

CSE 152 Section 7

## **HW3: Photometric Stereo and Optical Flow**

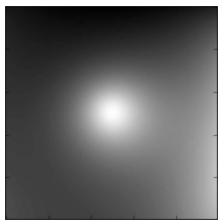
May 20, 2019

Owen Jow

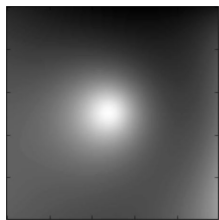
# [1a] Lambertian Photometric Stereo

## Input:

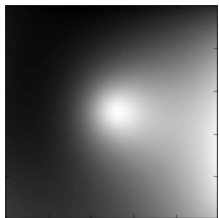
images and associated lighting information



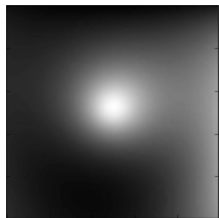
light source direction:  $[0, 0, 1]^T$



light source direction:  $[0.2, 0, 1]^T$



light source direction:  $[-0.2, 0, 1]^T$



light source direction:  $[0, 0.2, 1]^T$

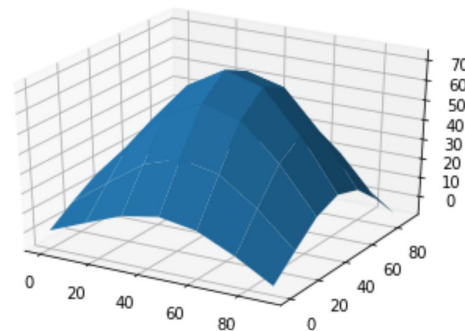


**Photometric  
Stereo**



## Output:

normals, albedo, depth



# [1a] Assumptions

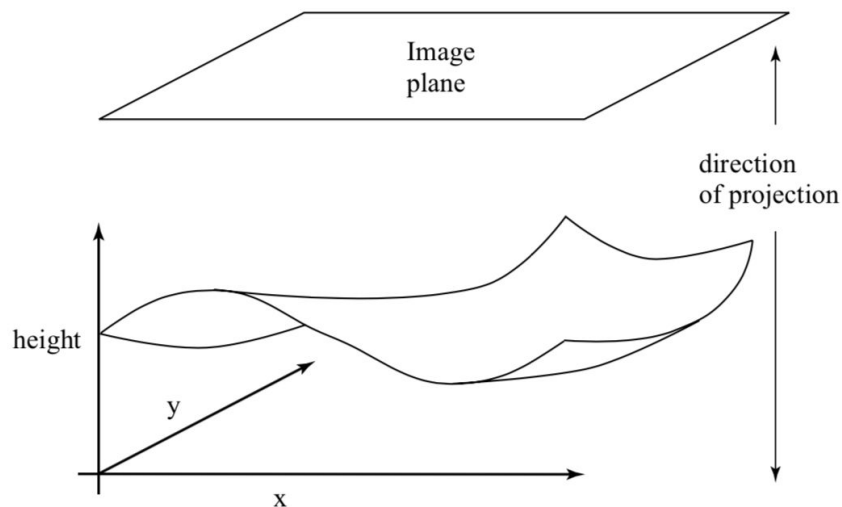


image source: Forsyth and Ponce

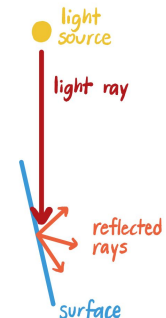
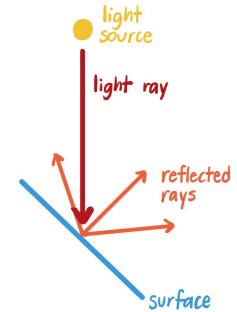
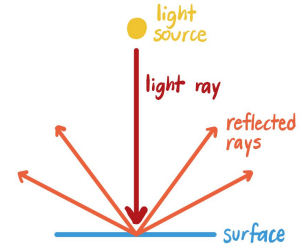
- **Orthographic camera model:**  
 $(x, y, z)$  projects to  $(x, y) \rightarrow$  our goal will be to recover the height/depth map  $z = f(x, y)$
- **Distant lighting:**  
treat every pixel in one image as sharing the same lighting direction/intensity
- **Static scene/viewpoint:**  
 $(x, y)$  in one image corresponds to  $(x, y)$  in all of the other images

