CSE 152 Section 7

HW3: Epipolar Geometry

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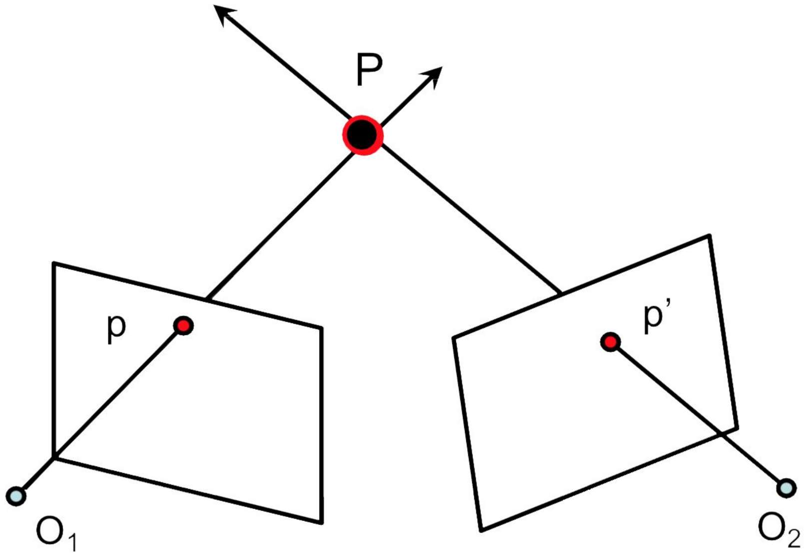
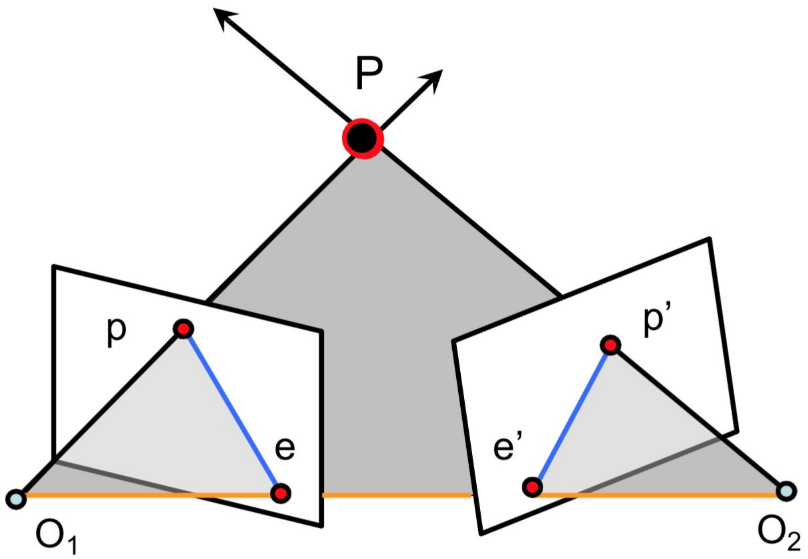
Epipolar Geometry



P	arbitrary 3D point
p	projection of **P** onto image 1
p'	projection of **P** onto image 2
O_1	pinhole (center of projection) of camera 1
O_2	pinhole (center of projection) of camera 2

