CSE 152 Section 7
HW3: Photometric Stereo and Optical Flow
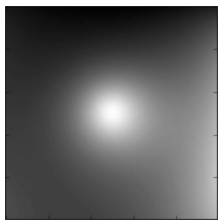
May 20, 2019

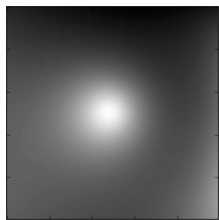Owen Jow

[1a] Lambertian Photometric Stereo
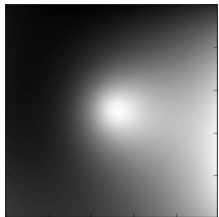
Input:
  images and associated lighting information

Output:
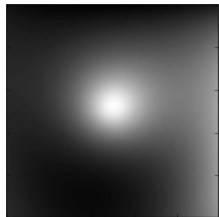 normals, albedo, depth

light source direction: $[0, 0, 1]^T$

light source direction: $[0.2, 0, 1]^T$

light source direction: $[-0.2, 0, 1]^T$

light source direction: $[0, 0.2, 1]^T$
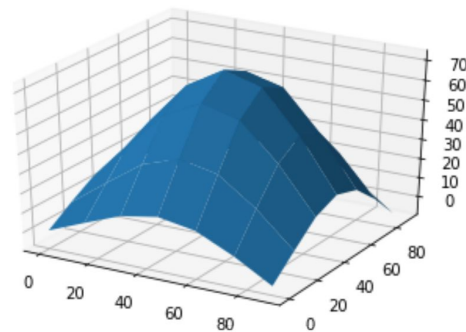
Photometric Stereo

[1a] Assumptions

Image plane

direction of projection

height

y

x

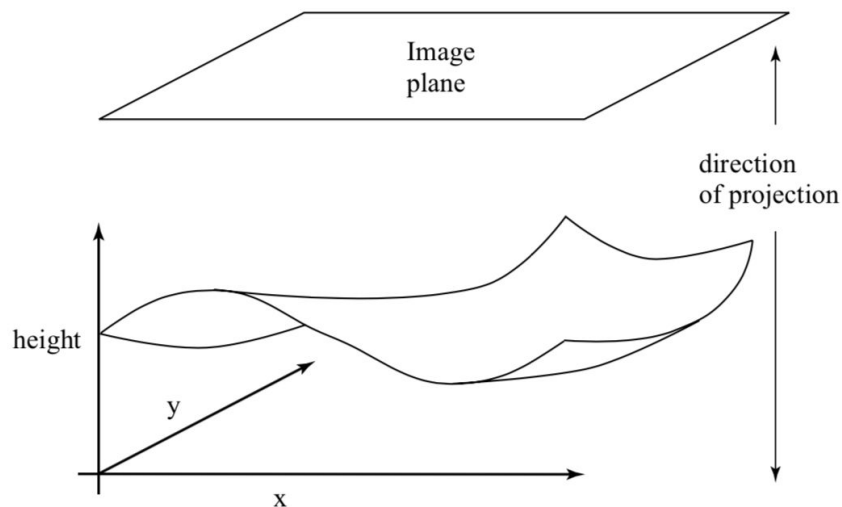image source: Forsyth and Ponce

- **Orthographic camera model:**
  (x, y, z) projects to (x, y) → our goal will be to recover the height/depth map z = f(x, y)

- **Distant lighting:**
  treat every pixel in one image as sharing the same lighting direction/intensity

- **Static scene/viewpoint:**
  (x, y) in one image corresponds to (x, y) in all of the other images

[1a] Assumptions, cont. (Lambertian)

- **Lambertian surface:**
  assume that the surface being imaged is Lambertian,
  i.e. at any point on the surface, there is equal reflectance in all directions

$$e(x, y) = [a(x, y) \, \mathbf{n}(x, y)] \cdot \mathbf{s}$$
$$= \mathbf{b}(x, y) \cdot \mathbf{s}$$

e(x, y)	intensity at pixel (x, y)	(known)
a(x, y)	albedo at the point on the surface corresponding to (x, y)	**(unknown)**
n(x, y)	unit normal at the point on the surface corresponding to (x, y)	**(unknown)**
s	unit direction to the light, <u>scaled</u> by the intensity of the light	(known)

