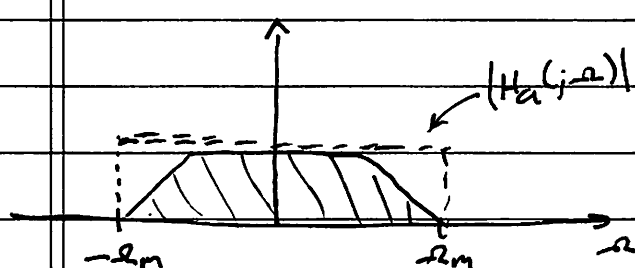
- System Design

Anti-Aliasing Filter

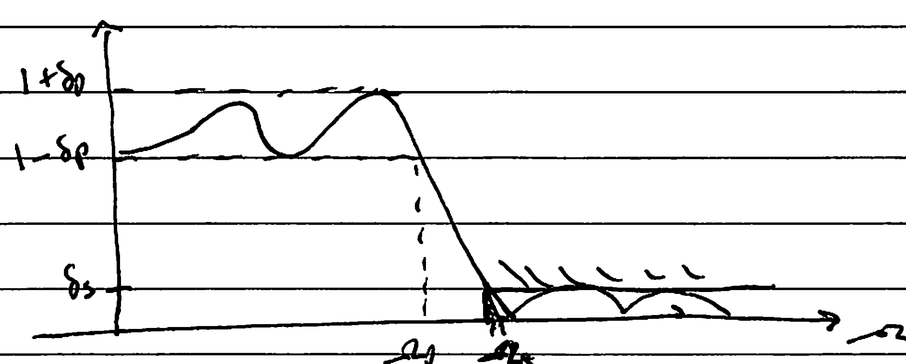
Sampling Theorem – A bandlimited CT signal can be fully recovered from its uniformly sampled version if $\Omega_s \geq 2\,\Omega_m$.

$$x_c(t) \longrightarrow \boxed{h_A(t)} \longrightarrow \boxed{\text{sampling}} \longrightarrow x[n]$$

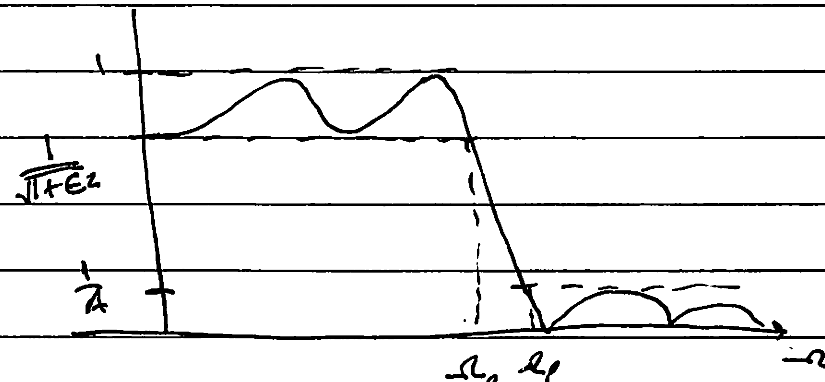
Ideal filter – $H_a(j\Omega) = \begin{cases} 1 & |\Omega| < \Omega_s/2 \\ 0 & |\Omega| \geq \Omega_s/2 \end{cases}$



Ideal filter is not realizable and must be approximated.



