



CS 61A

DISCUSSION NINE

November 10, 2016



TOPICS FOR TODAY

Iterators, generators, streams (, ...turtle graphics demo !?)

Overview

- ▶ Iterators are objects that sweep over a collection of items (in a specific order / step size). This happens via repeated application of the `next` method, which is necessarily defined on all iterators.
- ▶ Streams are lazily evaluated linked lists. Elements in the list aren't computed until you specifically ask for them, which means you can represent infinite sequences as streams.

ANNOUNCEMENTS

- ▶ No in-person lab on Thanksgiving week (instead, we'll have a “long, optional” one to be completed on your own)

ITERATORS

ITERATION (REVIEW)

Iterators step through a collection, item by item, via `next`

Iterables “are” the collection, and provide **iterators** via `iter`

If you have an **iterable**, you can get an **iterator** over it by calling `iter`. Then you can observe all of its elements by repeatedly calling `next` on the **iterator**.

Be warned: iterators are single-use only! (Once an iterator has gone through all the elements, it’s done; calling `next` on it will give `StopIteration` errors forever.) To get a fresh iterator, one would have to call `iter` on the iterable again.

- Note that depending on the implementation, the second iterator might be finished anyway (e.g. if every iterator modifies the same state variables)

ITERATION (REVIEW)

`iter(iterable) → iterator`

`next(iterator) → value, or a StopIteration error`

ITERATION (REVIEW)

Lots of **built-in** functions take or produce iterators! Be aware of these.

- ▶ `map(function, iterable)`
 - ▷ Returns an **iterator** over mapped elements in the iterable
- ▶ `filter(function, iterable)`
 - ▷ Returns an **iterator** over filtered elements from the iterable
- ▶ `zip(*iterables)`
 - ▷ Returns an **iterator** over aggregations of elements from each of the iterables

ITERATION (REVIEW)

To create an iterable, you could write a class that implements `__iter__` as a generator function.

```
class Primes:
    def __init__(self, n):
        self.n = n # upper limit
    def __iter__(self):
        P = [True for i in range(2, self.n + 1)]
        for i in range(2, self.n + 1):
            if not P[i - 2]: continue
            yield i
            if i > sqrt(self.n): continue
            for j in [i*i + k*i for k in range((self.n - i*i)//i + 1)]:
                P[j - 2] = False
```

```
list(Primes(30)) → [2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
```


ITERATION (REVIEW)

Behind the scenes, for-loops really just create iterators using `iter` and then call `next` a bunch of times on the iterator.

```
for x in <expr>:  
    <do stuff>  
– is equivalent to –  
iterator = iter(<expr>)  
try:  
    while True:  
        n = next(iterator)  
        <do stuff>  
except StopIteration:  
    pass
```

ITERATORS E1

```
>>> t = iter([0, 1, 2]) # t is now an iterator  
>>> u = iter(t) # does it error..?
```

What happens when you call `iter` on an iterator?

ITERATORS E1

```
>>> t = iter([0, 1, 2]) # t is now an iterator
>>> u = iter(t) # does it error..?
```

What happens when you call `iter` on an iterator?

You get the same iterator back. So... technically speaking, iterators also implement the iterable interface! (Unless they're user-defined iterators that didn't implement the `__iter__` method, but you don't need to worry about that this semester...)

```
>>> t == u
True
>>> t == iter(t)
True
```

ITERATORS E2

Let's implement the `list` function. Here we have the function specification:

```
def list(iterable):  
    """Creates a list.  
  
    >>> list(range(4))  
    [0, 1, 2, 3]  
    >>> list(iter(range(4)))  
    [0, 1, 2, 3]  
    """  
    # YOUR CODE HERE
```

ITERATORS E2

Let's implement the `list` function. Here we have the function specification:

```
def list(iterable):  
    iterator = iter(iterable)  
    result = []  
    try:  
        while True:  
            result.append(next(iterator))  
    except StopIteration:  
        return result
```

GENERATORS

GENERATORS (REVIEW)

Generator functions are functions containing `yield` statements.

- ▶ These functions return generators when called.

Generators are **iterators** obtained by calling a generator function.

- ▶ Every time you call `next` on a generator, it goes through the generator function body until it hits a `yield` – at which point it yields the specified value. The state of the function with respect to this generator is saved, so whenever `next` is called again on the generator, execution of the function body will continue from where it left off.