image source: <u>Wayne Brown</u>
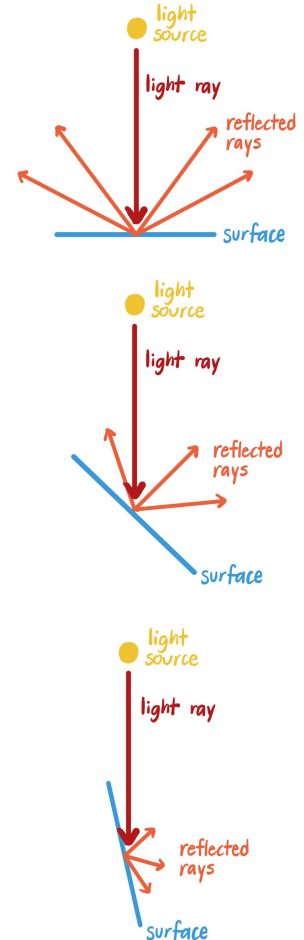
[1a] Solving for <u>b</u> at Each Pixel

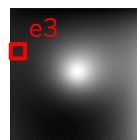**b** is a 3-vector, so we need at least three equations/images. Let's say we have n.

$$\begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1 \cdot \mathbf{b} \\ \vdots \\ \mathbf{s}_n \cdot \mathbf{b} \end{bmatrix}$$

$$\begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} - & \mathbf{s}_1 & - \\ & \vdots & \\ - & \mathbf{s}_n & - \end{bmatrix} \mathbf{b}$$

$$\underbrace{\mathbf{e}}_{n \times 1} = \underbrace{\mathbf{S}}_{n \times 3} \underbrace{\mathbf{b}}_{3 \times 1}$$

e1

s1 = [0, 0, 1]$^{\mathsf{T}}$

e2

s2 = [0.2, 0, 1]$^{\mathsf{T}}$

e3

s3 = [0, 0.2, 1]$^{\mathsf{T}}$

linear least squares
(see lecture)

$$\mathbf{b} = \left(\mathbf{S}^T \mathbf{S}\right)^{-1} \mathbf{S}^T \mathbf{e}$$

albedo $a = \|\mathbf{b}\|$

normal $\mathbf{n} = \dfrac{\mathbf{b}}{\|\mathbf{b}\|}$

[1a] **n** Encodes the Partial Derivatives of Depth

Note: this uses a left-handed coordinate system, whereas lecture uses a right-handed one.
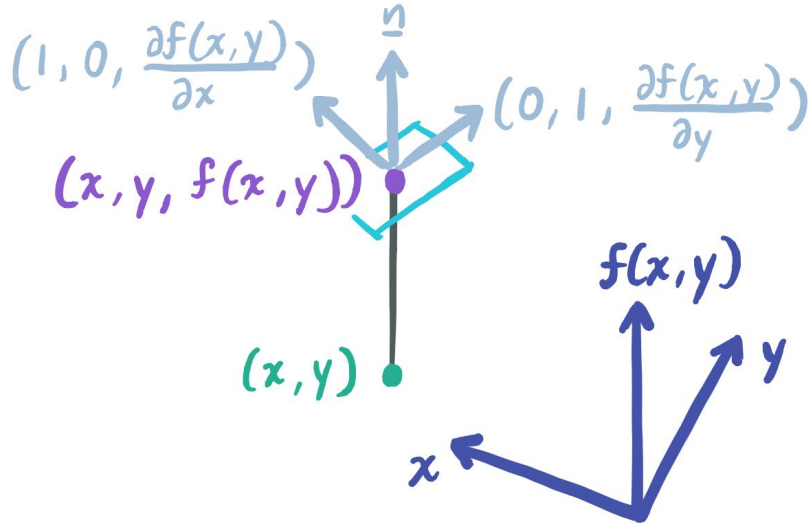
$$\mathbf{n} = \text{normalized} \left(\begin{bmatrix} 0 \\ 1 \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ \frac{\partial f(x,y)}{\partial x} \end{bmatrix} \right)$$

$$= \frac{1}{\sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 + 1}} \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \\ -1 \end{bmatrix}$$

$$\frac{\partial f(x, y)}{\partial x} = -\frac{n_1}{n_3}$$

$$\frac{\partial f(x, y)}{\partial y} = -\frac{n_2}{n_3}$$

[1a] Simple Scanline Integration 1

Once we have the partial derivatives, we can integrate to get depth.

