Homework #2 Solutions

Problem 2-1

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Let $h[n]$ be non zero for $0 \le n \le M-1$
Let $x[n]$ be non zero for $0 \le n \le N-1$

~~with~~

$$y[n] = \sum_{k=0}^{M-1} h[k]\, x[n-k]$$

$y[n]$ is non zero when $0 \le n-k \le N-1$.
This $n-k$ equals ϕ when $n=0$ and $k=0$.
It is maximum when $n-k = N-1$. This happens when $k = M-1$. So,

$$n-(M-1) = N-1$$
$$n = M+N-2$$

Hence, the total number of output samples is

$$M+N-2$$

Problem 2-2 $$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

where $x[n]$ is a periodic sequence
so, $x[n-m+kN] = x[n-m]$

$$y[n+kN] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m+kN]$$
$$= \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

Thus, $y[n]$ is also periodic with period N.

Problem 2-3 $$y_1[n] = x_1[n] * h_1[n]$$
$$= (3\delta[n-2] - 2\delta[n+1]) * (-\delta[n+2] + 4\delta[n] - 2\delta[n-1])$$
$$= -3\delta[n-2] * \delta[n+2] + 12\delta[n-2] * \delta[n] - 6\delta[n-2] * \delta[n-1]$$
$$+ 2\delta[n+1] * \delta[n+2] - 8\delta[n+1] * \delta[n] + 4\delta[n+1] * \delta[n-1]$$

$$= -3\delta[n] + 12\delta[n-2] - 6\delta[n-3] + 2\delta[n+3] - 8\delta[n+1] + 4\delta[n]$$

$$= 2\delta[n+3] - 8\delta[n+1] + \delta[n] + 12\delta[n-2] - 6\delta[n-3]$$

Problem 2-4 $h[n]$ is non-negative for $n \geq 0$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \sum_{k=0}^{n} h[k] \quad , n \geq 0$$

when $x[n] = u[n]$

$y[n] = 0$ for $n < 0$

Since $h[k]$ is non-negative for $n \geq 0$, $y[n]$ is monotonically increasing for $n \geq 0$, and is not oscillatory. Hence, it can have no overshoot.

Problem 2-5 $$x_a(t) = \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$$

$$= \int_{-\infty}^{\infty} (1)\, e^{j\Omega t}\, d\Omega = \frac{1}{j2\pi t}\left(-e^{-jt} + e^{jt}\right)$$

$$= \frac{1}{j2\pi t}\left(e^{jt} - e^{-jt}\right) = \frac{\sin(t)}{\pi t}$$

Problem 2-6

(a) $$V_a(j\Omega) = \int_{-\infty}^{\infty} \cos(\Omega_0 t)\, e^{-j\Omega t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left(e^{j\Omega_0 t} + e^{-j\Omega_0 t}\right) e^{-j\Omega t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \left(e^{-j(\Omega-\Omega_0)t} + e^{-j(\Omega_0+\Omega)t}\right) dt$$

$$= \frac{1}{2}\,\delta(\Omega-\Omega_0) + \frac{1}{2}\,\delta(\Omega+\Omega_0)$$

(b) $$V_a(j\Omega) = \int_{-\infty}^{\infty} e^{j\Omega_0 t}\, e^{-j\Omega t}\, dt = \int_{-\infty}^{\infty} e^{-j(\Omega-\Omega_0)t}\, dt$$

$$= \delta(\Omega-\Omega_0)$$

Notice: Complex spectrum of complex function is one-sided. Spectrum of real signal is two-sided.

Problem 2-7

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\Omega t}\, dt$$

$$= \left| X_a(j\Omega) \right| e^{j\theta_a(\Omega)}$$

where $\theta_a(\Omega) = \tan^{-1}\left\{ \frac{Im(X(j\Omega))}{Re(X(j\Omega))} \right\}$

Thus, $X_a(-j\Omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{j\Omega t}\, dt$

$$= \left| X_a(-j\Omega) \right| e^{+j\theta_a(-\Omega)}$$

Since $x_a(t)$ is real-valued, $X(j\Omega)$ and $X(-j\Omega)$ are complex conjugates. Therefore,

$$\left| X_a(-j\Omega) \right| = \left| X_a(j\Omega) \right|$$

and $\theta_a(-\Omega) = -\theta_a(\Omega)$

So, for real-valued function, the magnitude spectrum is an even function and the phase spectrum is an odd function.

<u>Problem 2-9</u> The impulse response is found by using a delta function as the input and then computing the output.

$$\delta[n] \longrightarrow \boxed{LTI} \longrightarrow h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k]\, h[n-k] = h[k]$$

The frequency response is the spectrum of the impulse response.

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$$