To create a fresh iterator, just call the generator function again.

GENERATORS (REVIEW)

Here we have a function that returns an iterator over the natural numbers:

```
def gen_naturals():  
    curr = 0  
    while True:  
        yield curr  
        curr += 1
```

```
>>> gen = gen_naturals()  
>>> gen  
<generator object gen at ...>  
>>> next(gen)  
0  
>>> next(gen)  
1
```


GENERATORS (REVIEW)

``yield from`` is like `yield`, except it yields **all values** from an **iterable**.

```
yield from <expr>
```

– *is equivalent to* –

```
for x in <expr>:  
    yield x
```

GENERATORS (REVIEW)

```
square = lambda x: x * x
def many_squares(s):
    for x in s:
        yield square(x)
    yield from [square(x) for x in s]
    yield from map(square, s)
```

```
>>> list(many_squares([1, 2, 3]))
[1, 4, 9, 1, 4, 9, 1, 4, 9]
```

GENERATORS E1

```
>>> def gen_y():  
...     yield (7, 8, 9)  
...  
>>> def gen_yf():  
...     yield from (7, 8, 9)  
...  
>>> next(gen_y())  
  
>>> next(gen_yf())
```

GENERATORS E1

```
>>> def gen_y():  
...     yield (7, 8, 9)  
...  
>>> def gen_yf():  
...     yield from (7, 8, 9)  
...  
>>> next(gen_y())  
(7, 8, 9)  
  
>>> next(gen_yf())  
7
```

GENERATORS E2

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup)
```

What would Python print?

GENERATORS E2

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup)
```

What would Python print?

Trick question, `countup` is just a generator function (NOT a generator; NOT an iterable)

(`TypeError: 'function' object is not iterable`)

GENERATORS E3

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup())
```

What would Python print?

GENERATORS E3

```
>>> def countup():  
...     yield from [1, 2, 3]  
...     return 4  
...     yield from [5, 6]  
...  
>>> list(countup())
```

What would Python print?

[1, 2, 3]

Return results in a **StopIteration** (incl. the implicit return at the end of the function). The value returned is not actually returned by the generator function.

GENERATORS E4

```
>>> def myst(iterable, n):  
...     if n <= 0: return  
...     iterator = iter(s0)  
...     yield next(iterator)  
...     if n % 2:  
...         yield from myst(iterable, n - 1)  
...     else:  
...         yield from myst(iterator, n - 1)  
...  
>>> gen = lambda: (yield from range(7))  
>>> [e for e in myst(gen(), 3)]
```

What would Python print?

GENERATORS E4

```
>>> def myst(iterable, n):  
...     if n <= 0: return  
...     iterator = iter(s0)  
...     yield next(iterator)  
...     if n % 2:  
...         yield from myst(iterable, n - 1)  
...     else:  
...         yield from myst(iterator, n - 1)  
...  
>>> gen = lambda: (yield from range(7))  
>>> [e for e in myst(gen(), 3)]
```

[0, 1, 2]. Note that ``yield from myst(iterable, n - 1)`` is equivalent to ``yield from myst(iterator, n - 1)`` in this case. (Why?)

GENERATORS E5

```
>>> def myst(iterable, n):  
...     if n <= 0: return  
...     iterator = iter(s0)  
...     yield next(iterator)  
...     if n % 2:  
...         yield from myst(iterable, n - 1)  
...     else:  
...         yield from myst(iterator, n - 1)  
...  
>>> [e for e in myst(range(7), 3)]
```

Now what would Python print?

(This is exactly the same code as last time, except we're calling `myst` on a different iterable.)

GENERATORS E5

```
>>> def myst(iterable, n):  
...     if n <= 0: return  
...     iterator = iter(s0)  
...     yield next(iterator)  
...     if n % 2:  
...         yield from myst(iterable, n - 1)  
...     else:  
...         yield from myst(iterator, n - 1)  
...  
>>> [e for e in myst(range(7), 3)]
```

[0, 0, 1]. Why? Because `iter` is returning a fresh iterator every time! (In the last case, it wasn't – because the iterable that `iter` was being called on was an iterator. *Note, again: iterators are technically iterables because you can call `iter` on them.*)

GENERATORS E6

