

CSE 152 Section 2

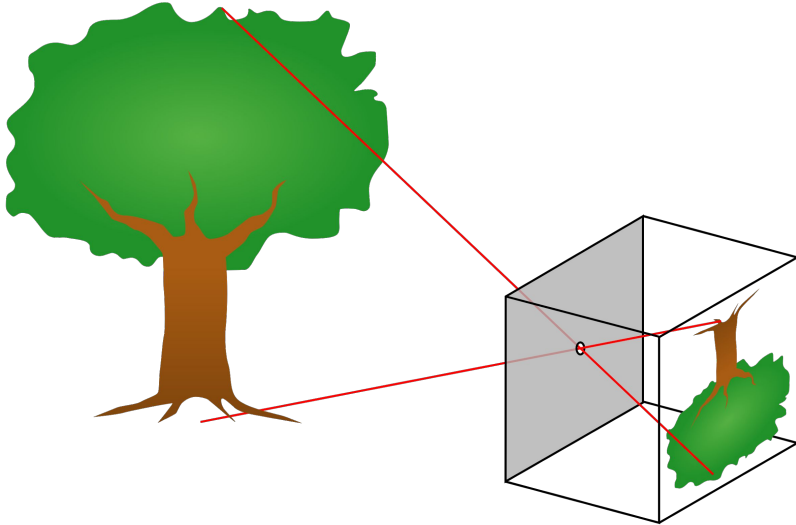
Recap: Geometric Image Formation

April 08, 2019

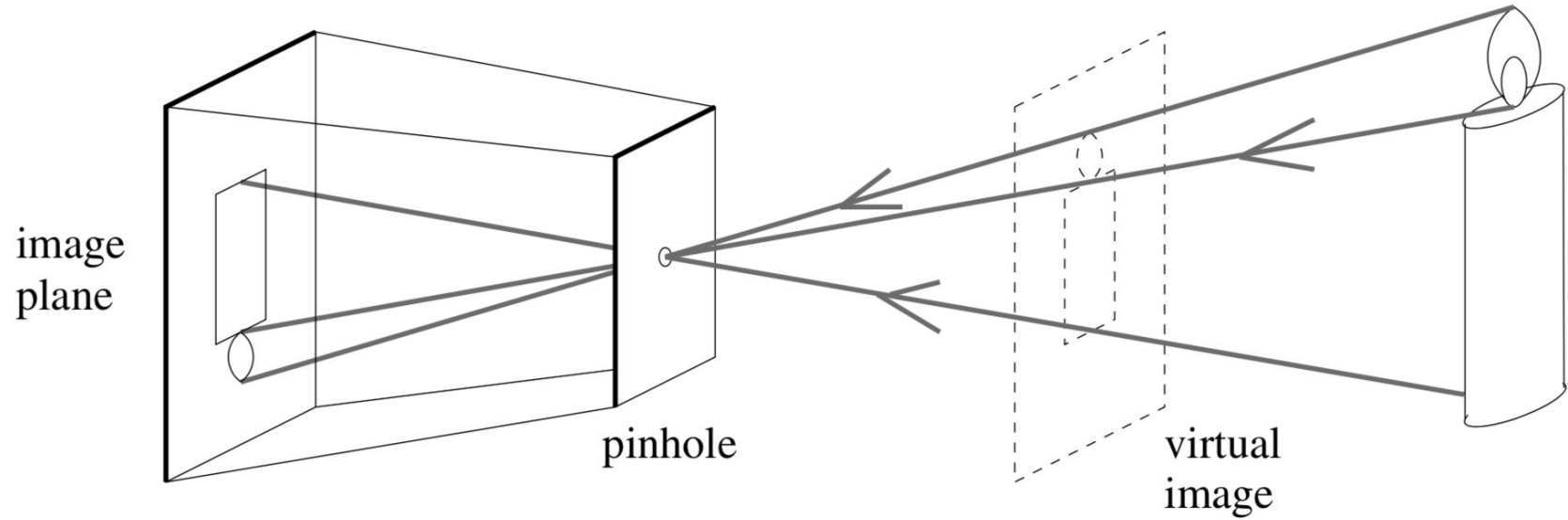
Owen Jow

Pinhole Perspective Projection

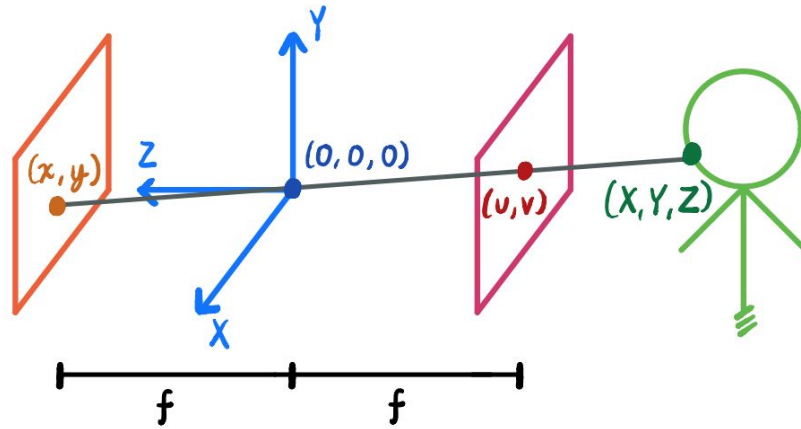
Light ray bounces off something in world, passes through pinhole, gets recorded on back wall of camera.



Virtual Image Plane



Relevant Equations

