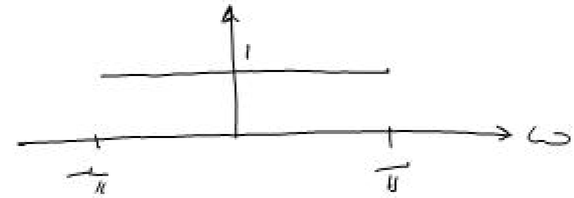
- Ideal LPF has real-valued frequency response.

$$H(e^{j\omega}) = |H(e^{j\omega})| \quad , \quad \theta(\omega) = 0 \qquad \text{"zero phase"}$$

We can build piecewise-constant frequency responses by combining ideal responses.

<u>Example</u>: Highpass filter

Allpass filter $\quad \delta[n] \Longrightarrow$



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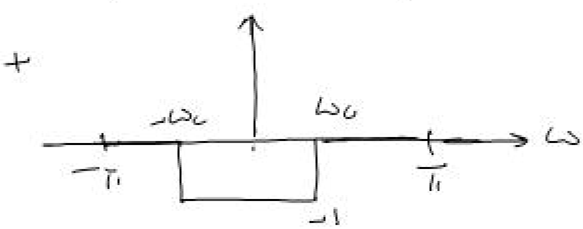
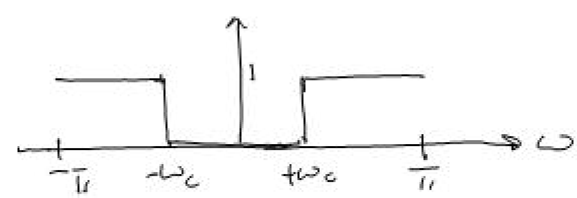
lowpass filter $\; -h_{LP}[n]$