```
>>> def gen_e6(n):  
...     yield n / (n - 1)  
...     yield from gen_e6(n - 1)  
...
```

```
>>> e6 = gen_e6(3)
```

```
>>> next(e6)
```

```
>>> next(e6)
```

```
>>> next(e6)
```

```
>>> next(e6)
```

GENERATORS E6

```
>>> def gen_e6(n):  
...     yield n / (n - 1)  
...     yield from gen_e6(n - 1)  
...  
>>> e6 = gen_e6(3)  
>>> next(e6)  
1.5  
  
>>> next(e6)  
2.0  
  
>>> next(e6)  
ZeroDivisionError: division by zero  
  
>>> next(e6)  
StopIteration
```

GENERATORS 1.5.1

Write a generator function that combines the elements of two input iterators using a given combiner function. When either iterator runs out of elements, the whole generator should also run out of elements.

```
def combiner(iterator1, iterator2, combiner):  
    # YOUR-CODE-HERE
```

```
>>> from operator import add  
>>> evens = combiner(gen_naturals(), gen_naturals(), add)  
>>> next(evens)  
0  
>>> next(evens)  
2  
>>> next(evens)  
4
```

GENERATORS 1.5.1/2

```
def combiner(iterator1, iterator2, combiner):  
    while True:  
        yield combiner(next(iterator1), next(iterator2))
```

WWPP?

```
>>> nats = gen_naturals()  
>>> doubled_nats = combiner(nats, nats, add)  
>>> next(doubled_nats)  
  
>>> next(doubled_nats)
```


GENERATORS 1.5.1/2

```
def combiner(iterator1, iterator2, combiner):  
    while True:  
        yield combiner(next(iterator1), next(iterator2))
```

WWPP?

```
>>> nats = gen_naturals()  
>>> doubled_nats = combiner(nats, nats, add)  
>>> next(doubled_nats)  
1  
>>> next(doubled_nats)  
5
```

GENERATORS 1.5.3

Write a generator function that goes through all subsets of the positive integers from 1 to n . Each call to this generator's `next` method will return a list of subsets of the set $[1, 2, \dots, n]$, where n is the number of times `next` was previously called.

```
def generate_subsets():
    """
    >>> subsets = generate_subsets()
    >>> for _ in range(3):
    ...     print(next(subsets))
    ...
    [[]]
    [[], [1]]
    [[], [1], [2], [1, 2]]
    """
    # YOUR-CODE-HERE
```

GENERATORS 1.5.3

```
def generate_subsets():  
    # YOUR-CODE-HERE
```

Thought process:

- ▶ Uh...
- ▶ Okay, well it's a generator function so we're going to have to yield stuff
- ▶ Looking at the doctest, it seems as if we always want the positive integers to be in order. We're just splitting them up
 - ▷ `[]`
 - ▷ `[[], [1]]`
 - ▷ `[[], [1], [2], [1, 2]]`
- ▶ What do you notice? Each successive yield is just *everything from before*, and also *everything from before* with the latest value of `n` tacked onto the end

GENERATORS 1.5.3

```
def generate_subsets():  
    # YOUR-CODE-HERE
```

In other words,

- ▶ `n = 0:`
`[]`
- ▶ `n = 1:`
`[], [1]`, which is really just
`[] AND [] + [1]`
- ▶ `n = 2:`
`[], [1], [2], [1, 2]`, which is really just
`[], [1] AND [] + [2], [1] + [2]`
- ▶ `n = 3:`
`[], [1], [2], [1, 2], [3], [1, 3], [2, 3], [1, 2, 3]`, or
`... AND [] + [3], [1] + [3], [2] + [3], [1, 2] + [3]`

GENERATORS 1.5.3

