HW4 - Normal Distributions

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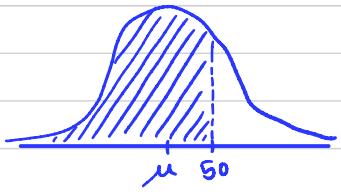
33). Mopeds (small motorcycles with an engine capacity below 50 cm^3) are very popular in Europe because of their mobility, ease of operation, and low cost. The article "Procedure to Verify the Maximum Speed of Automatic Transmission Mopeds in Periodic Motor Vehicle Inspections" (J. of Automobile Engr. 2008: 1615-1623) described a rolling bench test for determining maximum vehicle speed. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such moped.

a). What is the probability that maximum speed is at most 50 km/h?

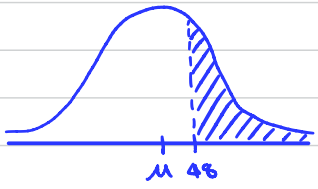


$$z = \frac{x-\mu}{\sigma} = \frac{50-46.8}{1.75} = 1.83$$

From a standard normal z-table,

$P(\text{max. spd.} \le 50 \text{ km/h}) = 0.9664$

b). What is the probability that maximum speed is at least 48 km/h?

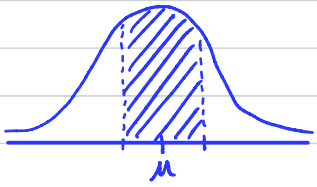


$$z = \frac{x-\mu}{\sigma} = \frac{48-46.8}{1.75} = 0.69$$

From a standard normal z-table,

$P(\text{max. spd.} \ge 48 \text{ km/h}) = 1 - 0.7549 = 0.2451$

c). What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?



range $= \mu \pm 1.5(\sigma) = 46.8 \pm 1.5(1.75) = (44.175, 49.425)$

$z_1 = (x_1 - \mu)/\sigma = (49.425 - 46.8)/1.75 = 1.5$

$z_2 = (x_2 - \mu)/\sigma = (44.175 - 46.8)/1.75 = -1.5$

From a standard normal z-table,

$P_1(z_1 = 1.5) = 0.9332$

$P_2(z_2 = -1.5) = 0.0668$

$P(\text{max. spd. differs from the mean value by at most 1.5 standard deviations})$

$= P_1(z_1 = 1.5) - P_2(z_2 = -1.5)$

$= 0.9332 - 0.0668$

$= 0.8664$

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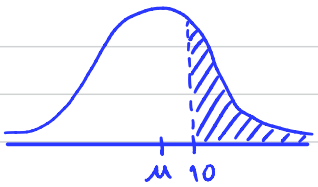
35). Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$, as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (Forest Products J., May 1997 : 36-41)

a). What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10 in.?



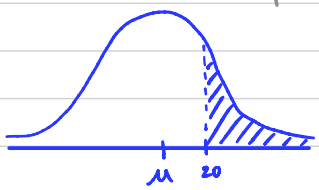
$z = (x-\mu)/\sigma = (10-8.8)/2.8 = 0.43$

From a standard normal z-table,

$P(\text{diameter} \geq 10 \text{ in.}) = 1 - 0.6664 = 0.3336$

$P(\text{diameter exceed } 10 \text{ in.}) = 0.3336$

b). What is the probability that the diameter of a randomly selected tree will exceed 20 in.?

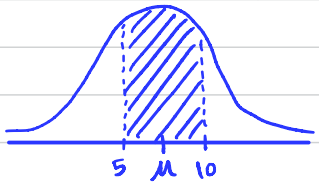


$z = (x-\mu)/\sigma = (20-8.8)/2.8 = 4$

From a standard normal z-table,

$P(\text{diameter} > 20 \text{ in.}) = 1 - 0.99997 = 0.00003$

c). What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?



$z_1 = (x_1-\mu)/\sigma = (10-8.8)/2.8 = 0.43$

$z_2 = (x_2-\mu)/\sigma = (5-8.8)/2.8 = -1.36$

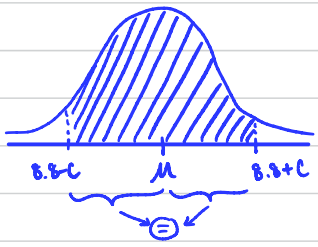
From a standard normal z-table,

$P_1(\text{dia.} \leq 10 \text{ in.}) = 0.6664$

$P_2(\text{dia.} \leq 5 \text{ in.}) = 0.0869$

$P(\text{dia. between 5 and 10 in.}) = P_1 - P_2 = 0.6664 - 0.0869 = 0.5795$

d). What value c in such that the interval $(8.8-c, 8.8+c)$ includes 98% of all diameter values?



$z_{8.8+c} = (8.8+c-\mu)/\sigma = (8.8+c-8.8)/2.8 = c/2.8$

$z_{8.8-c} = (8.8-c-\mu)/\sigma = (8.8-c-8.8)/2.8 = -c/2.8$

$P(8.8-c \leq x \leq 8.8+c) = P(c/2.8) - P(-c/2.8) = 0.98$ —①

As it is symmetric, $P(c/2.8) + P(-c/2.8) = 1$ —②

From ① and ②, $2P(c/2.8) = 1.98$

$P(c/2.8) = 0.99$; $0.99 \approx P(z = 2.33)$

$c/2.8 = 2.33$

$c = 6.524$

e). If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 in.?

From a), $P(\text{diameter} \geq 10 \text{ in.}) = 0.3336$

Let x = the number of trees that have diameter exceed 10 in.

