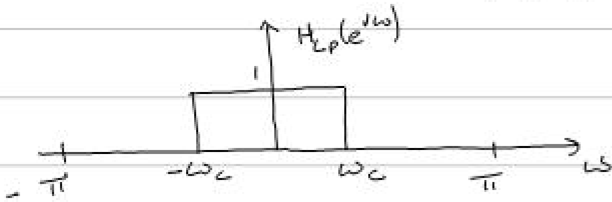
Midterm Solutions

Problem 1 (a)

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$



$$h_{LP}[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_{LP}(e^{j\omega})e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{e^{j\omega n}}{jn}\Bigg|_{-\omega_c}^{\omega_c} = \frac{1}{2\pi}\left(\frac{e^{j\omega_c n}}{jn} - \frac{e^{-j\omega_c n}}{jn}\right) = \frac{\sin(\omega_c n)}{\pi n}$$

for $-\infty < n < \infty,\ n \neq 0$

for $n = 0$

$$h_{LP}[0] = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1\,d\omega = \frac{\omega_c}{\pi}$$

so, $$h_{LP}[n] = \begin{cases} \frac{\omega_c}{\pi}, & n = 0 \\ \frac{\sin(\omega_c n)}{\pi n}, & n \neq 0 \end{cases}$$

(b) $h_{LP}[n] = \frac{\sin(\omega_c n)}{\pi n} w[n] \quad , -M \le n \le M$

Let $N = 2M+1$ shift right by M samples

$$h_{LP}[n] = \begin{cases} \frac{\omega_c}{\pi} & , n = M \\ \frac{\sin(\omega_c(n-M))}{\pi(n-M)} & , 0 \le n < N \\ 0 & , \text{otherwise} \end{cases}$$

(c)

(Ideal) $h_{HP}[n] = \begin{cases} 1 - \frac{\omega_c}{\pi} & , n = 0 \\ -\frac{\sin(\omega_c n)}{\pi n} & , |n| > 0 \end{cases}$

Note: This is just 1 minus the low pass filter.
To make it causal and finite, Let $N = 2M+1$

$$h_{HP}[n] \cdot w[n] = \begin{cases} 1 - \frac{\omega_c}{\pi} & , \text{for } n = M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)} & , n \ne M, 0 \le n < N \\ 0 & , \text{otherwise} \end{cases}$$

(c) continued...

(ideal) $h_{BP}[n] = \dfrac{\sin(\omega_{c2} n)}{\pi n} - \dfrac{\sin(\omega_{c1} n)}{\pi n}$, $|n| \geq 0$

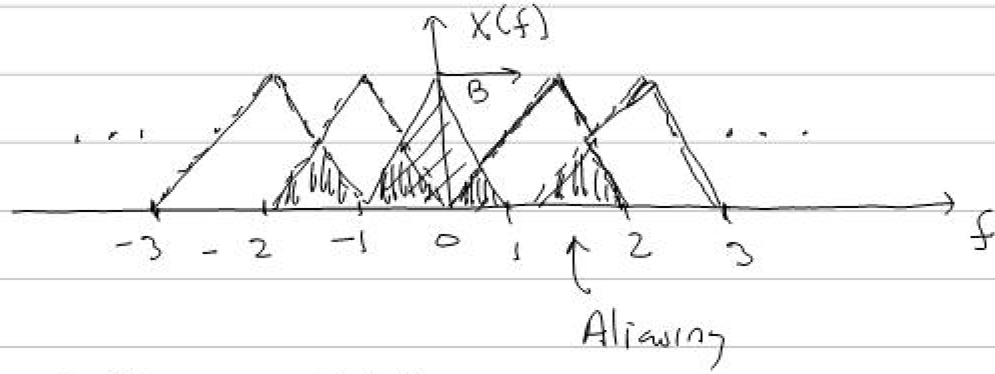
Note: This is a LPF plus a HPF.

$$h_{BP}[n] = \frac{\sin(\omega_{c2}(n-m))}{\pi(n-m)} - \frac{\sin(\omega_{c1}(n-m))}{\pi(n-m)}, \quad 0 \leq n < N$$

(d) see matlab solutions

(e) see matlab solutions.

Problem 2



Nyquist Freq. = 1 MHz

F_s = 1 MHz does not satisfy Nyquist Condition ($F_s \geq 2B$).

Nyquist Rate = 2B

$\Rightarrow$ Oversampling by 10x = $10 \cdot (2B) = 10(2\text{MHz}) = 20\text{MHz}$.

Problem 3 (a) $F_s = 100e^6$ Hz $\quad T_s = \frac{1}{100e^6 \text{Hz}} = .01$ microseconds

2x1024 FFT $\Rightarrow \frac{F_s}{N} = \frac{100e^6}{2048} = 4.883e^4 = 48.83$ kHz

(b) When computational resources are limited and/or for real-time processing applications.

(c) $H(z)\Big|_{z=re^{j\omega}, r=1} = H(e^{j\omega})$

(d) Lower sidelobes are achieved at the expense of a wider mainlobe. The rectangular (boxcar) window has the best resolution.