# [1a] Assumptions, cont. (Lambertian)

- **Lambertian surface:**  
assume that the surface being imaged is Lambertian,  
i.e. at any point on the surface, there is equal reflectance in all directions

$$\begin{aligned}e(x, y) &= [a(x, y) \mathbf{n}(x, y)] \cdot \mathbf{s} \\ &= \mathbf{b}(x, y) \cdot \mathbf{s}\end{aligned}$$

$e(x, y)$	intensity at pixel $(x, y)$	(known)
$a(x, y)$	albedo at the point on the surface corresponding to $(x, y)$	<b>(unknown)</b>
$\mathbf{n}(x, y)$	unit normal at the point on the surface corresponding to $(x, y)$	<b>(unknown)</b>
$\mathbf{s}$	unit direction to the light, <u>scaled</u> by the intensity of the light	(known)



# [1a] Solving for $\mathbf{b}$ at Each Pixel

$\mathbf{b}$  is a 3-vector, so we need at least three equations/images. Let's say we have  $n$ .

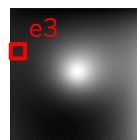
$$\begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1 \cdot \mathbf{b} \\ \vdots \\ \mathbf{s}_n \cdot \mathbf{b} \end{bmatrix}$$
$$\begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} \text{---} & \mathbf{s}_1 & \text{---} \\ & \vdots & \\ \text{---} & \mathbf{s}_n & \text{---} \end{bmatrix} \mathbf{b}$$
$$\underbrace{\mathbf{e}}_{n \times 1} = \underbrace{\mathbf{S}}_{n \times 3} \underbrace{\mathbf{b}}_{3 \times 1}$$



$$\mathbf{s}_1 = [0, 0, 1]^T$$



$$\mathbf{s}_2 = [0.2, 0, 1]^T$$



$$\mathbf{s}_3 = [0, 0.2, 1]^T$$

linear least squares  
(see lecture)

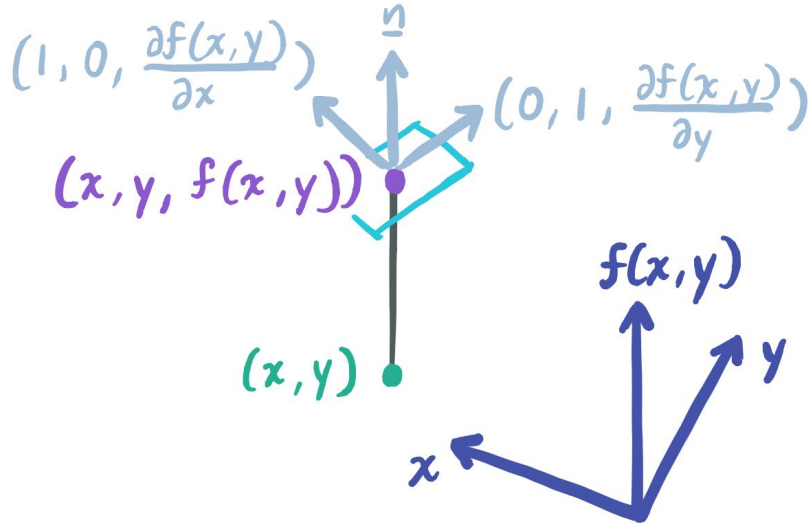
$$\mathbf{b} = (\mathbf{S}^T \mathbf{S})^{-1} \mathbf{S}^T \mathbf{e}$$

$$\text{albedo } a = \|\mathbf{b}\|$$

$$\text{normal } \mathbf{n} = \frac{\mathbf{b}}{\|\mathbf{b}\|}$$

# [1a] n Encodes the Partial Derivatives of Depth

Note: this uses a left-handed coordinate system, whereas lecture uses a right-handed one.



