Homework #6 Solutions

Problem 6-1

$$g_a(t) \longrightarrow \boxed{\text{sampling}} \longrightarrow g[n]$$

$$E_{g_a}(t) = \int_{-\infty}^{\infty} |g_a(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G_a(j\Omega)|^2 \, d\Omega$$

by Parseval's relationship

For a bandlimited signal $\Rightarrow$ $E_{g_a}(t) = \frac{1}{2\pi} \int_{-\Omega_m}^{\Omega_m} |G_a(j\Omega)|^2 \, d\Omega$

For the discrete-time signal -

$$E_{g[n]}[n] = \sum_{n=-\infty}^{\infty} |g[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |G(e^{j\omega})|^2 \, d\omega$$

Remember $\omega = \frac{\Omega}{F_s} = \Omega T_s$ $\qquad d\omega = d\Omega \, T_s$

$$G(e^{j\omega}) = G_a(j\Omega)\Big|_{\Omega = \omega/T_s}$$

so,
$$E_{g[n]}[n] = \frac{1}{2\pi} \int_{\Omega = -\pi/T_s}^{\Omega = \pi/T_s} |G(e^{j\Omega T_s})|^2 \, d\Omega \, T_s$$

$$= \frac{T_s}{2\pi} \int_{-\pi/T_s}^{\pi/T_s} \left| \frac{1}{T_s} G(j\Omega) \right|^2 d\Omega = \frac{1}{2\pi T_s} \int_{-\pi/T_s}^{\pi/T_s} |G_a(j\Omega)|^2 \, d\Omega$$

Let $\Omega_m = \pi/T_s$ $\qquad E_{g[n]}[n] = \frac{1}{2\pi T_s} \int_{-\Omega_m}^{\Omega_m} |G_a(j\Omega)|^2 \, d\Omega$

$$= \frac{1}{T_s} E_{g_a}(t) \;/\!/$$

Problem 6-2 $T_s = \frac{2.5}{5000}$ $F_s = \frac{1}{T_s} = 2000\,Hz$

Therefore, highest freq. component in CT signal is $< 1000\,Hz$.

Problem 6-3

$$x_a(t) \longrightarrow \underset{F_s = 2KHz}{\boxed{\text{Sampling}}} \longrightarrow x[n] \longrightarrow \underset{f_c = 900\,Hz}{\boxed{\text{LPF}}} \longrightarrow y_a(t)$$

$$X_p(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a(j(\Omega + k\Omega_T))$$

Positive Freq. only:

$F_1 = 300\,Hz \Rightarrow 300, 1700, 2300$, etc.

$F_2 = 500\,Hz \Rightarrow 500, 1500, 2500$, etc

$F_3 = 1200\,Hz \Rightarrow 1200, 800, 3200$, etc

$F_4 = 2150\,Hz \Rightarrow 2150, 150, 4150$, etc

$F_5 = 3500\,Hz \Rightarrow 3500, 1500, 5500$, etc

After the 900 Hz LPF, only 300, 500, 800, 150 Hz components remain, the rest are attenuated.

Problem 6-3 $\quad y[n] = x[n] + a\,x[n-1]$

Remember, for a distortionless system the frequency response has the form

$$H(e^{j\omega}) = c\,e^{-j\omega n_0}$$

The magnitude response is constant - $|H(e^{j\omega})| = c$
And, the phase response is linear - $\arg\{H(e^{j\omega})\} = -\omega n_0$

Transfer Function - $\quad H(z) = \dfrac{Y(z)}{X(z)} = 1 + a z^{-1}$

Impulse Response - $\quad h[n] = \delta[n] + a\,\delta[n-1]$

Freq. Response - $\quad H(e^{j\omega}) = 1 + a e^{-j\omega}$

Therefore, the system has both amplitude and phase distortion.

$$|H(e^{j\omega})|^2 = (1 - a e^{-j\omega})(1 + a e^{j\omega}) = 1 + 2a\cos(\omega) + a^2$$