0
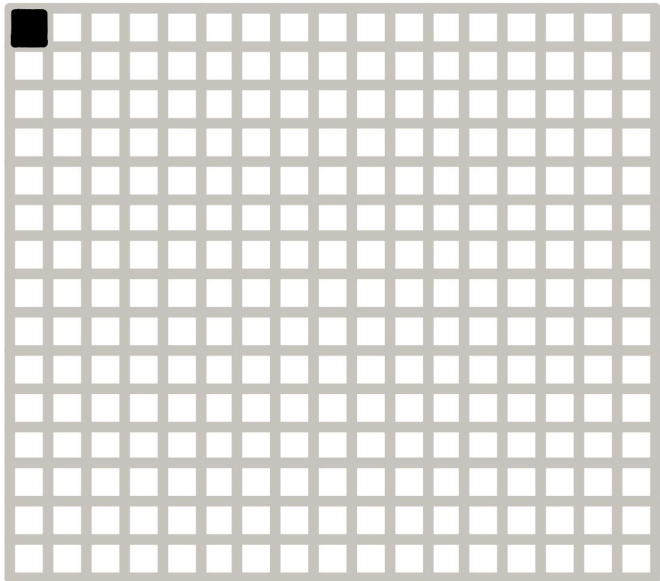
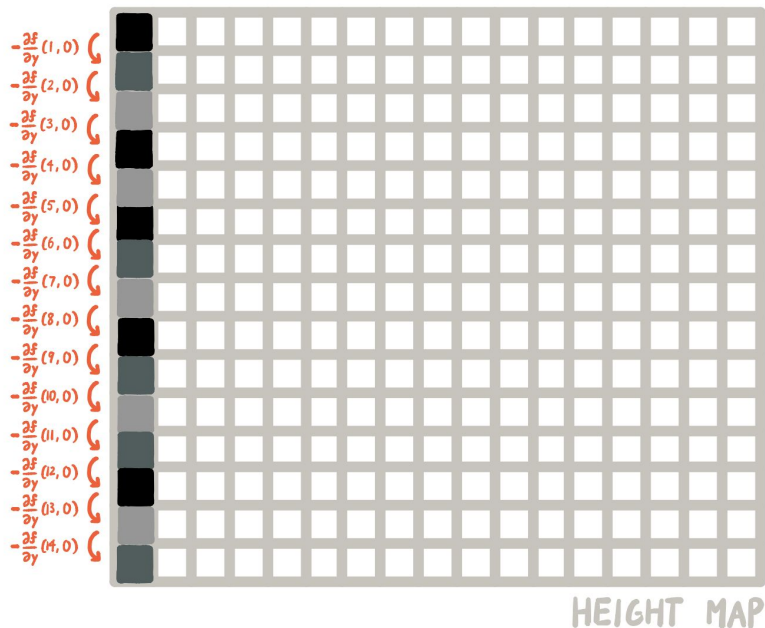**Initialize the top-left corner of the height map to 0**

HEIGHT MAP

[1a] Simple Scanline Integration 2

Once we have the partial derivatives, we can integrate to get depth.

$-\frac{\partial f}{\partial y}$ (1, 0) ↰
$-\frac{\partial f}{\partial y}$ (2, 0) ↰
$-\frac{\partial f}{\partial y}$ (3, 0) ↰
$-\frac{\partial f}{\partial y}$ (4, 0) ↰
$-\frac{\partial f}{\partial y}$ (5, 0) ↰
$-\frac{\partial f}{\partial y}$ (6, 0) ↰
$-\frac{\partial f}{\partial y}$ (7, 0) ↰
$-\frac{\partial f}{\partial y}$ (8, 0) ↰
$-\frac{\partial f}{\partial y}$ (9, 0) ↰
$-\frac{\partial f}{\partial y}$ (10, 0) ↰
$-\frac{\partial f}{\partial y}$ (11, 0) ↰
$-\frac{\partial f}{\partial y}$ (12, 0) ↰
$-\frac{\partial f}{\partial y}$ (13, 0) ↰
$-\frac{\partial f}{\partial y}$ (14, 0) ↰

HEIGHT MAP

Initialize the top-left corner of the height map to 0
For each pixel in the leftmost column [except (0, 0)]:
height = height of above pixel - ∂f/∂y