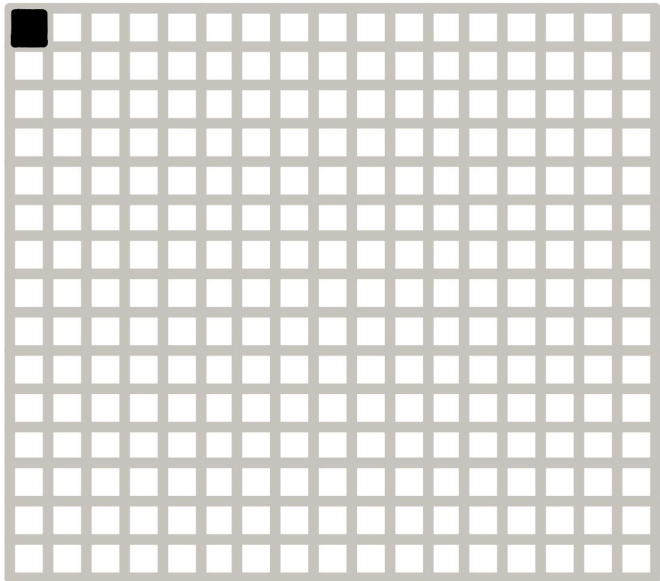
$$\mathbf{n} = \text{normalized} \left( \begin{bmatrix} 0 \\ 1 \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ \frac{\partial f(x,y)}{\partial x} \end{bmatrix} \right)$$
$$= \frac{1}{\sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2 + 1}} \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \\ -1 \end{bmatrix}$$

$$\begin{aligned} \frac{\partial f(x,y)}{\partial x} &= -\frac{n_1}{n_3} \\ \frac{\partial f(x,y)}{\partial y} &= -\frac{n_2}{n_3} \end{aligned}$$

## [1a] Simple Scanline Integration 1

Once we have the partial derivatives, we can integrate to get depth.

0

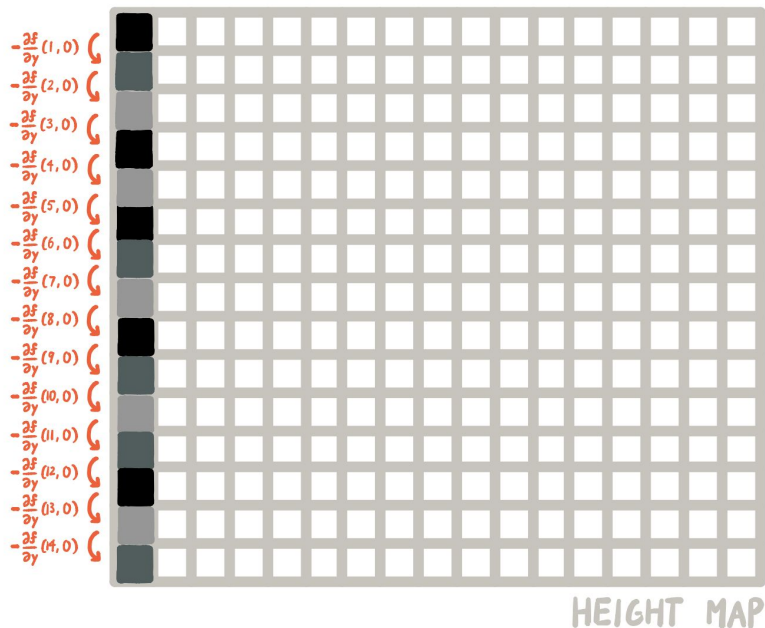


HEIGHT MAP

Initialize the top-left corner of the height map to 0

# [1a] Simple Scanline Integration 2

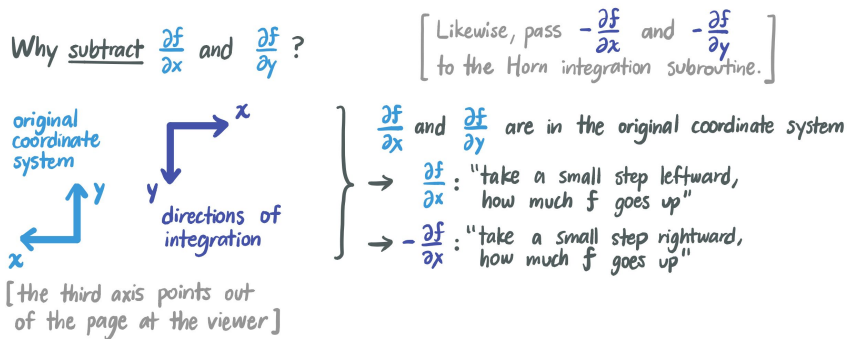
Once we have the partial derivatives, we can integrate to get depth.



