

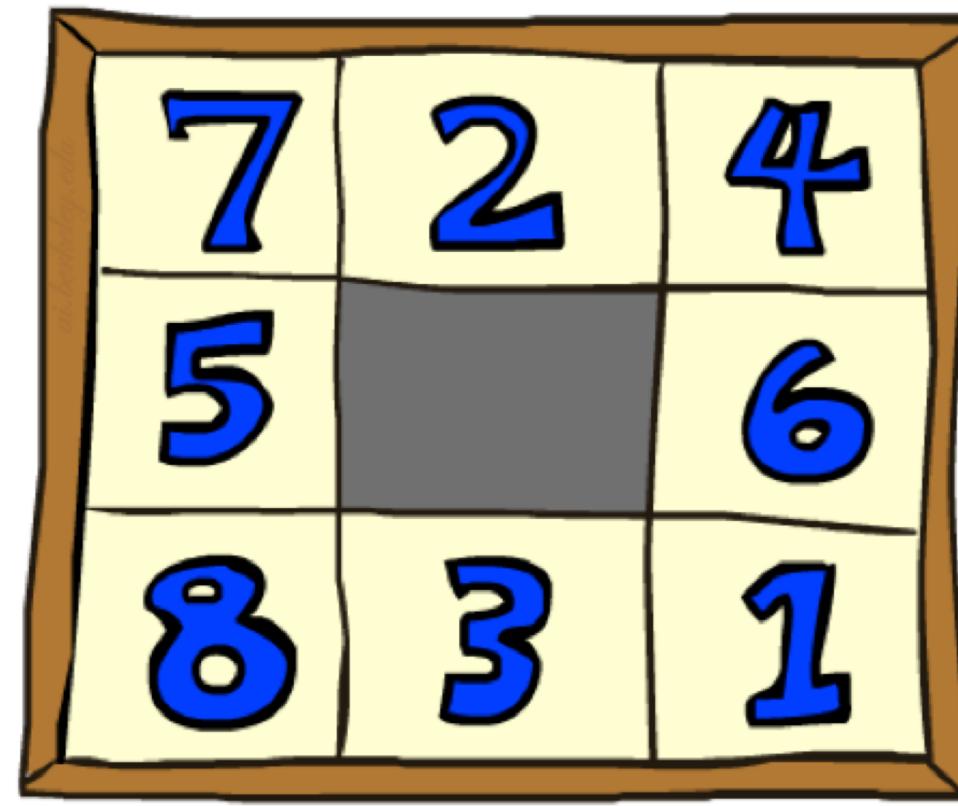
PURDUE CS47100

INTRODUCTION TO AI

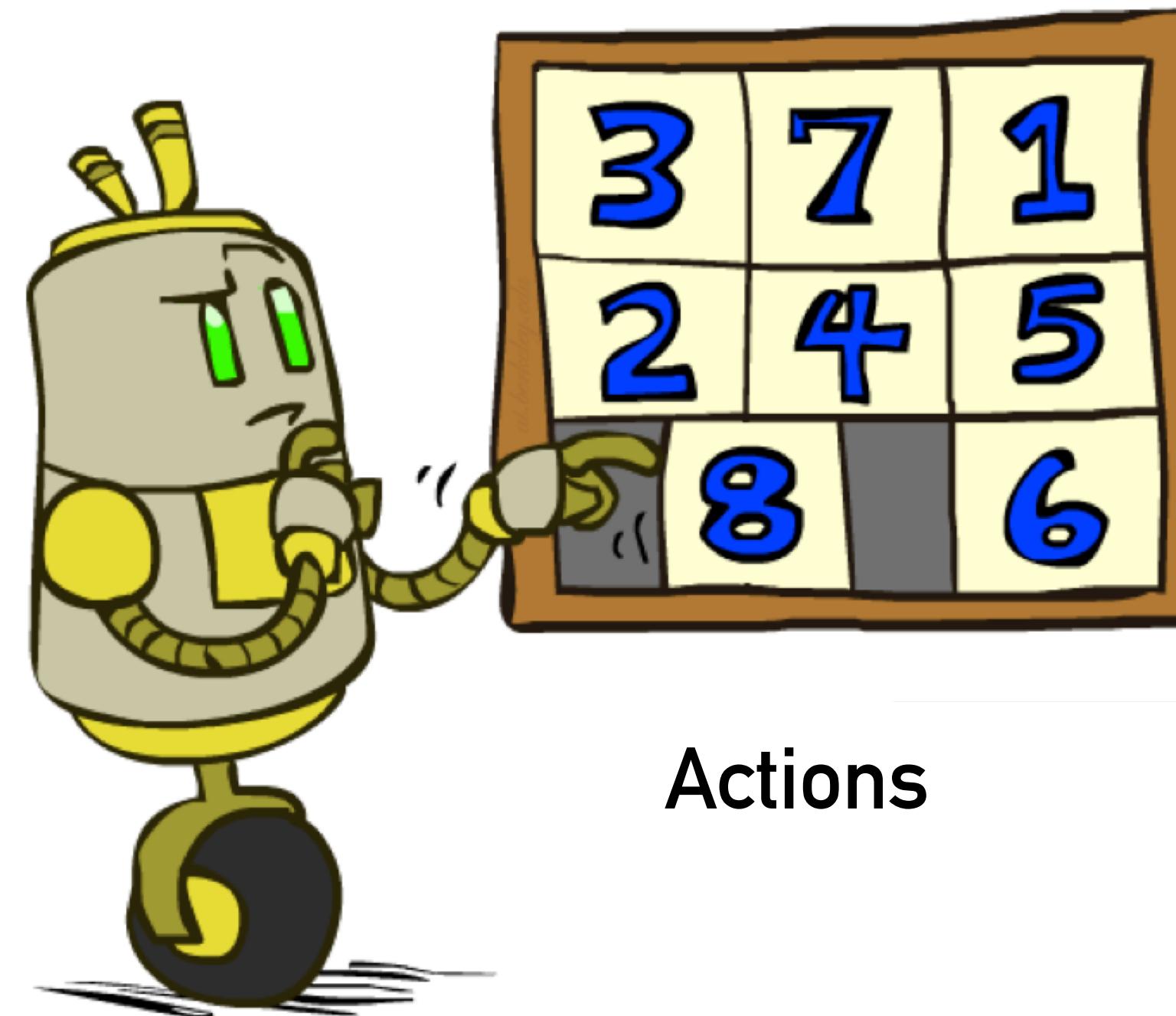
RECAP: COST-SENSITIVE SEARCH

- ▶ Uninformed search: Uniform cost search
 - ▶ Prioritize nodes with smaller cumulative cost $g(n)$
- ▶ Informed search
 - ▶ Greedy search: Prioritize nodes with smaller heuristic function value $h(n)$
 - ▶ A* search: Prioritize nodes with smaller evaluation function value $f(n) = g(n) + h(n)$

