Homework #3 Solutions

Problem 3-1

a.) $x[n]$ is real & even, so $X(e^{j\omega}) = X(e^{-j\omega}) = X(e^{j\omega})^*$

$$x[n] = \frac{1}{2}\left(x[n] + x[-n]\right) = \frac{1}{2}\left(\text{ICTFT}\{X(e^{j\omega})\} + \text{ICTFT}\{X(e^{-j\omega})\}\right)$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{+j\omega n}\, d\omega$$

$$x[-n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{-j\omega})\, e^{+j\omega(-n)}\, d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{-j\omega n}\, d\omega$$

so,

$$x[n] = \frac{1}{4\pi}\left(\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega + \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{-j\omega n}\, d\omega\right)$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\left(e^{j\omega n} + e^{-j\omega n}\right) d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\cos(\omega n)\, d\omega$$

$$= \frac{1}{\pi}\int_{0}^{\pi} X(e^{j\omega})\cos(\omega n)\, d\omega$$

Note: $X(e^{j\omega})\cos(\omega n)$ is even.

b.) $x[n]$ is real and odd, so $X(e^{j\omega}) = -X(e^{-j\omega})$ and $x[n] = -x[-n]$. Following same process as in (a) using $x[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$ we get

$$x[n] = \frac{j}{\pi}\int_{0}^{\pi} X(e^{j\omega})\sin(\omega n)\, d\omega$$

$\Rightarrow$ $X(e^{j\omega})\sin(\omega n)$ is even.

Problem 3-2

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

So, $X(e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega/2)n}$ and $X(-e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n](-1)^n e^{-j\omega/2 n}$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} = \frac{1}{2}\left\{ X(e^{j\omega/2}) + X(-e^{j\omega/2}) \right\}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left\{ x[n] + x[n](-1)^n \right\} e^{-j\omega/2 n}$$

Thus,

$$y[n] = \begin{cases} \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega/2 n} & \text{for } n \text{ even,} \\ 0 & \text{for } n \text{ odd.} \end{cases}$$

Problem 3-3 Take the inverse DTFT of $X(e^{j\omega})$ ....

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{k=-\infty}^{\infty} 2\pi\, \delta(\omega + 2\pi k)\, e^{j\omega n}\, d\omega$$

$$= \int_{-\pi}^{\pi} \delta(\omega)\, e^{j\omega n}\, d\omega = \int_{-\pi}^{\pi} e^{j(0)n}\, d\omega = 1$$

since integral is only over $-\pi$ to π, only $k=0$ is inside integration range.

<u>Problem 3-5</u>

$$u[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, z^{n-k}$$

$$= z^n \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$$

$$= z^n H(z) \qquad \text{where } H(z) = \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$$

input reproduced, so it is eigenfunction

If $v[n] = z^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, z^{n-k}\, u[n-k] = z^n \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]\, z^{-k}$$

$$= z^n \sum_{k=-\infty}^{n} h[k]\, z^{-k}$$

It is an eigenfunction.

Problem 3-6

$$y[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \pm 3L, \dots \\ 0, & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n]\, e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n/L]\, e^{-j\omega n}$$

Let $x[n/L] = x[m]$

$m = \frac{n}{L}$

$$= \sum_{m=-\infty}^{\infty} x[m]\, e^{-j\omega m L} = X(e^{j\omega L})$$

$$Y(e^{j\omega}) = X(e^{j\omega L})$$

Problem 3-7

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j\omega 2}$$

$$= h[0] + h[1]e^{-j\omega} + h[0]e^{-j\omega 2} \quad \text{since } h[0]=h[2]$$

so, $$H(e^{j\omega}) = h[0](1+e^{-j\omega 2}) + h[1]e^{-j\omega}$$

$$= e^{-j\omega}\left(h[0]e^{j\omega}(1+e^{-j\omega 2}) + h[1]\right)$$

$$= e^{-j\omega}\left(h[0](e^{j\omega}+e^{-j\omega}) + h[1]\right)$$

$$= e^{-j\omega}\left(2h[0]\cos(\omega) + h[1]\right)$$

We require that $$|H(e^{j0.3})| = 2h[0]\cos(\overset{=0.3}{\omega}) + h[1] = 1$$

and $$|H(e^{j0.6})| = 2h[0]\cos(0.6) + h[1] = 0$$

Solve two equations for 2 unknowns, $h[0]$ & $h[1]$ ...

$h[0] = 3.8461$
$h[1] = -6.3487$