University of Colorado Denver
College of Engineering and Applied Science
EE 4637/5637 – Digital Signal Processing

Master Copy

FINAL EXAM: DIGITAL SIGNAL PROCESSING
Saturday, 10 Dec 2011

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Student ID (last 4 digits only): Instructor

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Instructions:

Permitted - lecture notes, homework and solutions, textbook, calculator
Not Permitted - any other electronics, neighbors
Duration - 1hr 50 min

Problem Score:

1.(10 points)
2.(10 points)
3.(10 points)
4.(10 points)
5.(10 points)
Total(50 points)

DO NOT TURN PAGE TO START EXAM UNTIL INSTRUCTED

Problem 1. Linear Systems [10 points total]
A discrete-time signal $x[n]$ is applied to the input of a discrete-time LTI system described by the difference equation

$$y[n] = -\frac{1}{3}y[n-1] + x[n] - \frac{1}{4}x[n-1]$$

(a) Which of the following rational system functions describes this system? Provide analytic justification.

$$H_1(z) = \frac{1 - 1/4z^{-1}}{1 - 1/3z^{-1}}$$

$$H_2(z) = \frac{1 - 1/4z^{-1}}{1 + 1/3z^{-1}}$$

$$H_3(z) = \frac{1 + 1/4z^{-1}}{1 - 1/3z^{-1}}$$

$$H_4(z) = \frac{1 + 1/4z^{-1}}{1 + 1/3z^{-1}}$$

why?

(b) What are the poles and zeros for your choice? (c) Is this system stable? (d) Find the frequency response for this system. (e) What is the inverse transfer function for this system? (f) Is the inverse system stable?

a) $Y(z) = -\frac{1}{3}Y(z)z^{-1} + X(z) - \frac{1}{4}X(z)z^{-1} = -\frac{1}{3}Y(z)z^{-1} + X(z)(1 - \frac{1}{4}z^{-1})$

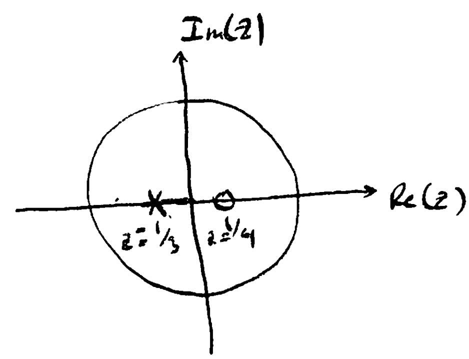
$\Rightarrow Y(z) + \frac{1}{3}Y(z)z^{-1} = X(z)(1 - \frac{1}{4}z^{-1})$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{4}z^{-1}}{1 + \frac{1}{3}z^{-1}}$

b)

poles : $z = -\frac{1}{3}$

zeros : $z = \frac{1}{4}$



c.) Poles are inside unit circle, so system is stable

d.) $H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}} = \frac{1 - \frac{1}{4}e^{-j\omega}}{1 + 1/3\, e^{-j\omega}}$

e.) $H^{inv}(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{4}z^{-1}}$

f.) $H^{inv}(z)$: poles - $z = \frac{1}{4}$

zeros - $z = -\frac{1}{3}$

Poles are inside unit circle, so $H^{inv}(z)$ is stable.

Problem 2. DFT [10 points total]
Find the N-point DFT of the discrete time signal $x[n] = \frac{1}{2}cos(n\omega_0)$, for $0 \leq n \leq N-1$, and when $\omega_0 = 2\pi k_0/N$.

$$x[n] = \frac{1}{2} \cdot \left[\frac{1}{2} e^{jn\omega_0} + \frac{1}{2} e^{-jn\omega_0} \right] = \frac{1}{4} e^{jn\omega_0} + \frac{1}{4} e^{-jn\omega_0}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k n}{N}} = \frac{1}{4} \sum_{n=0}^{N-1} e^{jn\omega_0} e^{-j\frac{2\pi k n}{N}} + \frac{1}{4} \sum e^{-jn\omega_0} e^{-j\frac{2\pi k n}{N}}$$

