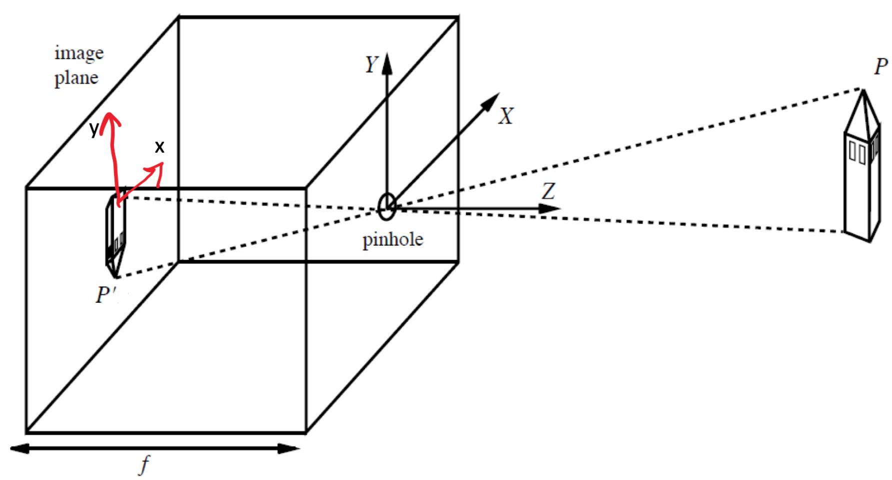
Geometric Image Formation 2

CSE 152: Intro to Computer Vision

April 08, 2019

1 Perspective Projection



1. Let $\mathbf{P} = (X, Y, Z)$ be a point in the camera frame shown above, and let $\mathbf{P}' = (x, y)$ be its perspective projection in the real image plane. Using similar triangles, derive the associated perspective projection equation(s), i.e. the equation(s) for $\mathbf{P}'$ in terms of $\mathbf{P}$.

Solution:

$$x = -\frac{fX}{Z},\ y = -\frac{fY}{Z}$$

2. If the Z-axis were pointing in the opposite direction, what would the perspective projection equations become? (Assume that the X- and Y-axes remain unchanged.)

Solution:

$$x = \frac{fX}{Z},\ y = \frac{fY}{Z}$$

3. If we place a virtual image plane in front of the camera (i.e. in the world) at a distance f' along the Z-axis, what are the projection equations for a point $\mathbf{Q}' = (x_v, y_v)$ on that virtual plane? Use the original coordinate system (the one depicted in the diagram).

Solution:

$$x_v = \frac{fX}{Z},\ y_v = \frac{fY}{Z}$$

2 Vanishing Points

1. We can express a line in 3D as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} + \lambda \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix}$$

where O is a point on the line and D is the direction of the line.

As we've learned, perspective projection can take 3D points at infinity (which are at the "ends" of 3D lines) to finite 2D **vanishing points**. What is the vanishing point (x, y) associated with the form of the line given above? *Hint: compute the perspective projection of the line and take the limit as λ goes to infinity.*

Solution:

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \lim_{\lambda \to \infty} \begin{bmatrix} -\frac{f(O_x + \lambda D_x)}{O_z + \lambda D_z} \\ -\frac{f(O_y + \lambda D_y)}{O_z + \lambda D_z} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{fD_x}{D_z} \\ -\frac{fD_y}{D_z} \end{bmatrix} \text{ (L'Hopital's)} \end{aligned}$$

2. Based on your answer to the previous question, how can you tell if two lines have the same vanishing point if you are only given the O and D vectors for each line?

Solution: If the directions of the lines are the same (up to scale), the lines have the same vanishing point. If their directions aren't the same, they don't.