```
def generate_subsets():  
    n, subsets = 1, [[]]  
    while True:  
        yield subsets  
        subsets += [s + [n] for s in subsets]  
        n += 1
```

A large blue geometric shape, resembling a parallelogram or a tilted rectangle, occupies the right half of the image. It is a solid medium blue color.

STREAMS

STREAMS

Streams are linked lists that are evaluated lazily:

- ▶ The rest won't be computed until we ask for it
- ▶ After we ask for it, the result will be **remembered**
- ▶ Rules [i.e. functions] will be used to compute the next element

Scheme stream interface:

- ▶ `car` gives us the first element of the stream
- ▶ `nil` is the empty stream
- ▶ `cons-stream`: like `cons`, except the rest isn't evaluated at first
- ▶ `cdr-stream`: like `cdr`, but tells the stream **to actually compute the rest** if it hasn't already. Note: don't use `cdr` with streams!

STREAMS

Non-required material, but you'll probably run into it so

Scheme promises (force/delay)

- ▶ How lazily evaluated expressions are actually implemented in Scheme
- ▶ [Spec description](#)
- ▶ **Promise:** a delayed expression
 - ▷ Can be “forced” (already evaluated and now cached)
 - ▷ or “not forced” (not evaluated yet)
- ▶ If you print a stream, you'll see this

```
scm> (cons-stream 1 2)
(1 . #[promise (not forced)])
```


STREAMS

What is the advantage of using a stream over a linked list?

STREAMS

What is the advantage of using a stream over a linked list?

Elements won't be evaluated unnecessarily if they are never used... meaning efficient space usage! Also, streams allow for the representation of infinite-length sequences.

On streams versus iterators:

Every time you call `next` on an iterator, it changes. The nice thing about streams is that they never change.

STREAMS E1

We attempt to define an infinite sequence of natural numbers.

```
(define (naturals n)
  (cons n (naturals (+ n 1))))
```

```
(define nat (naturals 0))
```

What happens?

STREAMS E1

We attempt to define an infinite sequence of natural numbers.

```
(define (naturals n)
  (cons n (naturals (+ n 1))))
```

```
(define nat (naturals 0))
```

Error: maximum recursion depth exceeded

STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

```
(define nat (naturals 0))
```

```
(car nat)
```

```
(car (cdr-stream nat))
```

```
(car (cdr-stream (cdr-stream nat)))
```

STREAMS E1

That didn't work, so we turn to streams instead.

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

```
(define nat (naturals 0)) → nat
```

```
(car nat) → 0
```

```
(car (cdr-stream nat)) → 1
```

```
(car (cdr-stream (cdr-stream nat))) → 2
```

STREAMS E1

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

```
(define nat (naturals 0))
```

```
(car nat)
```

```
(cdr nat)
```

```
(cdr-stream nat)
```

```
(cdr nat)
```

STREAMS E1

```
(define (naturals n)
  (cons-stream n (naturals (+ n 1))))
```

```
(define nat (naturals 0))
```

```
(car nat) → 0
```

```
(cdr nat) → #[promise (not forced)]
```

```
(cdr-stream nat) → (1 . #[promise (not forced)])
```

```
(cdr nat) → #[promise (forced)]
```


STREAMS 2.1.1 - WWSD?

```
scm> (define (has-even? s)
      (cond ((null? s) False)
            ((even? (car s)) True)
            (else (has-even? (cdr-stream s)))))
```

has-even?

```
scm> (define ones (cons-stream 1 ones))
```

ones

```
scm> (define twos (cons-stream 2 twos))
```

twos

```
scm> ones
```

```
scm> (cdr-stream ones)
```

```
scm> (has-even? ones)
```

```
scm> (has-even? twos)
```

STREAMS 2.1.1 - WWSD?

```
scm> (define (has-even? s)
      (cond ((null? s) False)
            ((even? (car s)) True)
            (else (has-even? (cdr-stream s)))))
```

has-even?

```
scm> (define ones (cons-stream 1 ones))
```

ones

```
scm> (define twos (cons-stream 2 twos))
```

twos

```
scm> ones
```

```
(1 . #[promise (not forced)])
```

```
scm> (cdr-stream ones)
```

```
(1 . #[promise (forced)])
```

```
scm> (has-even? ones)
```

Runs forever

```
scm> (has-even? twos)
```

True

STREAMS 2.1.2

Implement `map-stream`, which maps a function `f` to a stream `s`.

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

```
scm> (define evens (map-stream (lambda (x) (* x 2)) nat))
evens
```

```
scm> (car (cdr-stream evens))
2
```

```
scm> (car (cdr-stream (cdr-stream evens)))
4
```

STREAMS 2.1.2

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

Approach (what do we know?):

- ▶ We need to create a new stream
- ▶ We need to apply the function to every element
- ▶ We need to not think about the enormity of applying a function to every element in an infinite sequence, and instead consider the problem inductively

STREAMS 2.1.2