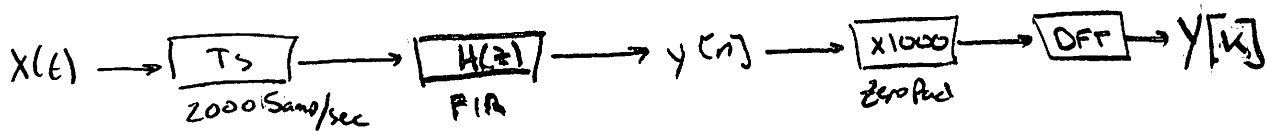
$$= \frac{1}{4} \sum_{n=0}^{N-1} e^{-jn\left(\frac{2\pi k}{N} - \omega_0\right)} + \frac{1}{4} \sum_{n=0}^{N-1} e^{-jn\left(\frac{2\pi}{N} k + \omega_0\right)}$$

If $\omega_0 = \frac{2\pi k_0}{N}$

$$X[k] = \frac{1}{4} \sum_{n=0}^{N-1} e^{-jn\frac{2\pi}{N}(k - k_0)} + \frac{1}{4} \sum_{n=0}^{N-1} e^{-jn\frac{2\pi}{N}(k + k_0)}$$

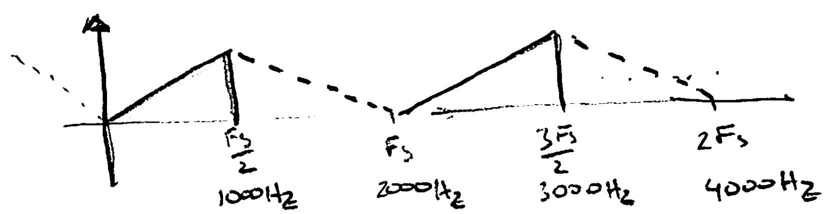
$$X[k] = \begin{cases} \frac{N}{2} & , k = k_0 \text{ or } k = N - k_0 \\ 0 & , \text{otherwise} \end{cases}$$

Problem 3. Sampling [10 points total]
A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 200 Hz, 600 Hz, 1.2 kHz, and 2.4 kHz. The signal is sampled at a rate of 2000 samples/second. The sampled sequence is passed through a digital linear phase FIR low pass filter with a stop band frequency of 900Hz, generating the signal $y[n]$. You then zero pad $y[n]$ by 1000X, compute the DFT, and plot the magnitude spectrum. What frequency components (in Hz) are present?



$|Y[k]|^2 \Rightarrow$ 

Aliasing from Sampling –



200 Hz, 1800 Hz, 2200 Hz, ...

600 Hz, 1400 Hz, 2600 Hz, ...

1200 Hz, 800 Hz, 2800 Hz, ...

2400 Hz, 400 Hz, 1600 Hz, ...

After FIR filter, only 200 Hz, 400 Hz, 600 Hz, 800 Hz components remain.

A large amount of zero-padding is used to approximate the DTFT from the DFT. You would see strong peaks in the magnitude spectrum at 200 Hz, 600 Hz, 800 Hz, and 800 Hz.

Problem 4. Convolution and Correlation [10 points total]
Given the discrete-time sequences $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$ and $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$, (a) compute the output of the convolution sum $y[n] = x[n] * h[n]$. (b) Compute the cross correlation between $x[n]$ and $y[n]$. (c) Manually plot both the convolution output and the cross correlation output.

