

(1) Elementary Algebra

1.1 $\frac{x^{32}}{x^9 \cdot x^2} \cdot \frac{x^7}{x^2} = \frac{x^{39}}{x^{13}} = x^{26}$

1.2 $8^2 \cdot 4^x \cdot 2^x = 8^4$
 $(4 \cdot 2)^x = \frac{8^4}{8^2}$
 $8^x = 8^2$
 $x = 2$

1.3 If $\frac{x}{y} = 3$, then:

$$x = 3y \quad x^{-4} y^4 = (3y)^{-4} \cdot y^4 = \frac{y^4}{3^4 \cdot y^4} = \frac{1}{3^4} = 81^{-1}$$

1.4 $\frac{\sqrt{4^{15}}}{\sqrt{16^7}} = \frac{4^{\frac{15}{2}}}{4^{\frac{14}{2}}} = 4^{\frac{1}{2}} = \sqrt{4} = 2$

- | | | |
|------------------------------------|---|-------|
| 1.5 A) $x + (y + z) = (y + x) + z$ | - | True |
| B) $y(x + z) = xy + zy$ | - | True |
| C) $x^{y+z} = x^z + x^y$ | - | False |
| D) $\frac{x^z}{x^y} = x^{y-z}$ | - | False |

1.6 $\ln(x) \geq e$

$\ln(x) = e$

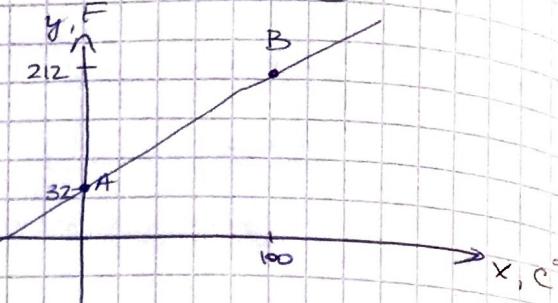
$x = e^{\ln x}$

$x \geq e^{\ln x}$

$x \in [e^{\ln x}, \infty)$

② Functions of one variable

2.1 A (0; 32)
B (100; 212)



Intercept: $x = 0$
 $y = 32$

Slope: $\frac{212 - 32}{100} = 1,8$

$$y = 1,8x + 32$$

Temperature is the same in C° and F
when $x = y$

$$y = 1,8y + 32$$

$$y = -40$$

$$-40 C^\circ = -40 F$$

2.2

$$f(x) = 3x - 12$$

$$f(y) = 0$$

$$3x - 12 = 0$$

$$3x = 12$$

$$x = 4$$

2.3

$g^{x^2 - 6x + 2}$	$= 81$
$g^{x^2 - 6x + 2}$	$= g^2$
$x^2 - 6x + 2 = 2$	
$x(x - 6) = 0$	
$x_1 = 0$	$x_2 = 6$

2.4

If GDP is X and t is the years, then

$$f(t) = X \cdot 1,03^t$$

$$3X = X \cdot 1,03^t$$

$$1,03^t = 3$$

$$t = \frac{\log 3}{\log 1,03}$$

$$t = 37,17$$

- years to triple GDP

$$2.5 \quad \log_{\pi} \left(\frac{1}{\pi^5} \right) = \log_{\pi} 1 - \log_{\pi} \pi^5 = \log_{\pi} 1 -$$

$$-5 \log_{\pi} \pi = \pi - 5\pi = \pi(1-5) \approx -46,2$$

③ Calculus

$$3.1 \quad \sum_{i=0}^{\infty} \left(\frac{1}{5^i} + 0,3^i \right) =$$

$$3.2 \quad \lim_{x \rightarrow 5} \frac{x^2 - 25}{x-5} = \lim_{x \rightarrow 5} \frac{(x-5)(x+5)}{x-5} = 5 + 5 = 10$$

$$3.3 \quad f(x) = x^3 - 4 \quad A(-2, -12)$$

$$f'(-2) = \lim_{x \rightarrow (-2)} \frac{f(x) + 12}{x + 2} = \lim_{x \rightarrow (-2)} \frac{x^3 + 8}{x + 2} =$$

$$= \lim_{x \rightarrow (-2)} \frac{(x+2)(x^2 - 2x + 4)}{x+2} = \lim_{x \rightarrow (-2)} x^2 + 4 - 2x = 4 + 4 + 4 = 12$$

$$3.4 \quad f(x) = \frac{x^5 + 3}{x^2 - 1}$$

$$f'(x) = \frac{5x^4(x^2 - 1) - 2x(x^5 + 3)}{(x^2 - 1)^2} = \frac{5x^6 - 5x^4 - 2x^6 - 6x}{(x^2 - 1)^2} =$$

$$= \frac{3x^6 - 5x^4 - 6x}{(x^2 - 1)^2}$$

$$3.5 \quad f(x) = x^9 + 3$$

$$f'(x) = 9x^8$$

$$f''(x) = 72x^7$$

3.6 $f(x) = \frac{1}{x}$ is NOT continuous at $x = 0$
 because denominator cannot be 0
 Therefore it is excluded from $f(x)$.

