2. 확률과 베이즈 정리

이경재

인하대학교 통계학과

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확률(Probability)

- Probabilities apply to processes with **unpredictable** outcomes ("random experiments")
  - Coin flips
  - The number of accidents
  - Scores
- Probability model:
  - X: Random variable (the result, or outcome).
  - $\mathcal{X}$: Sample Space (set of all possible outcomes).
  - P: Probability distribution over $\mathcal{X}$.

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확률(Probability)

- The probability of an event A is the sum of probabilities of all the points in A. It is denoted by $P(A)$.
  - 주사위를 던졌을 때, 짝수가 나올 확률
  - 손흥민이 다음 경기에서 공격포인트를 기록할 확률
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확률의 공리(Axioms of probability)

- 확률은 다음의 성질들을 반드시 만족해야 한다.
- 표본공간을 S, 그 위의 임의의 사건을 A라 하자.

1. $0 \le P(A) \le 1$
2. $P(S) = 1$
3. $A_1, A_2, \ldots$가 배반, 즉, $A_i \cap A_j = \phi, \forall i \neq j$이면,

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots.$$

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조건부 확률(Conditional probability)

- 임의의 사건 A와 B에 대해, $P(B) > 0$이면, B가 발생했을 때 A가 발생할 조건부 확률은 다음과 같이 정의한다:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}. \tag{1}$$

- $P(A \mid B)P(B) = P(A \cap B) = P(B \mid A)P(A)$

- (1)는 베이즈 정리의 특별한 경우로 볼 수 있다.

독립성(Independence)

- 두 사건 A와 B가 $P(A \cap B) = P(A)P(B)$를 만족하면, A와 B는 서로 독립이다. 이는 $P(A \mid B) = P(A)$와 동치이다.
- 사건 H에 대해 $P(H) > 0$라 하자. 두 사건 A와 B가

$$P(A \cap B \mid H) = P(A \mid H)P(B \mid H)$$

를 만족하면, A와 B를 H가 주어진 하에서 조건부 독립이라 한다.

Example: 조건부 확률

- A woman and a man (unrelated) each have two children. At least one of the woman's children is a boy, and the man's older child is a boy. Do the chances that the woman has two boys equal the chances that the man has two boys?
- Marilyn says: The probability that the woman has two boys is $\frac{1}{3}$, and the probability that the man has two boys is $\frac{1}{2}$.
- Many people argue that Marilyn is horribly wrong & obviously the chances are equal. Who is correct?

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Example: 조건부 확률

- **Assumptions**: For any family, the probability of a boy on one birth is $\frac{1}{2}$, and births are independent.
- **Notations**:
  1. $\mathcal{X} = \{(0,0),(0,1),(1,0),(1,1)\}$
  2. Events:

$$A = \{\text{older birth is a boy}\} = \{(0,1),(1,1)\}$$

$$C = \{\text{Exactly one boy in two births}\} = \{(1,0),(0,1)\}$$

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Example: 조건부 확률

- We are given that the man's older child is a boy. What is the probability of two boys, given the older is a boy?
- Mathematically, it is $P(D \mid A)$.
- Answer:

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\begin{aligned}
P(D \mid A) &= \frac{P(D \cap A)}{P(A)} \\
&= \frac{P(D)}{P(A)} = \frac{1/4}{1/2} = \frac{1}{2}.
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- We are also given that the woman has at least one boy.

  What is the probability of two boys, given at least one boy?
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$$P(D \mid C \cup A) = \frac{P(D \cap \{C \cup A\})}{P(C \cup A)} = \frac{P(D)}{P(C \cup A)} = \frac{1/4}{3/4} = \frac{1}{3}.$$

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Example: 조건부 확률(세 공장)

- Three plants, C_1, C_2 and C_3, produce respectively, 10, 50, and 40 percent of a company's output.
- Although plant C_1 is a small plant, its manager believes in high quality and only 1% of its products are defective. The other two C_2 and C_3, are worse and produce items that are 3 and 4% defective, respectively.
- $P(C_1) = 0.1$, $P(C_2) = 0.3$ and $P(C_3) = 0.4$
- All products are sent to a central warehouse.
- An item is randomly selected and observed to be defective, say event C.
- What is the probability that it comes from plant C_1?

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- We have $P(C \mid C_1) = 0.01$, $P(C \mid C_2) = 0.03$ and $P(C \mid C_3) = 0.04$.
- Thus,

$$
\begin{aligned}
P(C_1 \mid C) &= \frac{P(C_1 \cap C)}{P(C)} \\
&= \frac{.10 \times .01}{.10 \times .01 + .50 \times .03 + .40 \times .04} \\
&= \frac{1}{32}
\end{aligned}
$$

- Note that this is much smaller than $P(C_1) = 0.1$.