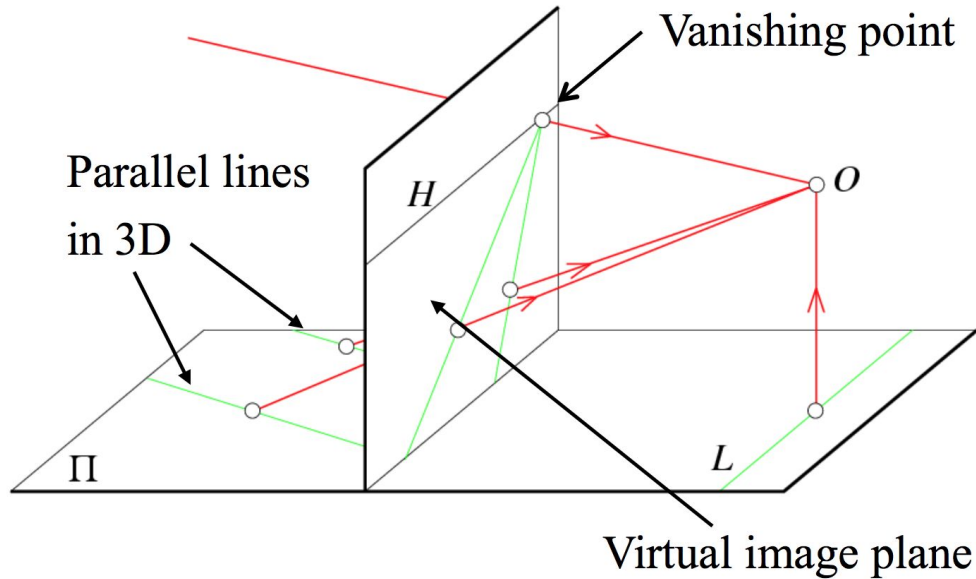


$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} fX/Z \\ fY/Z \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -fX/Z \\ -fY/Z \end{bmatrix}$$

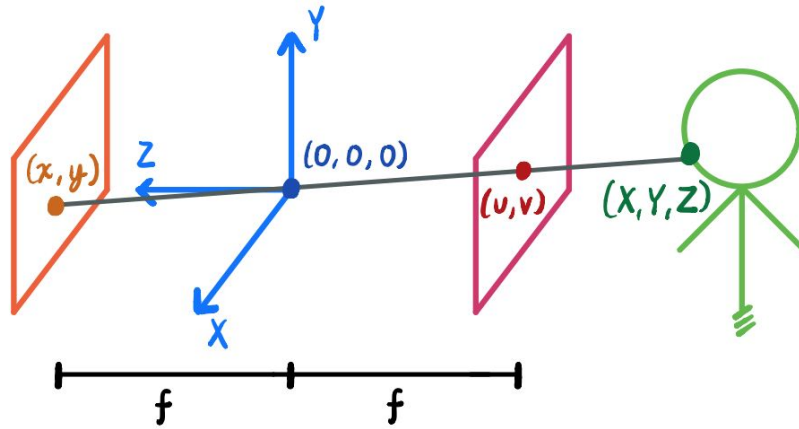
Vanishing Points

- Projected **point at infinity** (point to which line converges in projective space)



Homogeneous Coordinates

- Make translation and perspective projection “linear”