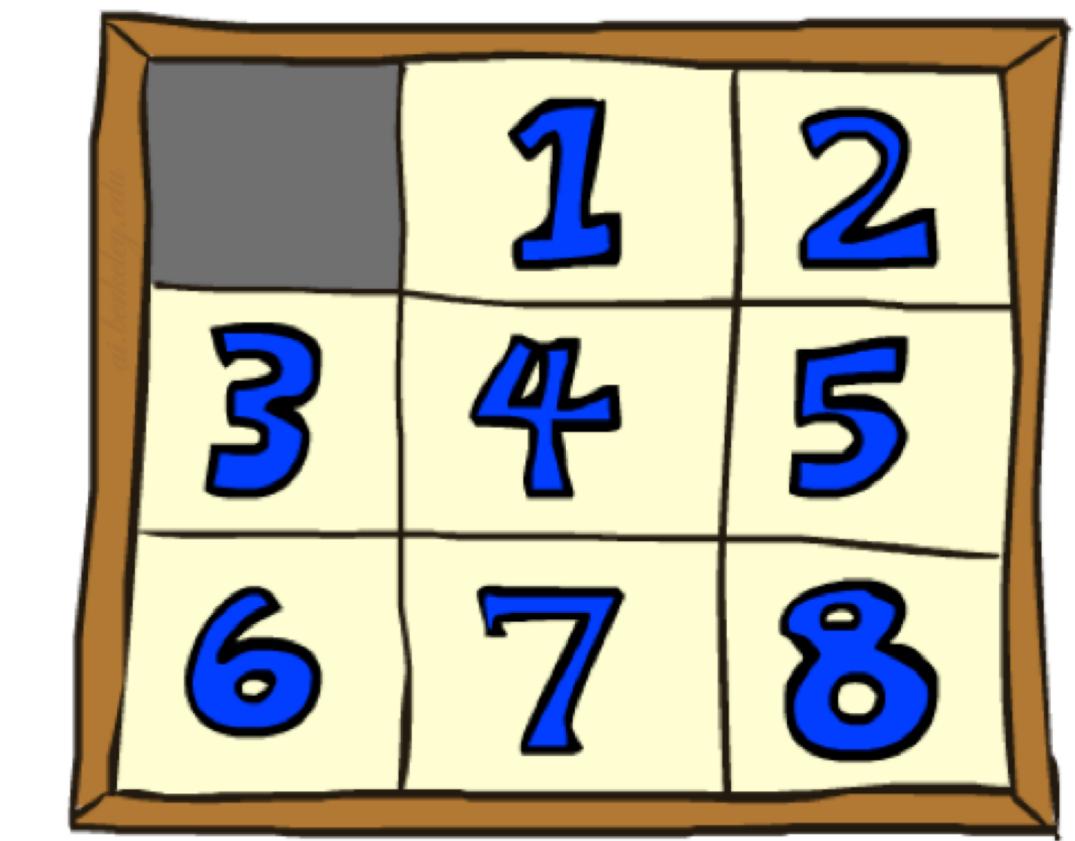
ADMISSIBLE HEURISTICS EXAMPLE: 8 PUZZLE



Start State



Actions



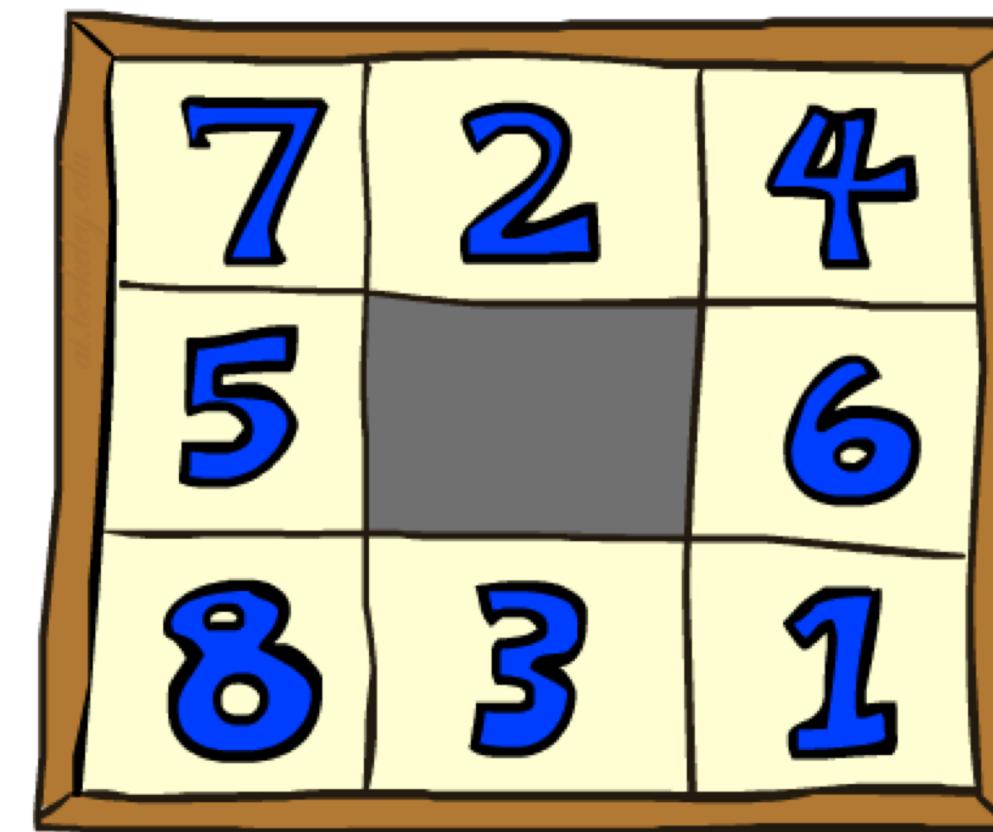
Goal State

What could be an admissible heuristic function for 8-puzzle problems?

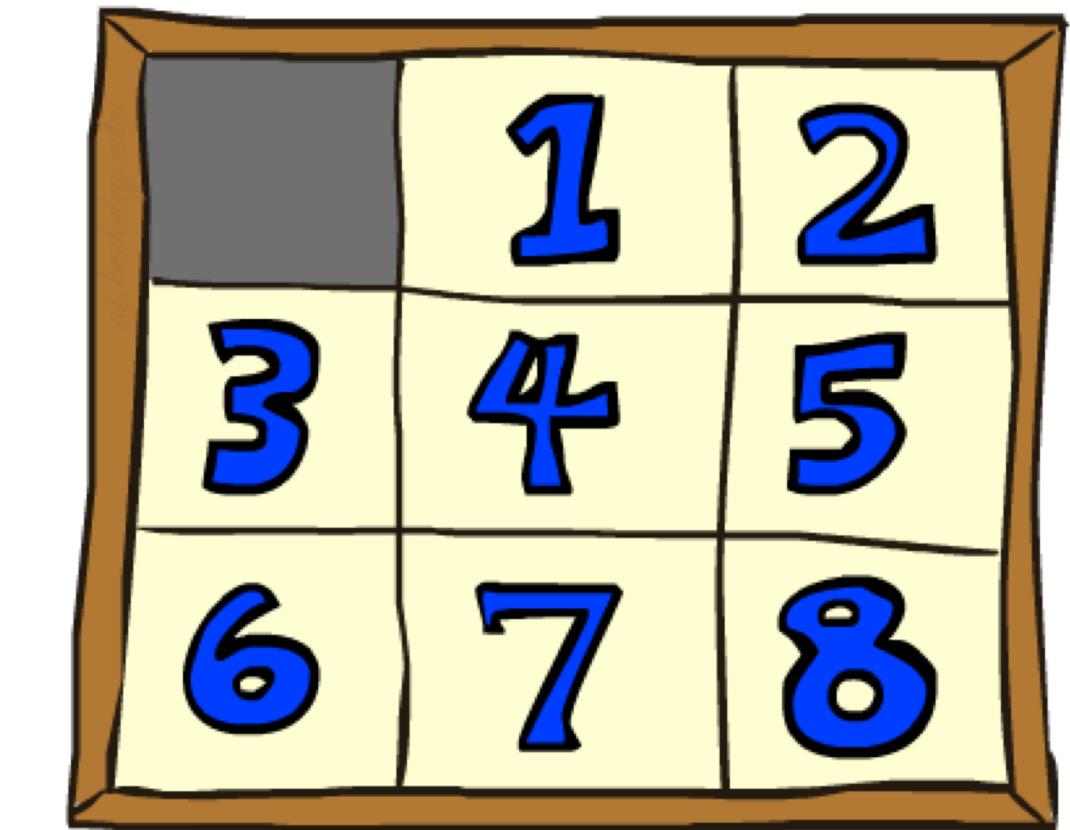
Hint: A tile can move from Location A to B if A and B are adjacent and B is blank...Relax it?

8 PUZZLE HEURISTIC I

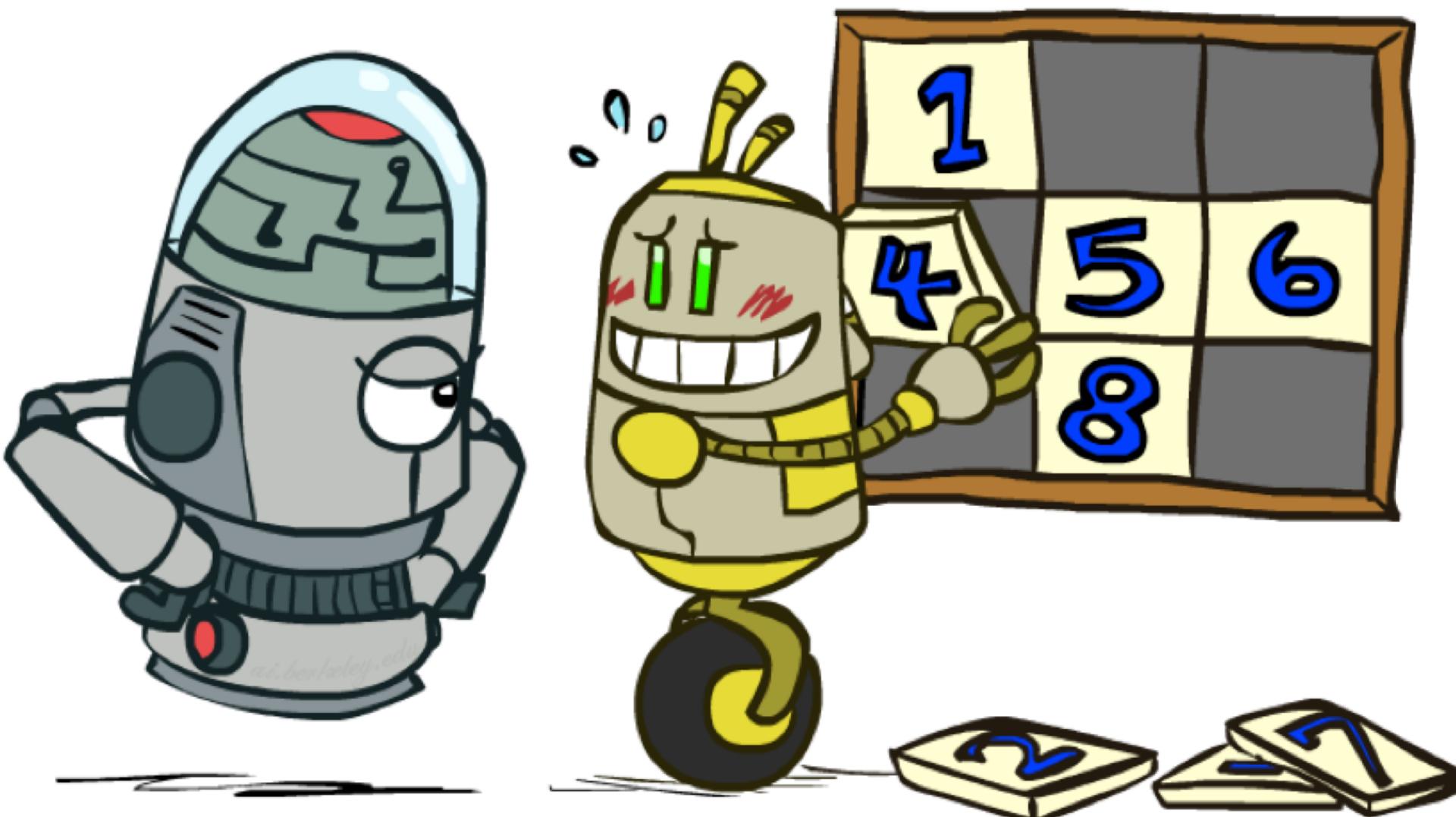
- ▶ Heuristic: Number of misplaced tiles
- ▶ Why is it admissible?
- ▶ $h(\text{start}) = 8$
- ▶ This is a *relaxed-problem* heuristic



Start State



Goal State

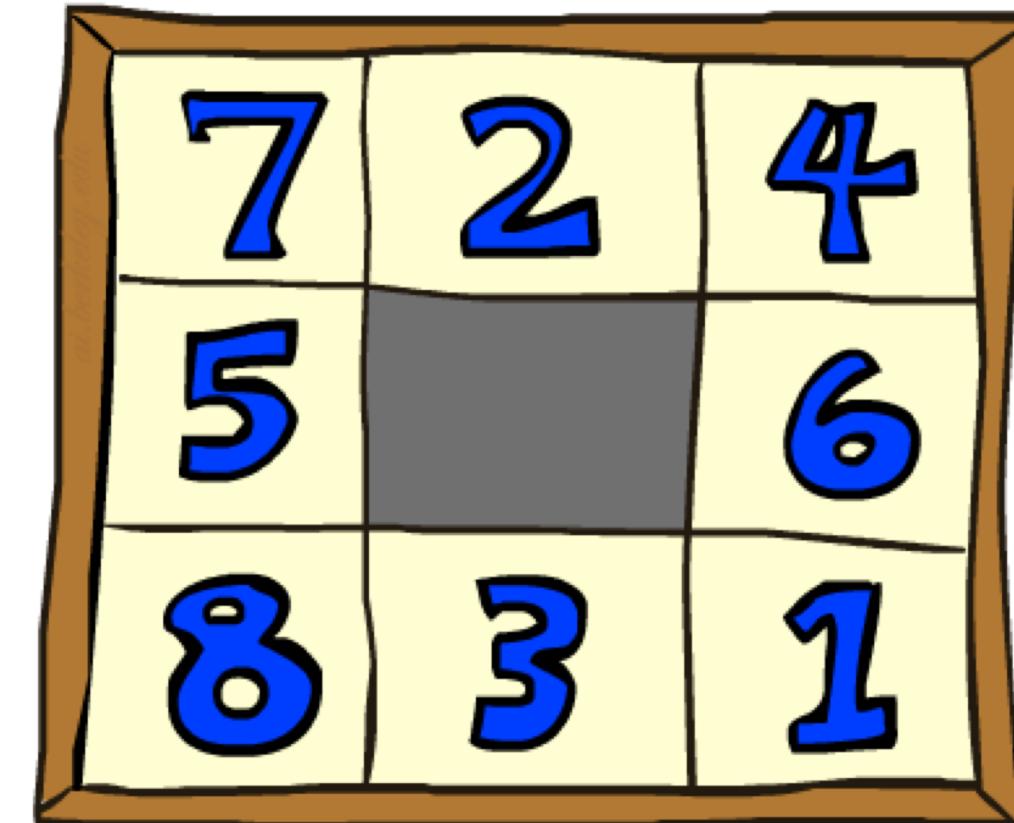


Average nodes expanded when the optimal path has...