Normalize -



Parameters

min stopband Atten. $= -20\log_{10}(1/A)$

Selectivity $\quad k = \frac{\Omega_p}{\Omega_s} \quad$ for lowpass $k < 1$

discrimination $\quad k_1 = \frac{\epsilon}{\sqrt{A^2-1}} \quad$ usually $k_1 \ll 1$

- Analog Filters
  - Butterworth $\quad |H_a(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$

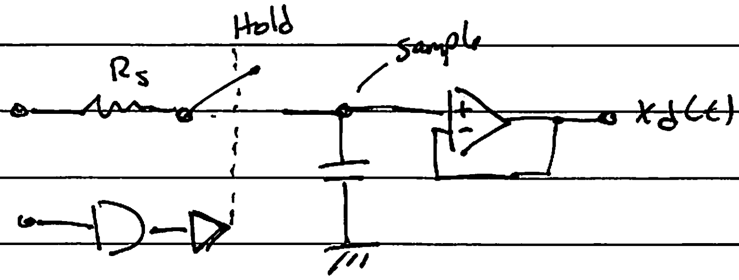
    Two parameters that characterize
    1. Ω_c -3dB cutoff freq.
    2. N order

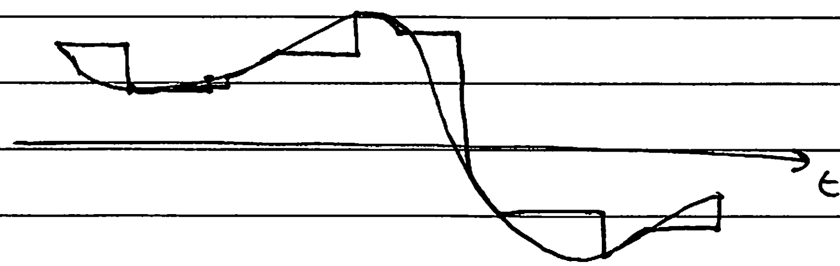
~~Show plot~~

Butterworth filter is maximally Flat -
See example 4.6 & matlab 4-1.m

- Chebyshev
  Two types
  - Type 1 - equiripple in passband
  - Type 2 - equiripple in stopband
- Elliptic → plot

- Sample and Hold circuit





- A/D converter
  output is a sequence of words
  wordlength is given by number of bits

example. 8-bit A/D   00000010
bit 7   bit 0

Accuracy of A/D is expressed as the "resolution" -
determined by discrete levels of output $= 2^N$

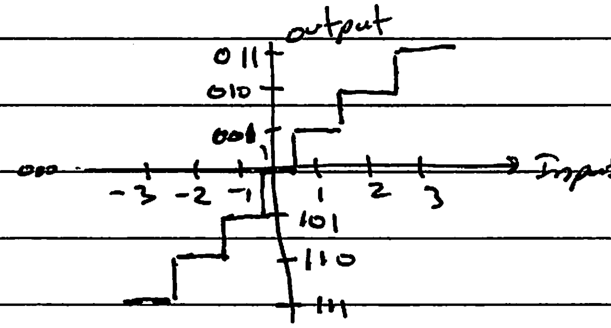
$$\text{resolution} = \frac{1}{2^N} \text{ or } \frac{100}{2^N} \% \text{ "percent"}$$

- Types of A/D converters:
  1 - Flash A/D — low resolution / high speed
  2 - Serial - Parallel -
  3 - Successive Approximation → high resolution / medium speed
  4 - Sigma - Delta (oversampling) - high resolution / high speed

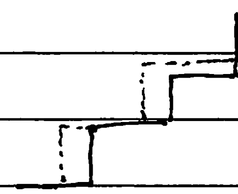
- A/D errors
  - quantization error

$$\delta = 2^{-N} = \text{quantization step}$$

$$-\frac{\delta}{2} < e[n] \le \frac{\delta}{2}$$



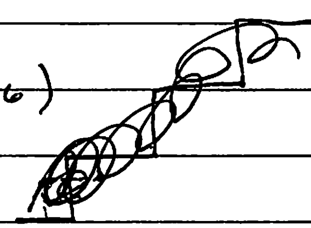
δ = LSB value



  - linearity error

Maximum value over range is called differential nonlinearity (DNL)