Initialize the top-left corner of the height map to 0

For each pixel in the leftmost column [except (0, 0)]:

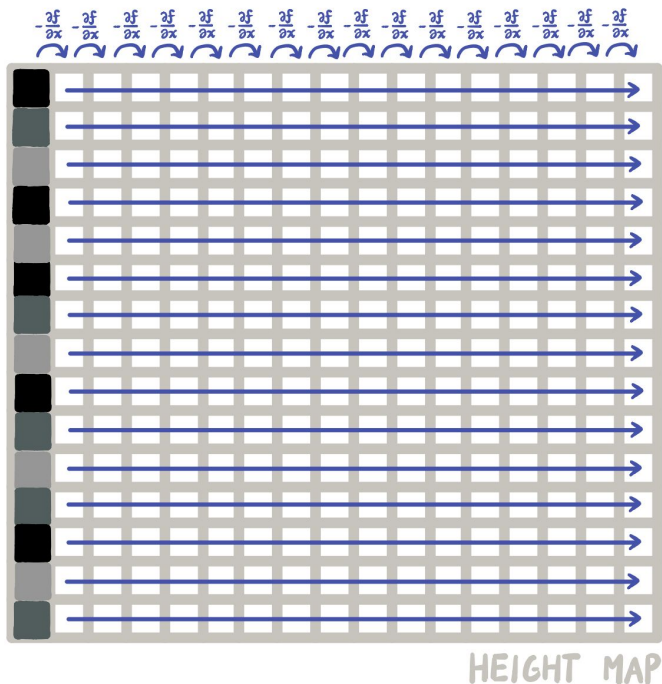
height = height of above pixel -  $\partial f / \partial y$





# [1a] Simple Scanline Integration 3

Once we have the partial derivatives, we can integrate to get depth.



Initialize the top-left corner of the height map to 0  
For each pixel in the leftmost column [except (0, 0)]:  
height = height of above pixel -  $\partial f / \partial y$   
**For each pixel (except the leftmost) in each row:**  
height = height of pixel to the left -  $\partial f / \partial x$

Why subtract  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  ? [ Likewise, pass  $-\frac{\partial f}{\partial x}$  and  $-\frac{\partial f}{\partial y}$  to the Horn integration subroutine. ]

original coordinate system

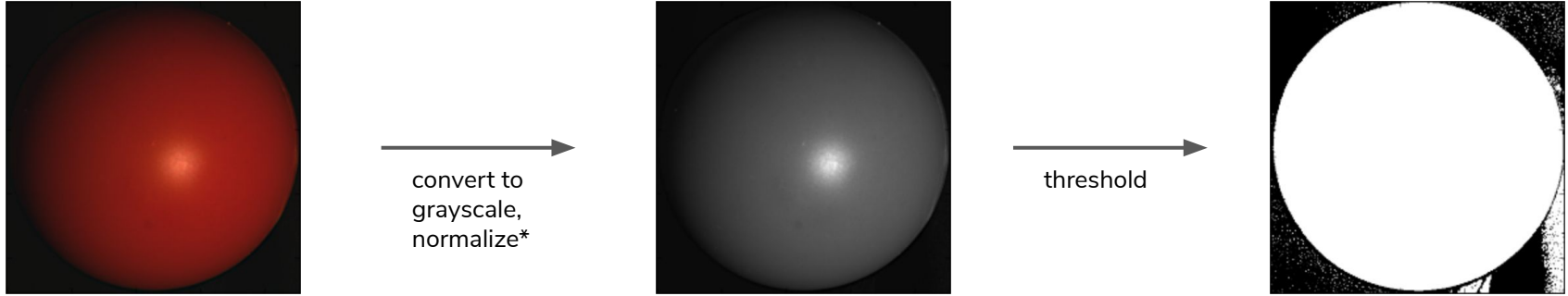
directions of integration

$\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  are in the original coordinate system

- $\frac{\partial f}{\partial x}$  : "take a small step leftward, how much f goes up"
- $-\frac{\partial f}{\partial x}$  : "take a small step rightward, how much f goes up"

[the third axis points out of the page at the viewer]

## [1a] The Mask Parameter



- 1s for locations to use during integration
- 0s for locations to ignore during integration
- Unnecessary for 1a, but can use to filter out the background in 1b

$$*(img - img.min()) / (img.max() - img.min())$$

## [1b] Applicability of the Lambertian Equation

The Lambertian equation does not apply for

- **shadowed regions**, where the view of the light is blocked
- **specularities**, which arise as a result of a different relationship

If we perform photometric stereo on such locations, we'll end up with noisy/pointy/bumpy artifacts.

