Week. 2
CS-E4850

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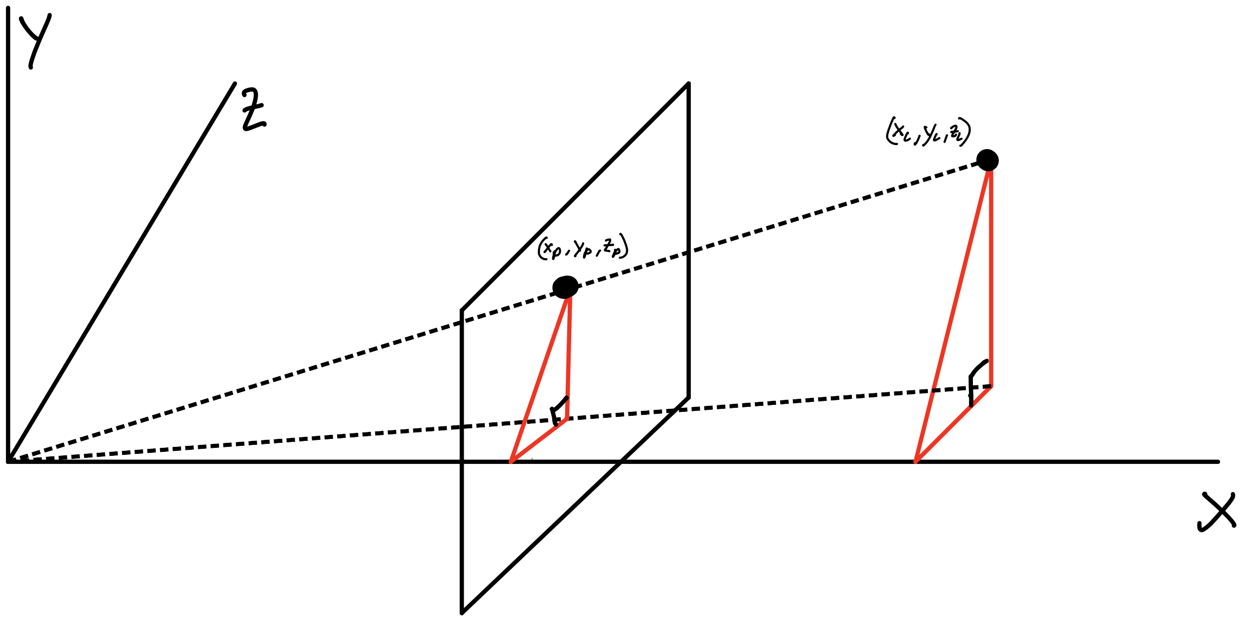
Ex 1.



$$\frac{X_c}{Z_c} = \frac{X_p}{f} \Rightarrow x_p = f\,\frac{X_c}{Z_c}, \quad \frac{Y_c}{Z_c} = \frac{Y_p}{f} \Rightarrow y_p = f\,\frac{Y_c}{Z_c}$$

When $z_p = f$

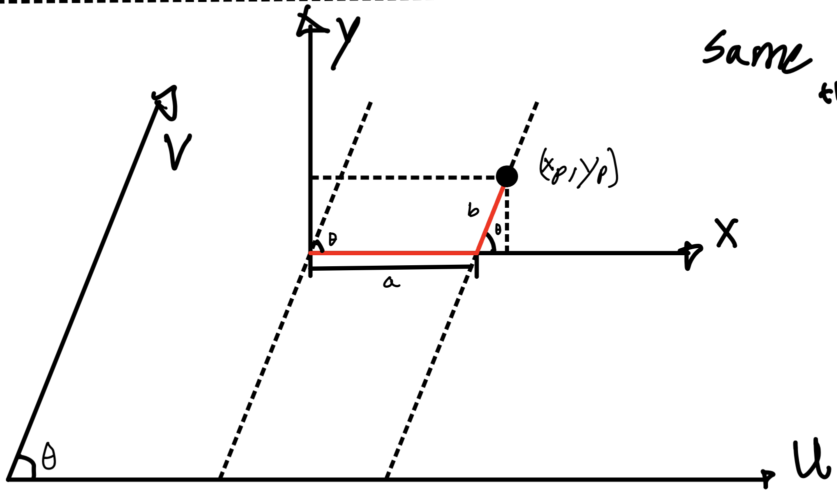
Ex. 2a Transforming image coordinates to pixel coordinates

$u \parallel x$
$v \parallel y$

$$\begin{bmatrix} m_u & 0 & 0 \\ 0 & m_v & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f\frac{X_c}{Z_c} + p_x \\ f\frac{Y_c}{Z_c} + p_y \\ 1 \end{bmatrix} = \begin{bmatrix} m_u X_p + m_u p_x \\ m_v Y_p + m_v p_y \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ V \\ 1 \end{bmatrix}$$

$$\Rightarrow \quad U = m_u X_p + U_0$$
$$V = m_v Y_p + V_0$$

Ex.2.b



Same as Ex 2a but with these constraints

$$x \parallel u$$

$$v, u \measuredangle = \theta$$

$$\sin\theta = \frac{y_p}{b} \Rightarrow b = \frac{y_p}{\sin\theta}$$

$$\tan\theta = \frac{y_p}{x_p - a} \Rightarrow a = x_p - \frac{y_p}{\tan\theta}$$

$$u = m_u a + u_0 \Rightarrow m_u\left(x_p - \frac{y_p}{\tan\theta}\right) + u_0$$

$$v = m_v b + v_0 \Rightarrow m_v \frac{y_p}{\sin\theta} + v_0$$

Ex 3.