```
(define (map-stream f s)
  ; YOUR-CODE-HERE
)
```

Approach (what do we know?):

- ▶ We need to create a new stream
 - ▷ So we'll use `cons-stream`
- ▶ We need to apply the function to every element
 - ▷ So we'll do `(f (car s))`
- ▶ We need to not think about the enormity of applying a function to every element in an infinite sequence, and instead consider the problem inductively
 - ▷ So we'll recurse, creating the rest of the stream via `(map-stream f (cdr-stream s))`. Our base case will be an empty stream.

STREAMS 2.1.2

```
(define (map-stream f s)
  (if (null? s) nil
      (cons-stream (f (car s))
                    (map-stream f (cdr-stream s)))))
)
```

STREAMS 2.1.3

Consider the following two implementations of `filter-stream`. One is correct and one is not. Which is the incorrect one, and why?

```
(define (filter-stream f s)
  (if (null? s) nil
      (let ((rest (filter-stream f (cdr-stream s))))
        (if (f (car s))
            (cons-stream (car s) rest)
            rest))))

(define (filter-stream f s)
  (cond ((null? s) nil)
        ((f (car s)) (cons-stream (car s) (filter-stream f (cdr-stream s))))
        (else (filter-stream f (cdr-stream s)))))
```

STREAMS 2.1.3

The first one is incorrect, because it recursively creates the rest of the stream either until it hits the end of `s` or until it hits the maximum recursion depth. We want lazy evaluation, so we shouldn't be constructing the entire stream at once!

```
(define (filter-stream f s)
  (if (null? s) nil
      (let ((rest (filter-stream f (cdr-stream s))))
        (if (f (car s))
            (cons-stream (car s) rest)
            rest)))))
```

```
(define (filter-stream f s)
  (cond ((null? s) nil)
        ((f (car s)) (cons-stream (car s) (filter-stream f (cdr-stream s))))
        (else (filter-stream f (cdr-stream s)))))
```


STREAMS 2.1.4

Write a function, `range-stream`, that takes in two integers `start` and `end` and returns the same thing that `range(start, end)` would... but as a stream.

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

```
scm> (define rs (range-stream 1 3))
rs
scm> (car rs)
1
scm> (car (cdr-stream rs))
2
scm> (cdr-stream (cdr-stream rs))
()
```

STREAMS 2.1.4

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We have to create a stream where the first element is `start`
- ▶ The “rest” of the stream should be another stream whose first element is `start + 1`
- ▶ ^ Sounds like recursion to me. We can just make the rest of the stream be the return value of a `range-stream` call
- ▶ The base case, then, will be when `start >= end`. In this case, there’s really no range to speak of – so we return an empty stream

STREAMS 2.1.4

```
(define (range-stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We have to create a stream where the first element is `start`
 - ▷ `(cons-stream start ...)`
- ▶ The “rest” of the stream should be another stream whose first element is `start + 1`
 - ▷ `(cons-stream (+ start 1) ...)`
- ▶ ^ Sounds like recursion to me. We can just make the rest of the stream be the return value of a `range-stream` call
 - ▷ `(cons-stream start (range-stream (+ start 1) end))`
- ▶ The base case, then, will be when `start >= end`. In this case, there’s really no range to speak of – so we return an empty stream
 - ▷ `(if (>= start end) nil ...)`

STREAMS 2.1.4

```
(define (range-stream start end)
  (if (>= start end) nil
      (cons-stream start (range-stream (+ start 1) end))))
)
```

STREAMS 2.1.5

Write a function, `slice`, that returns a **list** containing the elements of `stream` from index `start` to index `end - 1`. If you run out of elements in `stream`, just cut the list short.