image source: Hata, Savarese

Epipolar Geometry



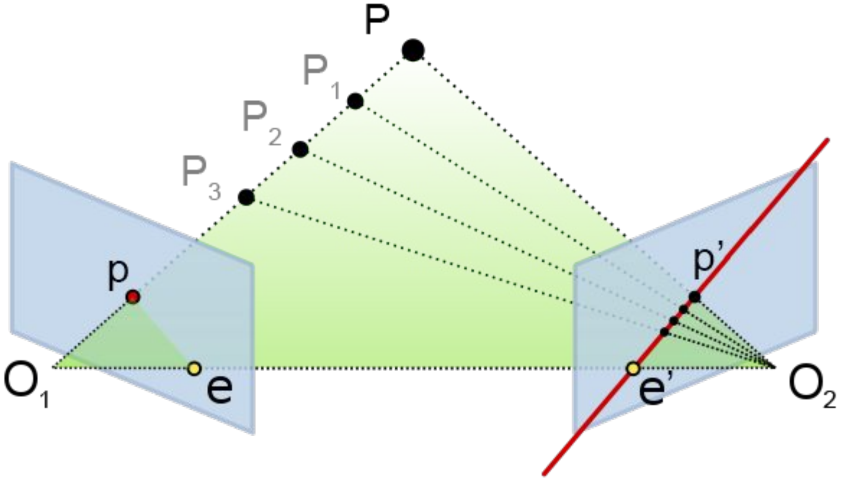
P arbitrary 3D point
p projection of **P** onto image 1
p' projection of **P** onto image 2
O_1 pinhole (center of projection) of camera 1
O_2 pinhole (center of projection) of camera 2

e epipole 1 (projection of **O_2** onto image 1)
e' epipole 2 (projection of **O_1** onto image 2)

gray plane epipolar plane (defined by **P**, **O_1**, **O_2**)
orange line baseline (defined by **O_1**, **O_2**)
blue line epipolar line (intersection of epipolar plane with image)

image source: Hata, Savarese

Epipolar Geometry



Epipolar constraint:
the point **p'** which corresponds to **p**
must lie on the epipolar line for image 2

an alternative interpretation of this epipolar line:
the projection of the line O_1 - P onto image 2

image source (modified): Arne Nordmann (norro) - Own work (Own drawing),

3D → 2D

Recall from the calibration lecture:

p = M P = K [R t] P

for **K** the 3x3 intrinsic (camera projection) matrix, **[R t]** the 3x4 extrinsic (camera pose) matrix

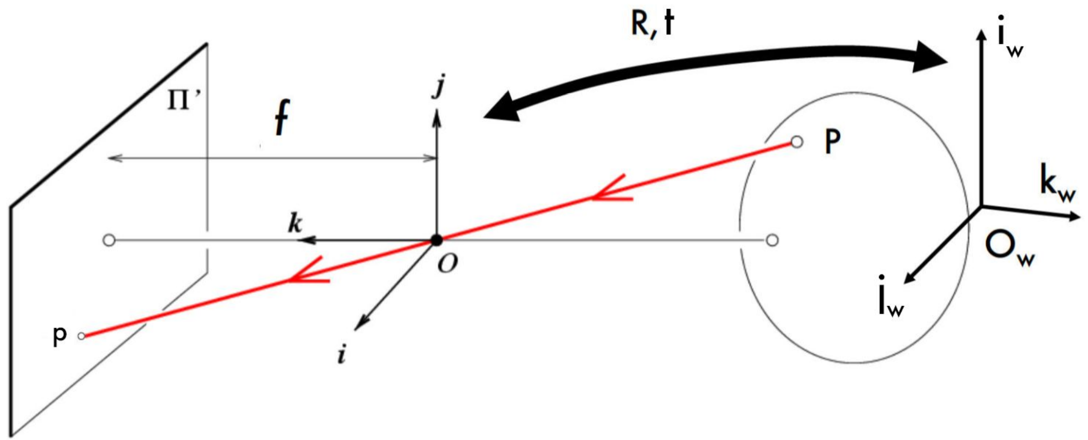


image source: Savarese

3D → 2D

Let

$$\mathbf{M} \text{ (mapping for camera 1)} = \mathbf{K} \; [\mathbf{I} \;\; \mathbf{0}]$$

$$\mathbf{M'} \text{ (mapping for camera 2)} = \mathbf{K'} \; [\mathbf{R} \;\; \mathbf{t}]$$

i.e.

