CS556 Fall 2022 Test 2

NAME:

Closed book: no textbook, no electronic devices, one sheet of paper with handwritten notes. *Read carefully before answering!* Write your answers on this test paper. Also hand in your note sheet, with your name on it.

Question 1 (20 points) Given $f(x) = (x^2 - 1)^{\ln(x)}$, find the derivative f'(x) using logarithmic differentiation.

Question 2 (10 points) Compute the derivative of $f(x) = \sqrt{3x^2 + 2}$.

Question 3 (10 points) Compute the gradient of $f(x,y) = (x+2) + xy\sin(y)$ at (1,0).

Question 4 (15 points) Owners of a car rental company have determined that if they charge customers p dollars per day to rent a car, where $50 \le p \le 200$, the number of cars n they rent per day can be modeled by the linear function n(p) = 1000 - 5p. If they charge \$50 per day or less, they will rent all their cars. If they charge \$200 per day or more, they will not rent any cars. Assuming the owners plan to charge customers between \$50 per day and \$200 per day to rent a car, how much should they charge to maximize their revenue?

Question 5 (5 points) Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, P(A|B) or P(B|A)? Why?

Question 6 (20 points) The Reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60% of them are short, 30% are medium, and the other 10% are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80% are in Word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.

- What is the probability that the selected review was submitted in Latex format?
- If the selected review was submitted in Latex format, what are the posterior probabilities of it being long?

Question 7 (10 points) The concentration of a substance is normally distributed with $\mu = 0.5$ and $\sigma = 0.1$. (Round up to two decimal points. For example you should round 2.176 to 2.18).

- 1. What is the probability that the concentration is at most 0.10?
- 2. What is the probability that the concentration exceeds 0.25?
- 3. How would you characterize the largest 5% of all concentration values?

Question 8 (10 points) This question considers the wind speeds of Hurricane Katrina, which affected New Orleans, Louisiana, in August 2005.

Hours after Midnight, August 26	1	5	11	29	49	58	73	81	85
Wind Speed (mph)	45	75	100	115	145	175	155	125	95

Using the table above, estimate the derivative of the wind speed at hour 77 using the forward, backward and central difference methods.

Question 9 (5 points) **BONUS** Complete the sentence. Be creative. In the rest of the semester, I am hoping for \dots