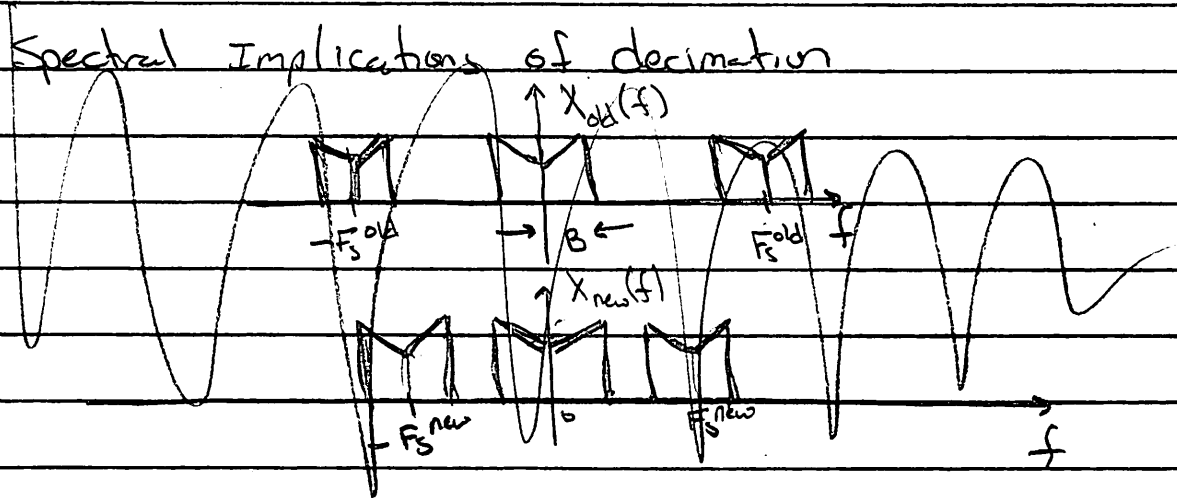
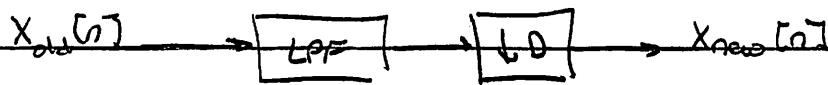


Spectral Implications of decimation

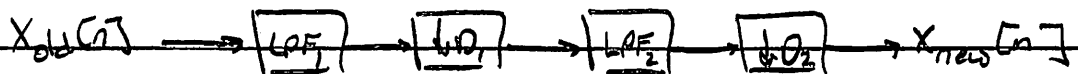


Decimation is limited by the requirement that $F_s^{\text{new}} > 2B$

If application requires $F_s^{\text{new}} < 2B$, then low pass filter $X_{\text{old}}[n]$.



Q: What if decimation factor, D , is large - $D > 10$
- Computational savings by doing it in stages.

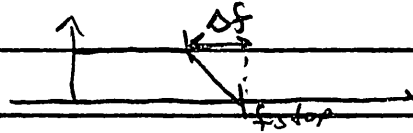


$$D = D_1 D_2$$

Q: What should D_1 & D_2 be to minimize number of taps in low pass FIR filters LPF_1 & LPF_2 ?

$$D_{1,\text{opt}} \approx 2D \frac{1 - \sqrt{0.5F/(2-F)}}{2 - F(D+1)}$$

where $F = \frac{\Delta f}{f_{\text{stop}}}$



Now, $D_2 = D / D_{opt}$

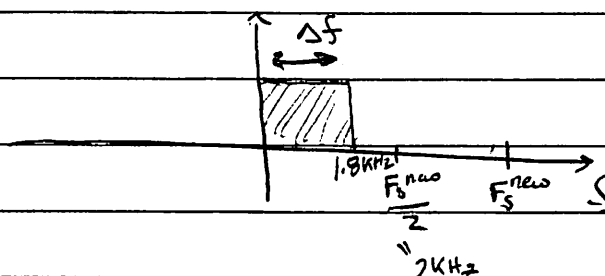
Example

$$F_s^{old} = 400 \text{ kHz}$$

$$D = 100$$

$$F_s^{new} = 4 \text{ kHz}$$

Baseband freqs of interest are: 0 to 1.8 kHz
"B"



Taps for single stage FIR lowpass filter

$$T = K \frac{F_s^{old}}{(f_{stop} - B')} = K \frac{F_s^{old}}{\Delta f}$$

filter transition Band

where $2 < K < 3$ depends on passband ripple & stopband atten.

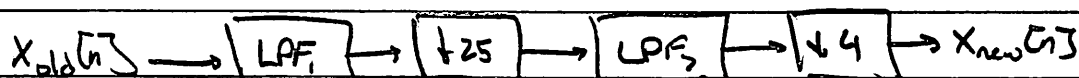
so, let $K=2 \Rightarrow T = 2 \left(\frac{400}{0.2} \right) = 4000$ Ouch!!

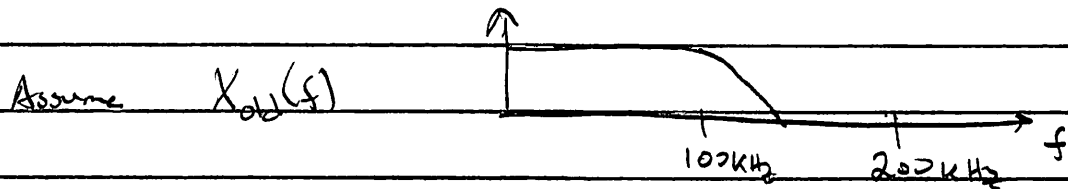
- Partition into 2 stages -

$$D_{opt} \approx 31.9 \quad \text{choose submultiple of 100}$$

$$D_{opt} = 25$$

so, $D_2 = 4$





LPF₁ has cutoff of 1.8 kHz w/ $f_{\text{stop}} = 8 \text{ kHz}$
 LPF₂ has cutoff of 1.8 kHz w/ $f_{\text{stop}} = 2 \text{ kHz}$

$$\text{Use } T \approx k \frac{F_{\text{old}}}{\Delta f}$$

$$\Rightarrow T_{\text{LPF}_1} = 2 \frac{400}{(8 - 1.8)} =$$

$$T_{\text{LPF}_2} = 2 \frac{16}{(2 - 1.8)} =$$

$$T_{\text{Total}} = 289 \text{ taps} \quad \text{Much Better!}$$

Note: If our coeffs are symmetrical we can reduce the computations further.

Note: A decimator is not time-invariant.

Note: Decimation does not cause time-domain loss, but it does induce magnitude loss by factor of D in Freq. domain.

Q: Why decimate our signals