Homework Assignment 2

Problem 2-1
Show that the convolution of a length-M sequence with a length-N sequence leads to a sequence of length $(M + N - 1)$.

Problem 2-2
A periodic sequence $x[n]$ with a period N is applied as an input to an LTI discrete-time system characterized by an impulse response $h[n]$ generating an output $y[n]$. Is $y[n]$ a periodic sequence? If it is, what is its period?

Problem 2-3
Consider the following sequences:
$x[n] = 3\delta[n-2] - 2\delta[n+1]$
$h[n] = -\delta[n+2] + 4\delta[n] - 2\delta[n-1]$

Determine the folowing linear convolution: $y[n] = x[n] \star h[n]$.

Problem 2-4
A causal LTI discrete-time system is said to have an overshoot in its step response if the response exhibits an oscillatory behavior with decaying amplitude around a final constant value. Show that the system has no overshoot in its step reponse if the impulse response $h[n]$ of the system is nonnegative for all $n \geq 0$.

Problem 2-5
The finite-energy function $x_a(t) = sin(t)/\pi t$ is not absolutely summable. Show that its CTFT is given by

$$X_a(j\Omega) = \begin{cases} 1, |\Omega| \leq 1 \\ 0, |\Omega| > 1 \end{cases}$$

Problem 2-6
Determine the CTFT of the folowing continuous-time functions defined for $-\infty < t < \infty$:
(a) $y_a(t) = cos(\Omega_0 t)$, (b) $v_a(t) = \exp(j\Omega_0 t)$

Problem 2-7
Let $X_a(j\Omega)$ denote the CTFT of a real-valued continuous-time function $x_a(t)$. Show that the magnitude spectrum $|X_a(j\Omega)|$ is an even function of Ω and the phase spectrum $\theta(\Omega)$ is an odd function of Ω.

Problem 2-8
How do you find the impulse response $h[n]$ of an LTI discrete-time system? Given the impulse response $h[n]$, how do you find the frequency response?