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - P(\text{Tree 1} < 10 \text{ in}) \cdot P(\text{Tree 2} < 10 \text{ in}) \cdot P(\text{Tree 3} < 10 \text{ in}) \cdot P(\text{Tree 4} < 10 \text{ in})$

$= 1 - (1-0.3336)(1-0.3336)(1-0.3336)(1-0.3336)$

$= 1 - (1-0.3336)^4$

$= 0.8028$

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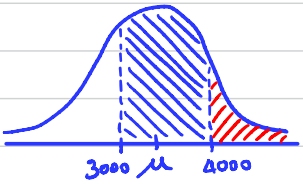
49). Consider babies born in the "normal" range of 37-43 weeks gestational age. Extensive data supports the assumption that for such babies born in the United States, birth weight is normally distributed with mean 3432 g. and standard deviation 482 g. [The article "Are Babies Normal?" (The American Statistician, 1999: 298-302) analyzed data from a particular year; for a sensible choice of class intervals, a histogram did not look at all normal, but after further investigations it was determined that this was due to some hospitals measuring weight in grams and others measuring to the nearest ounce and then converting to grams. A modified choice of class intervals that allowed for this gave a histogram that was well described by a normal distribution.]

a). What is the probability that the birth weight of a randomly selected baby of this type exceeds 4000 g? Is between 3000 and 4000?



$z_{4000} = (x_{4000} - \mu)/\sigma = (4000 - 3432)/482 = 1.18$

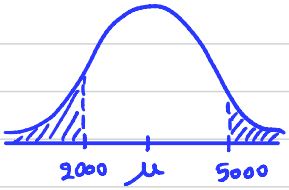
$z_{3000} = (x_{3000} - \mu)/\sigma = (3000 - 3432)/482 = -0.90$

From z-table, $P(z = 1.18) = P(x \le 4000) = 0.881$, $P(z = -0.90) = P(x \le 3000) = 0.18406$

$P(x > 4000) = 1 - P(x \le 4000) = 1 - 0.881 = 0.119$ //.

$P(3000 < x < 4000) = P(x \le 4000) - P(x \le 3000) = 0.881 - 0.18406 = 0.69694$ //.

b). What is the probability that the birth weight of a randomly selected baby of this type is either less than 2000g or greater than 5000g?



$z_{5000} = (x_{5000} - \mu)/\sigma = (5000 - 3432)/482 = 3.25$

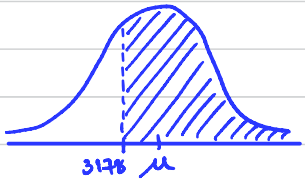
$z_{2000} = (x_{2000} - \mu)/\sigma = (2000 - 3432)/482 = -2.97$

From z-table, $P(z = 3.25) = P(x \le 5000) = 0.99942$, $P(z = -2.97) = P(x \le 2000) = 0.00149$

$P(x < 2000 \text{ or } x > 5000) = P(x < 2000) + (1 - P(x \le 5000))$

$= 0.00149 + (1 - 0.99942) = 0.00207$ //.

c). What is the probability that the birth weight of a randomly selected baby of this type exceeds 7 lb? (1 lb ≈ 454 g ⇒ 7 lb ≈ 3,178 g)

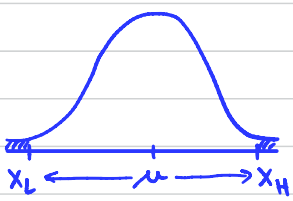


$z_{3178} = (x_{3178} - \mu)/\sigma = (3178 - 3432)/482 = -0.53$

From z-table, $P(z = -0.53) = P(x \le 3178) = 0.29806$

$P(x > 3178) = 1 - P(x \le 3178) = 1 - 0.29806 = 0.7019$ //.

d). How would you characterize the most extreme .1% of all birth weights?



$P(x < x_L \text{ or } x > x_H) = 1 - P(x_H) + P(x_L) = 0.001 \rightarrow P(x_H) - P(x_L) = 0.999$ —①

As it is symmetric, $P(x_H) + P(x_L) = 1$ —②

From ① and ②, $2P(x_H) = 1.999$

$P(x_H) = 0.9995 \xrightarrow{\text{From z-table}} z_{x_H} = 3.29$

From $P(x_H) = 0.9995$, $P(x_L) = P(x_H) - 0.999 = 0.0005 \xrightarrow{\text{From z-table}} z_{x_L} = -3.29$

$x_H = (z_{x_H})(\sigma) + \mu = (3.29)(482) + 3432 = 5017.78$ g

$x_L = (z_{x_L})(\sigma) + \mu = (-3.29)(482) + 3432 = 1846.22$ g

Therefore, most extreme 0.1% weights are either less than 1,846.22 g or more than 5,017.78 g //.

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Question 49). continued.

e). If X is a random variable with a normal distribution and a is a numerical constant ($a \neq 0$), then $Y = aX$ also has a normal distribution. Use this to determine the distribution of birth weights expressed in pounds (shape, mean, and standard deviation), and then recalculate the probability from part c). How does this compare to your previous answer?
(1 lb ≈ 454 g ⇒ 7 lb ≈ 3,178 g)

From Expected Value Properties, $Y = aX + b$

$$E(Y) = E(aX + b)$$
$$= aE(X) + b$$

hence, multiply a random variable (X) by a constant (a) does not change the shape of distribution

Convert g to lb : $\mu = 3432/454 = 7.559$ lb
$\sigma = 482/454 = 1.062$ lb

$$Z = (X - \mu)/\sigma = (7 - 7.559)/1.062 = -0.53$$

From z-table, $P(z = -0.53) = P(X \leq 7\text{ lb}) = 0.29806$

$P(X > 7\text{ lb}) = 1 - P(X \leq 7\text{ lb}) = 1 - 0.29806 = 0.7019$ (result is same as part c)

Negative z-table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

Positive z-table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998