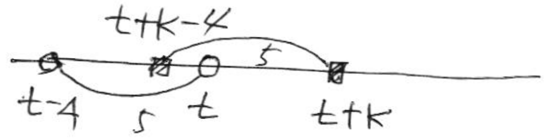
TS Mid-term Exam Solution Set (2015)

1\.

1\) $M_t = \sum_{j=0}^{4} Z_{t-j}/5 = (Z_t + Z_{t-1} + \cdots + Z_{t-4})/5$

$M_t - E[M_t] = \sum_{j=0}^{4} a_{t-j}/5$

$\Rightarrow Var(M_t) = (\frac{1}{5})^2 (5\sigma^2) = \frac{\sigma^2}{5}$



i) $Cov(M_t, M_{t+k}) = E[(\sum_{j=0}^{4} a_{t-j})(\sum_{i=0}^{4} a_{t+k-i})] \cdot (\frac{1}{5})^2$

$= E[a_{t+k-4}^2 + \cdots + a_t^2]/(5)^2$

$= (\frac{1}{5})^2 \cdot E[\sum_{j=0}^{4-k} a_{t-j}^2] = (\frac{1}{5})^2 \cdot \sigma^2 \cdot (5-k) \leftarrow 4-k+1$

$\Rightarrow Corr(M_t, M_{t+k}) = \begin{cases} \frac{1}{5}(5-k) = 1 - k/5, & k = 0, 1, \cdots, 4 \\ 0 & o.w. \end{cases}$

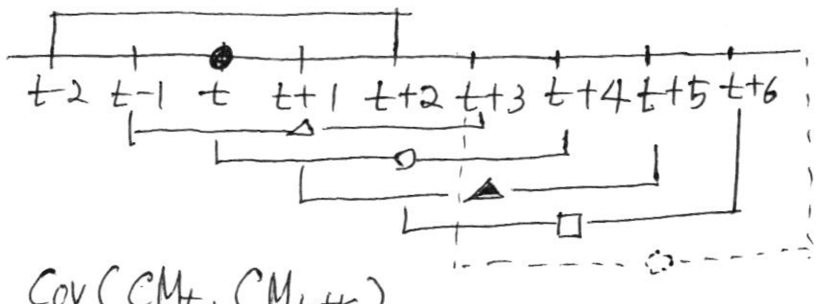
ii) $e_n(2) = Z_{n+2} - M_n$

$Var(e_n(2)) = Var(Z_{n+2}) + Var(M_n)$ since $Z_{n+2} \perp\!\!\!\perp M_n$

$= \sigma^2 + \sigma^2/5 = \sigma^2(1 + 1/5)$

$= \sigma^2(\frac{6}{5}) \leftarrow \sigma^2(\frac{m+1}{m})$

2\)



$Cov(CM_t, CM_{t+k})$

$= (\frac{1}{5})^2 \cdot E[(a_{t-2} + a_{t-1} + a_t + a_{t+1} + a_{t+2})$

$\times (a_{t+k-2} + a_{t+k-1} + a_{t+k} + a_{t+k+1} + a_{t+k+2})]$

$= (\frac{1}{5})^2 \cdot E(a_{t+k-2}^2 + \cdots + a_{t+2}^2)$

$= \begin{cases} (\frac{1}{5})^2 \cdot (5-k) \cdot \sigma^2 & k = 0, 1, \cdots, 4 \\ 0 & o.w. \end{cases}$

$Corr(CM_t, CM_{t+k}) = \begin{cases} \frac{1}{5}(5-k) = 1 - k/5, & k = 0, 1, \cdots, 4 \\ 0 & o.w. \end{cases}$

M_t와 CM_t의 Corr은 같다.

1-3)

i) $S_{n+1} = \omega Z_{n+1} + (1-\omega)S_n$

$= S_n + \omega(Z_{n+1} - S_n) = S_n + \omega \cdot e_n(1)$

$(n+1)$ 시점 평활값 S_{n+1}은 n 시점 평활값 S_n에 예측오차의 (오차) 일부가 반영된 것

ii) $\omega = \frac{2}{m+1}$

$m=5$의 MA의 경우 : 같은 정도의 ES의 평활 상수는

$\omega = \frac{2}{m+1} = \frac{2}{5+1} = \frac{2}{6} = 0.33$

$\omega = 0.2 < 0.33 (\Leftarrow \omega(m=5))$ 이므로

ES의 평활 상수가 MA보다 더 작으므로, ES가 MA 보다 more smoother!

2. ASM　　　　　MSM

$Z_{n+l} = T_{n+l} + S_{n+l} + I_{n+l}$　　　$Z_{n+l} = T_{n+l} * S_{n+l} + I_{n+l}$

$S_i = S_{i+s} = S_{i+2s} = \cdots$

$\sum_{i=1}^{s} S_i = 0$　　　$\sum_{i=1}^{s} S_i = s$

3. $Z_t = 1.0 + 0.5 Z_{t-2} + a_t$

$= 1.0 + (0.5)(1.0) + (0.5)^2 Z_{t-4} + a_t + 0.5 a_{t-2}$

$= 1.0 + 0.5 + 0.5^2 + (0.5)^3 Z_{t-6} + a_t + 0.5 a_{t-2} + 0.5^2 a_{t-4}$

$= \sum_{k=0}^{\infty} (0.5)^k + \sum_{k=0}^{\infty} (0.5)^k a_{t-2k}$

i) $\mu = \sum_{k=0}^{\infty} (0.5)^k = \frac{1}{1-0.5} = 2.0$

$\psi_k = \begin{cases} 0 & \text{if } k = \text{홀수} \\ (0.5)^{k/2} & \text{if } k = \text{짝수} \end{cases}$

ii) Z_t가 linear process form으로 표현되고,

$\sum_0^{\infty} \psi_k^2 = \sum_0^{\infty} \left((0.5)^{k/2}\right)^2 = \sum_0^{\infty} (0.5)^k = \frac{1}{1-0.5} = 2 < \infty$

$\Rightarrow$ stationary process 이다!

4. $Z_t = 2.0 + a_t - 0.5\, a_{t-2}$

$E(Z_t) = 2.0$ since $E(a_{t-j}) = 0$

$Var(Z_t) = Var(2.0 + a_t - 0.5\, a_{t-2})$

$= Var(a_t) + Var(-0.5\, a_{t-2})$, since $a_t \perp\!\!\!\perp a_{t-k}$

$= \sigma^2 + (0.5)^2 \sigma^2 = 1.25 \cdot \sigma^2$

$\gamma_K = Cov(Z_t, Z_{t-k})$

$= Cov(a_t - 0.5\, a_{t-2},\ a_{t-k} - 0.5\, a_{t-k-2})$

$$= \begin{cases} 1.25\,\sigma^2 & \text{if } K=0 \\ 0 & \text{if } K=1 \\ E(-0.5\, a_{t-2} * a_{t-2}) = -0.5\,\sigma^2 & \text{if } K=2 \\ 0 & \text{if } K>3 \end{cases}$$

$\leftarrow \mu, \sigma^2, \gamma_K$: free of t

$\therefore$ stationay process !