Derivatives

① $h(x) = \cos(5x^2)$

$g(x) = 5x^2$

$f(x) = \cos(x)$

$h'(x) = f'(g(x)) = f'(g(x)) \cdot g'(x)$

$h'(x) = -\sin(5x^2) \cdot 10x$

$= -10x \sin(5x^2)$

② $h(x) = (2x+1)^5 (3x-2)^7$

$h'(x) = \frac{d}{dx}\left((2x+1)^5\right) \times (3x-2)^7 + (2x-1)^5 \times \frac{d}{dx}\left((3x-2)^7\right)$

$= 10(2x+1)^4 \times (3x-2)^7 + (2x-1)^5 \times 7(3x-2)^6 \times 3$

$= 10(2x+1)^4 (3x-2)^7 + 21(2x-1)^5 (3x-2)^6$

③ $y = (2x^4+1)^{\cos x}$

$\ln(y) = \ln(2x^4+1)^{\cos x}$

$\ln(y) = \cos x \ln(2x^4+1)$

$\frac{1}{y}\frac{dy}{dx} = -\sin x \ln(2x^4+1) + \cos x \frac{8x^3}{2x^4+1}$

$\frac{dy}{dx} = (2x^4+1)^{\cos x} \cdot \left(-\sin x \ln(2x^4+1) + \cos x \frac{8x^3}{2x^4+1}\right)$

⑨ $y = \dfrac{x\sqrt{2x+1}}{e^x \sin^3 x}$

$$\ln y = \ln x\sqrt{2x+1} - \ln e^x \sin^3 x$$

$$\ln y = \ln x + \frac{1}{2}\ln 2x+1 - x\ln e - 3\ln \sin x$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{2}{2(2x+1)} - 1 - 3\frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{1}{2x+1} - 1 - 3\frac{\cos x}{\sin x}\right)\frac{x\sqrt{2x+1}}{e^x \sin^3 x}$$