Taylor Series Expansion

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A Taylor series is a representation of a function as a weighted sum of itself [1] and its derivatives evaluated at a point. The function must be analytic over the relevant domain.[2] Note that if we only care about the neighborhood around the point, we can approximate the function by dropping higher-order terms. Often we will do this to obtain linear or quadratic approximations. The greater the number of terms and/or the smaller the neighborhood, the more accurate the approximation will be.

1 Single-Variable Scalar-Valued Functions

$$f(x + \Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2!}f''(x)\Delta x^2 + \frac{1}{3!}f'''(x)\Delta x^3 + \dots$$

2 Multi-Variable Scalar-Valued Functions

$$\begin{aligned} f(\mathbf{x} + \Delta\mathbf{x}) &= f(\mathbf{x}) + \mathbf{J}(\mathbf{x})\Delta\mathbf{x} + \frac{1}{2!}\Delta\mathbf{x}^T\mathbf{H}(\mathbf{x})\Delta\mathbf{x} + \dots \\ &= f(\mathbf{x}) + \begin{bmatrix} \frac{\partial f(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f(\mathbf{x})}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} + \frac{1}{2!} \begin{bmatrix} \Delta x_1 & \cdots & \Delta x_n \end{bmatrix} \begin{bmatrix} \frac{\partial^2 f(\mathbf{x})}{\partial x_1^2} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f(\mathbf{x})}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f(\mathbf{x})}{\partial x_n^2} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \vdots \\ \Delta x_n \end{bmatrix} + \dots \end{aligned}$$

References

[1] Ruye Wang, *Taylor series expansion*. http://fourier.eng.hmc.edu/e176/lectures/NM/node45.html.

[1] technically the function is its own zeroth derivative

[2] as far as I'm concerned, this just means smooth ("infinitely differentiable") over the domain