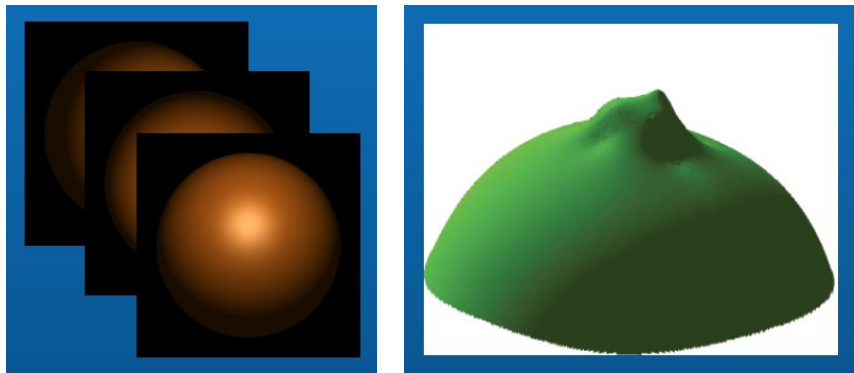


image source: Prof. Kriegman's lecture slides

## [1b] Simple Shadow/Specularity Removal

Identify shadowed/specular locations based on brightness. Then, either...

1. ...clamp each associated brightness to a threshold value.
2. ...set each associated brightness to the median of a sizable surrounding window.

Do this before solving for **b**. You should be able to mitigate the artifact(s) to some degree.

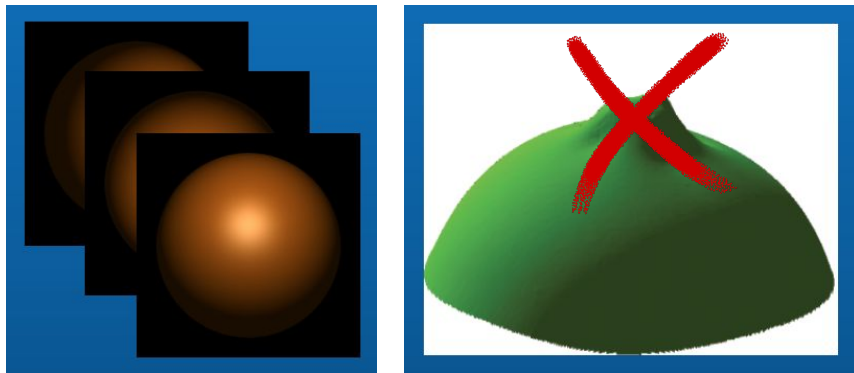


image source: Prof. Kriegman's lecture slides

## [2a] Optical Flow 1

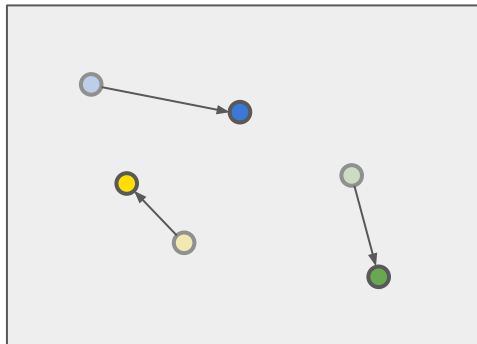
Estimate the apparent motion of each pixel from frame A to frame B.



frame A

## [2a] Optical Flow 2

Estimate the apparent motion of each pixel from frame A to frame B.