표본사건의 분할

- 임의의 사건열 $\{A_1, \ldots, A_k\}$가 다음을 만족하면, 이를 S의 분할(partition)이라 한다:
  1. $A_1, \ldots, A_k$가 서로 배반이다.
  2. $\dot\bigcup_{i=1}^{k} A_i = S$
- 임의의 사건 A에 대해, $\{A, A^c\}$는 S의 분할이다.
- (e.g.) {음의 정수, 0, 양의 정수}는 정수의 분할

전확률 법칙(Law of total)

- S의 분할 $\{A_1, \ldots, A_k\}$은 다음을 만족한다:

  임의의 사건 B에 대해,

$$P(B) = P(B \cap A_1) + \cdots + P(B \cap A_k).$$

Example: 전확률 법칙

- Suppose that two factories supply light bulbs to the market. Factory X's bulbs work for over 5000 hours in 99% of cases, whereas factory Y's bulbs work for over 5000 hours in 95% of cases.
- It is known that factory X supplies 60% of the total bulbs available.
- What is the chance that a purchased bulb will work for longer than 5000 hours?

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Example: 전확률 법칙

- Applying the law of total probability, we have:

$$\begin{aligned}\Pr(A) &= \Pr(A \mid B_X) \cdot \Pr(B_X) + \Pr(A \mid B_Y) \cdot \Pr(B_Y) \\ &= \frac{99}{100} \cdot \frac{6}{10} + \frac{95}{100} \cdot \frac{4}{10} = \frac{594 + 380}{1000} = \frac{974}{1000}\end{aligned}$$

- Thus each purchased light bulb has a 97.4% chance to work for more than 5000 hours.

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베이즈 정리(Bayes' theorem)

- 주어진 분할 $\{A_1, \ldots, A_k\}$와 임의의 사건 E에 대해 다음이 성립한다:

$$P(A_i \mid E) = \frac{P(E \mid A_i)P(A_i)}{\sum_{j=1}^{k} P(E \mid A_j)P(A_j)}.$$

- 이 공식은 전확률 법칙과 조건부 확률의 정의로부터 쉽게 유도된다. (Why?)

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베이즈 정리 유도

- 전확률 법칙에 의해,

$$\begin{aligned} P(E) &= P(A_1 \cap E) + \cdots + P(A_k \cap E) \\ &= P(E \mid A_1)P(A_1) + \cdots + P(E \mid A_k)P(A_k). \end{aligned}$$

- 따라서,

$$\begin{aligned} P(A_i \mid E) &= \frac{P(E \cap A_i)}{P(E)} \\ &= \frac{P(E \mid A_i)P(A_i)}{P(E \mid A_1)P(A_1) + \cdots + P(E \mid A_k)P(A_k)}. \end{aligned}$$

베이즈 정리의 해석

- Let D be the observed data and denote $A_1, \ldots, A_k$ by $\theta_1, \ldots, \theta_k$. Then we can interpret this as a "posterior probability".
- 관심모수 θ가 $\{\theta_1, \ldots, \theta_k\}$ 중 하나의 값을 가질 수 있다고 하면, $\theta = \theta_i$일 사후분포 확률은

$$P(\theta_i \mid D) = \frac{P(\theta_i)P(D \mid \theta_i)}{\sum_{j=1}^{k} P(\theta_j)P(D \mid \theta_j)} = \frac{P(\theta_i)P(D \mid \theta_i)}{P(D)}.$$

- $P(D)$ is the marginal distribution of the data.
- Note if the values of θ are portions of the continuous real line, the sum may be replaced by an integral.

Example: 베이즈 정리

- 1975년, “영국이 European Economic Community(EEC)의 일원으로 남아야 하는가?”에 대한 국민투표가 있었다.
- 투표자 중 52%는 노동당, 48%는 보수당을 지지했다. 노동당 지지자 중 55%, 보수당 지지자 중 85%는 찬성 의사를 가지고 있었다.
- Let **L** denote “Labour” and **Y** denote “Yes”.
- What is the probability that a person voting “Yes” to remaining in EEC is a Labour voter?

$$P(L \mid Y) = \frac{P(Y \mid L)P(L)}{P(Y)}.$$

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전확률 법칙에 의해,

$$P(Y) = P(Y, L) + P(Y, L^c) = P(Y \mid L)P(L) + P(Y \mid L^c)P(L^c).$$

따라서, 구하고 싶은 확률은 다음과 같다:

$$\begin{aligned} P(L \mid Y) &= \frac{P(Y \mid L)P(L)}{P(Y \mid L)P(L) + P(Y \mid L^c)P(L^c)} \\ &= \frac{(.55)(.52)}{(.55)(.52) + (.85)(.48)} \\ &= 0.41. \end{aligned}$$

Example: 베이즈 정리(4 Classes)

- In the 1996 General Social Survey, for males (age 30+):
  - 11% of those in the lowest income quartile were college graduates.
  - 19% of those in the second-lowest income quartile were college graduates.
  - 31% of those in the third-lowest income quartile were college graduates.
  - 53% of those in the highest income quartile were college graduates.
- What is the probability that a college graduate falls in the lowest income quartile (11%)?

