

Homework #6 Solutions

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Problem 6-1

$$g_a(t) \xrightarrow{\text{Sampling}} g[n]$$

$$E_{g_a(t)} = \int_{-\infty}^{\infty} |g_a(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G_a(j\Omega)|^2 d\Omega$$

by Parseval's relationship

$$\text{For a bandlimited signal} \Rightarrow E_{g_a(t)} = \frac{1}{2\pi} \int_{-\Omega_m}^{\Omega_m} |G_a(j\Omega)|^2 d\Omega$$

For the discrete-time signal -

$$E_{g[n]}[n] = \sum_{n=-\infty}^{\infty} |g[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |G(e^{j\omega})|^2 d\omega$$

$$\text{Remember } \omega = \frac{\Omega}{F_s} = \Omega T_s \quad d\omega = d\Omega T_s$$

$$G(e^{j\omega}) = G_a(j\Omega) \Big|_{\substack{\Omega = \omega/T_s \\ \omega = \pi/T_s}}$$

$$\text{so, } E_{g[n]}[n] = \frac{1}{2\pi} \int_{-\pi/T_s}^{\pi/T_s} |G(e^{j\Omega T_s})|^2 d\Omega T_s$$

$$= \frac{T_s}{2\pi} \int_{-\pi/T_s}^{\pi/T_s} \left| \frac{1}{T_s} G(j\Omega) \right|^2 d\Omega = \frac{1}{2\pi T_s} \int_{-\pi/T_s}^{\pi/T_s} |G_a(j\Omega)|^2 d\Omega$$

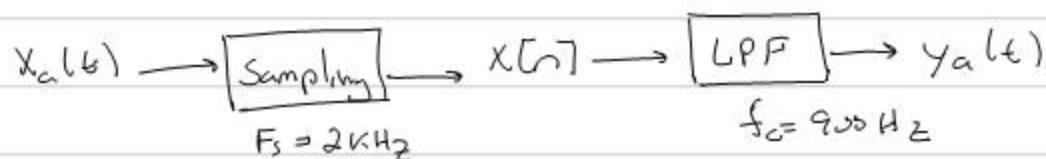
$$\text{Let } \Omega_m = \pi/T_s \quad E_{g[n]}[n] = \frac{1}{2\pi T_s} \int_{-\Omega_m}^{\Omega_m} |G_a(j\Omega)|^2 d\Omega = \frac{1}{T_s} E_{g_a(t)} =$$

Problem 6-2

$$T_s = \frac{2.5}{5000}$$

$$F_s = \frac{1}{T_s} = 2000 \text{ Hz}$$

Therefore, highest freq. component in CT signal is $< 1000 \text{ Hz}$.

Problem 6-3

$$X_p(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a(j(\Omega + k\Omega_T))$$

Positive Freqs only:

$$F_1 = 300 \text{ Hz} \Rightarrow 300, 1700, 2300, \text{etc.}$$

$$F_2 = 500 \text{ Hz} \Rightarrow 500, 1500, 2500, \text{etc.}$$

$$F_3 = 1200 \text{ Hz} \Rightarrow 1200, 800, 3200, \text{etc.}$$

$$F_4 = 2150 \text{ Hz} \Rightarrow 2150, 150, 4150, \text{etc.}$$

$$F_5 = 3500 \text{ Hz} \Rightarrow 3500, 1500, 5500, \text{etc.}$$

After the 900 Hz LPF, only 300, 500, 800, 150 Hz components remain, the rest are attenuated.

Problem 6-3

$$y[n] = x[n] + a x[n-1]$$

Remember, for a distortionless system the frequency response has the form

$$H(e^{j\omega}) = c e^{-j\omega n_0}$$

The magnitude response is constant - $|H(e^{j\omega})| = c$

And, the phase response is linear - $\arg\{H(e^{j\omega})\} = -\omega n_0$

Transfer Function - $H(z) = \frac{y(z)}{x(z)} = 1 + a z^{-1}$

Impulse Response - $h[n] = \delta[n] + a \delta[n-1]$

Freq. Response - $H(e^{j\omega}) = 1 + a e^{-j\omega}$

Therefore, the system has both amplitude and phase distortion.

$$|H(e^{j\omega})|^2 = (1 - a e^{-j\omega})(1 + a e^{j\omega}) = 1 + 2a \cos(\omega) + a^2$$