highpass filter $\; h_{HP}[n] \qquad =$

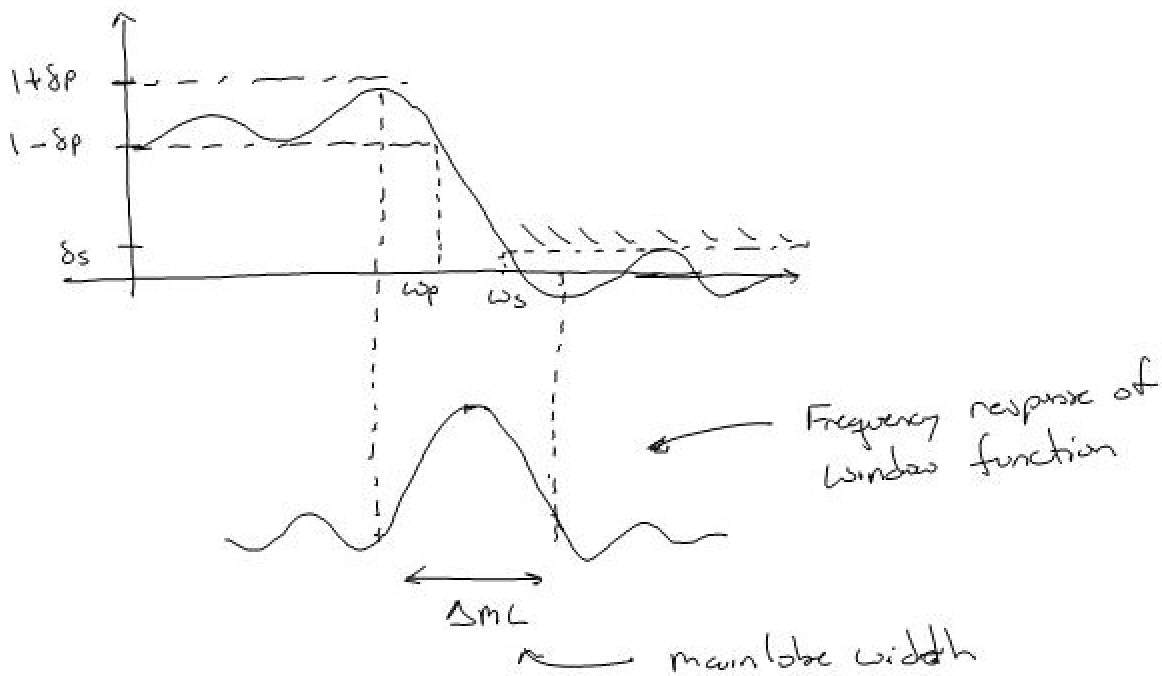


Therefore, $\qquad h_{HP}[n] = \delta[n] - \dfrac{\sin(\omega_c n)}{\pi n}$

- FIR Filter order estimation (see 10.1.2)

Kaiser's Formula
Bellanger's Formula
Hermann's Formula

- Window function and filter magnitude response



Note: Smaller ΔML results in faster transition from ω_p to ω_s.

Reducing passband and stopband ripple requires area under window sidelobes to be smaller.

- Designing FIR filters by windowing method

1- determine $\omega_c = (\omega_p + \omega_s)/2$
2- Estimate M (e.g. $N = 2M+1$)

$\Delta\omega = (\omega_s - \omega_p) \approx \frac{c}{M}$ from 10.2 table in book.

<u>Example</u>: $\omega_p = 0.3\pi$ $\omega_s = 0.5\pi$

stopband attenuation needs to be at least 40dB

$\Rightarrow \alpha_s = 40\,dB$.

1. $\omega_c = (\omega_p + \omega_s)/2 = 0.4$

2. $\Delta\omega = \omega_s - \omega_p = 0.2\pi$

Using Table 10.2 we see α_s can be achieved by the three windows: Hann, Hamming, Blackman.

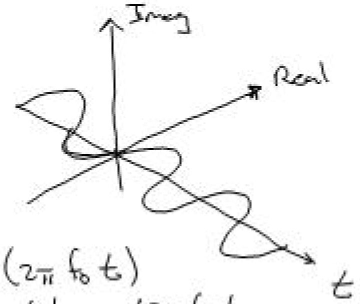
Lets use a Hann window function.