Linear

$x[n] = [1\ 1\ 1\ 1]$
$h[n] = [1\ 2\ 3\ 4]$

a.) Convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$n=-3$: 1 1 1 1 / 4 3 2 1 → / 1 = 1

$n=-2$: 1 1 1 1 / 4 3 2 1 / 2+1 = 3

$n=-1$: 1 1 1 1 / 4 3 2 1 / 3+2+1 = 6

$n=0$: 1 1 1 1 / 4 3 2 1 / 4+3+2+1 = 10

$n=1$: 1 1 1 1 / 4 3 2 1 / 4+3+2 = 9

$n=2$: 1 1 1 1 / 4 3 2 1 / 4+3 = 7

$n=3$: 1 1 1 1 / 4 3 2 1 / 4 = 4

$y[n] = [1, 3, 6, 10, 9, 7, 4]$

Note: $M = 2N - 1 = 2(4) - 1 = 7$

b.) $$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]\, h[n-\ell]$$

$\ell=-3$: 1 1 1 1 / 1 2 3 4 / 4 = 4

$\ell=-2$: 1 1 1 1 / 1 2 3 4 / 3+4 = 7

$\ell=-1$: 1 1 1 1 / 1 2 3 4 / 2+3+4 = 9

$\ell=0$: 1 1 1 1 / 1 2 3 4 / 1+2+3+4 = 10

$\ell=1$: 1 1 1 1 / 1 2 3 4 / 1+2+3 = 6

$\ell=2$: 1 1 1 1 / 1 2 3 4 / 1+2 = 3

$\ell=3$: 1 1 1 1 / 1 2 3 4 / 1 = 1

$r_{xy}[\ell] = [4, 7, 9, 10, 6, 3, 1]$

Problem 5. Basic Concepts [10 points total]
Answer the following questions:
(a) What is an analytic signal? How can you generate one from a real-valued signal?
(b) If you have 100,000 samples of a discrete-time band-limited signal, sampled at a rate of 100MHz, what is the time duration of your data record? What is the time period between samples? If you take a 32,768 point FFT of the sampled signal, what will be the frequency spacing between the output samples of the FFT?
(c) Draw a block diagram for a FIR filter that has an impulse response, $h[n]$, of length 6 (i.e., 6 coefficients)?
(d) How can you compute the frequency response of a system from the impulse response?
(e) When designing linear phase FIR filters using the window method, what is the trade-off between the width of the transition band (i.e., frequency spacing between pass band to stop band in the frequency magnitude response) and the pass band and stop band ripple?

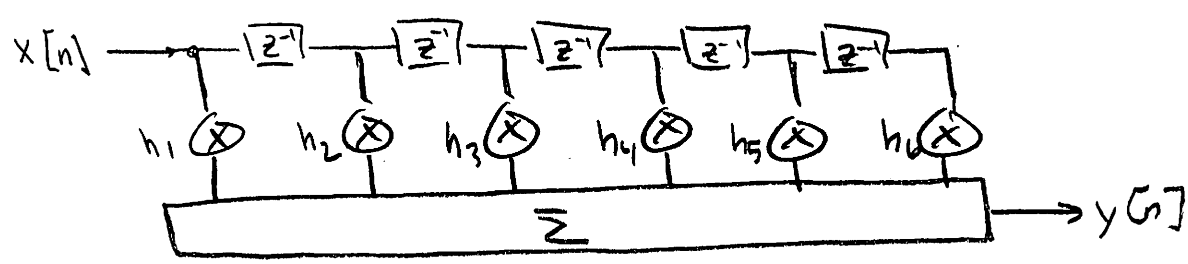
a) An analytic signal is complex valued and has a one-sided spectrum.

b) $F_S = 100$ Msamples/sec $\Rightarrow T_S = 1/F_S = 1.0e^{-8}$ seconds

$T_S \times N = T = (1e^{-8})(100e^{3}) = 1e^{-3}$ sec $=$ 1 millisecond

$F_S/N_{fft} = \frac{100e^{6}}{32,768} = 3051.76$ Hz.

c) $h[n] = [h_1\ h_2\ h_3\ h_4\ h_5\ h_6]$



d) Take the Fourier Transform of the impulse response to get the frequency response. $H(e^{j\omega}) = DTFT\{h[n]\}$

e) The sharper the transistion band, the more ripple on the pass band and stop band.