Homework #8 Solutions

Problem 8-1 $H(e^{j\omega}) = a_1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega} + a_4 e^{-j3\omega} + a_5 e^{-j4\omega} + a_6 e^{-j5\omega}$

from - $H(z) = a_1 + a_2 z^{-1} + a_3 z^{-2} + a_4 z^{-3} + a_5 z^{-4} + a_6 z^{-5}$

$$H(z)\Big|_{z=e^{j\omega}} = H(e^{j\omega})$$

$$H(e^{j\omega}) = e^{-j\frac{5\omega}{2}}\left[\left(a_1 e^{j\frac{5\omega}{2}} + a_6 e^{-j\frac{5\omega}{2}} \right) + \left(a_2 e^{j\frac{3\omega}{2}} + a_5 e^{-j\frac{3\omega}{2}} \right) + \left(a_3 e^{j\omega/2} + a_4 e^{-j\omega/2} \right) \right]$$

$$= e^{-j\frac{5\omega}{2}}\left[2(a_1+a_6)\cos\left(\tfrac{5\omega}{2}\right) + 2j(a_1-a_6)\sin\left(\tfrac{5\omega}{2}\right) + 2(a_2+a_5)\cos\left(\tfrac{3\omega}{2}\right) + 2j(a_2-a_5)\sin(3\omega/2) + 2(a_3+a_4)\cos\left(\tfrac{\omega}{2}\right) + 2j(a_3-a_4)\sin(3\omega/2) \right]$$

$H(e^{j\omega})$ will have linear phase if imaginary parts are zero. So, $a_1 = a_6$, $a_2 = a_5$ and $a_3 = a_4$

Problem 8-2

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega}$$

$$h[0] = h[2] \Rightarrow H(e^{j\omega}) = h[0](1 + e^{-j2\omega}) + h[1]e^{-j\omega}$$

$$H(e^{j\omega}) = e^{-j\omega}\left(h[0]\left(e^{+j\omega} + e^{-j\omega}\right) + h[1]\right)$$

$$= e^{-j\omega}\left(2h[0]\cos(\omega) + h[1]\right)$$

We require that -

$$\left|H(e^{j0.3})\right| = 2h[0]\cos(0.3) + h[1] = 1$$

and

$$\left|H(e^{j0.6})\right| = 2h[0]\cos(0.6) + h[1] = 0$$

Solve linear equations to get

$$h[0] = 3.8461 \quad , \quad h[1] = -6.3487$$

Problem 8-3 a) $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} - h[1]e^{-j3\omega} - h[0]e^{-j4\omega}$

$= e^{-j2\omega}\left(h[0]\left(e^{j2\omega} - e^{-j2\omega}\right) + h[1]\left(e^{j\omega} - e^{-j\omega}\right)\right)$

$= j2e^{-j2\omega}\left(h[0]\sin(2\omega) + h[1]\sin(\omega)\right)$

$\Rightarrow |H(e^{j\omega})| = 2\left(h[0]\sin(2\omega) + h[1]\sin(\omega)\right)$

So,

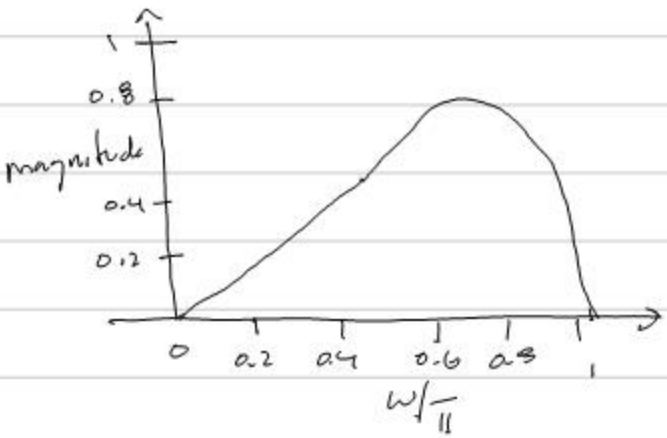
$|H(e^{j0.3\pi})| = 2\left(h[0]\sin(0.6\pi) + h[1]\sin(0.3\pi)\right) = 0.3$

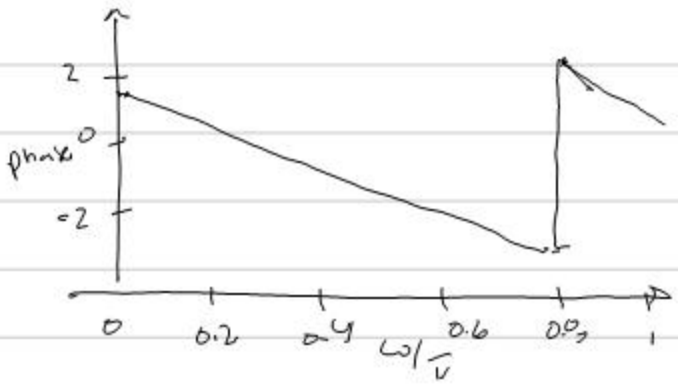
$|H(e^{j0.6\pi})| = 2\left(h[0]\sin(1.2\pi) + h[1]\sin(0.6\pi)\right) = 0.8$

Solve two equations for two unknowns –

$h[0] = -0.131118 \qquad h[1] = 0.339549$

b) $H(e^{j\omega}) = j2e^{-j2\omega}(-0.131118\sin(2\omega) + 0.339549\sin(\omega))$





Problem 8-4 See matlab solution problem 8_4.m

Problem 8-5 Depends on your choice ....