Homework Assignment 4 - Due Saturday Sept. 24, 2011

Problem 4-1
We call two signals $\psi_m[n], \psi_k[n]$ orthogonal on an interval $[N1, N2]$ if they satisfy the condition

$$\sum_{n=N_1}^{N_2} \psi_m[n]\psi_k^*[n] = \begin{cases} 0, & m \neq k, \\ \alpha, & m = k. \end{cases}$$

Show that the set of complex exponential sequences $\psi_k[n] = e^{jk(2\pi/N)n}$ for $k = 0, 1, ..., N-1$ is orthogonal on any interval of length N. You will need to use the closed form series solution:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N, & \alpha = 1, \\ \frac{1-\alpha^N}{1-\alpha}, & \alpha \neq 1 \end{cases}$$

Problem 4-2
Find the frequency-shifting property for the DTFT of a discrete-time signal $x[n]$. Hint: Follow the derivation in the lecture notes for the CT case.

Problem 4-3
Find the inverse fourier transform $x(t)$ of $X(j\Omega) = 2\pi\delta(\Omega - \Omega_0)$.

Problem 4-4
Find the differentiation in frequency property for the discrete-time case. Hint: Follow the derivation in the lecture notes for the CT case.

Problem 4-5
Find the time-shifting property for the discrete-time case.

Problem 4-6
Write a matlab program to generate a complex sinusoid with a frequency of $1200Hz$. (a) Compute the spectrum of this signal (look at the matlab cose specsin.m posted on the course blackboard site) and plot its magnitude spectrum. (b) Using the frequency shift property, shift the frequency of this signal to $720Hz$ and plot the magnitude spectrum. (c) Take the real part of the frequency shifted signal and plot its magnitude spectrum. What do you notice different about the magnitude spectrums of the complex signal versus the real part of the signal?

Problem 4-7
Consider two different moving-average systems described by the input-output equations

$$y_1[n] = 1/2(x[n] + x[n-1])$$

$$y_2[n] = 1/2(x[n] - x[n-1])$$

The first system averages successive inputs, while the second system forms the difference. The impulse responses are

$$h_1[n] = 1/2\delta[n] + 1/2\delta[n-1]$$

$$h_2[n] = 1/2\delta[n] - 1/2\delta[n-1]$$

Find the frequency response of each system and use Matlab to plot the magnitude responses.

Problem 4-8
Play around with the Matlab demos posted on the course blackboard site to improve your understanding of the course concepts. Nothing needs to be turned in for this problem - just for your learning enjoyment.