$$K_{3\times3}\begin{bmatrix} x_c \\ y_c \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

using homogeneous to represent Ex.2.a

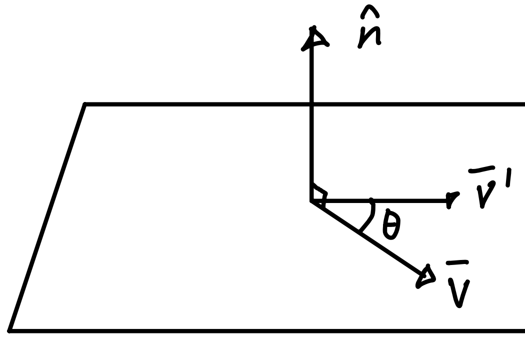
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} m_u & 0 & u_0 \\ 0 & m_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} = \begin{bmatrix} m_u & 0 & u_0 \\ 0 & m_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} f x_c \\ f y_c \\ f z_c \end{bmatrix}$$

Ex 4 Real world => pixel camera matrix

$$\begin{vmatrix} u \\ v \\ 1 \end{vmatrix} = K X_c = K(R x_w + t) =$$

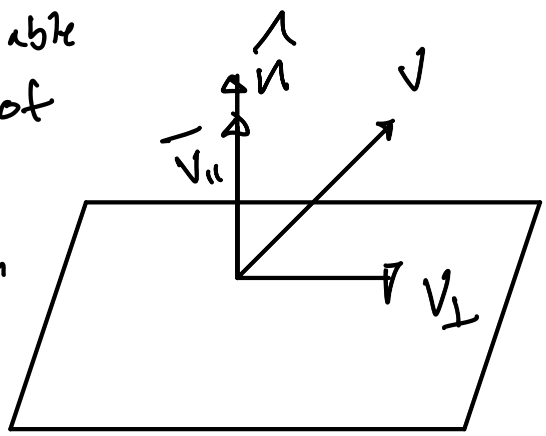
$$K \begin{bmatrix} R & t \end{bmatrix} \begin{bmatrix} x_w \\ 1 \end{bmatrix} = P \begin{bmatrix} x_w \\ 1 \end{bmatrix}$$

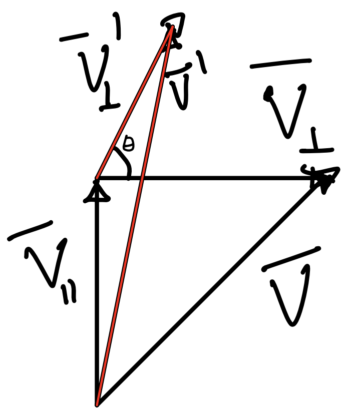
Ex 5.a



$$\bar{V}' = \cos\theta\,\bar{V} + \sin\theta(\hat{n} \times v)$$

Using variable V instead of X for less confussion





$$\bar{V} = \bar{V}_{\parallel} + \bar{V}_{\perp} \qquad \bar{V}' = \bar{V}_{\parallel} + \bar{V}_{\perp}'$$

$$\bar{V}' = \bar{V}_{\parallel} + \cos(\theta)\,\bar{V}_{\perp} + \sin\theta\,(\hat{n} \times \bar{V}_{\perp})$$

$$\bar{V}' = \bar{V}_{\parallel} + \cos\theta\,(\bar{V} - \bar{V}_{\parallel}) + \sin\theta\,(\hat{n} \times \bar{V})$$

$$\bar{v}' = (1 - \cos\theta)\,\bar{v}_{\parallel} + \cos\theta\,\bar{v} + \sin\theta\,(\hat{n} \times \bar{v})$$

$$\hat{n} \times \bar{V} = \hat{n} \times (\bar{V}_{\parallel} + \bar{V}_{\perp})$$
$$= \underbrace{\cancel{\hat{n} \times \bar{V}_{\parallel}}}_{0} + \hat{n} \times \bar{V}_{\perp}$$

$$\bar{V}_{\parallel} = (\bar{V} \cdot \hat{n})\,\hat{n}$$

Rodriges formula

$$\bar{V}' = (1 - \cos\theta)(\bar{V} \cdot \hat{n})\,\hat{n} + \cos\theta\,\bar{v} + \sin\theta\,(\hat{n} \times \bar{v})$$

$$\bar{x}' = (1 - \cos\theta)(\bar{x} \cdot \hat{u})\,\hat{u} + \cos\theta\,\bar{x} + \sin\theta\,(\hat{u} \times \bar{x})$$

simillar variable names as exercise

Ex 5.b

$$u \times X = \begin{vmatrix} u_2 X_3 - u_3 X_2 \\ u_3 X_1 - u_1 X_3 \\ u_1 X_2 - u_2 X_1 \end{vmatrix} = \underbrace{\begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x}$$

$\Rightarrow Ux$

Rodriges formula

$$U = I + \sin\theta\, U + (1 - \cos\theta)\, U^2$$