Computer vision - Week 5

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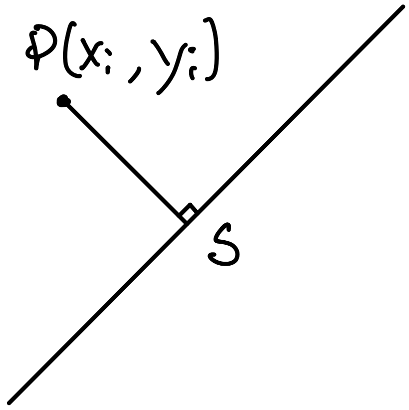
Ex 1.1

$$ax + by - d = 0 \qquad a^2 + b^2 = 1$$



$l = ax + by - d$

PS is perpendicular to line l

$$K_{PS} \cdot K_l = -1 \Rightarrow K_{PS} = \frac{b}{a}$$

$$\Rightarrow \quad y - y_i = \frac{b}{a}(x - x_i)$$

Itersection between line l and PS

$$\left(\frac{b^2 x_i - ab y_i + ad}{a^2 + b^2} , \frac{a^2 y_i - ab x_i + bd}{a^2 + b^2} \right)$$

$$\|PS\| = \sqrt{\left(\frac{b^2 x_i - ab y_i + ad}{a^2 + b^2} - x_i\right)^2 + \left(\frac{a^2 y_i - ab x_i + bd}{a^2 + b^2} - y_i\right)^2}$$

$$= \frac{|a x_i + b y_i - d|}{\sqrt{a^2 + b^2}}$$

when $a^2 + b^2 = 1$

$$\Rightarrow |a x_i + b y_i - d|$$

Ex 1.2

$$(x_i, y_i) \quad i = 1, \dots, n$$

$$E = \sum_{i=1}^{n} (ax_i + by_i + d)^2$$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - c) = 0$$

$$nc = \sum_{i=1}^{n} ax_i + by_i$$

$$c = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\underline{c = a\bar{x} + b\bar{y}}$$

Ex 1.3

$$E = \sum_{i=1}^{n} (ax_i + by_i + d)^2$$

substituting d

$$E = \sum_{i=1}^{n} (ax_i + by_i - a\bar{x} - b\bar{y})^2$$

$$E = \sum_{i=1}^{n} \left(a(x_i - \bar{x}) + b(y_i - \bar{y})\right)^2$$

$$\left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ x_2 - \bar{x} & y_2 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = ((a\ b)U)^T(ab\,U)$$

$$\Rightarrow (a \;\; b) U^T U (a \;\; b)^T$$

Ex 1.4

$$E = (a \;\; b) \; U^T U (a \; b)^T$$

$$= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{dE}{dN} = 2(U^T U) \begin{bmatrix} a & b \end{bmatrix} = 0$$

$$(U^T U) \begin{bmatrix} a & b \end{bmatrix} = 0 \qquad \| a \;\; b \|^2 = 1$$

Eigen vector $U^T U$ associated with smallest eigenvalue, Least square solution to homogenous linear system $UN = 0$