Numerical optimization III

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The method of steepest descent

Newton-Raphson method requires the hessian matrix !

Let $F : R^n \to R$ be differentiable.
Consider

$$\phi(t) = F(x_0 + tv)$$

where $||v|| = 1$.

Q) Show that $\phi^{'}(0)$ is minimized when $v = -\frac{\nabla F(x_0)}{||\nabla F(x_0)||}$.

Therefore, our remaining problem is to find one-dimensional variable t for the choice of v.

Let t_0 be the minimizer for $\phi_0(t) = F(x_0 - t\nabla F(x_0))$.

Then, set

$$x_1 = x_0 - t_0 \nabla F(x_0).$$

By repeating this process, we have

$$x_2 = x_1 - t_1 \nabla F(x_1).$$

The method of steepest descent is: Given x_0,

$$x_{k+1} = x_k - t_k \nabla F(x_k)$$

where t_k minimizes

$$F(x_k - t\nabla F(x_k)).$$

Q) Apply the method of steepest descent to

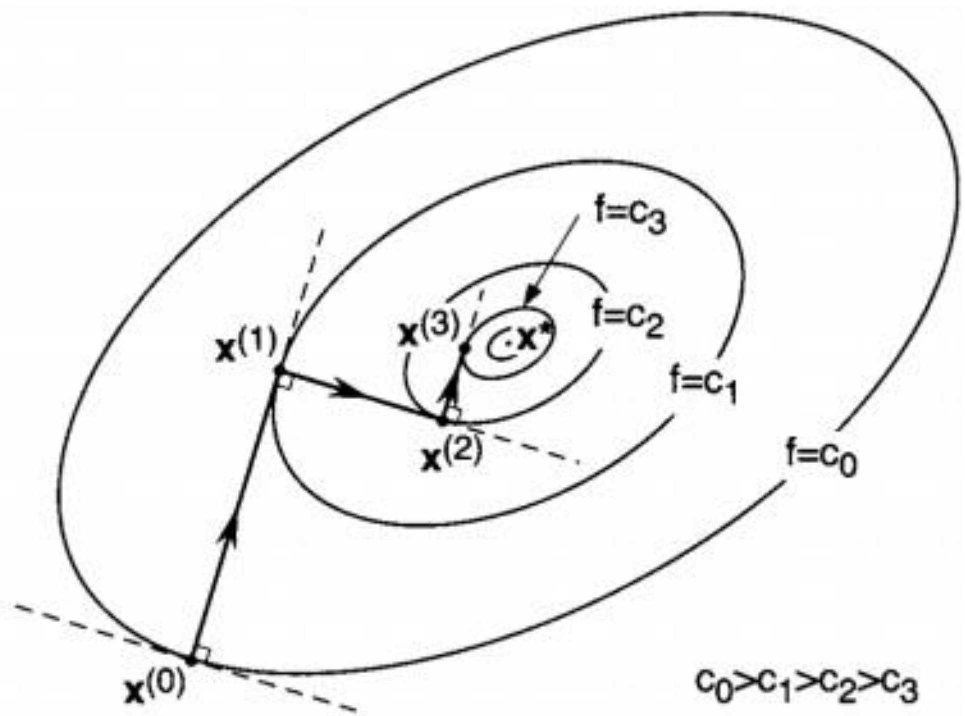
$$f(x, y) = 4x^2 - 4xy + 2y^2$$

with $x_0 = (2, 3)$.

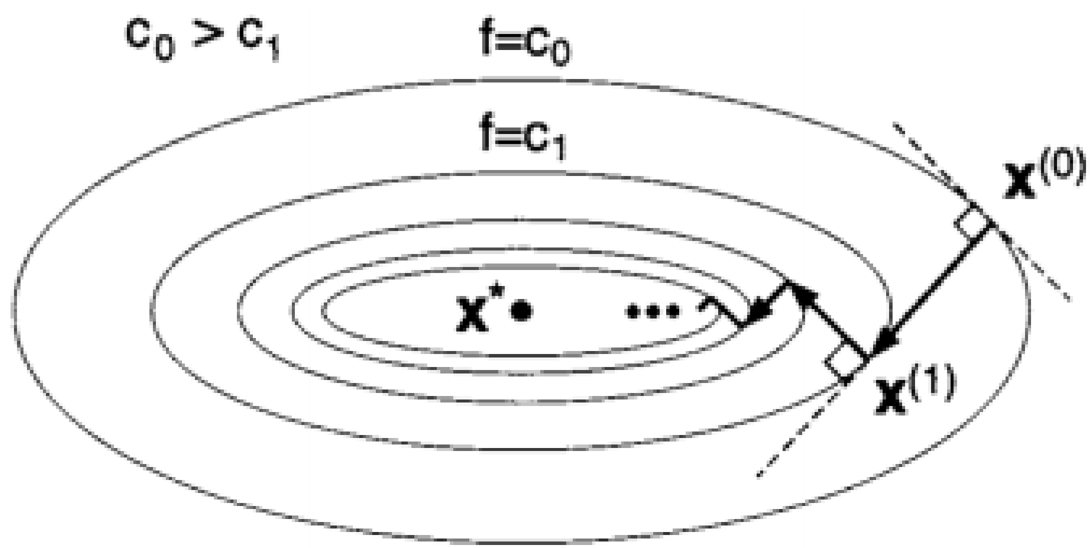
Theorem. Let $F : R^n \to R$ be continuously differentiable function. Suppose that x_k and x_{k+1} are two consecutive iterates given by the method of steepest descent. Then,

$$(\nabla F(x_k))^T \nabla F(x_{k+1}) = 0.$$

Q) Discuss the trajectory of (x_k) given by the method of steepest descent.



Proposition 8.1 *If $\{\boldsymbol{x}^{(k)}\}_{k=0}^{\infty}$ is a steepest descent sequence for a given function $f : \mathbb{R}^n \to \mathbb{R}$, then for each k the vector $\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}$ is orthogonal to the vector $\boldsymbol{x}^{(k+2)} - \boldsymbol{x}^{(k+1)}$.* □


$c_0 > c_1$
$f=c_0$
$f=c_1$
$x^{(0)}$
$x^{(1)}$
x^*