Why <u>subtract</u> $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

[Likewise, pass $-\frac{\partial f}{\partial x}$ and $-\frac{\partial f}{\partial y}$
to the Horn integration subroutine.]

original coordinate system

x
y

y
coordinate system during integration
x

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are in the original coordinate system

→ $\frac{\partial f}{\partial x}$: "take a small step leftward, how much f goes up"

→ $-\frac{\partial f}{\partial x}$: "take a small step rightward, how much f goes up"

[in both cases, the third axis points out of the page at the viewer]

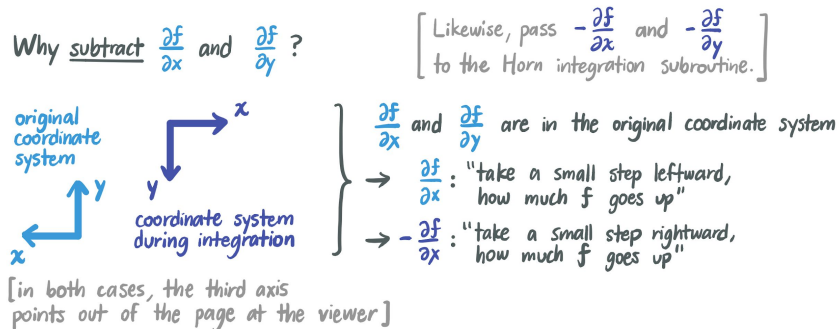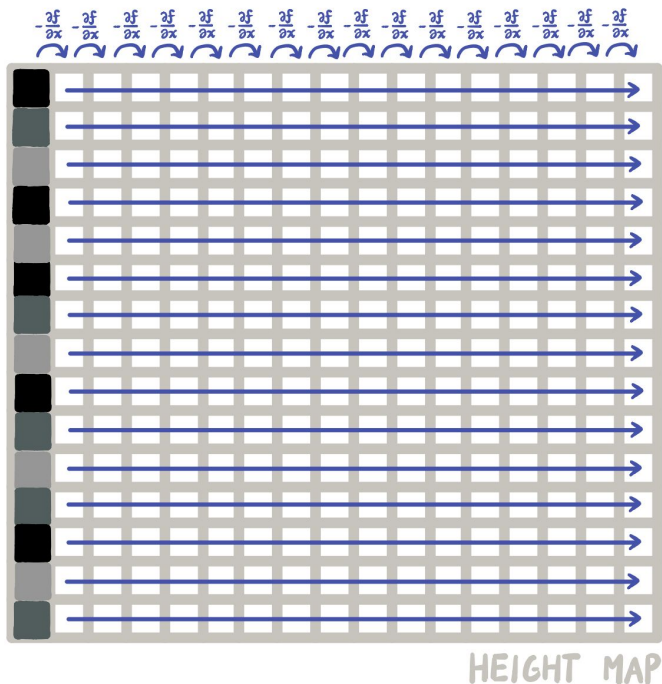# [1a] Simple Scanline Integration 3

Once we have the partial derivatives, we can integrate to get depth.

HEIGHT MAP

Initialize the top-left corner of the height map to 0

For each pixel in the leftmost column [except (0, 0)]:
height = height of above pixel - ∂f/∂y

For each pixel (except the leftmost) in each row:
 height = height of pixel to the left - ∂f/∂x

Why subtract $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$?

[Likewise, pass $-\frac{\partial f}{\partial x}$ and $-\frac{\partial f}{\partial y}$ to the Horn integration subroutine.]