frame B + optical flow

## [2a] The Brightness Constancy Equation

**Assumption 1:** the brightness/color of each pixel remains constant as it moves.

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t)$$

**Assumption 2:** pixels don't move too far between frames. Linearizing via Taylor expansion:

$$I(x, y, t) + \Delta x \frac{\partial I}{\partial x} + \Delta y \frac{\partial I}{\partial y} + \Delta t \frac{\partial I}{\partial t} = I(x, y, t)$$

$$\Delta x \frac{\partial I}{\partial x} + \Delta y \frac{\partial I}{\partial y} + \Delta t \frac{\partial I}{\partial t} = 0$$

$$\frac{\Delta x}{\Delta t} \frac{\partial I}{\partial x} + \frac{\Delta y}{\Delta t} \frac{\partial I}{\partial y} + \frac{\partial I}{\partial t} = 0$$

$$u I_x + v I_y + I_t = 0$$

**Goal: solve for  $u, v$  at every pixel.**  
Can compute  $I_x, I_y, I_t$  from the images.

## [2a] The Lucas-Kanade Method

Problem: one equation, two unknowns.

**Assumption 3:** flow is constant in the neighborhood around each pixel.

→ Get one equation for every point in a window around each pixel.

$$\underbrace{\begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}}_{n \times 2} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{2 \times 1} = \underbrace{\begin{bmatrix} -(I_t)_1 \\ \vdots \\ -(I_t)_n \end{bmatrix}}_{n \times 1}$$

$\xrightarrow{\text{linear least squares}}$

$$\begin{aligned}
 &\underbrace{\begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}^T \begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}}_{2 \times 2} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{2 \times 1} = \underbrace{\begin{bmatrix} (I_x)_1 & (I_y)_1 \\ \vdots & \vdots \\ (I_x)_n & (I_y)_n \end{bmatrix}^T}_{2 \times 1} \underbrace{\begin{bmatrix} -(I_t)_1 \\ \vdots \\ -(I_t)_n \end{bmatrix}}_{n \times 1} \\
 &\underbrace{\left( \sum_{x,y \in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)}_{2 \times 2} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{2 \times 1} = \underbrace{\sum_{x,y \in W} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix}}_{2 \times 1}
 \end{aligned}$$

the second moment  
matrix strikes again



## [2a] Notes

- Use the pseudoinverse to solve for  $[\mathbf{u}, \mathbf{v}]^T$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \left( \sum_{x,y \in W} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)^\dagger \left( \sum_{x,y \in W} \begin{bmatrix} -I_x I_t \\ -I_y I_t \end{bmatrix} \right)$$

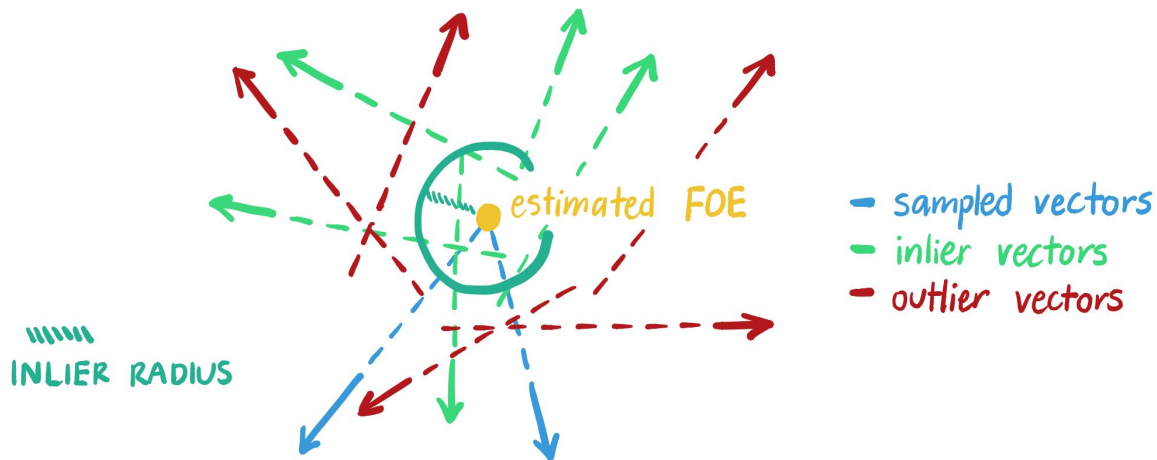
$\dagger$  : pseudoinverse

- Run this for every pixel (loops are fine)
- Make sure you compute the y-gradient with respect to an upward axis
  - If you use `np.gradient` to compute the image gradients, negate the y-gradient you get back