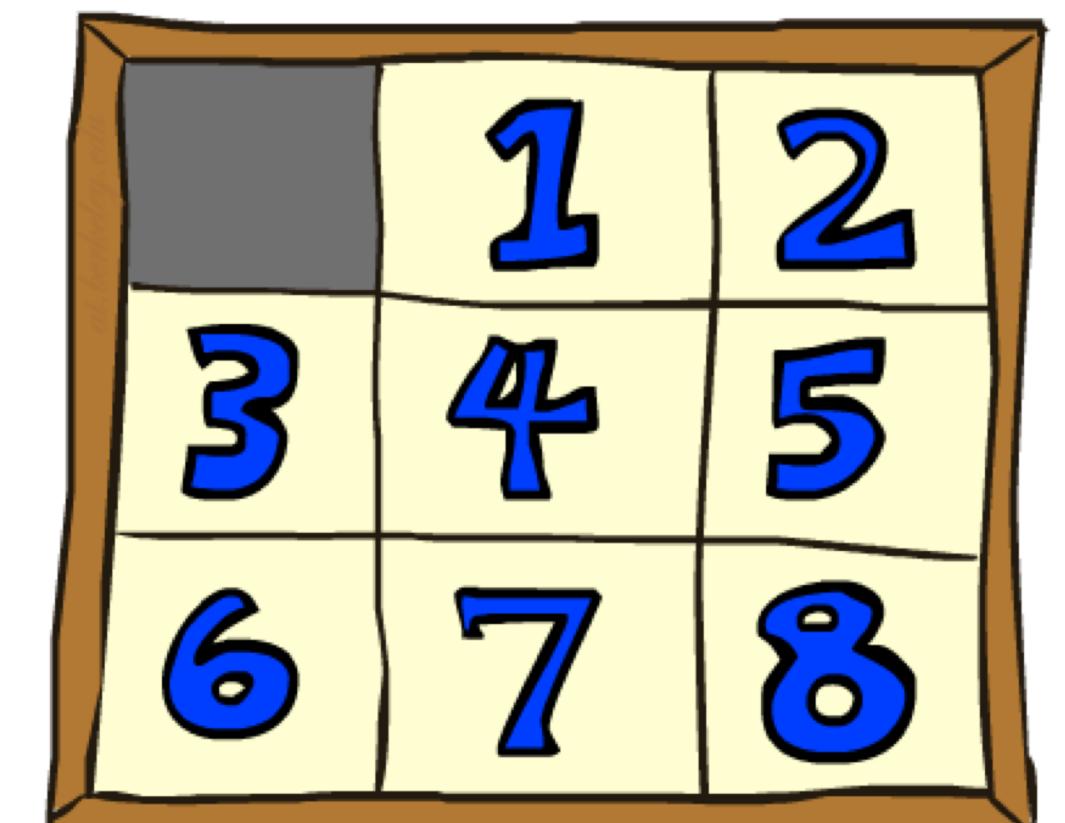
	...4 steps	...8 steps	...12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

8 PUZZLE HEURISTIC II

- ▶ Heuristic: Sum of Manhattan distance for each tile
- ▶ Why is it admissible?
- ▶ $h(\text{start}) = \mathbf{3 + 1 + 2 + \dots = 18}$
- ▶ This is still a *relaxed-problem* heuristic



Start State



Goal State

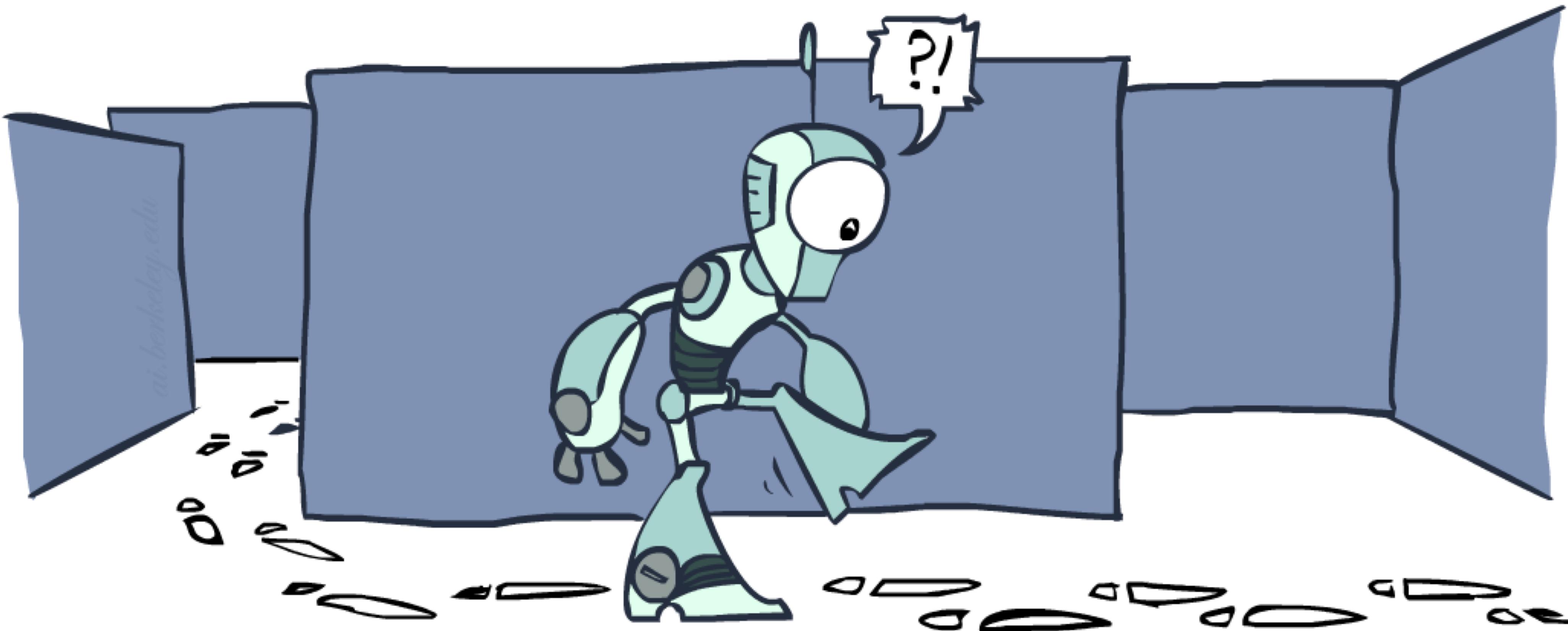
Average nodes expanded when the optimal path has...

	...4 steps	...8 steps	...12 steps
TILES	13	39	227
MANHATTAN	12	25	73

8 PUZZLE III

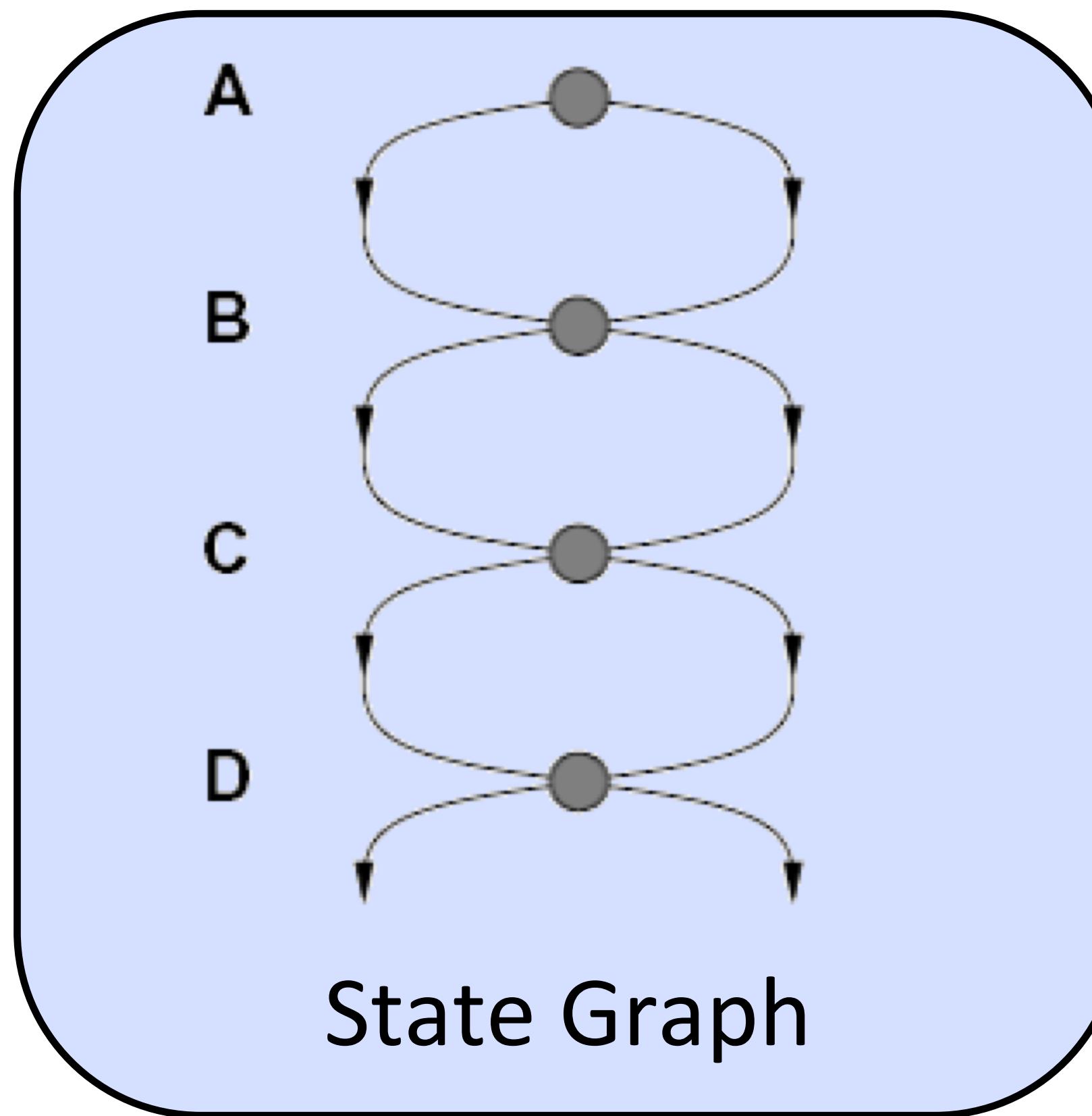
- ▶ How about using the *actual cost* as a heuristic?
 - ▶ Would it be admissible?
 - ▶ Would we save on nodes expanded?
 - ▶ What's wrong with it?
- ▶ With A*: a trade-off between quality of estimate and work per node
 - ▶ As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself
 - ▶ Max of admissible heuristics is admissible, and it is a "better" heuristic