  - Gain error (fig. 4.46)
  - offset error



- D/A Converter - digital to Analog
  (see book)
  resolution, linearity error, gain error, etc
- Reconstruction filter

$$H_r(j\Omega) = \begin{cases} T, & |\Omega| \le \Omega_s/2 \\ 0, & |\Omega| > \Omega_s/2 \end{cases}$$

$$y[n] \rightarrow \boxed{D/A} \rightarrow \boxed{H_r} \rightarrow y_a(t)$$

from before $\rightarrow$ $$y_a(t) = \sum_{n=-\infty}^{\infty} y[n] \, \frac{\sin\left(\pi(t-nT)/T_s\right)}{\pi(t-nT)/T_s}$$

Low-pass filter

$$H_r(j\Omega) = \begin{cases} \dfrac{\Omega T/2}{\sin(\Omega T/2)} & |\Omega| \le \Omega_c \\ 0 & |\Omega| \ge \Omega_c \end{cases}$$

- Digital Data Formats
  - Fixed-point binary $\qquad 101101 \qquad$ 6-bits

    msb $\qquad$ LSB

$$\overset{32}{(1\cdot 2^5)} + \overset{16}{(0\cdot 2^4)} + \overset{8}{(1\cdot 2^3)} + \overset{4}{(1\cdot 2^2)} + (0\cdot 2^1) + (1\cdot 2^0) = 45$$

  - Hexadecimal

$$10101001 \rightarrow 1010 \mid 1001$$

A	B	C	D	E	F
10	11	12	13	14	15

so, $1010_2 = 10_{10} = A_{16}$ & $1001_2 = 9_{10} = 9_{16}$

$$\Rightarrow 10101001_2 = A9_{16}$$

– Signed binary

$0011_2 = 3_{10}$ $\qquad$ $1011_2 = -3_{10}$

↑ sign bit

unsigned range $\quad 0$ to $2^{b}-1$

signed range $\quad 0$ to $2^{b-1}-1$

example – 8-bit A/D $\quad 2^{8}-1 = 255$ unsigned

$\pm 2^{8-1}-1 = \pm 127$ signed

– Two's Complement $\quad 0011$

↑ sign bit

To get negative of positive, change one to zero / zero to one and add one.

example. $0011_2 = 3_{10}$

$$
\begin{array}{rl}
 & 0011_2 \\
\Rightarrow & 1100 \\
+ & 0001 \\
\hline
 & 1101 = -3_{10}
\end{array}
$$

Range $\Rightarrow$ -2^{b-1} to $2^{b-1}-1$

- Binary number precision & Dynamic Range.

$$\text{dynamic range}_{dB} = 20 \cdot \log_{10}\left(\frac{\text{largest possible word value}}{\text{smallest " " "}}\right)$$

For signed

$$= 20 \cdot \log_{10}\left(\frac{2^{b-1}}{1}\right) = 20 \cdot \log_{10}(2^b - 1)$$

If $2^b >> 1$

$$= 20 \cdot \log_{10}(2^b) = 20 \cdot \log_{10}(2) \cdot b = 6.02 \cdot b \text{ dB}$$

example

8 bit ~~A/D~~ → $6.02 \cdot 7 = 42.14$ dB
2's comp

- Effects of finite ~~un~~fixed-point binary word length

Exampl 8 bit A/D, Input of −1 to +1 volts

$$\text{lsb value} = \frac{2 \text{ volts}}{2^8} = 7.81 \text{ millivolts}$$

Can represent integers of 7.81mV perfectly, anything else is approximated ⇒ "quantization errors"

This is a roundoff noise ⇒ Ideally it is always less than $\pm\frac{1}{2}$ lsb or ± 3.905 mV.