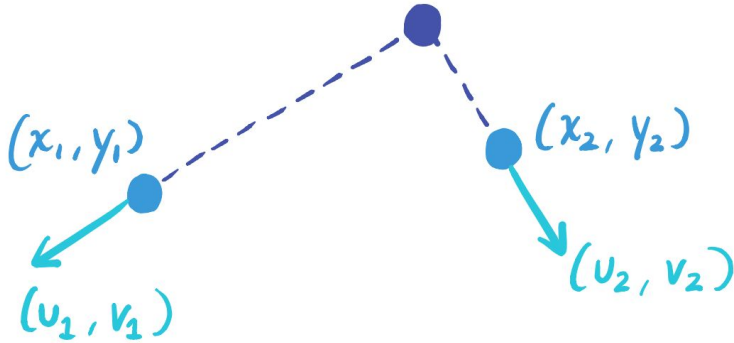
## [3b] RANSAC for Estimating the Focus of Expansion

Idea: repeatedly

- Sample two flow vectors
- Estimate the focus of expansion as their intersection
- Check the consistency of the estimate across all flow vectors



## [3b] Ray-Ray Intersection



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + t_1 \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + t_2 \begin{bmatrix} u_2 \\ v_2 \end{bmatrix}$$

- Solve for  $t_1$  and  $t_2$ . Derivation of the exact solution is left as an exercise. :)
- Note that there are other ways to compute intersections. Use whichever method you like.

## [3b] Distance from a Point to a Ray

When checking consistency, you'll need to compute the perpendicular distance from your estimated focus of expansion to each of the rays (or to some subset of them).

[Here's a reference for that.](#)

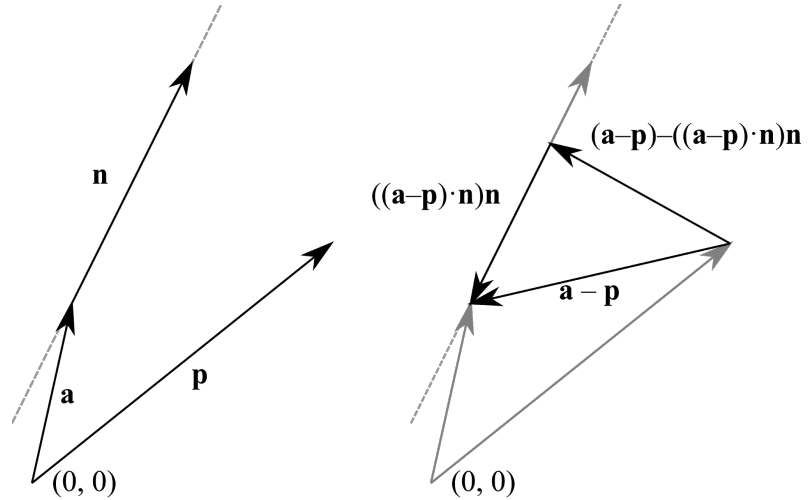


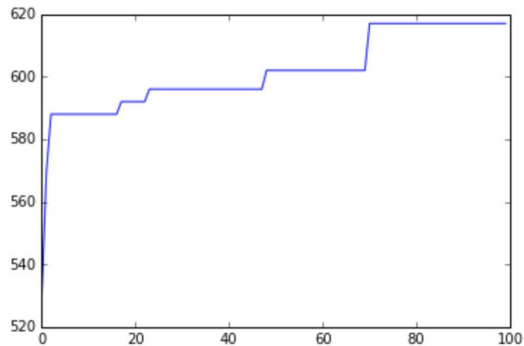
image source: [Wikipedia](#)

## [3b] Example Results

My parameters:

Distance threshold	100
Number of iterations	100
Random seed	15

My bestInliersNumList plot:



Estimated focus of expansion is  $(y, x) = (137, 107)$   
Flow vector at that location is  $(u, v) = (0.174968, 0.094821)$

No need to have exactly these results, just try to get something in the same ballpark