Computer vision - week 6

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Ex 1.a

$$E = \sum_{i=1}^{n} \| x_i' - M x_i - t \|^2$$

$$= \sum_{i=0}^{n} \left\| \begin{bmatrix} x_i' \\ y_i' \end{bmatrix} - \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} - \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \right\|^2$$

$$= \sum_{i=0}^{n} \left\| \begin{bmatrix} x_i' \\ y_i' \end{bmatrix} - \begin{bmatrix} m_1 x_i + m_2 y_i \\ m_3 x_i + m_4 y \end{bmatrix} - \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \right\|^2$$

$$\frac{\partial E}{\partial m_1} = \sum_{i=0}^{n} 2x_i \left(x_i' - m_1 x_i + m_2 y_i - t_1 \right)$$

$$\frac{\partial E}{\partial m_2} = \sum_{i=0}^{n} 2y_i \left(x_i' - m_1 x_i + m_2 y_i - t_1 \right)$$

$$\frac{\partial E}{\partial m_3} = \sum_{i=0}^{n} 2x_i \left(y_i' - m_3 x_i - m_4 y_i - t_2 \right)$$

$$\frac{\partial E}{\partial m_4} = \sum_{i=0}^{n} 2y_i \left(y_i' - m_3 x_i - m_4 y_i - t_2 \right)$$

$$\frac{\partial E}{\partial t_1} = \sum_{i=0}^{n} 2 \left(x_i' - m_1 x_i + m_2 y_i - t_1 \right)$$

$$\frac{\partial E}{\partial t_2} = \sum_{i=0}^{n} 2 \left(y_i' - m_3 x_i - m_4 y_i - t_2 \right)$$

Ex 1.b $\nabla E = \frac{\partial E}{\partial m_1} + \frac{\partial E}{\partial m_2} + \frac{\partial E}{\partial m_3} + \frac{\partial E}{\partial m_4} + \frac{\partial E}{\partial t_1} + \frac{\partial E}{\partial t_2}$

$\nabla E = 0$

Changing to form $Sh = u$

$$\begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & 0 & 0 & \sum_{i=1}^{n} x_i & 0 \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & 0 & 0 & \sum_{i=1}^{n} y_i & 0 \\ 0 & 0 & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & 0 & \sum_{i=1}^{n} x_i \\ 0 & 0 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & 0 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i & 0 & 0 & n & 0 \\ 0 & 0 & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & 0 & n \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i x_i' \\ \sum_{i=1}^{n} y_i x_i' \\ \sum_{i=1}^{n} x_i y_i' \\ \sum_{i=1}^{n} y_i y_i' \\ \sum_{i=1}^{n} x_i' \\ \sum_{i=1}^{n} y_i' \end{bmatrix}$$

Ex 1.c

	x_i y_i		x_i' y_i'
$i=1$	$(0,0)$	$\to$	$(1,2)$
$i=2$	$(1,0)$	$\to$	$(3,2)$
$i=3$	$(0,1)$	$\to$	$(1,4)$

$\sum_{i=1}^{n} x_i x_i' = 3$

$\sum_{i=1}^{n} y_i x_i' = 1$

$\sum_{i=1}^{n} x_i y_i' = 2$

$\sum_{i=1}^{n} y_i y_i' = 4$

$\sum_{i=1}^{n} x_i' = 5$

$\sum_{i=1}^{n} y_i' = 8$

$\sum_{i=1}^{n} x_i^2 = 1$ $\quad \sum_{i=1}^{n} x_i y_i = 0$ $\quad \sum_{i=1}^{n} y_i = 1$

$\sum_{i=1}^{n} y_i^2 = 1$ $\quad \sum_{i=1}^{n} x_i = 1$ $\quad n = 3$

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{vmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \\ 5 \\ 8 \end{bmatrix} \Rightarrow \begin{aligned} m_1 &= -\frac{6}{7} \\ m_2 &= \frac{8}{7} \\ m_3 &= 0 \\ m_4 &= 2 \\ m_5 &= \frac{11}{7} \\ m_6 &= 2 \end{aligned}$$