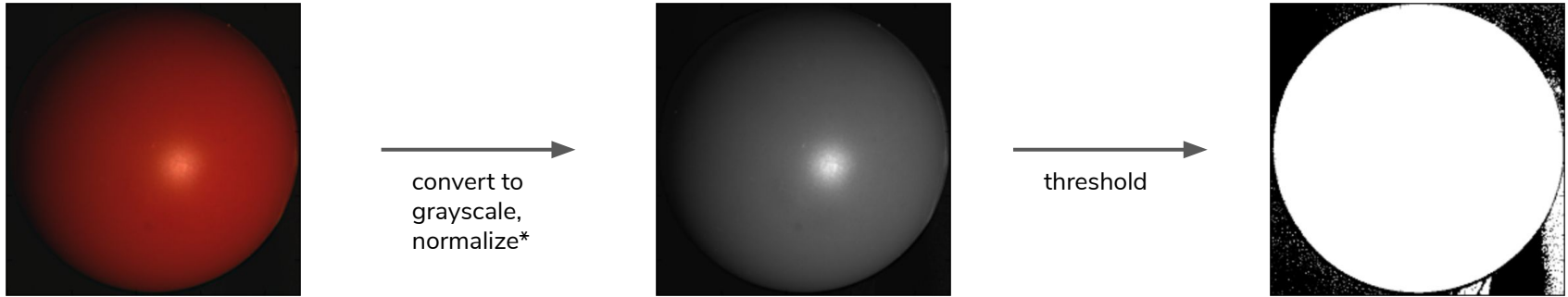original coordinate system

coordinate system during integration

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are in the original coordinate system

→ $\frac{\partial f}{\partial x}$: "take a small step leftward, how much f goes up"

→ $-\frac{\partial f}{\partial x}$: "take a small step rightward, how much f goes up"

[in both cases, the third axis points out of the page at the viewer]

[1a] The Mask Parameter

convert to grayscale, normalize*

threshold

- 1s for locations to use during integration
- 0s for locations to ignore during integration
- Unnecessary for 1a, but can use to filter out the background in 1b

*(img - img.min()) / (img.max() - img.min())

[1b] Applicability of the Lambertian Equation

The Lambertian equation does not apply for

- **shadowed regions**, where the view of the light is blocked
- **specularities**, which arise as a result of a different relationship

If we perform photometric stereo on such locations, we'll end up with noisy/pointy/bumpy artifacts.
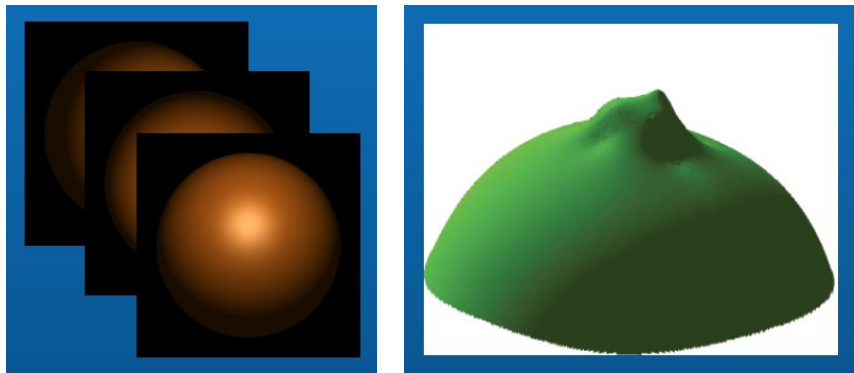

image source: Prof. Kriegman's lecture slides

[1b] Simple Shadow/Specularity Removal

Identify shadowed/specular locations based on brightness. Then, either...

1. ...clamp each associated brightness to a threshold value.
2. ...set each associated brightness to the median of a sizable surrounding window.

Do this before solving for **b**. You should be able to mitigate the artifact(s) to some degree.
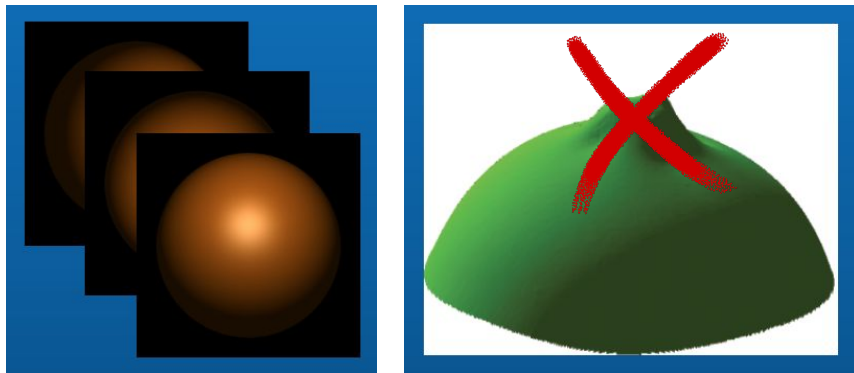

