Homework 4 Solutions

<u>Problem 4-1</u> $\psi_k[n] = e^{jk(2\pi/N)n}$ $\quad k = 0, 1, \ldots, N-1$

$$\sum_{n=0}^{N-1} \psi_k[n]\,\psi_m^*[n] = \sum_{n=0}^{N-1} e^{jk\frac{2\pi n}{N}} e^{-jm\frac{2\pi n}{N}}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n(k-m)} \qquad \text{Let } \alpha = e^{j\frac{2\pi}{N}(k-m)}$$

$$\Rightarrow \sum_{n=0}^{N-1} \alpha^n = \begin{cases} \dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \\ N & \alpha = 1 \end{cases}$$

so,

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi n}{N}(k-m)} = \begin{cases} \dfrac{1 - e^{j\frac{2\pi}{N}(k-m)N}}{1 - e^{j\frac{2\pi}{N}(k-m)}} & k \neq m \\ N & k = m \end{cases}$$

notice that $e^{j\frac{2\pi}{N}(k-m)N} = e^{j2\pi(k-m)} = e^{j2\pi k} e^{-j2\pi m} = 1$

Therefore,

$$\sum_{n=0}^{N-1} \psi_k[n]\,\psi_m^*[n] = \begin{cases} 0 & k \neq m \\ N & k = m \end{cases}$$

Problem 4-2

$$\text{DTFT}\left\{ e^{j\omega_0 n} x[n] \right\} = \sum_{n=-\infty}^{\infty} e^{j\omega_0 n} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, e^{-j(\omega-\omega_0)n} = X(e^{j(\omega-\omega_0)})$$

Problem 4-3

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)\, e^{+j\Omega t}\, d\Omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\, \delta(\Omega-\Omega_0)\, e^{+j\Omega t}\, d\Omega$$

$$= \int_{-\infty}^{\infty} \delta(\Omega-\Omega_0)\, e^{+j\Omega t}\, d\Omega = e^{j\Omega_0 t}$$

Problem 4-4

$$n\,x[n] \xleftrightarrow{\text{DTFT}} j\frac{dX(e^{j\omega})}{d\omega}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\frac{dX(e^{j\omega})}{d\omega} = \frac{d}{d\omega} \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n] \frac{d}{d\omega} e^{-j\omega n}$$

$$= -j \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

Multiply both sides by j

$$j\frac{dX(e^{j\omega})}{d\omega} = \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

Problem 4-5

$$\text{DTFT}\{x[n-n_0]\} = \sum_{n=-\infty}^{\infty} x[n-n_0] e^{-j\omega n}$$

Let $m = n - n_0$

$$\Rightarrow \sum_{m=-\infty}^{\infty} x[m] e^{-j\omega(m+n_0)}$$

$$= e^{-j\omega n_0} \sum_{m=-\infty}^{\infty} x[m] e^{-j\omega m}$$

$$= e^{-j\omega n_0} X(e^{j\omega})$$

so,

$$x[n-n_0] \longleftrightarrow e^{-j\omega n_0} X(e^{j\omega})$$

Problem 4-6 see matlab code for Hw 4

Problem 4-7 The freq. Resp. is the DTFT of the impulse response, so

$$H_1(e^{j\omega}) = \frac{1}{2} + \frac{1}{2} e^{-j\omega} = e^{-j\frac{\omega}{2}}\left(\frac{e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}}{2}\right)$$

$$= e^{-j\frac{\omega}{2}} \cos(\omega/2)$$

$$|H_1(e^{j\omega})| = |\cos(\omega/2)| = \cos^2(\omega/2)$$

Also,

$$H_2(e^{j\omega}) = \frac{1}{2} - \frac{1}{2} e^{-j\omega} = j e^{-j\frac{\omega}{2}}\left(\frac{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}{2j}\right) = j e^{-j\frac{\omega}{2}} \sin(\omega/2)$$

$$|H_2(e^{j\omega})| = |\sin(\omega/2)| = \sin^2(\omega/2)$$

see matlap plots for Hw 4