References

[1] Hugues Hoppe et al. "Surface reconstruction from unorganized points". In: *Proceedings of the 19th annual conference on Computer graphics and interactive techniques*. 1992, pp. 71–78.