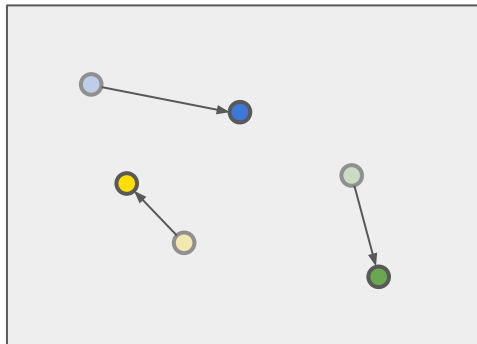image source: Prof. Kriegman's lecture slides

[2a] Optical Flow 1

Estimate the apparent motion of each pixel from frame A to frame B.

frame A

[2a] Optical Flow 2

Estimate the apparent motion of each pixel from frame A to frame B.

frame B + optical flow

[2a] On the Brightness Constancy Equation

Assumption 1: the brightness/color of each pixel remains constant as it moves.

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$$

Assumption 2: pixels don't move too far between frames. Linearizing via Taylor expansion:

$$I(x, y, t) + \Delta x \frac{\partial I}{\partial x} + \Delta y \frac{\partial I}{\partial y} + \Delta t \frac{\partial I}{\partial t} = I(x, y, t)$$

$$\Delta x \frac{\partial I}{\partial x} + \Delta y \frac{\partial I}{\partial y} + \Delta t \frac{\partial I}{\partial t} = 0$$

$$\frac{\Delta x}{\Delta t} \frac{\partial I}{\partial x} + \frac{\Delta y}{\Delta t} \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = 0$$

$$u I_x + v I_y + I_t = 0$$

Goal: compute u, v at every pixel.
Can compute Ix, Iy, It from the images.

[2a] The Lucas-Kanade Method

Problem: one equation, two unknowns.

Assumption 3: flow is constant in the neighborhood around each pixel.

→ **Get one equation for every point in a window around each pixel.**

$$\underbrace{\begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}}_{n \times 2} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{2 \times 1} = \underbrace{\begin{bmatrix} -(I_t)_1 \\ \vdots \\ -(I_t)_n \end{bmatrix}}_{n \times 1}$$

linear least squares →

$$\begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}^T \begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}^T \begin{bmatrix} -(I_t)_1 \\ \vdots \\ -(I_t)_n \end{bmatrix}$$

$$\underbrace{\left(\sum_{x,y \in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)}_{2 \times 2} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{2 \times 1} = \underbrace{\sum_{x,y \in W} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix}}_{2 \times 1}$$

the second moment
matrix strikes again

[2a] Notes

- Use the pseudoinverse to solve for **[u, v]**$^\mathsf{T}$

$$
\begin{bmatrix} u \\ v \end{bmatrix} = \left(\sum_{x,y \in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)^{\dagger} \left(\sum_{x,y \in W} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix} \right)
$$

$\dagger : \texttt{pseudoinverse}$

- Run this for every pixel (loops are fine)
- Make sure you compute the y-gradient with respect to an upward axis
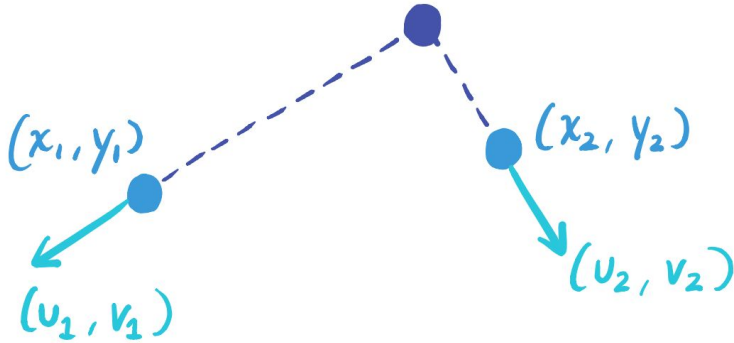 - If you use `np.gradient` to compute the image gradients, negate the y-gradient you get back

