Homework Assignment 5 - Due Oct. 8, 2011

Problem 5-1 (5 points)
You have already learned how to represent a periodic signal using the Fourier Series (FS), and you know that multiplication in the time domain is equivilent to convolution in the frequency domain:

$$y(t) = g(t)x(t) \overset{FT}{\longleftrightarrow} Y(j\Omega) = \frac{1}{2\pi} G(j\Omega) * X(j\Omega).$$

If $x(t)$ is periodic, we can use the FT of the FS representation to get

$$y(t) = g(t)x(t) \overset{FT}{\longleftrightarrow} Y(j\Omega) = G(j\Omega) * \sum_{k=-\infty}^{\infty} X[k]\delta(\Omega - k\Omega_0).$$

This results in multiple, frequency shifted versions of the original function. Hence, we have

$$y(t) = g(t)x(t) \overset{FT}{\longleftrightarrow} Y(j\Omega) = \sum_{k=-\infty}^{\infty} X[k]G(j(\Omega - k\Omega_0)).$$

So, multiplication of $g(t)$ with a periodic function $x(t)$ gives an FT consisting of a weighted sum of shifted versions of $G(j\Omega)$. When you learned about sampling you used the periodic impulse train as $x(t)$. However, any periodic function can be considered. Consider the periodic discrete-time signal

$$x[n] = cos(\frac{7\pi}{16}n) + cos(\frac{9\pi}{16}n).$$

(a) Find the FS coefficients of $x[n]$. (b) Obtain the DTFT of $x[n]$ using the coefficients and the knowledge you learned above. (c) What are the frequencies of each sinusoid in $x[n]$? Using Matlab, compute 25 seconds of the discrete-time signal $x[n]$ using an appropriate sample period T_s. Compute and plot the magnitude spectrum using specplot.m. Multiply $x[n]$ by

$$w[n] = \begin{cases} 1, & n \leq 16, \\ 0, & n > 16 \end{cases}$$

and compute the magnitude spectrum again. What happened? Can you explain what you see using the information above?

Problem 5-2 (3 points)
Determine the N-point DFT of the following length-N sequences defined for $0 \leq n \leq N-1$

$$x[n] = sin(8\pi n/N)$$
$$x[n] = cos^2(8\pi n/N)$$

Compute and plot the magnitude spectrums for both signals using Matlab and the fft(N,x) function for $N = 8, 16, 32, 64$. Comment on the changes you notice for each value of N.

Problem 5-3 (5 points)
Write a matlab function to computer the DFT (do not use the built in Matlab functions for DFT or FFT). Use your matlab function in the demo macro specgong.m instead of the Matlab fft function call. Compare the results of specgong.m when using your function and when using the fft function (Note: The results should be the same, but the DFT is much slower to compute). Work on your code until you get it correct.

Problem 5-4 (2 points)
Read the paper on the Blackboard site about bandpass sampling. Write down three things you learn from the paper.

Problem 5-5 (1 points)
Use the FDA tool in matlab to design a low pass filter for a bass guitar processor that only passes frequencies below $250Hz$. Comment on what you learn.