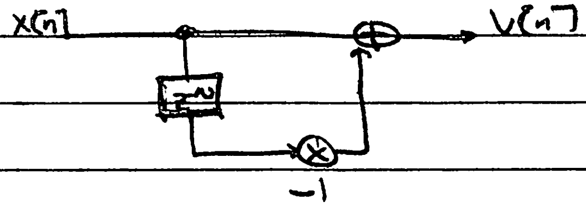
Comb Filter -




$$y[n] = x[n] - x[n-N]$$

$$Y(z) = X(z) - X(z)z^{-N} = X(z)\left(1 - z^{-N}\right)$$

$$H_{comb}(z) = \frac{Y(z)}{X(z)} = 1 - z^{-N}$$

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = 1 - e^{-j\omega N}$$

$$= e^{-j\omega N/2}\left(e^{+j\omega\frac{N}{2}} - e^{-j\omega\frac{N}{2}}\right)$$

$$= e^{-j\omega\frac{N}{2}}\left(2j\sin(\omega N/2)\right)$$

$$= e^{-j\omega\frac{N}{2}} e^{j\frac{\pi}{2}}\left(2\sin(\omega N/2)\right)$$

$$= e^{-j(\omega N - \pi)} \quad 2\sin(\omega N/2)$$

$$|H(e^{j\omega})| = 2\left|\sin(\omega N/2)\right|$$

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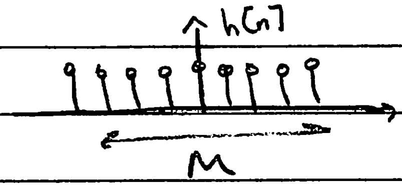
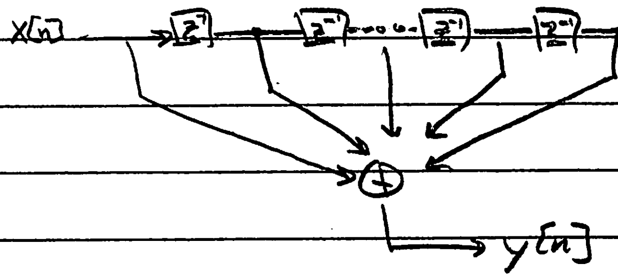
- Box Car Filter




$$\text{Delay} = \frac{M}{2}$$

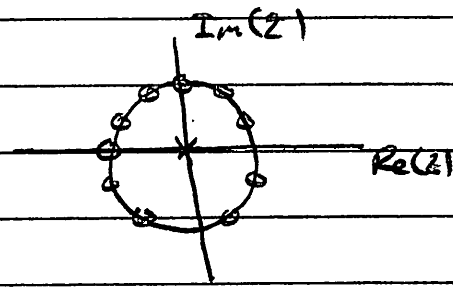
$$y[n] = \sum_{k=0}^{M-1} x[n-k]$$

$$Y(z) = \sum_{k=0}^{M-1} X(z)\, z^{-k} \implies H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{M-1} z^{-n}$$

Use Geometric Series.
$$\sum_{n=0}^{M-1} \beta^n = \begin{cases} \dfrac{1-\beta^M}{1-\beta}, & \beta \neq 1 \\ M, & \beta = 1 \end{cases}$$

so,
$$H(z) = \frac{1 - z^{-M}}{1 - z^{-1}} = \frac{z^{-M}}{z^{-M}} \cdot \frac{z^M - 1}{z^M - z^{M-1}} = \frac{z^{-M} z^M - 1}{z^{-M} \cdot z^{M-1}(z-1)}$$

$$H(z) = \frac{z^M - 1}{z^{M-1}(z-1)}$$




$$= \left(1 - z^{-M}\right)\left(\frac{1}{1 - z^{-1}}\right)$$

Comb

Integrator