Technical details for the SEIR graphical model of the Open SEIR-Graph project

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1 SEIR graphical model

We consider N nodes on the graph, $\mathcal{G}$ indexed by an integer $i \in \{1, ..., N\}$. Each node may be in one of four states, Susceptible (0), Exposed (1), Infectious(2) or Removed(3). Denoting the state of any particular node by $X_{i,t}$ for discrete time index, $t \geq 0$, then we define the four time series

$$\{S_t\}_{t\geq 0} = \sum_{i=1}^{N} \mathcal{I}\{X_{i,t} = 0\}$$
$$\{E_t\}_{t\geq 0} = \sum_{i=1}^{N} \mathcal{I}\{X_{i,t} = 1\}$$
$$\{I_t\}_{t\geq 0} = \sum_{i=1}^{N} \mathcal{I}\{X_{i,t} = 2\}$$
$$\{R_t\}_{t\geq 0} = \sum_{i=1}^{N} \mathcal{I}\{X_{i,t} = 3\}$$

Where I is the indicator function that the argument is true. Since the state is categorical, we have that at every time point $S_t + E_t + I_t + R_t = N$.

Starting in the intial configuration $X_{0,0} = 2$ and $X_{i,0} = 0$ for $i \neq 0$, the dynamics proceeds in the manner below, where we use $n_{i,t}(x, \mathcal{G})$ to denote the number of neighbors of the node i, which are in a state x at time t.

At each time point T, iterate through each node i:

- If $X_{i,T} = 0$, simulate $A \sim \text{Binomial}(n_{i,T}(1, \mathcal{G}), p_E)$ and $B \sim \text{Binomial}(n_{i,T}(2, \mathcal{G}), p_I)$, then if $A + B \geq 1$ set $X_{i,T+1}$ to 1.
- If $X_{i,T} = 1$ and $X_{i,T-\delta_I}$ then set $X_{i,T+1}$ to 2
- If $X_{i,T} = 2$ and $X_{i,T-\delta_R}$ then set $X_{i,T+1}$ to 3

This process is parameterised by $\theta = (p_E, p_I, \delta_I, \delta_R)$. The first may be interpreted as the probabilities of transmission, on any given day, from an exposed node to a susceptible neighbor, from an infectious node to a susceptible neighbor. The final two parameters are the incubation and recovery periods in days.

We notice that this process necessarily leads to a state where all nodes are either in state 0 or state 3. This can be used as a stopping condition in the iterative process.

2 Dynamic Interventions

In the SEIR model in the previous section, the Graph, $\mathcal{G}$ was static throughout the state evolution. We define a dynamic intervention as those with additional rules to modify $\mathcal{G}$. Denoting the state of the graph at a particular time point by $\mathcal{G}_t$ then an SEIR model with a dynamic intervention modifies the first step if the iteration to

- If $X_{i,T} = 0$, simulate $A \sim \text{Binomial}(n_{i,T}(1, \mathcal{G}_T), p_E)$ and $B \sim \text{Binomial}(n_{i,T}(2, \mathcal{G}_T), p_I)$, then if $A + B \geq 1$ set $X_{i,T+1}$ to 1.

Another step is included at each time point (after the node iteration is complete), which obtains $\mathcal{G}_{T+1}$ conditional the histories of $\{X_{i,t}\}$ and on $\mathcal{G}_t$ up to time point T

3 Estimands

The desired estimand of interest is the expected peak proportion of infected individuals given an intial graph $\mathcal{G}$ and dynamic intervention rule $\mathcal{D}$.

$$\psi(\mathcal{G}, \mathcal{D}) = E\left[\max_{t \geq 0}\{S_t\}\right] \tag{1}$$

Such an estimand can be made using the Monte Carlo estimate

$$\hat{\psi}(\mathcal{G}, \mathcal{D}) = M^{-1} \sum_{j=1}^{M} \left(\max_{t \geq 0}\{S_t\}\right)_j \tag{2}$$

where $\left(\max_{t \geq 0}\{S_t\}\right)_j$ is a set of M simulated samples.