GRAPH SEARCH

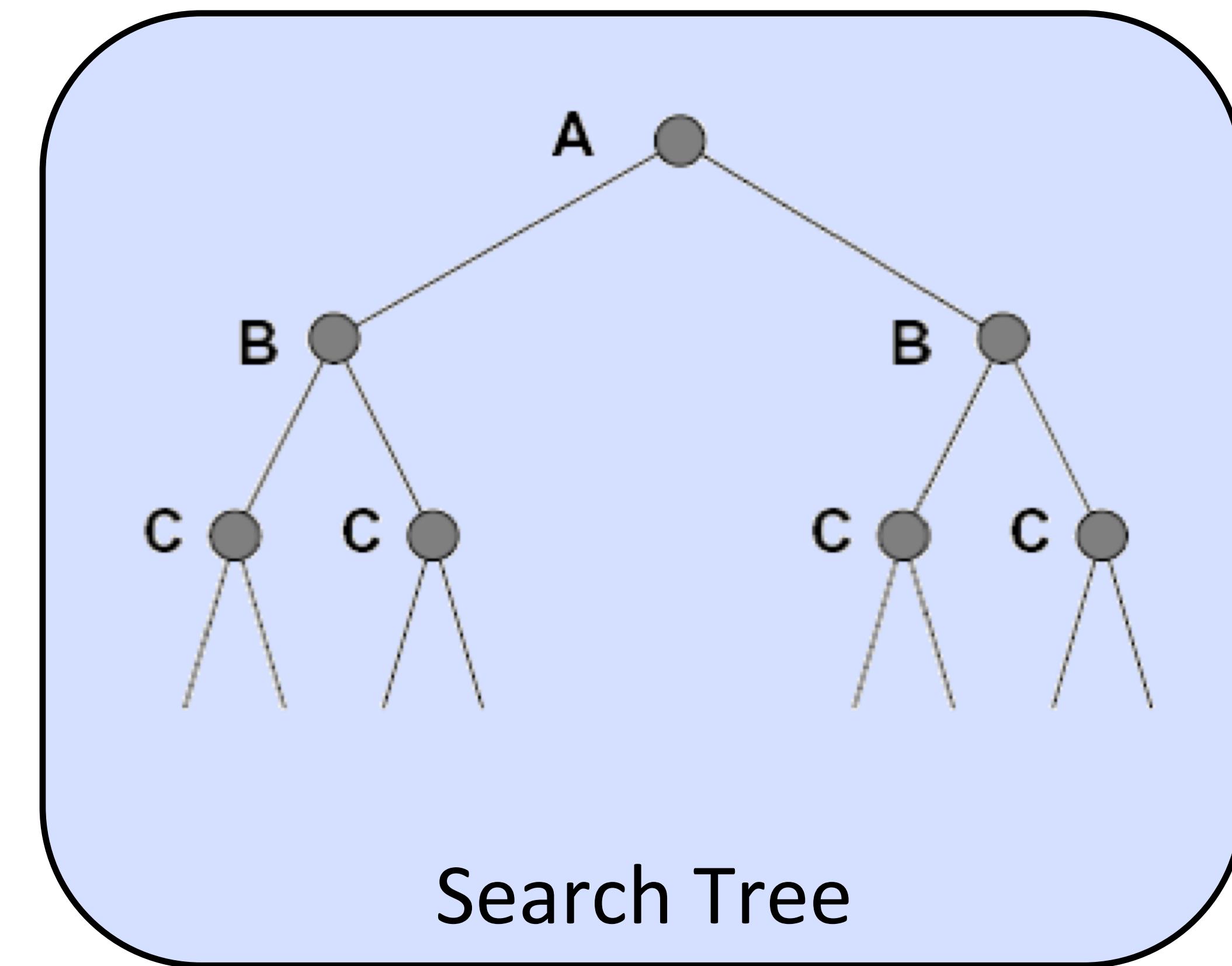


TREE SEARCH: EXTRA WORK

- ▶ Failure to detect repeated states can cause exponentially more work



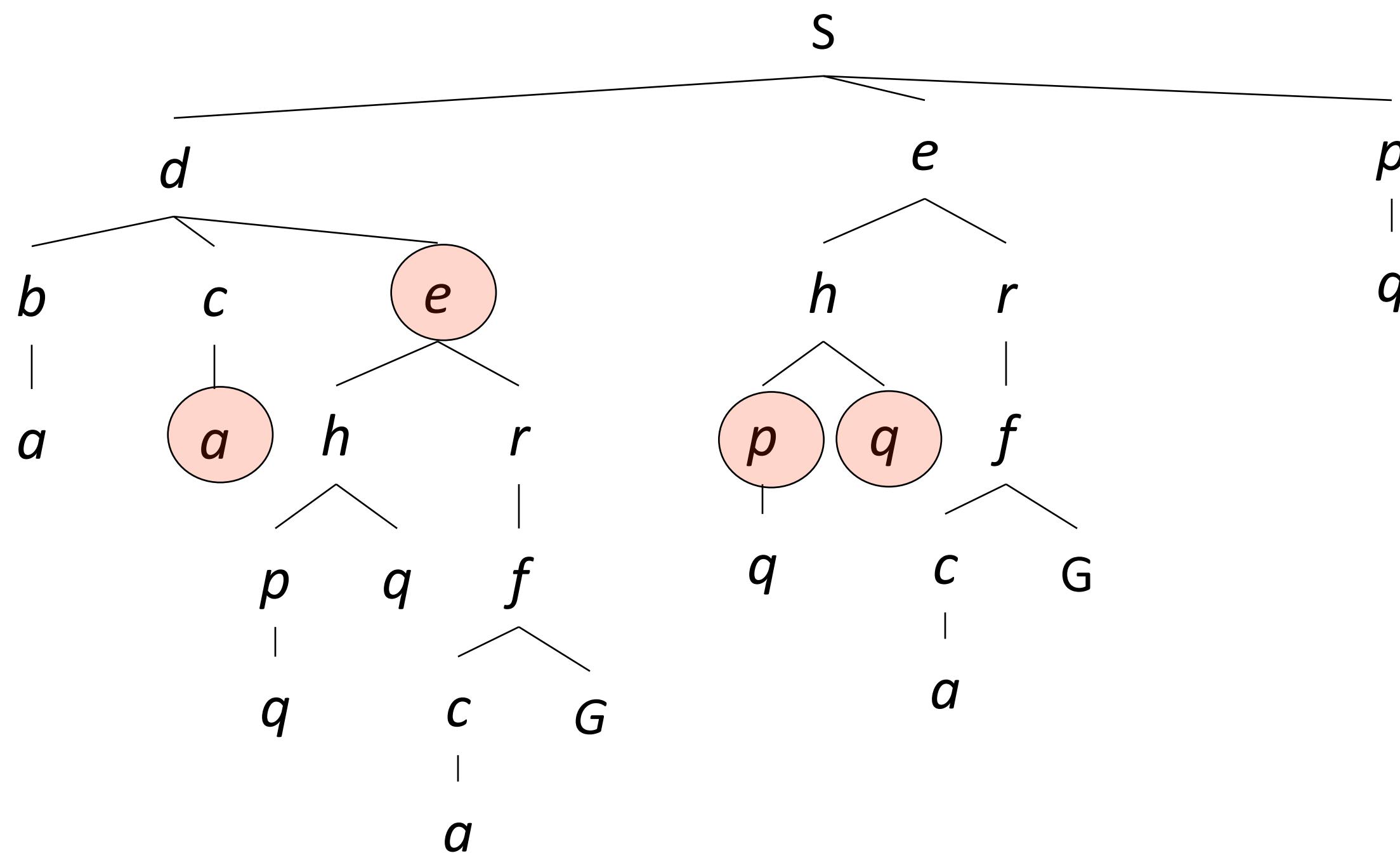
State Graph



Search Tree

GRAPH SEARCH

- ▶ In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



GRAPH SEARCH

- ▶ Idea: never **expand** a state twice
- ▶ How to implement:
 - ▶ Tree search + set of expanded states ("**closed set**")
 - ▶ Expand the search tree node-by-node, but...
 - ▶ Before expanding a node, check to make sure its state has never been expanded before (i.e., not in "closed set")
 - ▶ If not new, skip it, if new add to closed set
- ▶ Important: **store the closed set as a set**, not a list

GRAPH SEARCH ALGORITHMS

```
function GRAPH_SEARCH (problem) return a solution, or failure
```

```
fringe ← {initial_state}; explored ← Ø
```

```
repeat
```

```
    if fringe = Ø then return failure
```

```
    node ← POP(fringe)
```

```
    if GOAL_TEST(node) then return the corresponding solution
```

```
    if node not in explored:
```

```
        explored ← explored ∪ {node}
```

```
        for a in Action(node):
```