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

STREAMS 2.1.5

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process:

- ▶ We're returning a list! Back to `cons` and `list` and `append...` and no lazy evaluation. We need everything *now*
- ▶ Let's just recurse our way through `stream` decrementing `start` until it hits 0, and *then* start adding elements to our output list
- ▶ How will we know when to stop adding stuff? We want `end - start` elements overall... oh, we should also decrement `end` in that last step. Then, when `start` reaches 0, we'll know we want to add `end - 0` elements
- ▶ So we continue, decrementing `end` and adding elements to our list, until `end` is equal to 0. At this point, we'll be done

STREAMS 2.1.5

```
(define (slice stream start end)
  ; YOUR-CODE-HERE
)
```

Thought process, continued:

- ▶ We can use `cons` since it fits: we have an element and a “rest” list to use in the `(cons element rest)` formula
 - ▷ Specifically, we have `(cons (car stream) <recursive call>)`
- ▶ We should also check if `stream` is empty, since it's not guaranteed that our indices are in-range

STREAMS 2.1.5

Bringing it all together, we have

```
(define (slice stream start end)
  (cond ((or (= end 0) (null? stream)) nil)
        ((> start 0) (slice (cdr-stream stream)
                              (- start 1)
                              (- end 1)))
        (else (cons (car stream)
                      (slice (cdr-stream stream)
                              0
                              (- end 1))))))
)
```


STREAMS 2.1.6

Let's combine infinite-length streams using `zip-with`, which accepts a function `f` and two streams (`xs` and `ys`) and returns a single stream containing every pair of corresponding elements combined with `f`.

If the elements in `xs` were `x1`, `x2`, `x3`, ... and the elements in `ys` were `y1`, `y2`, `y3`, ..., then `(zip-with f xs ys)` would return a stream with the elements `(f x1 y1)`, `(f x2 y2)`, `(f x3 y3)`, ...

```
(define (zip-with f xs ys)
  (if (or (null? xs) (null? ys)) nil
      (cons-stream (f (car xs) (car ys))
                    (zip-with f (cdr-stream xs) (cdr-stream ys)))))
)
```

STREAMS 2.1.6

As an example, we can create the even numbers by adding every natural number to itself (s.t. the output sequence is $0 + 0$, $1 + 1$, $2 + 2$, ...).

```
scm> (define evens (zip-with + (naturals 0) (naturals 0)))  
evens  
scm> (slice evens 0 10)  
(0 2 4 6 8 10 12 14 16 18)
```

STREAMS 2.1.6

How would we define a stream containing the factorials of all numbers in order (starting at 0!) ? **Use zip-with!**

```
(define factorials  
  ; YOUR-CODE-HERE  
)
```

```
scm> (slice factorials 0 10)  
(1 1 2 6 24 120 720 5040 40320 362880)
```

STREAMS 2.1.6

```
(define factorials  
  ; YOUR-CODE-HERE  
)
```

Thought process:

- ▶ `zip-with`? What does that do again? Oh yeah...
- ▶ Factorials? What are those again? Oh yeah...
 - ▷ $0! = 1$
 - ▷ $1! = 1 * 0! = 1 * 1$
 - ▷ $2! = 2 * 1! = 2 * 1 * 1$
 - ▷ $3! = 3 * 2! = 3 * 2 * 1 * 1$
- ▶ Interesting. It seems like we'd want to combine elements with `*` (multiplication)
- ▶ Well, based on the recurrences above, we'd want to multiply every natural number with the previous factorial. But we're *computing* the factorials! This is going to be trippy...

STREAMS 2.1.6

```
(define factorials  
  (cons-stream 1 (zip-with * factorials (naturals 1)))  
)
```

- ▶ `cons-stream` is kind of like a base case (it's the anchor; the first element of the output stream)
- ▶ Observing the sequence:
 - ▷ First element is 1 (obvious)
 - ▷ Second element is $[\wedge \text{that element}] * 1 = 1 * 1 = 1$
 - ▷ Third element is $[\wedge \text{that element}] * 2 = 1 * 2 = 2$
 - ▷ Fourth element is $[\wedge \text{that element}] * 3 = 2 * 3 = 6$
 - ▷ Fifth element is $[\wedge \text{that element}] * 4 = 4 * 6$
 - ▷ ...
- ▶ I always thought this was neat af

STREAMS 2.1.6

Last one. How would we define a stream containing the Fibonacci numbers, starting with 0 and 1? **Use zip-with!**

```
(define fibs  
  ; YOUR-CODE-HERE  
)
```

```
scm> (slice fibs 0 10)  
(0 1 1 2 3 5 8 13 21 34)
```