3.7. $f(x) = 4x^3 - 12x$

$$4x^3 - 12x = 0$$

$$4x(x^2 - 3) = 0$$

$$x_1 = 0$$

$$x^2 = 3$$

$$x_2 = \sqrt{3}$$

$$x_3 = -\sqrt{3}$$

$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$$

} inflection

$$f'(x) = 12x^2 - 12 = 0$$

$$12x^2 = 12$$

$$x^2 = 1$$

$$x = \pm 1$$

$$f''(x) = 24x$$

$$f''(1) = 24 - \text{Minima point}$$

$$f''(-1) = -24 - \text{Maxima point}$$

3.8 $f(x, y) = x^3 - y^2$

$$f(2, 3) = 2^3 - 3^2 = 8 - 9 = -1$$

3.9 $f(x, y) = \ln(x - 3y)$

$$x - 3y > 0$$

$$x > 3y$$

$$y < \frac{x}{3}$$

3.10 $\frac{\partial}{\partial x} \left(x^5 y^7 + \frac{x^2}{y^3} \right) = \frac{\partial}{\partial x} (x^5 y^7 + x^2 y^{-3})$

$$\frac{\partial}{\partial x} = 5x^4 y^7 + 2x y^{-3}$$

$$3.11 \quad f(x,y) = \sqrt{xy} - x - y$$

$$f'(x) = \frac{1}{2} y x^{-\frac{1}{2}}$$

$$f'(y) = \frac{1}{2} x y^{-\frac{1}{2}}$$

$$3.12 \quad \max \quad x^2 y^2$$

$$\text{s.t.} \quad 2x + y = 9$$

$$L(x, y, \lambda) = x^2 y^2 - \lambda(2x + y - 9)$$

$$\frac{\partial}{\partial x} = 2x y^2 - 2\lambda$$

$$\frac{\partial}{\partial y} = 2y x^2 - \lambda$$

$$\frac{\partial}{\partial \lambda} = 2x + y - 9$$

$$\begin{cases} 2x y^2 - 2\lambda = 0 \\ 2y x^2 - \lambda = 0 \\ 2x + y - 9 = 0 \end{cases}$$

④ Linear Algebra

$$4.1 \quad A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \\ 7 & 6 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 9 & 1 & 5 \end{bmatrix}$$

$$\begin{array}{c|cc|cc} BA & & & & \\ \hline & 1 & 0 & 1 & 9 & 11 \\ & 9 & 1 & 5 & 55 & 76 \end{array}$$

$$BA = \begin{bmatrix} 9 & 11 \\ 55 & 76 \end{bmatrix}$$

$$4.2 \quad A = \begin{bmatrix} 5 & 3 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 8 & 4 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\begin{array}{c|ccc} & 8 & 4 & 0 \\ & 2 & 1 & 2 \\ \hline AB & 5 & 3 & 46 & 23 & 6 \\ 0 & 1 & & 2 & 1 & 2 \\ 1 & 2 & & 12 & 6 & 4 \end{array}$$

$$AB = \begin{bmatrix} 46 & 23 & 6 \\ 2 & 1 & 2 \\ 12 & 6 & 4 \end{bmatrix}$$

$$4.3 \quad T = \begin{bmatrix} e & 2 & 4 \\ 93 & 6,1 & \pi \\ 4,7 & 4,22 & 0 \end{bmatrix}$$

$$4.4 \quad \begin{bmatrix} 2 & 6 \\ 2 & 8 \end{bmatrix}$$

$$\det = 2 \cdot 8 - 2 \cdot 6 = 16 - 12 = 4$$

5 Probability theory

$$5.1 \quad P = 6^2 = 36$$

5.2 $P(A) = 0,98$ - probability among drug-users to get a positive result

$P(B) = 0,003$ - probability among non drug-users to get a positive test result

$$P = 0,98 + 0,003 = 0,983$$

$P = \frac{0,98}{0,983} = 0,996$ - probability of someone having a positive test result and being a drug-user