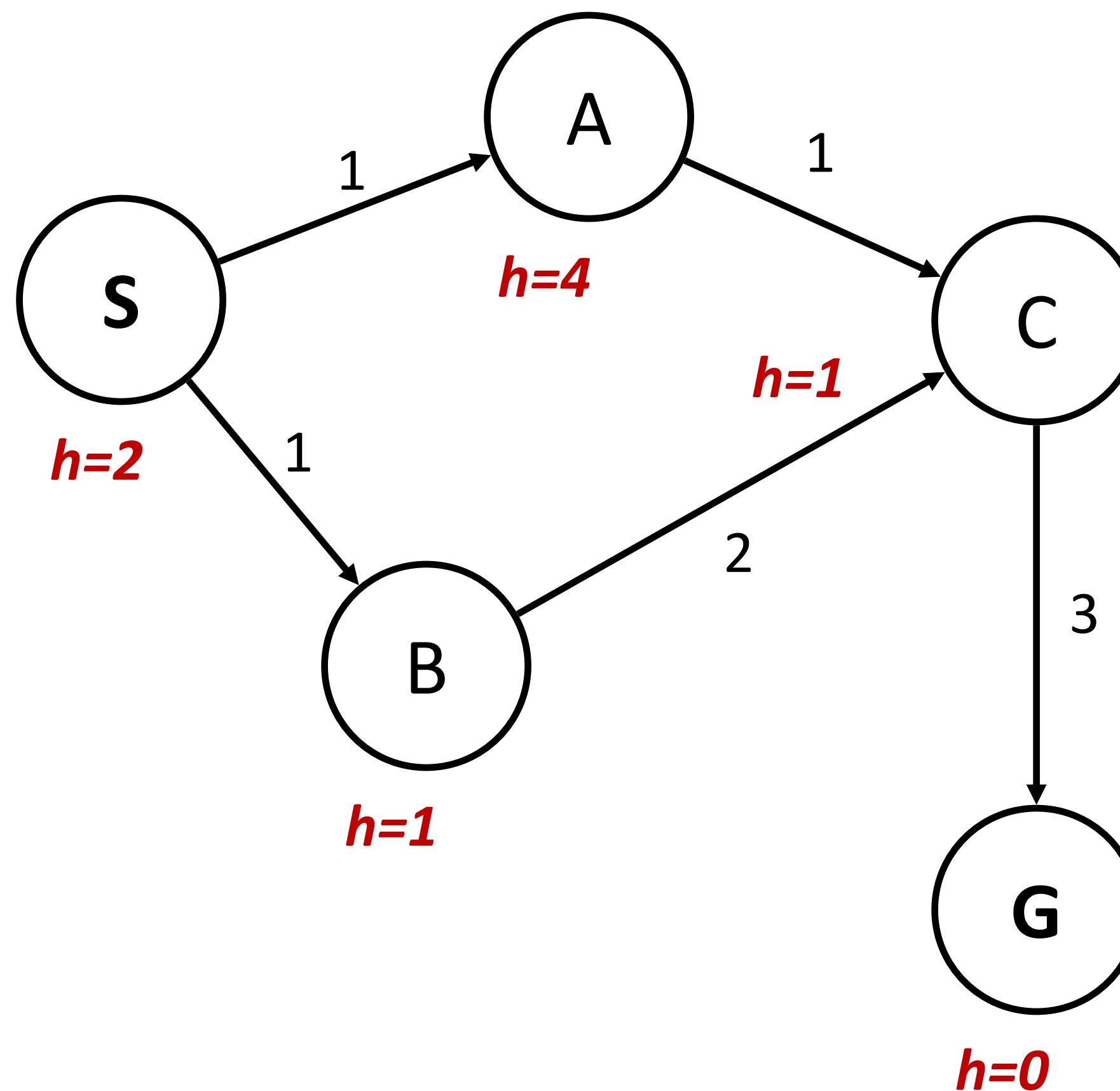
```
            a' = Result(node, a)
```

```
            INSERT(fringe, a')
```

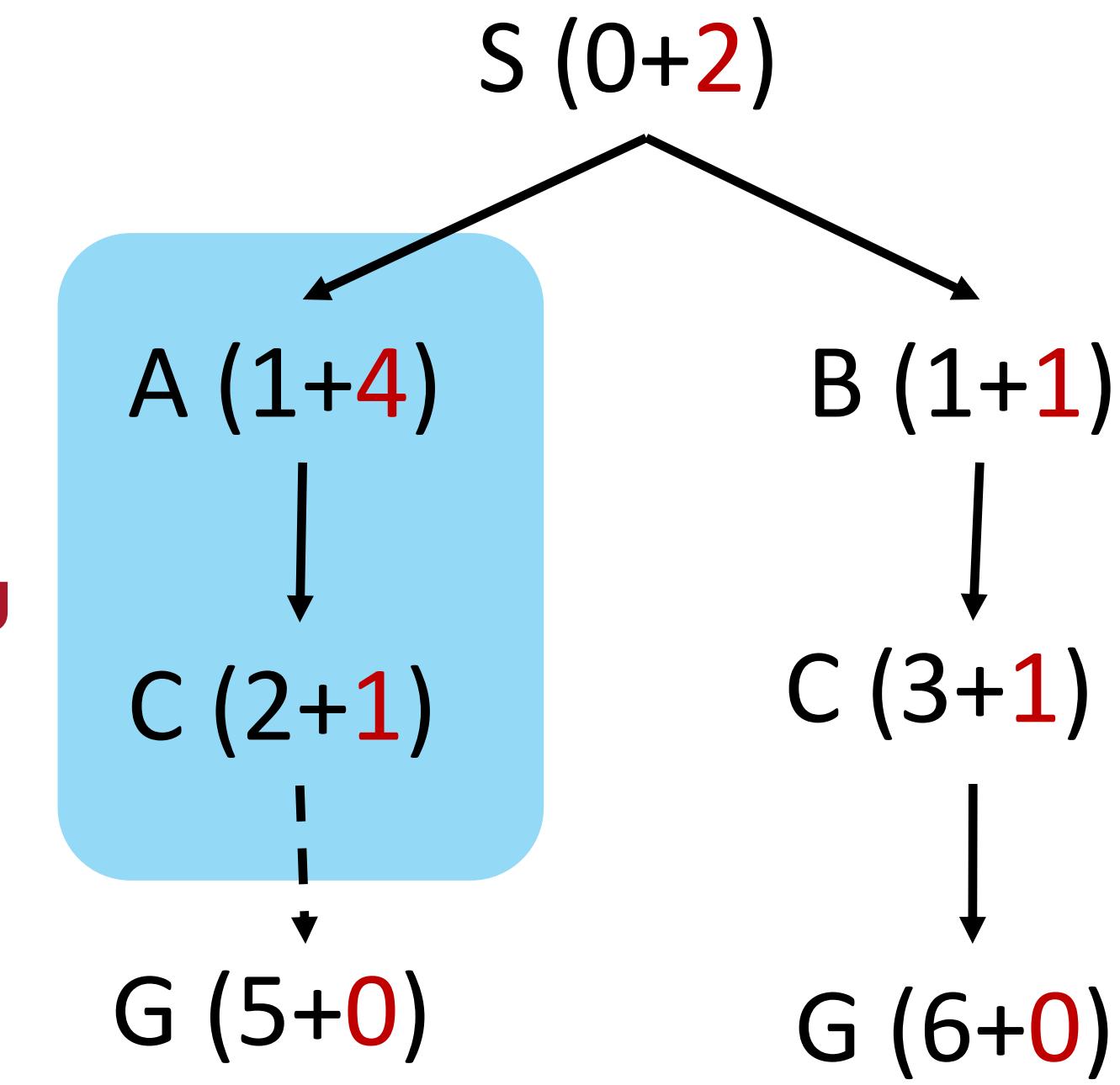
Can graph search wreck completeness?

How about optimality?

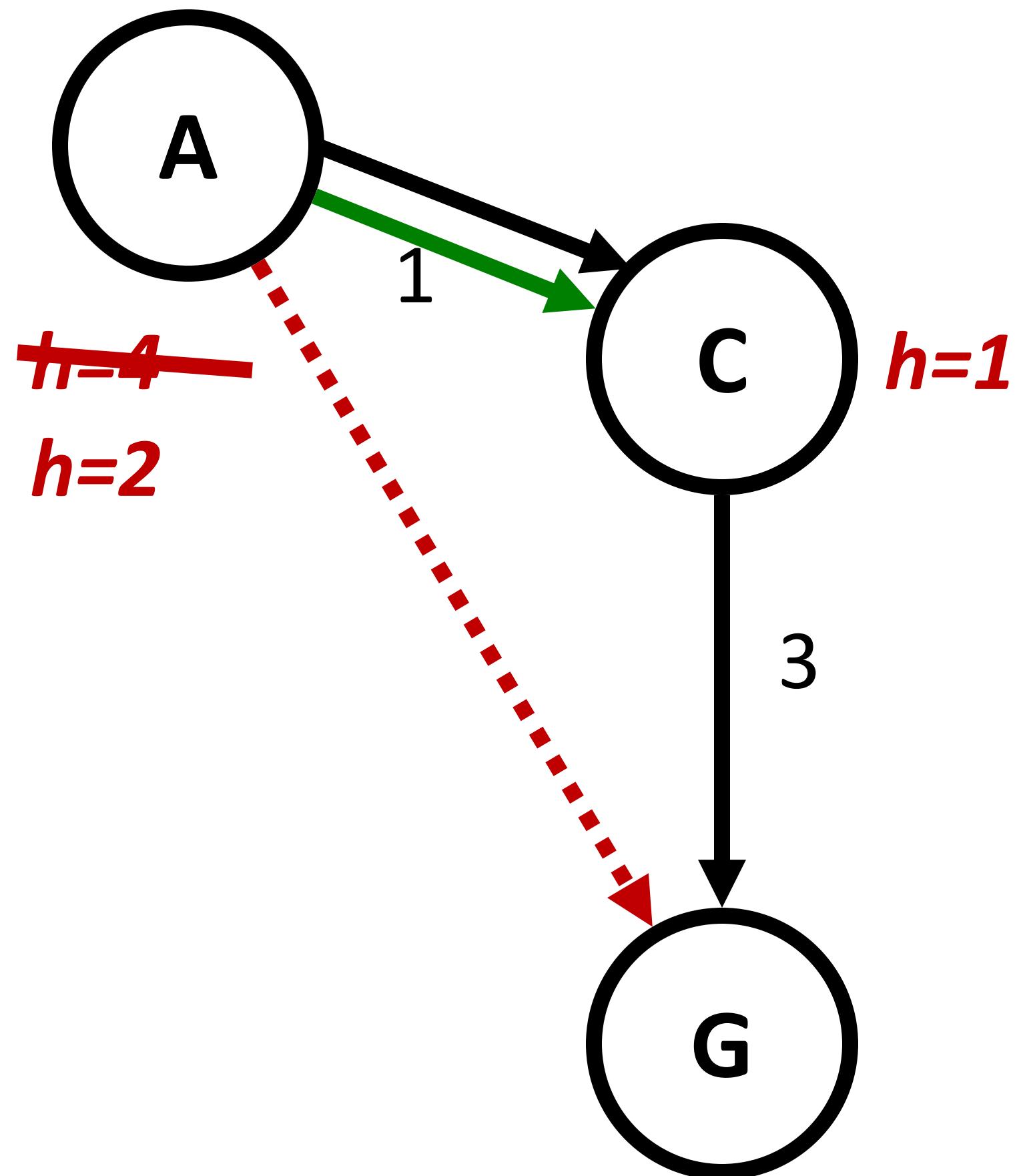
A* GRAPH SEARCH GONE WRONG?