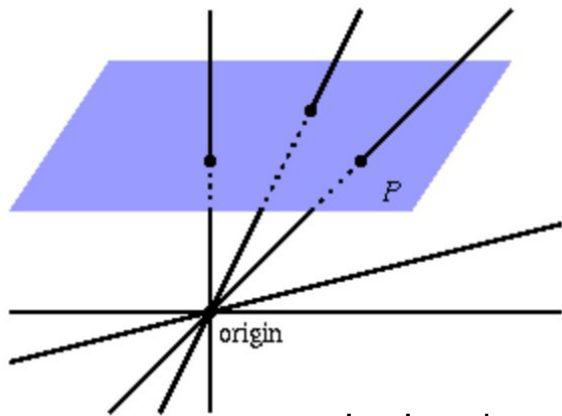
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} -fX \\ -fY \\ Z \end{bmatrix}$$

Projective Geometry

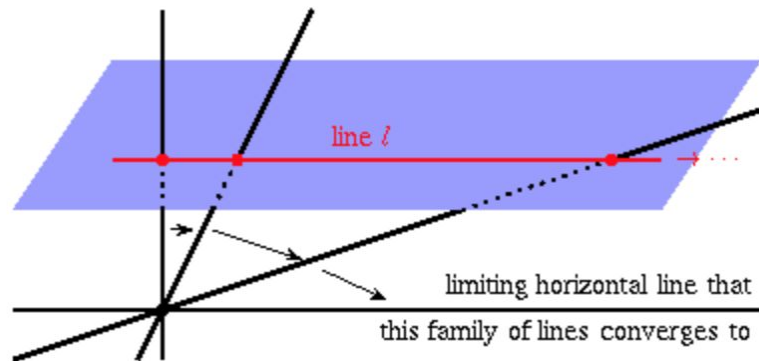
- Geometry in **projective space**

- Euclidean space + points at infinity
- Allows for transformations between Euclidean points and points at infinity



projective plane

Euclidean plane + points at infinity



projective line

Euclidean line + point at infinity

3D Rotation

- 3D rotation matrix $\in SO(3)$
 - 3x3 orthogonal matrix \rightarrow transpose is inverse
 - If you consider points as fixed and frame as changing, rows are new axes in current frame

$${}^B_A R = \begin{bmatrix} \text{---} & {}^A i_B & \text{---} \\ \text{---} & {}^A j_B & \text{---} \\ \text{---} & {}^A k_B & \text{---} \end{bmatrix}$$

${}^B_A R$: rotation matrix, transforms points in frame A to points in frame B

${}^A i_B, {}^A j_B, {}^A k_B$: frame B axes in coordinate system A

Rigid Transformation

- Rotation, then translation
- Preserves distances between pairs of points

$${}^B P = {}^B_A R {}^A P + {}^B O_A$$

frame A coordinates \rightarrow frame B coordinates

${}^A P, {}^B P$: point in frame A, point in frame B

${}^B_A R$: rotation matrix (frame A coords \rightarrow frame B coords)

${}^B O_A$: frame A origin in frame B coordinates



Full Camera Projection Process

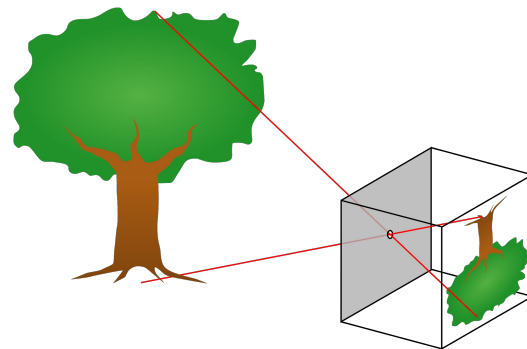
$$\underbrace{\mathbf{p}}_{\text{image point}} = \underbrace{\begin{bmatrix} f_x & s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsic matrix}} \underbrace{\left[\begin{array}{ccc|c} - & - & - & - \\ - & \mathbf{R} & - & \mathbf{t} \\ - & - & - & - \end{array} \right]}_{\text{extrinsic matrix}} \underbrace{\mathbf{P}}_{\text{world point}}$$

homogeneous 3x1 point in image coords inhomogeneous* 3x1 point in camera coords homogeneous 4x1 point in world coords

- **Extrinsic matrix:** world coords (rigid transf.) → camera coords
- **Intrinsic matrix:** camera coords (projection) → image coords

Section Takeaways

- How to:
 - perform perspective projection
 - compute locations of vanishing points
- Intuition about:
 - homogeneous coordinates and projective geometry
 - 3D rotations and rigid transformations
 - full camera projection process



Q & A