- world coordinates = camera 1 coordinates
- the transformation from camera 1 to camera 2 is **R, t**
- **p'** in camera 1 coordinates is $\mathbf{R^T[(K')^{-1}p' - t]}$

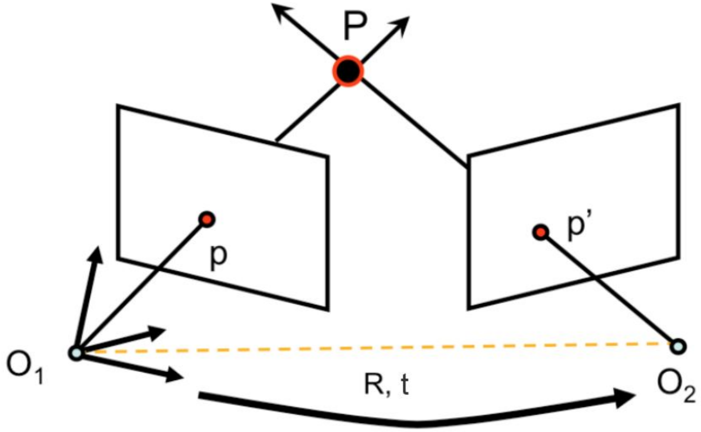


image source: Hata, Savarese

Fundamental Matrix

Then the fundamental matrix is

$$\mathbf{F} = (\mathbf{K'})^{-T}\, \mathbf{T}_\times\, \mathbf{R}\, \mathbf{K}^{-1}$$

where $\mathbf{T}_\times$ is a skew-symmetric matrix corresponding to the translation vector **t**: $\begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$

- 3x3, rank 2, seven degrees of freedom
- gives epipolar line in image 1 as $\mathbf{F}^T\mathbf{p'}$ (i.e. $\{\alpha \mathbf{F}^T\mathbf{p'} : \alpha$ is a scalar$\}$)
- gives epipolar line in image 2 as $\mathbf{Fp}$ (i.e. $\{\alpha \mathbf{Fp} : \alpha$ is a scalar$\}$)
- $\mathbf{F}^T\mathbf{e'} = 0$, $\mathbf{Fe} = 0$
- relates corresponding points **p, p'** according to $(\mathbf{p'})^T\mathbf{Fp} = 0$ (epipolar constraint)
  - note that **p** is in homogeneous image 1 coords, **p'** is in homogeneous image 2 coords

Eight-Point Algorithm

We can estimate the fundamental matrix using the **eight-point algorithm.**

Input: 8+ pairs of corresponding points $\mathbf{p_i} = (\mathbf{u_i}, \mathbf{v_i}, \mathbf{1})$, $\mathbf{p_i}' = (\mathbf{u_i}', \mathbf{v_i}', \mathbf{1})$
Output: fundamental matrix **F**

each correspondence is 1 equation $(\mathbf{p_i}')^T \mathbf{F} \mathbf{p_i} = \mathbf{0}$

$$\begin{bmatrix} u_i u_i' & v_i u_i' & u_i' & u_i v_i' & v_i v_i' & v_i' & u_i & v_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Eight-Point Algorithm

We use 8+ equations to solve for the 8 independent entries in **F** (the ninth is a scaling factor).

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ u_3u_3' & v_3u_3' & u_3' & u_3v_3' & v_3v_3' & v_3' & u_3 & v_3 & 1 \\ u_4u_4' & v_4u_4' & u_4' & u_4v_4' & v_4v_4' & v_4' & u_4 & v_4 & 1 \\ u_5u_5' & v_5u_5' & u_5' & u_5v_5' & v_5v_5' & v_5' & u_5 & v_5 & 1 \\ u_6u_6' & v_6u_6' & u_6' & u_6v_6' & v_6v_6' & v_6' & u_6 & v_6 & 1 \\ u_7u_7' & v_7u_7' & u_7' & u_7v_7' & v_7v_7' & v_7' & u_7 & v_7 & 1 \\ u_8u_8' & v_8u_8' & u_8' & u_8v_8' & v_8v_8' & v_8' & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

$$W\mathbf{f} = 0$$

Eight-Point Algorithm

Approach: find a least-squares solution to this system of equations. **Can use SVD for this!**
Might also want to normalize each $\mathbf{p}_i$ and $\mathbf{p}_i'$ for better results (must de-normalize resulting **F** as well!).

