Homework Assignment 6 - Due Oct. 22, 2011

Problem 6-1 (3 points)
A finite-energy continuous-time signal $g_a(t)$ is sampled at a rate satisfying the Nyquist condition, generating a discrete-time sequence $g[n]$. Develop the relation between the total energy $E_{ga(t)}$ of the continuous-time signal $g_a(t)$ and the total energy $E_{g[n]}$ of the discrete-time signal $g[n]$. Assume a bandlimited continuous-time signal.

Problem 6-2 (2 points)
A 2.5 second long segment of a continuous-time signal is uniformly sampled without aliasing, generating a finite-length sequence containing 5000 samples. What is the highest frequency component that could be present in the continuous-time signal?

Problem 6-3 (3 points)
A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300 Hz, 500 Hz, 1.2 kHz, 2.15 kHz, and 3.5 kHz. The signal is samples at a 2 kHz rate, and the sampled sequence is passed through an ideal lowpass filter with a cutoff frequency of 900 Hz, generating a continuous-time signal $y_a(t)$. You look at the signal $y_a(t)$ with a spectrum analyzer. What frequency components are present?

Problem 6-4 (3 points)
The matlab M-file called fourdfilt.m is posted on the course blackboard site. It creates a narrow-band lowpass signal that is corrupted with a high-frequency random noise. Your job is to filter out the high frequency noise using Fourier domain filtering. Generate a frequency response vector for an ideal filter and use multiplication in the frequency domain. Plot the before and after time-domain signals.

Problem 6-5 (3 points)
In practice, the conditions for distortionless transmission can be satisfied only approximately. There is always a certain amount of distortion present in the output of a physical LTI system. We can distinguish two components of linear distortion produced by transmitting a signal through an LTI system:

- Amplitude distortion — the frequency components of the input signal are transmitted through the system with different amounts of gain or attenuation.

- Phase distortion — the phase response is not linear with frequency inside the frequency band of interest. Different frequency components are subject to different delay in passing through the system.

In class, we introduced the multipath propagation model

$$y[n] = x[n] + ax[n-1].$$

In general, the model parameter a can be real or complex, so long as $|a| < 1$.

(a) What form of linear distortion is introduced by this channel? Justify your answer.