CS-E4850
Edvard Ohlström
729 255

Ex 1.a

$$ax + by + c = 0$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \qquad l = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\begin{pmatrix} x & y & 1 \end{pmatrix} l = 0 \qquad \begin{pmatrix} x & y & 1 \end{pmatrix} = x$$

$$x^T l = 0$$

Ex 1.b

$$l = \begin{pmatrix} a & b & c \end{pmatrix}^T$$
$$l' = \begin{pmatrix} d & e & f \end{pmatrix}^T$$

$$l \times l' = \begin{pmatrix} ec - fb \\ fa - dc \\ db - ea \end{pmatrix}^T \Rightarrow l' \cdot (l \times l') = d(ec - fb) + e(fa - dc) + f(db - ea)$$

$$= dec - dfb + efa - edc + fdb - fea = 0$$

$$l' \times l = \begin{pmatrix} bf - ce \\ dc - fa \\ ea - db \end{pmatrix}^T \Rightarrow l \cdot (l' \times l) = a(bf - ce) + b(dc - fa) + c(ea - db)$$

$$= abf - ace + bdc - bfa + cea - cdb$$

$$= 0$$

Therefore one can conclude there is a point x that intersects the line l and l'

Ex1.c

x and x' lie on the same line, then
$x^T l = 0$ and $x'^T l = 0$.
Considering triple scalar product $x(x' \times x) = x'(\cancel{x} \times x')$, we can see $l = x \times x'$

Ex1.d

$$y^T l = 0$$

$$y^T (x \times x') = 0$$

$$(\alpha x + (1-\alpha) x')^T (x \times x') = 0$$

$$\alpha x^T (x \times x') + (1-\alpha) x'^T (x \times x') = 0$$

Accordingly to $x(x' \times x) = x'(x \times x')$ theorem
y lies on the line

Ex 2 a

2D translation can be written as

$$X_c = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} X_w$$

Euclidean transformation (rotation + translation)

$$X_c = \begin{bmatrix} \cos\alpha & -\sin\alpha & t_1 \\ \sin\alpha & \cos\alpha & t_2 \\ 0 & 0 & 1 \end{bmatrix} X_w$$

Similarity transformation (rotation + translation + scalling)

$$X_c = s \begin{bmatrix} \cos\alpha & -\sin\alpha & t_1 \\ \sin\alpha & \cos\alpha & t_2 \\ 0 & 0 & 1 \end{bmatrix} X_w$$

Affine transformation

$$x^c = Ax$$

$$x^c = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ 0 & 0 & 1 \end{bmatrix} X^w$$

Projective transformation

$$x_c = H x_w$$

$$x^c = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & 1 \end{bmatrix} x^w$$

Ex.2.b

translation	2	(t_1, t_2)
Euclidean	3	(t_1, t_2, θ)
similarity	4	(t_1, t_2, θ, s)
affaine	6	$(a_{00}, a_{01}, a_{10}, a_{11}, a_{02}, a_{20})$
projective	8	

Ex.2.c

Because there is one element in the 3x3 matrix that is number 1. It means this will not change. Therefore there are 8 elements instead of 9 which will affect the picture

Ex 3.a

H transforms points

$$l'^T H^{-1} \underbrace{Hx} = 0$$

All points lie on the line $l'^T H^{-1}$

$\Longrightarrow$

line transformation

$$\underline{l'^T H^{-1}}$$

Ex 3 b