- Quantization Noise

$$x_a(t) \longrightarrow \boxed{\text{Ideal Sampler}} \xrightarrow{x[n]=x_a(nT)} \boxed{\text{Quantizer}} \xrightarrow{\hat{x}[n]=Q(x[n])} \boxed{\text{Coder}} \longrightarrow \hat{x}_{eq}[n]$$

Output Word Length = $b+1$ (including sign bit)

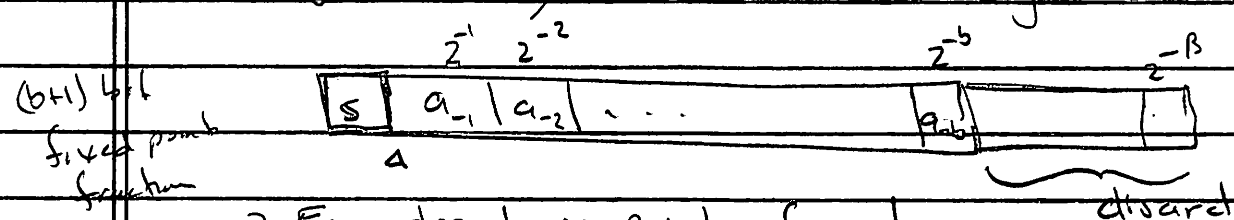
# of discrete levels = 2^{b+1}

Quantization step = $\delta = 2^{-(b+1)}$ (for bipolar A/D converter)

Quantization uses either truncation or rounding

Example Fixed-Point

To truncate a fixed point number from $(\beta+1)$ bits to $(b+1)$ bits, we discard least significant $(\beta-b)$ bits



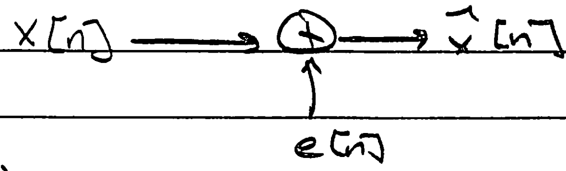
$\Rightarrow$ Error depends on number format

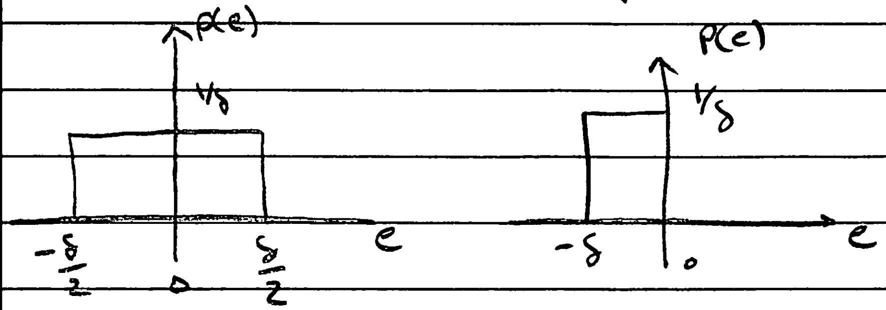
In case of rounding, number is quantized to nearest level.

$\Rightarrow$ Error does not depend on number format

$$-\frac{\delta}{2} < e_r \le \frac{\delta}{2}$$

$$e[n] = Q(x[n]) - x[n] = \hat{x}[n] - x[n]$$





rounding

2's complement Truncation

- Mean & variance for rounding case

$$m_e = \frac{\delta/2 - \delta/2}{2} = 0$$

$$\sigma_e^2 = \int_{-\delta/2}^{\delta/2} e^2 p(e)\, de = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} e^2 de = \frac{1}{\delta}\frac{e^3}{3}\Bigg|_{-\delta/2}^{+\delta/2}$$

$$= \frac{1}{\delta}\left(\frac{\delta^3/8}{3} + \frac{\delta^3/8}{3}\right) = \frac{2\delta^2}{24} = \frac{\delta^2}{12}$$

Signal-to-Quantization Noise Ratio

$$SNR_{A/D} = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right)$$

Case - Bipolar (b+1) bit A/D converter with 1 Vp. Input

$$\delta = 2^{-(b+1)} \cdot R_{FS} \qquad \text{where } R_{FS} = \text{Full scale Range}$$

So,

$$\sigma_e^2 = \frac{\left(2^{-(b+1)} R_{FS}\right)^2}{12} = \frac{2^{-2b} R_{FS}^2}{48}$$

And,

$$SNR_{A/D} = 10\log_{10}\left(\frac{48\,\sigma_x^2}{2^{-2b} R_{FS}^2}\right)$$

$$= 6.02b + 16.81 - 20\log_{10}\left(\frac{R_{FS}}{\sigma_x}\right)$$

σ_x = rms value of input signal amplitude

$$\cancel{V_{rms} = V_p/\sqrt{2} = \frac{LF \cdot R_{FS}}{V_p\sqrt{2}}}$$

$$\cancel{SNR_{A/D} = 6.02b + 16.81 - 20\log_{10}\left(\frac{\sqrt{2}\,LF\sqrt{2}\,V_p}{V_p}\right)}$$

$$\cancel{= 6.02b + 16.81 - 12.041 - 20\log}$$

Let $R_{FS} = K\sigma_x$

$\Rightarrow SNR_{A/D} = 6.02b + 16.81 - 20\log_{10}(K)$

<u>Example</u>

	<u>b=7</u>	<u>b=15</u>
K=4	46.91 dB	95.08 dB
K=8	40.89 dB	89.05 dB

choosing $4\sigma_x < R_{FS} < 8\sigma_x$ is a good idea!

Note - effective wordlength of A/D is usually about 1-2 bits less than theoretical, due to variety of errors.

<u>Example</u> We want to digitize a CT signal whose SNR = 55 dB.

Approx # of bits $\Rightarrow$ $6.02b = 55$

$b \approx 10$ bits

Assume A/D R_{FS} is ± 1 volt = 2 Volts

Lets choose $R_{FS} = 4\sigma_x$ $\Rightarrow \sigma_x = \frac{2\text{ Volts}}{4} = .5$ volts

Must set input $V_p = .707$ volts

Should choose ~ 12 bit A/D