STREAMS 2.1.6

```
(define fibs
  ; YOUR-CODE-HERE
)
```

Let's do this

- ▶ We've been computing Fibonacci numbers since day 0
 - ▷ $F(0) = 0$
 - ▷ $F(1) = 1$
 - ▷ $F(2) = F(0) + F(1)$
 - ▷ $F(3) = F(1) + F(2)$
 - ▷ ...
- ▶ Looks like we're gonna be combining with + (addition)
- ▶ Two "base cases", 0 and 1, and then we'll (zip-with +) a fibs and a staggered fibs

STREAMS 2.1.6

```
(define fibs
  (cons-stream 1 (cons-stream 2
    (zip-with + fibs (cdr-stream fibs))))
)
```

It's so beautiful...

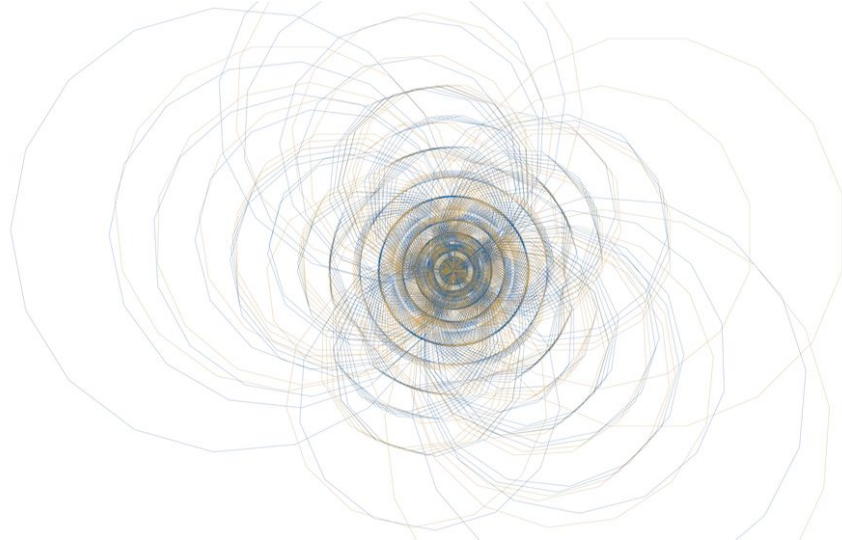
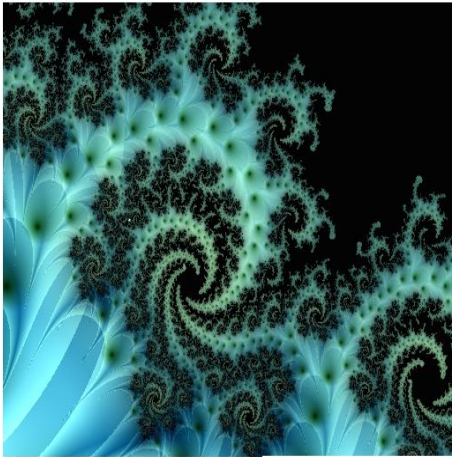
A large blue geometric shape, resembling a parallelogram or a large triangle, occupies the right side of the image. It is composed of two shades of blue, with a darker blue border or shadow effect.

TURTLE

RECURSIVE ART CONTEST

It's optional... but it's also cool. Even if you don't enter the contest, try playing around with the turtle graphics system for a bit!

Note: you can probably sell a lot of things as a recursive process



VECTOR GRAPHICS

Represent images using **geometric primitives** (points, lines, curves, shapes, polygons...) instead of **pixel values**. Based on **vectors** passing through control points, hence the name

As it happens, turtle graphics are vector-based!

DEMO: IMAGE VECTORIZATION

Something you can do with turtle graphics:

- ▶ Take a photograph (or grab any image you like)
- ▶ Convert the image to SVG format (<https://www.vectorizer.io/>)
- ▶ Implement turtle functions for all primitives in the VG format you're using. This may only be path (a composite Bézier curve)!
- ▶ Convert the vector image into turtle code (using the primitives you just wrote) via a parser
- ▶ Let the turtle do its thing
- ▶ Profit

As I can show...

DISCUSSION ATTENDANCE

<http://tiny.cc/5184>

QUIZ 9