$$m = \frac{3.11\pi}{0.2\pi} = 15.55 \approx 16$$

so, $N = 2m + 1 = 32$

- Hilbert Transform

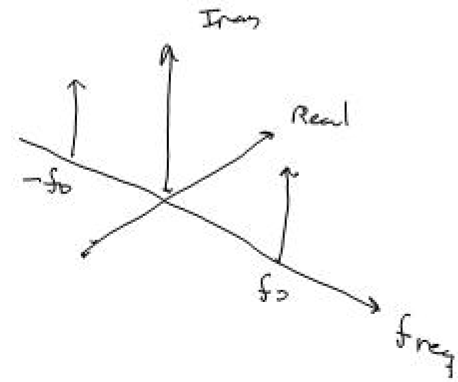
Complex Signals -

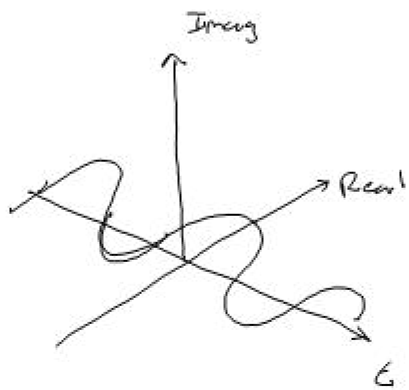


$\sin(2\pi f_0 t)$

$= j\frac{e^{j2\pi f_0 t}}{2} - j\frac{e^{j2\pi f_0 t}}{2}$

FT $\Longrightarrow$

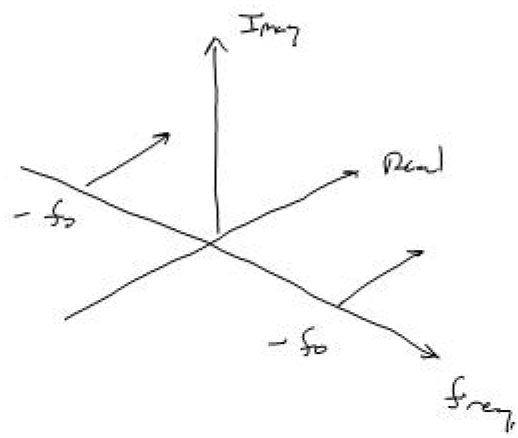




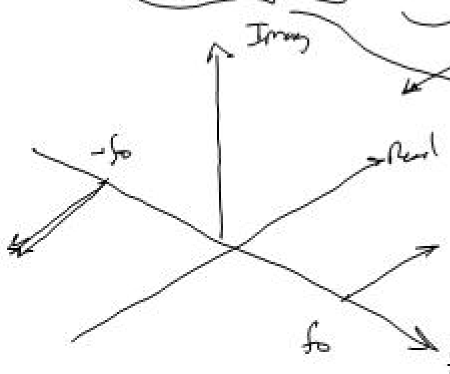
$\cos(2\pi f_0 t) =$

$\frac{e^{j2\pi f_0 t}}{2} + \frac{e^{-j2\pi f_0 t}}{2}$

FT $\Longrightarrow$



$z(t) = \underbrace{\cos(2\pi f_0 t)} + j\underbrace{\sin(2\pi f_0 t)}$



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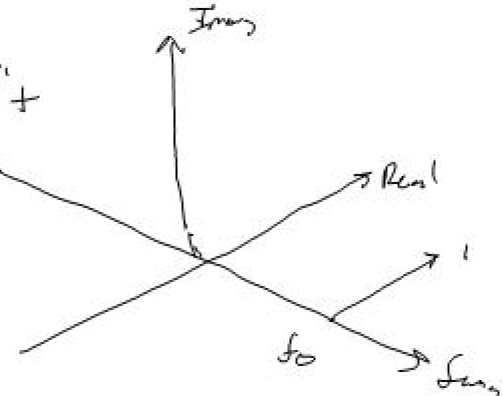
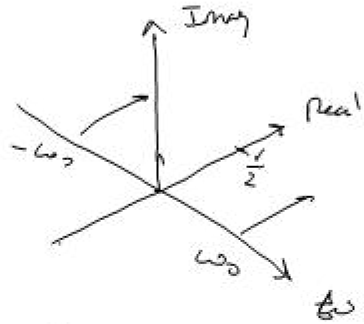


HT is process of changing real signals $x_r(t)$ into a new real signal $x_{HT}(t)$ where positive freq components are shifted in phase by $+90°$ and negative components are shifted in phase by $-90°$.

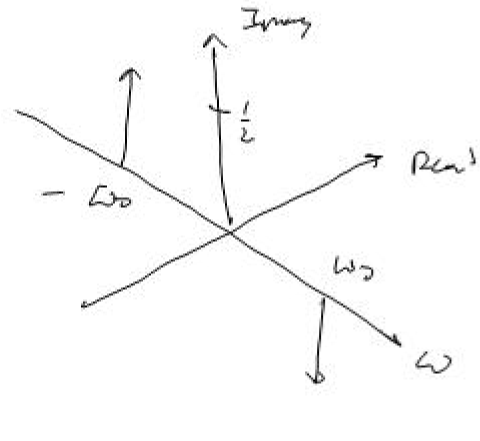
$$H_{HT}(e^{j\omega}) \begin{cases} +j & -\pi < \omega < 0 \\ -j & 0 < \omega < \pi \end{cases}$$

$$X_{HT}(e^{j\omega}) = H_{HT}(e^{j\omega}) \, X_r(e^{j\omega})$$

<u>Example</u> $x_r(t) = \cos(\omega t)$

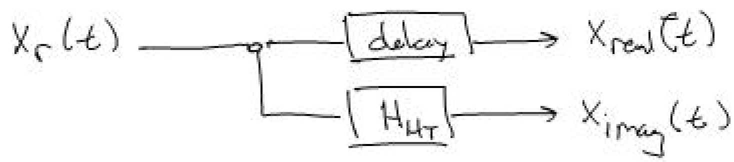


$x_{HT}(t) = \sin(\omega t)$



Q: Why do we care?

