## Homework Assignment 3 - Due Saturday, Sept. 17, 2011.

**Problem 3-1**
Let $X(e^{j\omega})$ denote the DTFT of a real sequence $x[n]$.

(a) Show that if x[n] is even, then it can be computed from $X(e^{j\omega})$ using $x[n] = 1/\pi \int_0^\pi X(e^{j\omega}) cos(\omega n) d\omega$.

(b) Show that if x[n] is odd, then it can be computed from $X(e^{j\omega})$ using $x[n] = 1/\pi \int_0^\pi X(e^{j\omega}) sin(\omega n) d\omega$.

**Problem 3-2**
Let $X(e^{j\omega})$ denote the DTFT of a real sequence $x[n]$. Define $Y(e^{j\omega}) = \frac{1}{2}\{X(e^{j\omega/2}) + X(-e^{j\omega/2}\}$. Determine the inverse DTFT $y[n]$ of $Y(e^{j\omega})$.

**Problem 3-3**
Show that the DTFT of the sequence $x[n] = 1$, $-\infty < n < \infty$, is given by $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$.

**Problem 3-4**
Using Parseval's relation, evaluate the following integral: $\int_0^\pi \frac{4}{5+4\cos(\omega)} d\omega$.

**Problem 3-5**
Show that the function $u[n] = z^n$, where $z$ is a complex constant, is an eigenfunction of an LTI discrete-time system. Is $v[n] = z^n u[n]$ with $z$ a complex constant also an eigenfunction of an LTI discrete-time system?

**Problem 3-6**
Determine the input-output relation of a factor-of-L up-sampler in the frequency domain.

**Problem 3-7**
An FIR filter of length 3 is defined by a symmetric impulse response; that is, $h[0] = h[2]$. Let the input to this filter be a sum of two cosine sequences of angular frequencies 0.3 rad/samples and 0.6 rad/samples, respectively. Determine the impulse response coefficients so that the filter passes only the low-frequency component of that input.

**Problem 3-8**
Use Matlab to determine and plot the real and imaginary parts and the magnitude and phase spectra of the following DTFT for $r = 0.9$ and $\theta = 0.75$:

$$G(e^{i\omega}) = \frac{1}{1 - 2r(\cos(\theta))e^{-j\omega} + r^2 e^{-j2\omega}}, 0 < r < 1$$