Ex 2.a

$$V' = X_2' - \cancel{X}_1' \qquad \text{and} \qquad V = \cancel{X}_2 - X_1 = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \end{vmatrix} = \begin{vmatrix} V_x \\ V_y \end{vmatrix}$$

$$= \begin{vmatrix} x_2' - x_1' \\ y_2' - y_1' \end{vmatrix} = s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = sMv$$

$$\cos\theta = \frac{V'V}{\|V'\| \, \|V\|} = \frac{(x_2'-x_1')(x_2-x_1) + (y_2'-y_1')(y_2-y_1)}{\sqrt{(x_2'-x_1')^2 + (y_2'-y_1')^2 \cdot (x_2-x_1)^2 + (y_2-y_1)^2}}$$

$$\theta = \cos^{-1}\left(\frac{(x_2'-x_1')(x_2-x_1) + (y_2'-y_1')(y_2-y_1)}{\sqrt{(x_2'-x_1')^2 + (y_2'-y')^2 \cdot (x_2-x_1)^2 + (y_2-y_1)^2}} \right)$$

Ex 1. b

$$V' = sMV$$

$$s = \frac{\|V'\|}{\|V\|} = \frac{\sqrt{(x_2'-x_1')^2 \cdot (y_2-y_1)^2}}{\sqrt{(x_2-x_1)^2 \cdot (y_2-y_1)^2}}$$

Ex 1. c

$$\begin{cases} x' = s \cdot \cos\theta (x_2 - x_1) - s \sin\theta (y_2 - y_1) + t_x \\ y' = s \cdot \sin\theta (x_2 - x_1) - s \cos\theta (y_2 - y_1) + t_y \end{cases}$$

$$t_x = x' - s \cdot \cos\theta (x_2 - x_1) + s \sin\theta (y_2 - y_1)$$
$$t_y = y' - s \cdot \sin\theta (x_2 - x_1) + s \cos\theta (y_2 - y_1)$$

Ex 2.2

$$\{(\tfrac{1}{2},0) \rightarrow (0,0)\} \rightarrow V = \begin{pmatrix} 0 - \frac{1}{2} \\ \frac{1}{2} - 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

with $(x_1, y_1) = (\tfrac{1}{2},0)$, $(x_1', y_1') = (0,0)$

$$\{(0,\tfrac{1}{2}) \rightarrow (-1,-1)\} \rightarrow V' = \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} V_x' \\ V_y' \end{pmatrix}$$

with $(x_2, y_2) = (0,\tfrac{1}{2})$, $(x_2', y_2') = (-1,-1)$

$$\theta = \cos^{-1}\left(\frac{(x_2'-x_1')(x_2-x_1) + (y_2'-y_1')(y_2-y_1)}{\sqrt{[(x_2'-x_1')^2+(y_2'-y_1')^2]\cdot[(x_2-x_1)^2+(y_2-y_1)^2]}}\right)$$

$$V = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \end{vmatrix} = \begin{pmatrix} V_x \\ V_y \end{pmatrix} \qquad V' = \begin{vmatrix} x_2' - x_1' \\ y_2' - y_1' \end{vmatrix} = \begin{pmatrix} V_x' \\ V_y' \end{pmatrix}$$

$$\theta = \cos^{-1}\left(\frac{V_x' V_x + V_y' V_y}{\sqrt{(V_x')^2+(V_y')^2 \cdot V_x^2+V_y^2}}\right) = \frac{\pi}{2}$$

$$s = \frac{\|V'\|}{\|V\|} = 2$$

$$t_x = 0 - \left(2 \cdot 0 \cdot \tfrac{1}{2}\right) + (2 \cdot 0 \cdot 1) = 0$$

$$t_y = 0 - \left(2 \cdot 1 \cdot \tfrac{1}{2}\right) + (2 \cdot 0 \cdot 0) = -1$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = 2 \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$