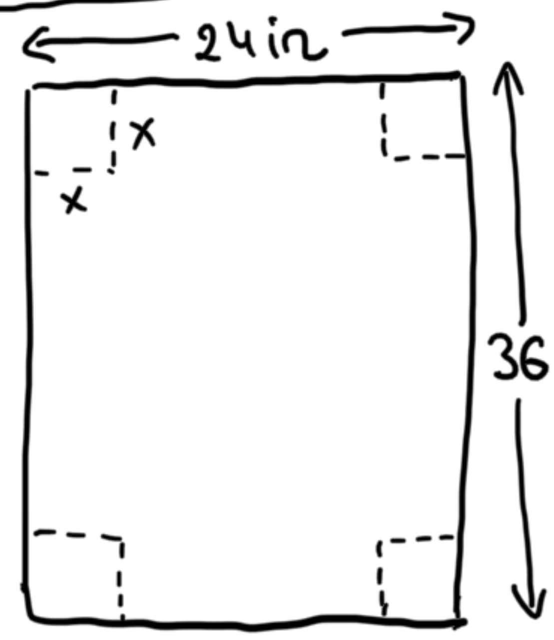
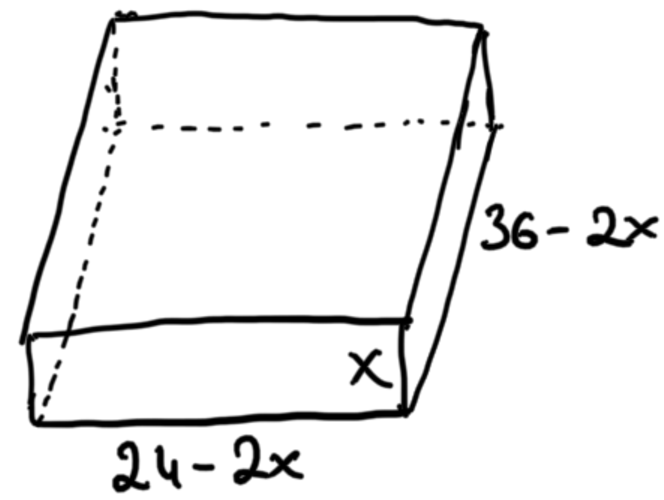
APPLIED OPTIMIZATION

Problem 1 – Maximizing the volume of a box.





$x \to$ side length of the square

$V \to$ volume of box

$V = (36-2x)(24-2x) \cdot x = 4x^3 - 120x^2 + 864x$

Constraints

(1) $x > 0$

(2) $x < 12 \to$ half of shorter side

- We aim to find the maximum volume for x over the open interval $(0, 12)$
- Since V is a continuous function over the closed interval $[0, 12]$, we know V will have an absolute maximum over the closed interval.

Since $V(x) = 0$ at the endpoints and $V(x) > 0$ for $0 < x < 12$, the maximum must occur at a critical point.