$f(n)$ is
decreasing



CONSISTENCY OF HEURISTICS



- ▶ Main idea: estimated heuristic costs \leq actual costs
- ▶ Admissibility: heuristic cost \leq actual cost to goal

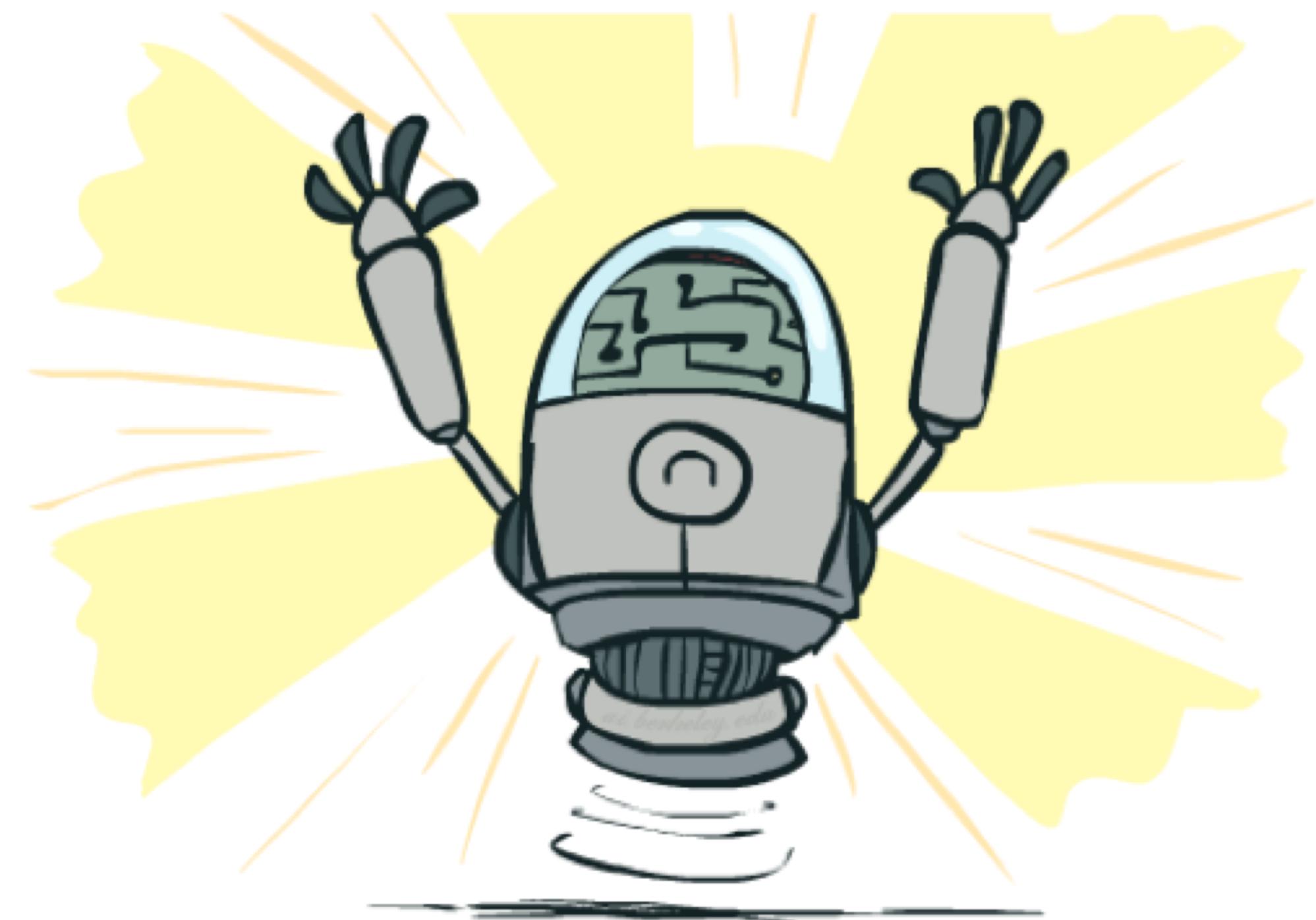
$$h(A) \leq \text{actual cost from } A \text{ to } G$$
- ▶ Consistency: heuristic "arc" cost \leq actual cost for each arc

$$h(A) - h(C) \leq \text{cost}(A \text{ to } C)$$

$$h(A) \leq \text{cost}(A \text{ to } C) + h(C)$$
- ▶ Consequences of consistency:
 - ▶ The f value along a path never decreases
 - ▶ A* graph search is optimal

OPTIMALITY

- ▶ Tree search:
 - ▶ A* is optimal if heuristic is admissible
 - ▶ UCS is a special case ($h = 0$)
- ▶ Graph search:
 - ▶ A* optimal if heuristic is consistent
 - ▶ UCS optimal ($h = 0$ is consistent)
- ▶ Consistency implies admissibility
- ▶ In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

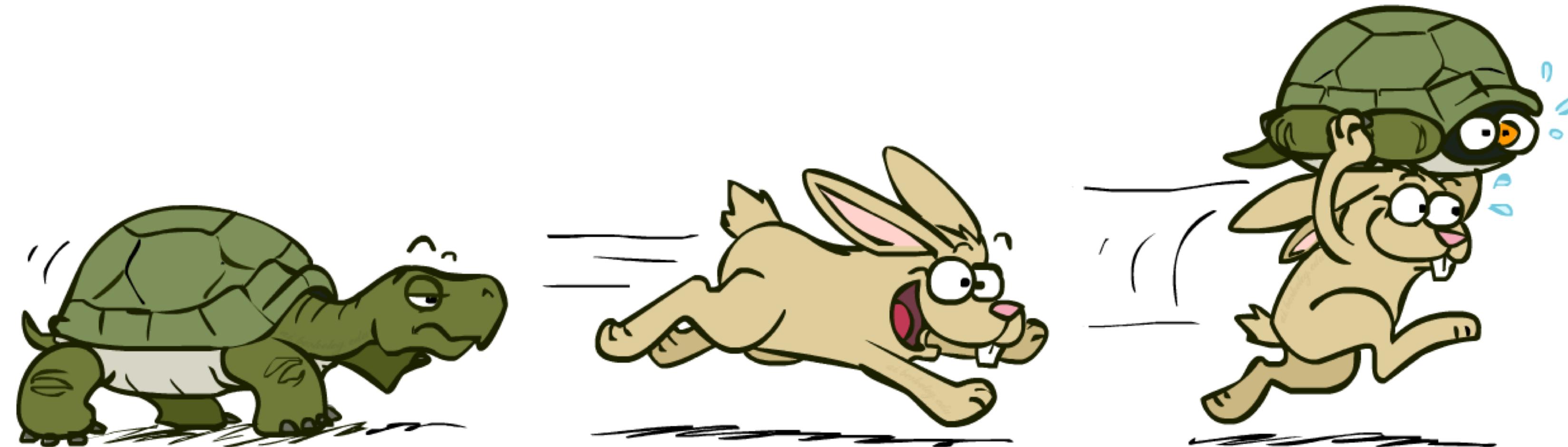


A*: SUMMARY



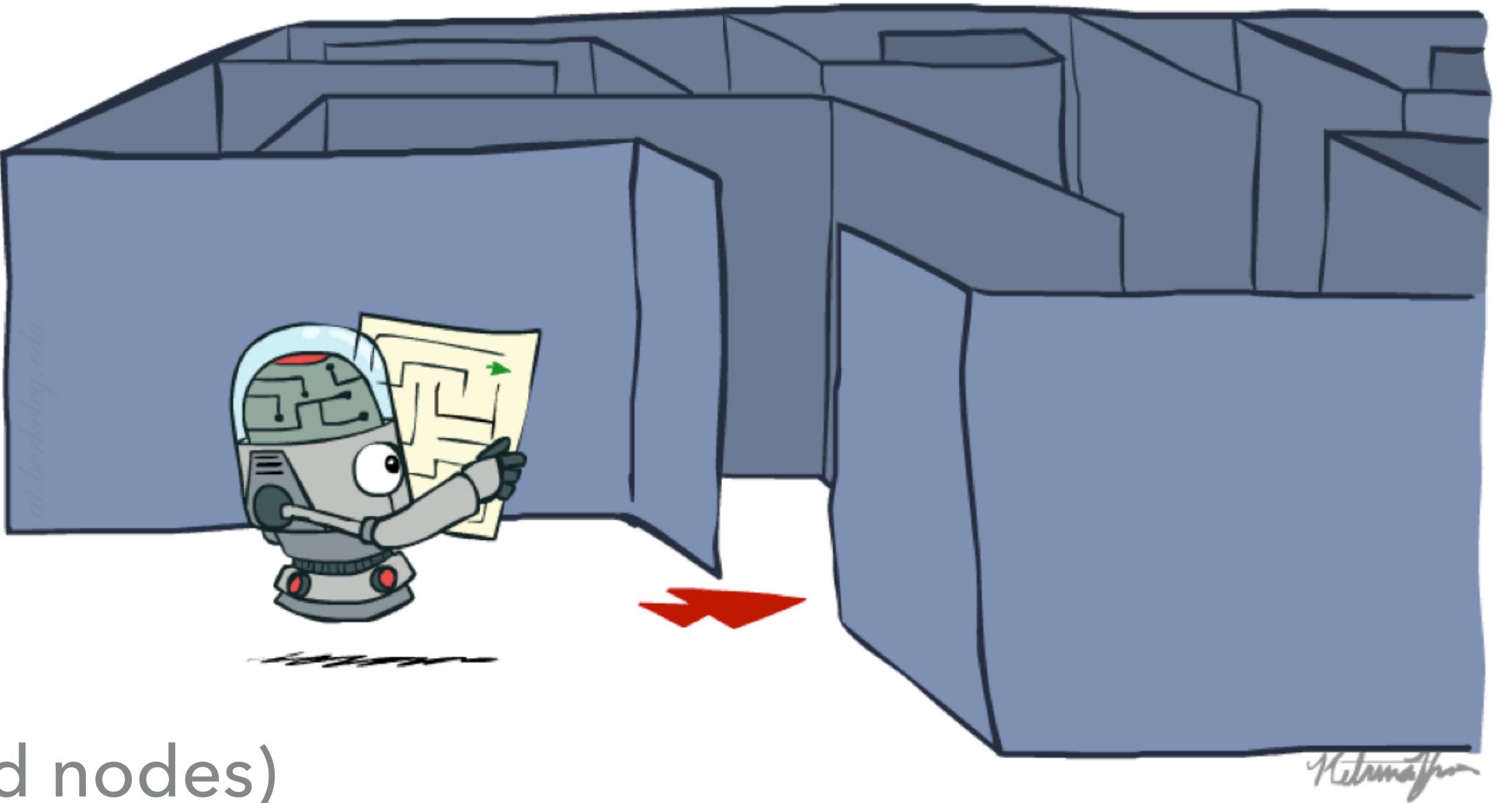
A*: SUMMARY

- ▶ A* uses both backward costs and (estimates of) forward costs
- ▶ A* is optimal with admissible / consistent heuristics
- ▶ Heuristic design is key: often use relaxed problems