- Quantization Noise - SNR

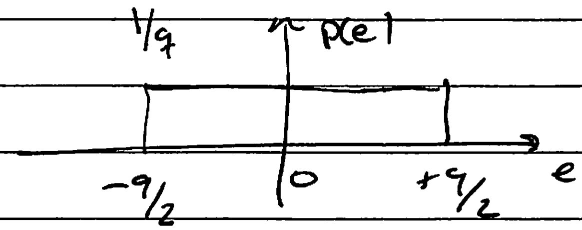
$$SNR_{A/D} = 10 \cdot \log_{10}\left(\frac{\text{input signal variance}}{\text{A/D quantization noise variance}}\right)$$

$$= 10 \cdot \log_{10}\left(\frac{\sigma^2_{signal}}{\sigma^2_{A/D\ noise}}\right)$$

quantization level $= q = 2V_p / 2^b$

probability of any quantization error value

$p(e)$ - probability density function



$$\sigma^2_{AD} = \int_{-q/2}^{-q/2} e^2 p(e)\, de = \frac{1}{q}\int_{-q/2}^{q/2} e^2\, de = q^2/12$$

$$\Rightarrow \sigma^2_{AD} = \frac{(2V_p/2^b)^2}{12} = \frac{V_p^2}{3 \cdot 2^{2b}}$$

Now lets get numerator for SNR_{AD}

$$\text{Loading Factor} = LF = \frac{\text{rms of input signal}}{V_p} = \frac{\sigma_{signal}}{V_p}$$

$\Rightarrow$ So,

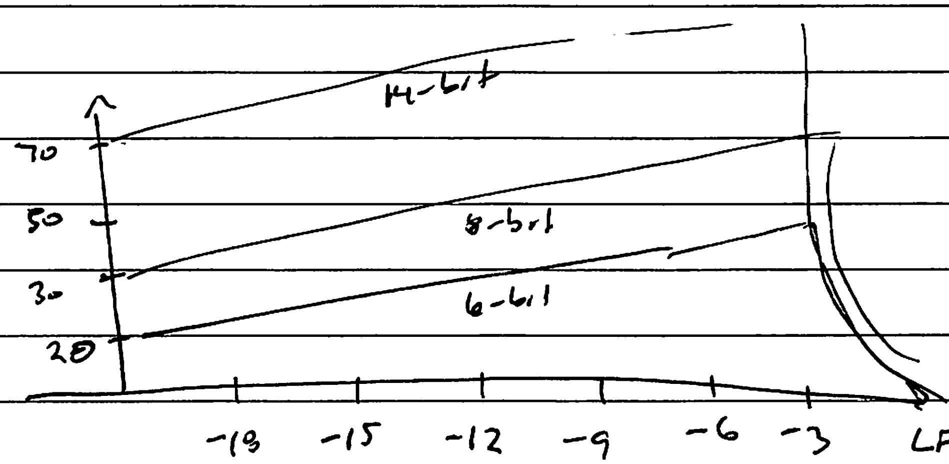
$$\sigma^2_{signal} = (LF)^2 V_p^2$$

And,

$$SNR_{A/D} = 10\log_{10}\left(\frac{(LF)^2 V_p^2}{V_p^2 / (3\cdot 2^{2b})}\right)$$

$$= 10\log_{10}\left((LF)^2\cdot(3\cdot 2^{2b})\right)$$

$$= 6.02\cdot b + 4.77 + 20\log_{10}(LF)$$



$V_p/\sqrt{2} = V_{rms}$ $\rightarrow$ 3dB below V_p

Note:

1. Any CT signal we are digitizing will never have $SNR_{A/D}$ greater than equation predicts after A/D conversion.

$$x(t) \longrightarrow \boxed{\text{8 bit A/D}} \longrightarrow \quad SNR_{A/D} = 6.02 \cdot 8 + 1.76 = 49.9\,dB$$

$SNR = 55\,dB$

Full scale

In practice – Assume $SNR_{A/D\,max}$ is 3 – 6 dB lower than ideal.

2. It is not wise to force input of A/D to full scale.

3. Try not to use extra bits when not needed. Let input CT signal determine this.

- Next Time → overflow, Truncation, floating-point