Lecture 4a

- CTFT

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$$

- DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

- DFT

$$X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{N}} = X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k n}{N}} \qquad 0 \le k \le N-1$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi k n}{N}} \qquad 0 \le n \le N-1$$

- Fourier Series (FS)

$$X[k] = \frac{1}{T} \int_{0}^{T} x(t)\, e^{-jk\Omega_0 t}\, dt \qquad \Omega_0 = 2\pi/T$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]\, e^{jk\Omega_0 t}$$

- Discrete-Time Fourier Series (DTFS)

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} \qquad \omega_0 = \frac{2\pi}{N}$$

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\omega_0 n}$$

- Properties of signal

Time Property	Periodic	Nonperiodic
CT	FS	FT
DT	DTFS	DTFT or DFT

- Symmetry Properties

	real-valued Signal	Imaginary Valued Signal
DTFT	$X^*(e^{j\omega}) = X(e^{-j\omega})$	$X^*(e^{j\omega}) = -X(e^{-j\omega})$

~~Recall~~

For real-valued signal -

$$Re\{X(j\Omega)\} = Re\{X(-j\Omega)\} \rightarrow \text{real part is even}$$

$$Im\{X(j\Omega)\} = -Im\{X(-j\Omega)\} \rightarrow \text{imag part is odd}$$

$\Rightarrow$ Mag. spectrum is even, phase spectrum is odd.

<u>Example:</u> $x(t) = A\cos(\omega t - \phi)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$x(t) = \frac{A}{2}\left(e^{j(\omega t - \phi)} + e^{-j(\omega t - \phi)}\right)$$

$$= \frac{A}{2}e^{j(\omega t - \phi)} + \frac{A}{2}e^{-j(\omega t - \phi)}$$

Use linearity property - $a\,x(t) + b\,y(t) \longrightarrow \boxed{LTI} \longrightarrow a\,X(j\Omega) + b\,Y(j\Omega)$

$$\Rightarrow\quad y(t) = \frac{A}{2}\left|H(j\Omega)\right| e^{j(\omega t - \phi + \theta)} + \frac{A}{2}\left|H(j\Omega)\right| e^{j(\omega t - \phi + \theta)}$$

where $\theta = \arg\left\{H(j\Omega)\right\}$

Note: system alters amplitude and phase of input signal.

$$y(t) = \left|H(j\Omega)\right| A\cos\left(\omega t - \phi + \arg\{H(j\Omega)\}\right)$$

Q: What if input signal, $x(t)$ is purely imaginary?

$x^*(t) = -x(t)$, $X^*(j\Omega) = -X(-j\Omega)$

Real part of spectrum is <u>odd</u>

Imag part of spectrum is <u>even</u>