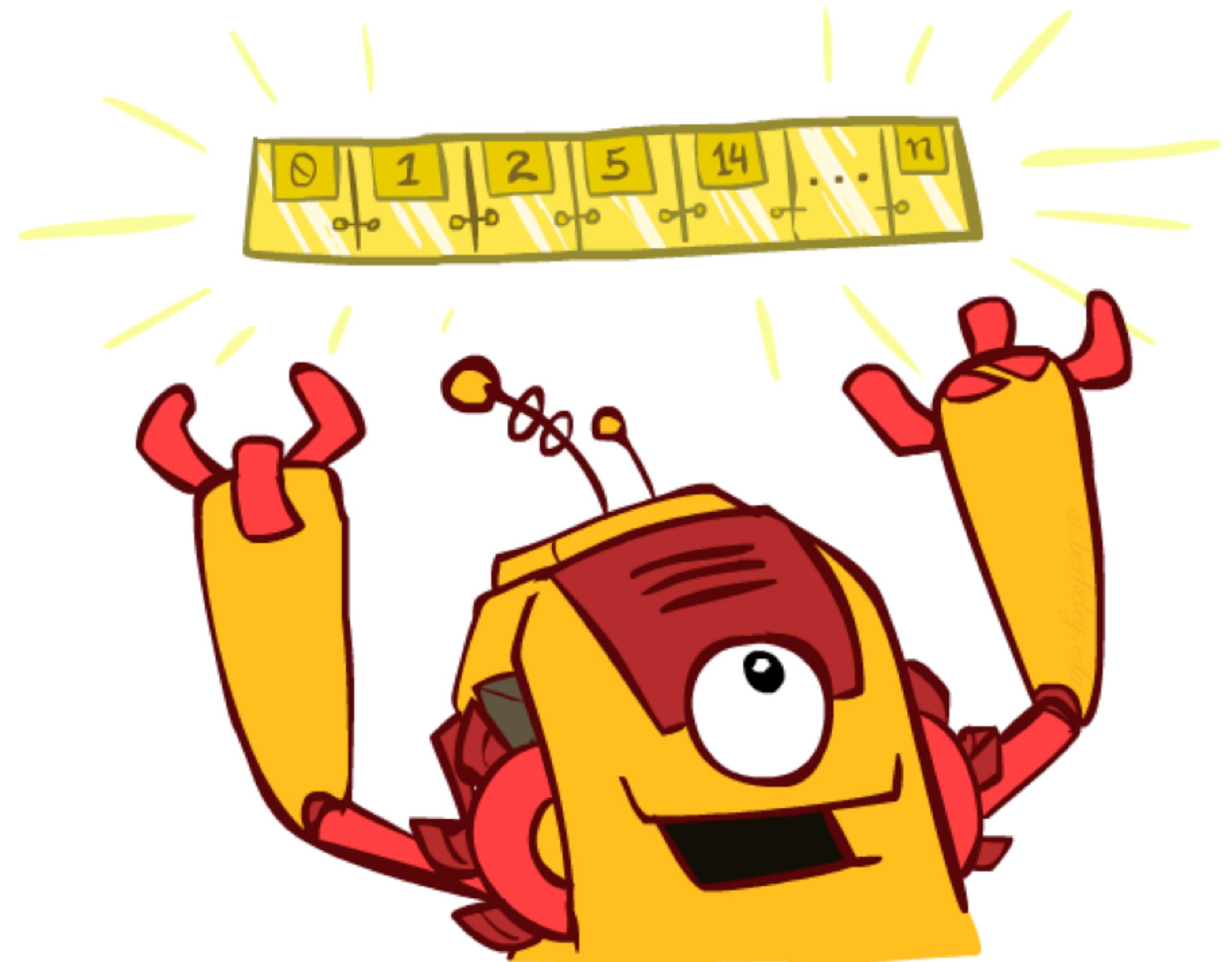
RECAP: SYSTEMATIC SEARCH

- ▶ Search tree:
 - ▶ Nodes: represent partial plans
 - ▶ Plans have costs (sum of action costs)
- ▶ Search algorithm:
 - ▶ Systematically builds a search tree
 - ▶ Chooses an ordering of the fringe (unexplored nodes)
 - ▶ Optimal: finds least-cost plans



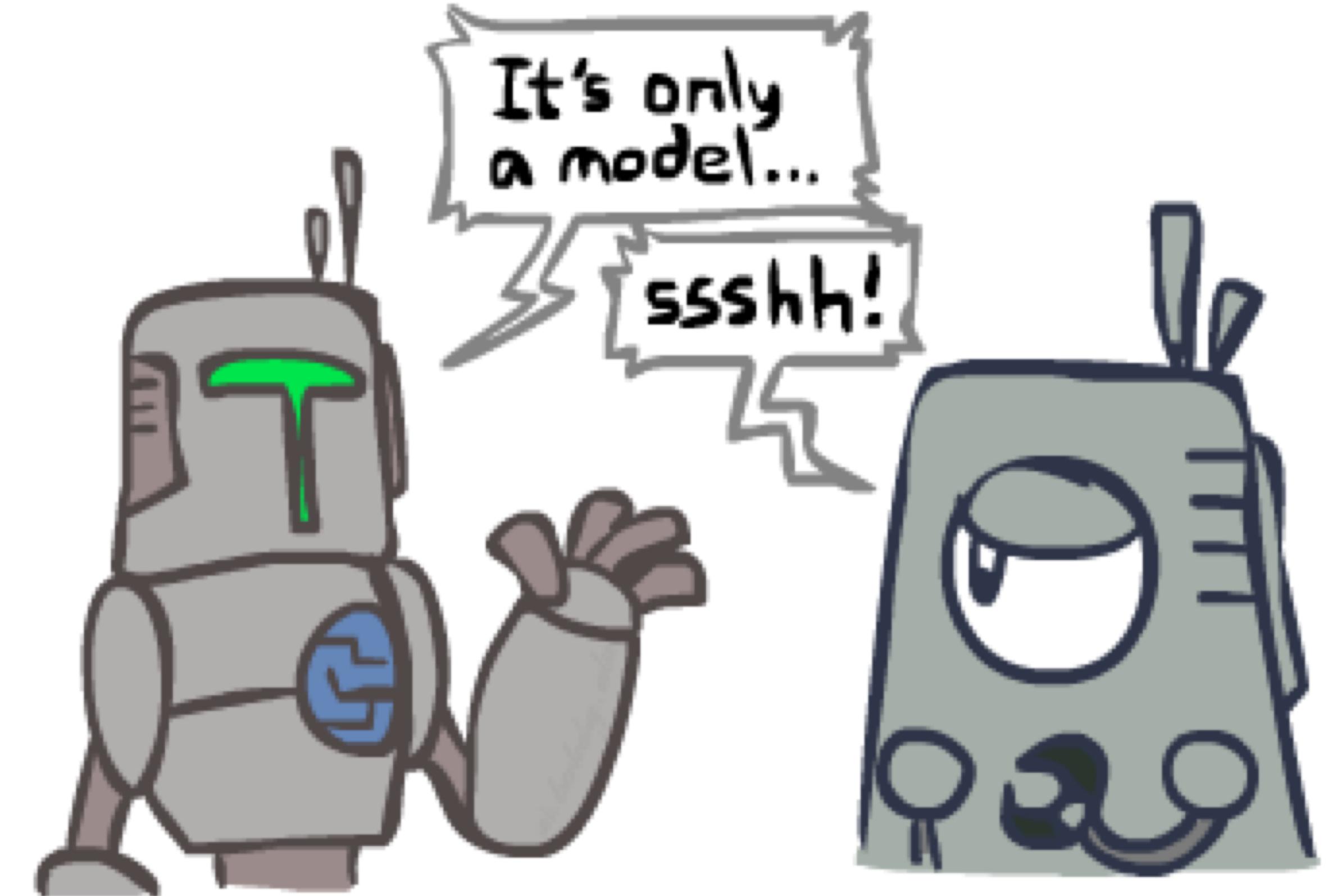
ONE SEARCH QUEUE

- ▶ All these search algorithms are the same except for fringe strategies
 - ▶ Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
 - ▶ Practically, for DFS and BFS, you can avoid the $\log(n)$ overhead from an actual priority queue, by using stacks and queues
 - ▶ Can even code one implementation that takes a variable queuing object



SEARCH AND MODELS

- ▶ Search operates over models of the world
 - ▶ The agent doesn't actually try all the plans out in the real world
 - ▶ Planning is all "in simulation"
 - ▶ Your search is only as good as your models...



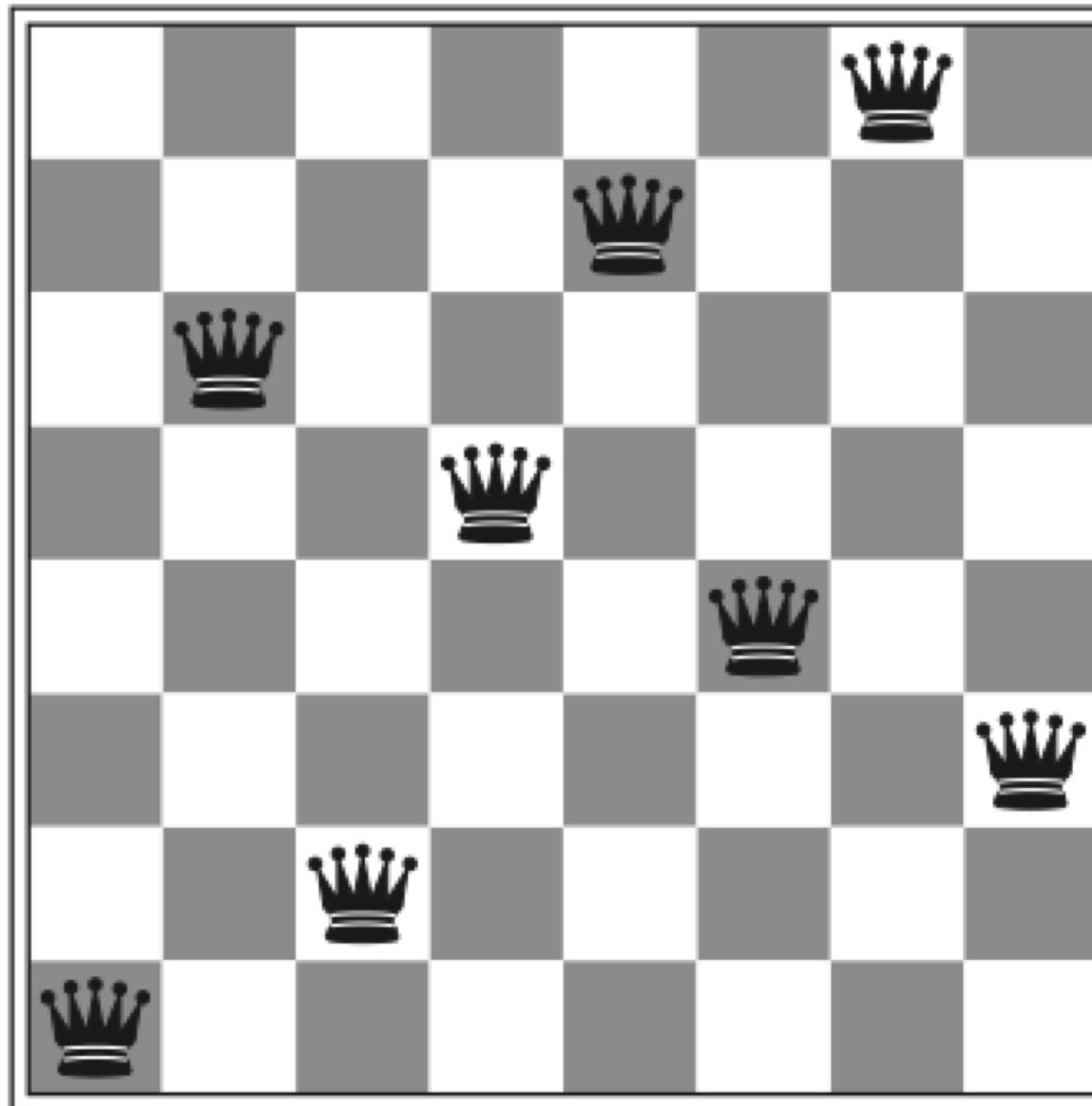
FROM PATH-BASED TO LOCAL SEARCH

- ▶ So far we have considered search methods that:
 - ▶ Systematically explore the full search space
 - ▶ Return the path from start node to the goal
- ▶ But in many problems, path is irrelevant; the goal state itself is the only thing you care about. Example applications:
 - ▶ 8-queens problems
 - ▶ Sometimes the goal test itself is unclear, e.g., you are really solving an optimization problem. Example applications:
 - ▶ Traveling salesperson