Many applications in DSP are easier to implement for analytic signals.



$$x_c(t) = x_r(t) + j\, x_{imag}(t)$$

AM Modulation - $x_r(t)$ is amplitude modulated - envelope contains information.

$$E(t) = |x_c(t)| = \sqrt{x_r^2(t) + x_i^2(t)}$$

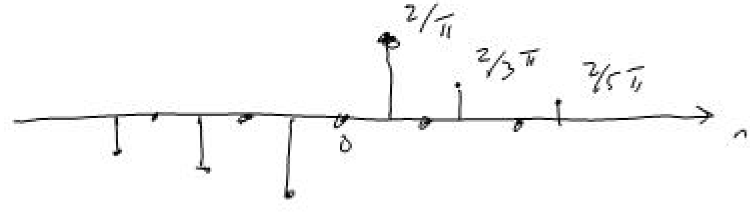
Instantaneous phase - $\theta(t) = \tan^{-1}\left(\frac{x_i(t)}{x_r(t)}\right)$

FM Demodulation. $F(t) = \frac{d}{dt}\left\{ \tan^{-1}\left(\frac{x_i(t)}{x_r(t)}\right)\right\}$

HT filtering can make filtering easier.

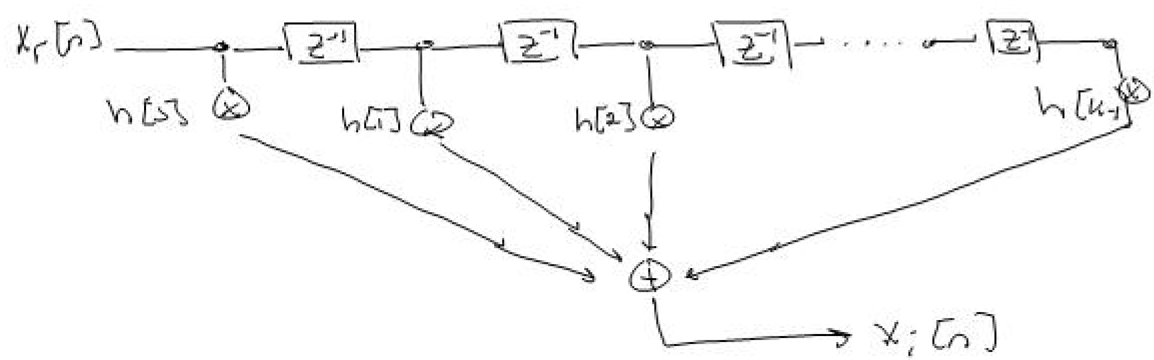
- HT impulse response

$$h[n] = \frac{1}{\pi n}\left(1 - \cos(\pi n)\right) = \frac{2\sin^2(\pi n/2)}{\pi n} = \begin{cases} 0, & n \text{ even} \\ \frac{2}{\pi n}, & n \text{ odd} \end{cases}$$



- HT FIR Filter structure.

$$x_i[n] = \sum_{k=-\infty}^{\infty} h_{HT}[k]\, x_r[n-k]$$



$h_{HT}[n]$ length $\rightarrow$	odd (Type III)	Even (Type IV)
	$\lvert H(e^{j0})\rvert = 0$	$\lvert H(e^{j0})\rvert = 0$
	$\lvert H(e^{j\pi})\rvert = 0$	no restriction

Note. Hard to design HT filter for low frequencies.

- Frequency Domain HT

1. Take N-point DFT of real, even length-N signal $x_r[n]$ to get $X_r[k]$ spectrum.
2. Set negative freqs. components to zero and scale positive freqs components by 2.
3. Inverse DFT to get $x_c[n]$

Note: only works for block processing applications.