$V'(x) = 12x^2 - 240x + 864$

To find critical points, we need to solve the equation

$12x^2 - 240x + 864 = 0 \to x^2 - 20x + 72 = 0$

Using quadratic formula

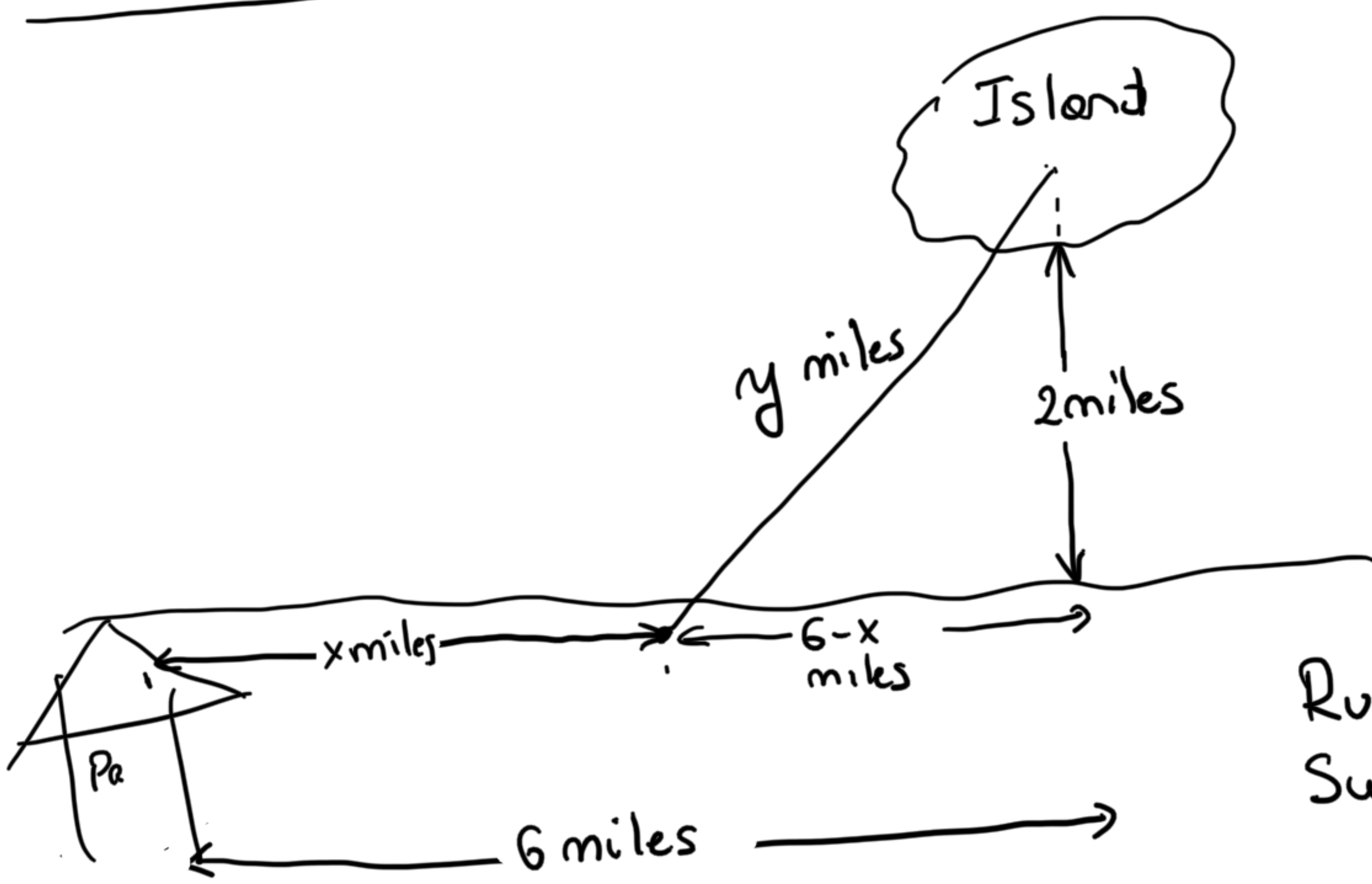
$$x = \frac{20 \pm \sqrt{(-20)^2 - 4 \cdot 1 \cdot 72}}{2} = 10 \pm 2\sqrt{7}$$

$\downarrow$

... the volume $V(x) \approx 1825 \text{ in}^3$

$x = 10 - 2\sqrt{7}$ maximizes the ...

Problem 2: Minimizing Travel Time



Run 8 mph
Swim 3 mph

Cabin

Goal → Minimize travelling time

$$T_R = \frac{D_R}{R_R} = \frac{x}{8} \qquad T_S = \frac{D_S}{R_S} = \frac{y}{8}$$

$$T = \frac{x}{8} + \frac{y}{3}$$

$$y^2 = 2^2 + (6-x)^2$$

$$y = \sqrt{(6-x)^2 + 4}$$

$$T = \frac{x}{8} + \frac{\sqrt{(6-x)^2+4}}{3} \qquad 0 \le x \le 6$$

Look at critical points

$$T'(x) = \frac{1}{8} - \frac{1}{2}\frac{[(6-x)^2-4]^{-1/2}}{3} \cdot 2(6-x) = \frac{1}{8} - \frac{6-x}{\sqrt{(6-x)^2+4}}$$

$$T'(x) = 0$$

$$\frac{1}{8} = \frac{6-x}{3\sqrt{(6-x)^2+4}}$$

$$3\sqrt{(6-x)^2-4} = 8(6-x)$$

$$9\left[(6-x)^2+4\right] = 64(6-x)^2$$

$$55(6-x)^2 = 36$$

$$(6-x)^2 = 36/55$$

$$x = 6 \pm \frac{6}{\sqrt{55}}$$

$x = 6 - \frac{6}{\sqrt{55}}$ is a critical point

$$T\left(6 - 6/\sqrt{55}\right) \approx 1.368\,h.$$