LOCAL SEARCH

- ▶ Useful when path to goal state does not matter / solving pure optimization problem
- ▶ Basic idea:
 - ▶ Only keep a single “current” state
 - ▶ Try to improve iteratively
 - ▶ Don’t save paths followed
- ▶ Characteristics
 - ▶ Low memory requirements—usually constant
 - ▶ Effective—can often find good solutions in extremely large state spaces

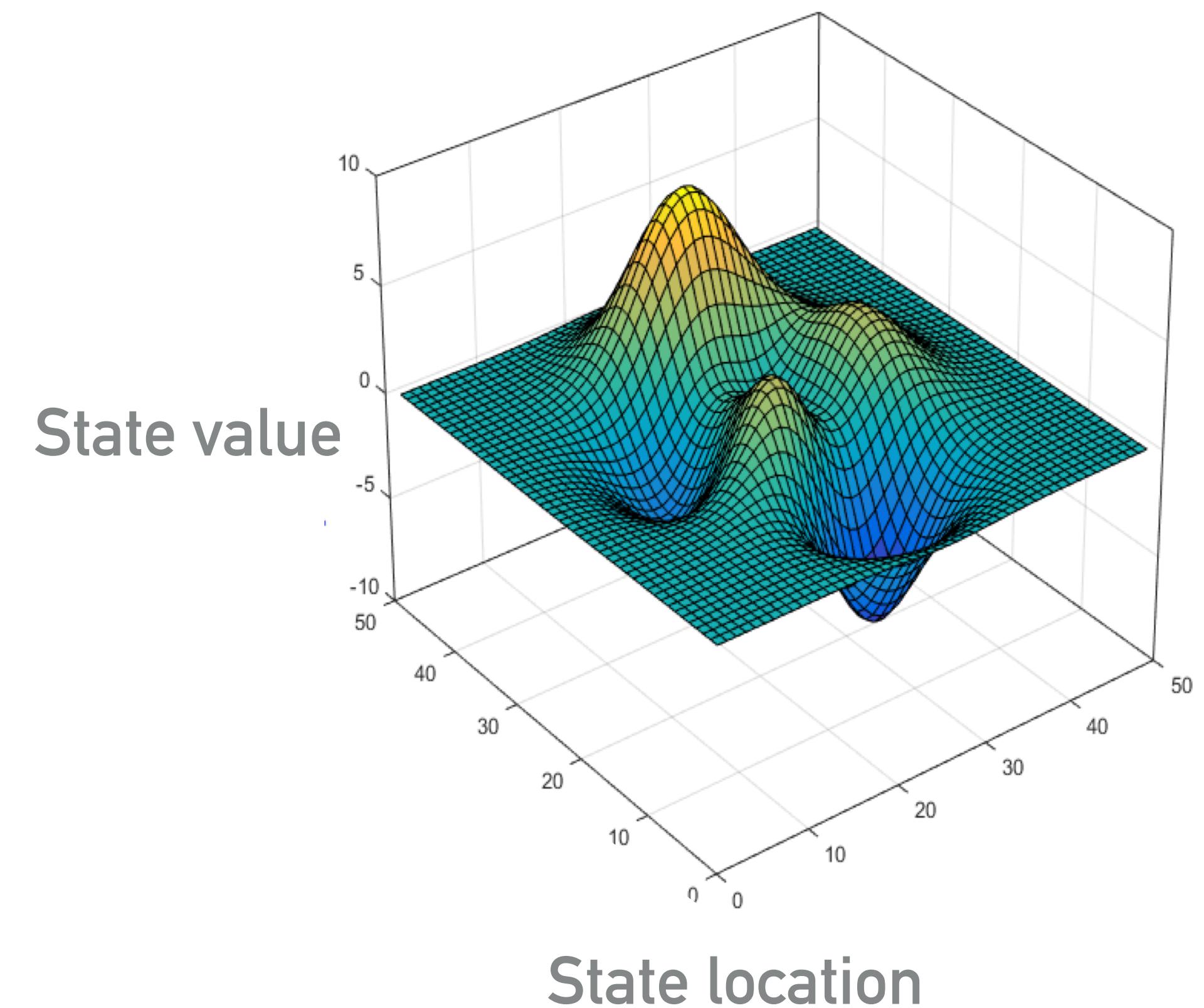
EXAMPLE: EIGHT-QUEENS PROBLEM



- ▶ Place eight queens on a chessboard so that no queen can attack another
 - ▶ Queen can move in the same row, column, or diagonally
- ▶ How to formulate search?
 - ▶ Incremental formulation
 - ▶ **Complete-state formulation**
- ▶ Heuristic/Objective: number of pairs of attacking queens

HILL-CLIMBING SEARCH

- ▶ Consider state-space landscape where:
 - ▶ Location=state
 - ▶ Elevation=evaluation of state (objective function value)
- ▶ Continually move in direction of increasing value (if trying to maximize objective function)



HILL-CLIMBING SEARCH

```
function HILL_CLIMBING (problem) return a state that is a local maximum
    current ← initial_state;
    repeat:
        best_neighbor ← current
        for state in Neighbors(current):
            if state.value > best_neighbor.value:
                best_neighbor ← state
            if best_neighbor.value > current.value:
                current ← best_neighbor
        else:
            return current
```

HILL-CLIMBING EXAMPLE: EIGHT-QUEENS PROBLEM

Heuristic/Objective:
number of pairs of
attacking queens

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	14	13	16	13	16
14	14	17	15	14	16	16	16
17	14	16	18	15	14	15	15
18	14	14	15	15	14	16	16
14	14	13	17	12	14	12	18

CHALLENGES FOR HILL-CLIMBING

- ▶ Local maxima
 - ▶ Once a local maximum is reached, there is no way to backtrack or move out of that maximum



Image: http://www.ndsu.nodak.edu/instruct/juell/vp/cs724s00/hill_climbing/

CHALLENGES FOR HILL-CLIMBING

- ▶ Plateaux
 - ▶ Hillclimbing can have difficult time finding its way off of a flat portion of the state space landscape



Image: http://www.ndsu.nodak.edu/instruct/juell/vp/cs724s00/hill_climbing/

CHALLENGES FOR HILL-CLIMBING

- ▶ Ridges
 - ▶ Ridges can produce a series of local maxima that are difficult to navigate out of

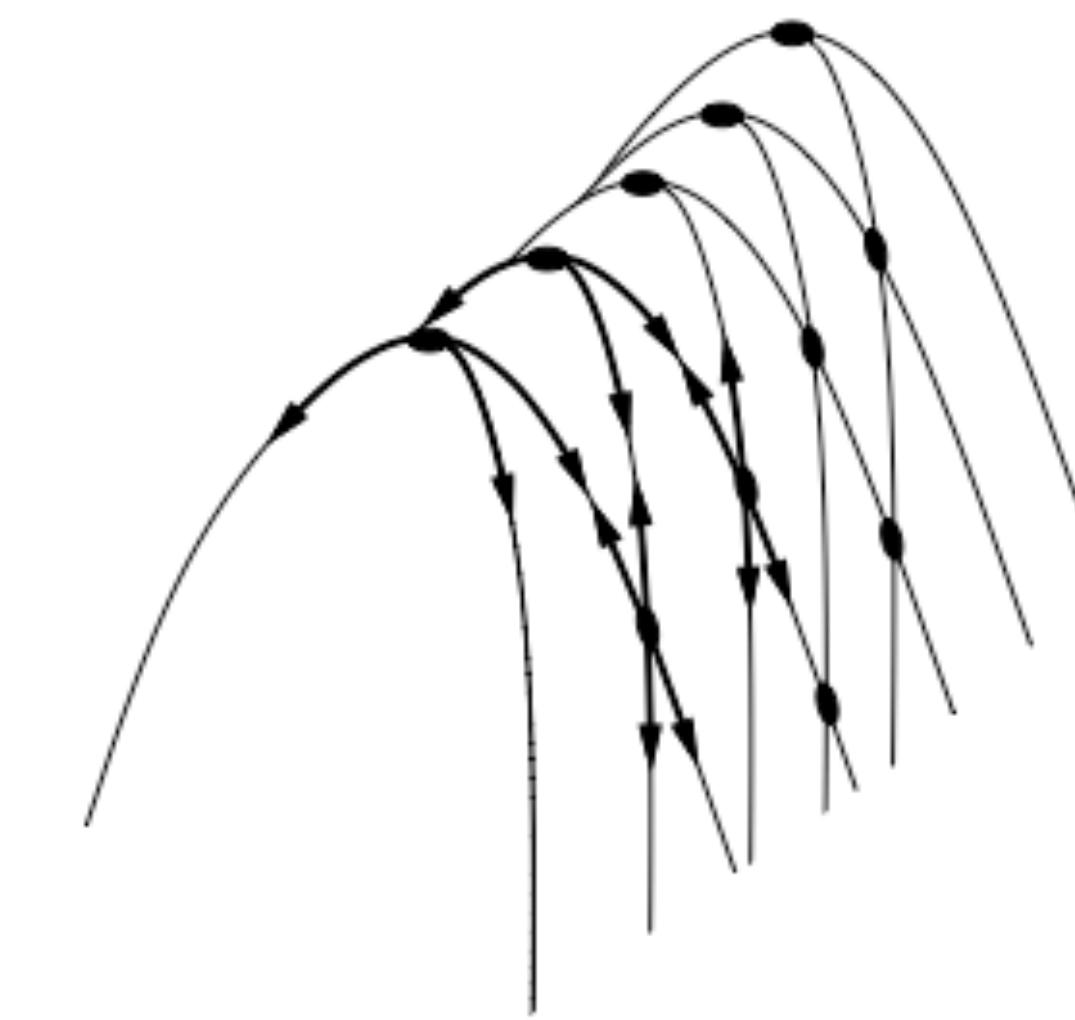


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STATE SPACE LANDSCAPES IN PRACTICE (FOR NEURAL NETWORKS)