[3b] RANSAC for Estimating the Focus of Expansion

Idea: repeatedly

- Sample two flow vectors
- Estimate the focus of expansion as their intersection
- Check the consistency of the estimate across all flow vectors
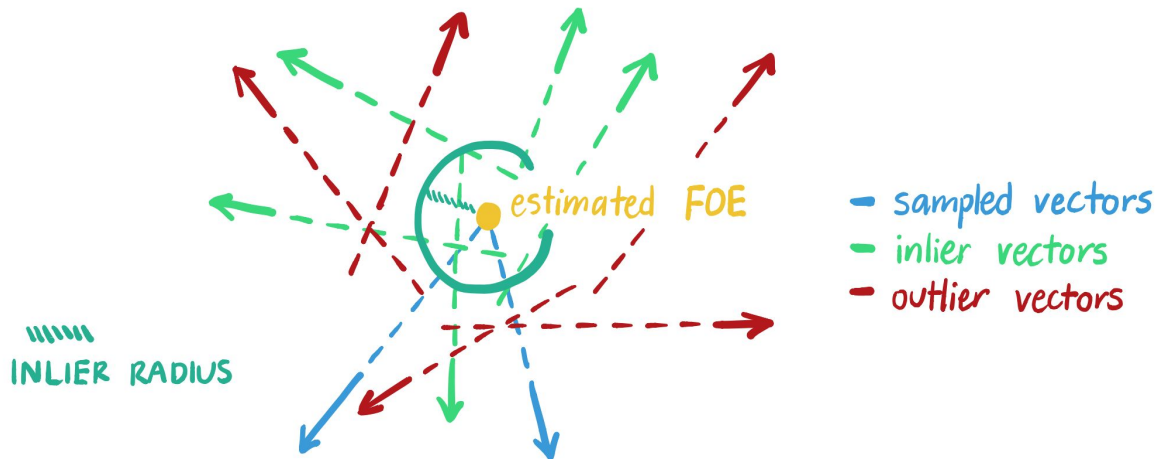
[3b] Ray-Ray Intersection

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + t_1 \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + t_2 \begin{bmatrix} u_2 \\ v_2 \end{bmatrix}$$

- Solve for t1 and t2. Derivation of the exact solution is left as an exercise. :)
- Note that there are other ways to compute intersections. Use whichever method you like.

[3b] Distance from a Point to a Ray

When checking consistency, you'll need to compute the perpendicular distance from your estimated focus of expansion to each of the rays (or to some subset of them).
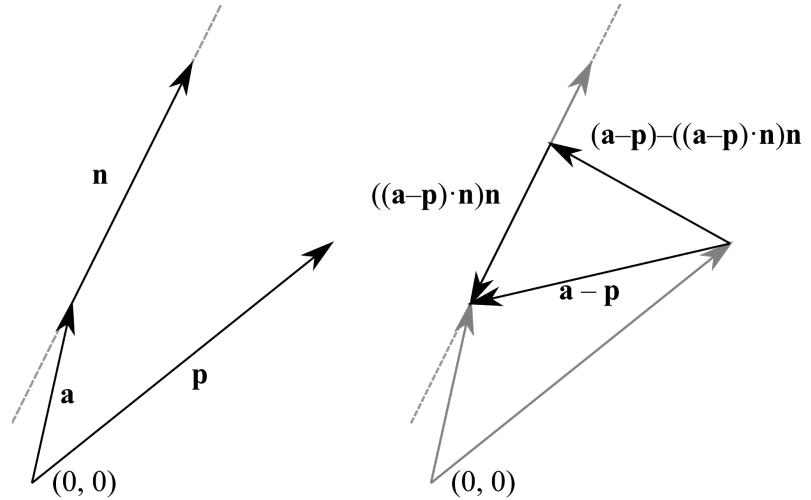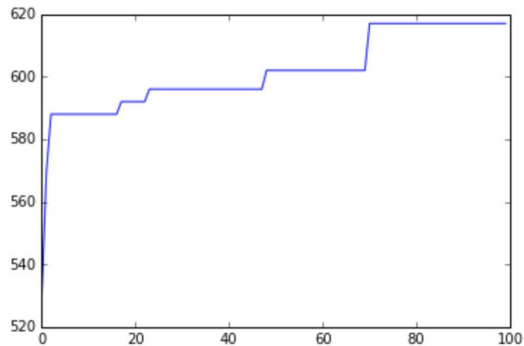
Here's a reference for that.

image source: Wikipedia

[3b] Example Results

My parameters:

Distance threshold	100
Number of iterations	100
Random seed	15

My bestInliersNumList plot:

Estimated focus of expansion is (y, x) = (137, 107)
Flow vector at that location is (u, v) = (0.174968, 0.094821)