1. Normalize points in each image according to **T** and **T'**, use normalized points to construct **W**.
2. Compute the SVD of **W**, reshape right singular vector into initial estimate of **F**.
    a. As reference, see sections 3 and 4 of this document.
3. Enforce rank = 2 by taking another SVD, this time of **F**, and zeroing out the last singular value.

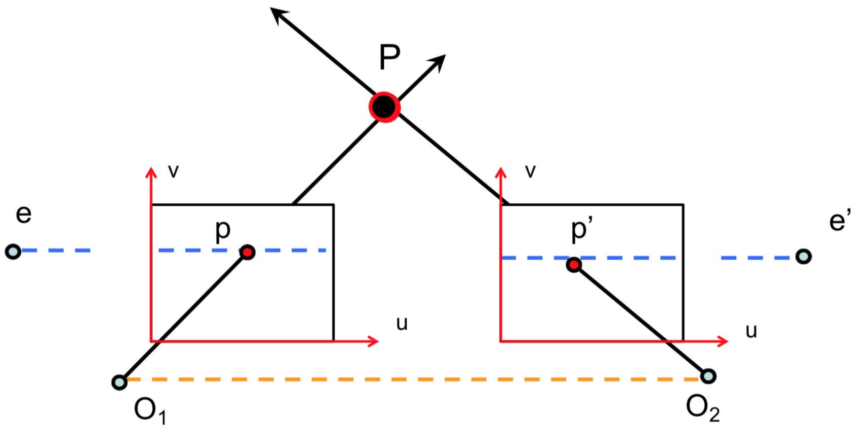
$$F = U \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$

4. De-normalize **F**.
    a. Currently, $\mathbf{(T'p')^T F(Tp) = 0} \rightarrow \mathbf{(p')^T (T')^T FTp = 0}$, so $\mathbf{(T')^T FT}$ is the true fundamental matrix.

Recap :)

courtesy Daniel Wedge

Question 1.1



Notes:

- no rotation, only translation
- can assume intrinsic matrices are the same

Suggestions:

- **try to compute the essential matrix**
  check: does this work?
  - what is the rotation matrix?
  - what is the translation vector?
- reason geometrically

image source: Hata, Savarese

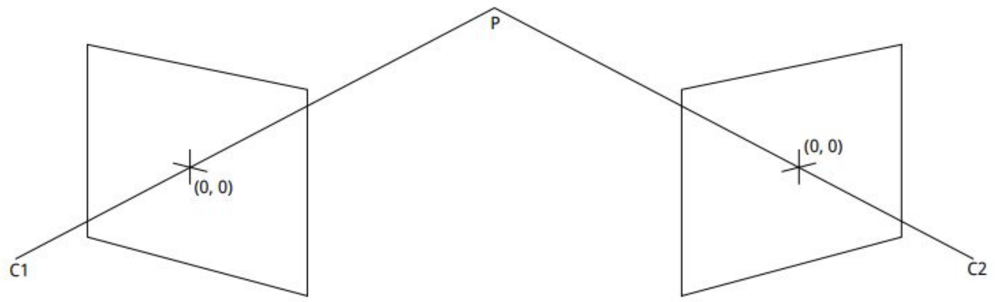
Question 1.2

Notes:

- x

Suggestions:

- x



Question 2

1. **Eight-point algorithm**
   Estimate the fundamental matrix given point correspondences.

2. **Metric reconstruction**
   Estimate the camera matrices, triangulate and visualize the 3D points.

3. **3D correspondence**
   Estimate corresponding points given the fundamental matrix.

2.1. Eight-Point Algorithm

2.2. Metric Reconstruction

2.3. 3D Correspondence

Additional Readings

- CS 231A course notes
- How to use SVD to solve homogeneous linear least-squares