Example: 베이즈 정리(4 Classes)

- G: college graduate, Q_j: j^{th} quartile.

$$
\begin{aligned}
P(Q_1 \mid G) &= \frac{P(Q_1)P(G \mid Q_1)}{\sum_{j=1}^{4} P(Q_j)P(G \mid Q_j)} \\
&= \frac{.11 \cdot .25}{.11 \cdot .25 + .19 \cdot .25 + .31 \cdot .25 + .53 \cdot .25} \\
&= 0.09.
\end{aligned}
$$

- Find $P(Q_2 \mid G)$. (Exercise)
- How does this **conditional** distribution differ from the **unconditional** distribution $P(Q1), ..., P(Q4)$?

Example: 베이즈 정리(수지)

- 갓 태어난 수지의 친아빠가 누구인지 밝혀내는 소송이 진행중이다.
- 엄마의 혈액형은 O형이며 아빠로 지목된 남자 Albert의 혈액형은 AB형이다.
- 소송을 진행하는 과정에서 수지의 혈액형을 조사하니 B형으로 나타났다.
- 수지의 혈액형이 B라는 사실을 사건 B로, Albert가 수지의 아빠일 사건을 F라고 하자.

Example: 베이즈 정리(수지)

- 우리가 구하고자 하는 것은 $P(F \mid B)$이다.
- 그런데 혈액형 이외에, 그동안 수지의 엄마와 Albert와의 관계 또는 주변사람들의 증언 등에 의하여 Albert가 수지의 아빠일 가능성 $P(F)$를 추측할 수 있을 것이다.
- 이때 $P(F)$는 자료, 즉 혈액형을 측정하기 이전의 확률이므로 사전확률(Prior)이라고 한다.
- 반면 $P(F \mid B)$는 자료측정 이후의 확률이므로 사후확률 (Posterior)이라고 한다.
- 베이즈 정리에 의하여 $P(F \mid B)$를 구하면

$$P(F \mid B) = \frac{P(B \mid F)P(F)}{P(B \mid F)P(F) + P(B \mid F^c)P(F^c)}$$

Example: 베이즈 정리(수지)

- 이제 $P(B \mid F)$와 $P(B \mid F^c)$를 구하자.
- Albert가 수지의 아빠일 경우 멘델의 법칙에 의하여 $P(B \mid F) = 0.5$이다. 또 $P(B \mid F^c)$를 생각해 보면 Albert가 수지의 아빠가 아닐 경우에도 수지의 혈액형이 B가 나올 수 있는데 이를 전체 중에서 B형인 비율인 9%로 놓기로 하자.
- 이들을 위의 식에 대입하면,

$$P(F \mid B) = \frac{0.5 \times P(F)}{0.5 \times P(F) + 0.09 \times P(F^c)} = \frac{50P(F)}{41P(F) + 9}.$$

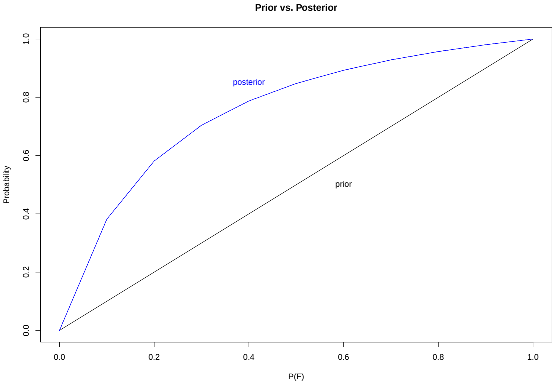
Example: 베이즈 정리(수지)

- $P(F)$와 $P(F \mid B)$를 비교해보자(R simulation):

```
> aa <- function(pf){
+   posterior = 50*pf/ (41*pf +9)
+   return(posterior)
+ }
> x <- seq(0,1, by = 0.1)
> x
[1] 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
> prob <- aa(x)
> prob
[1] 0.0000000 0.3816794 0.5813953 0.7042254 0.7874016
0.8474576 0.8928571 0.9283820 0.9569378 0.9803922 1.000000
```

Example: 베이즈 정리(수지)

```
> plot(x, x, type ="l", xlab ="P(F)", ylab="Probability")
> lines(x, prob, lty = 1, col=4)
> text(0.6,0.5,"prior"); text(.4,.85,"posterior",col=4)
> title("Prior vs. Posterior");
```



Example: 베이즈 정리(수지)

- 사전확률 $P(F)$가 0 혹은 1에 가깝지 않은 경우 $P(F)$에 비하여 $P(F \mid B)$가 상당히 크다.
- 이는 사전증거가 확실치 않은 경우 혈액형에 의한 증거가 상당한 영향을 미침을 의미한다.