Homework #5 solutions

Problem 5-1

$$x[n] = \cos\left(\frac{7\pi}{16}n\right) + \cos\left(\frac{9\pi}{16}n\right)$$

$$= \frac{e^{j\frac{7\pi}{16}n}}{2} + \frac{e^{-j\frac{7\pi}{16}n}}{2} + \frac{e^{j\frac{9\pi}{16}n}}{2} + \frac{e^{-j\frac{9\pi}{16}n}}{2}$$

a) compare to IDTFS $\quad x[n] = \sum_{k=0}^{N-1} X[k]\, e^{jk\omega_0 n}$

$\omega_0 = \frac{\pi}{16}$

$$\Rightarrow \quad x[n] = \sum_{k=0}^{N-1} X[k]\, e^{jk\frac{\pi}{16}n}$$

$$= X[7]\, e^{j\frac{7\pi}{16}n} + X[9]\, e^{j\frac{9\pi}{16}n}$$

$$\Rightarrow \quad X[k] = \begin{cases} \frac{1}{2}, & k = 7 \\ 1/2, & k = -7 \\ 1/2, & k = 9 \\ 1/2, & k = -9 \\ 0, & \text{otherwise} \end{cases}$$

(spectrum is symmetric)

b) $x[n] = \sum_{k=0}^{N-1} X[k]\, e^{jk\omega_0 n} \xrightarrow{\text{DTFT}} X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\,\delta(\omega - k\omega_0)$

so, $$X(e^{j\omega}) = \pi\,\delta\left(\omega + \frac{9\pi}{16} + 2\pi k\right) + \pi\,\delta\left(\omega + \frac{7\pi}{16} + 2\pi k\right)$$

$$+ \pi\,\delta\left(\omega - \frac{9\pi}{16} - 2\pi k\right) + \pi\,\delta\left(\omega - \frac{7\pi}{16} - 2\pi k\right)$$

c) $\omega_1 = 2\pi f_1 = \frac{7\pi}{16}$, $\quad \omega_2 = 2\pi f_2 = \frac{9\pi}{16}$

$f_1 = \frac{7}{32} \qquad\qquad f_2 = \frac{9}{32}$