University of Colorado Denver
College of Engineering and Applied Science
EE 4637/5637 – Digital Signal Processing

MIDTERM EXAM: DIGITAL SIGNAL PROCESSING
DUE SATURDAY, OCTOBER 29, 2011

Name: ____________________

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page, or staple additional pages to this exam.

Instructions:

Permitted - lecture notes, homework and solutions, textbook, calculator, computer
Not Permitted - help from others
Duration - 1 week

Problem Score:

1.(15 points)
2.(6 points)
3.(4 points)
Total(25 points)

Problem 1. Practical FIR Filter Design [15 points total]
The frequency response of an ideal low-pass filter with linear phase and a cutoff frequency ω_c is

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_c, \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

(a) Use the inverse DTFT to derive the impulse response.
Because the filter is unrealizable (noncausal and unstable), it is necessary to relax the ideal constraints on the frequency response and allow some deviation from the ideal response. The specifications for a low-pass filter will typically have the form

$$1 - \delta_p < |H(e^{j\omega})| \le 1 + \delta_p \qquad 0 \le |\omega| < \omega_p$$

$$|H(e^{j\omega})| \le \delta_s \qquad \omega_s \le |\omega| \le \pi$$

where ω_p is the passpand cutoff frequency, w_s is the stopband cutoff frequency, δ_p is the passband deviation, and δ_s is the stopband deviation. The interval between ω_p and ω_s is called the transition band. Once the filter specifications have been defined, the next step is to design a filter that meets these specifications.

Because the impulse response you derived in part a is infinite in length, it is necessary to find the FIR approximation by windowing, or truncating, the impulse response

$$h[n] = h_{lp} w[n]$$

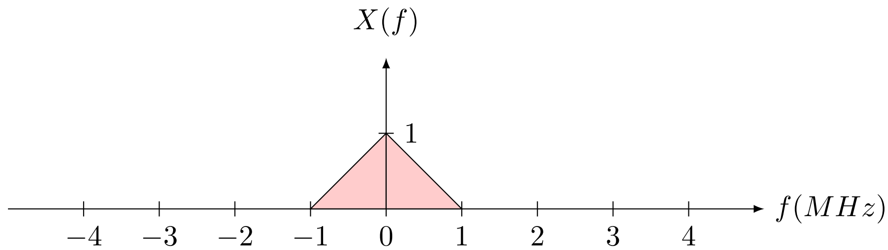
where $w[n]$ is a finite-length window that is equal to zero outside the interval $-M \le n \le M$. By applying the finite window we arrive at a finite-length noncausal approximation of length $N = 2M + 1$, which needs to be shifted to the right to yield the coefficients of a causal FIR lowpass filter. (b) Perform this shifting and write down the impulse response for the causal FIR lowpass filter. (c) Derive an expression for the causal FIR highpass filter and causal FIR bandpass filter. (d) Write a Matlab program to compute the coefficients for the lowpass, highpass, and bandpass FIR filters. (e) Compute and plot the magnitude response for the lowpass, highpass, and bandpass filters using each of the window sequences listed below.

$$w[n]_{rect} = \begin{cases} 1, & 0 \le n \le N, \\ 0, & otherwise \end{cases}$$

$$w[n]_{Hann} = \begin{cases} 0.5 - 0.5cos(2\pi n/N), & 0 \le n \le N, \\ 0, & otherwise \end{cases}$$

$$w[n]_{Hamming} = \begin{cases} 0.54 - 0.46cos(2\pi n/N), & 0 \le n \le N, \\ 0, & otherwise \end{cases}$$

Problem 2. Sampling [6 points total]
The actual spectrum for a real-valued baseband continuous-time signal $x(t)$ is depicted below. Sketch the spectrum after sampling by a sample rate of $F_s = 1MHz$. What is the Nyquist frequency? Does the $1MHz$ sample rate satisfy the Nyquist condition? What would be the sample rate if you oversampled the Nyquist rate by a factor of 10?



Problem 3. Basic Concepts [4 points total]
Answer the following questions:
(a) If you have 1024 samples of a discrete-time band-limited signal sampled at a rate of 100MHz what is the time period between samples? If you take a 2048 point FFT of the sampled signal, what will be the frequency spacing between the output samples?
(b) When might IIR filters be preferred over FIR filters?
(c) How can you compute the frequency response of a system from the transfer function?
(d) When using window functions for spectral analysis, what is the trade-off between mainlobe width and sidelobe levels?