Solutions for Homework 4:

Ans 1.

a)

$$f'(x) = lim_{h->0}\frac{\sqrt{x+h+4}-\sqrt{x+4}}{h}$$

$$= lim_{h->0}\frac{\sqrt{x+h+4}-\sqrt{x+4}}{h} \times \frac{\sqrt{x+h+4}+\sqrt{x+4}}{\sqrt{x+h+4}+\sqrt{x+4}}$$

$$= lim_{h->0}\frac{h}{h(\sqrt{x+h+4}+\sqrt{x+4})}$$

Now putting $h = 0$ after canceling h we get,

$$f'(x) = \frac{1}{2\sqrt{x+4}}$$

b)

$$f'(x) = lim_{h->0}\frac{\frac{3}{x+h}-\frac{3}{x}}{h}$$

$$= lim_{h->0}\frac{-3h}{h(x)(x+h)}$$

Now putting $h = 0$ after canceling h we get,

$$= f'(x) = \frac{-3}{x^2}$$

Ans 2.

a)

$$f'(x) = 3 \times 3x^2 - (\frac{-2 \times 4}{x^3})$$

$$= 9x^2 + \frac{8}{x^3}$$

b)

$$f'(x) = 3 \times (4-x^2)^2 \times (-2x)$$

$$= -6x(4-x^2)^2$$

c)

$$f'(x) = e^{sin(x)} \times cos(x)$$
$$= e^{sin(x)}cos(x)$$

d)

$$f'(x) = \frac{1}{x+2} \times 1 = \frac{1}{x+2}$$

e)

$$f'(x) = 2x \times cos(x) + (-sin(x) \times x^2) + x \times sec^2(x) + tan(x)$$
$$= 2xcos(x) - x^2sin(x) + xsec^2(x) + tan(x)$$

f)

$$f'(x) = \frac{1}{2} \times \frac{1}{\sqrt{3x^2+2}} \times 3 \times 2x$$
$$= \frac{3x}{\sqrt{3x^2+2}}$$

g)

$$f'(x) = \frac{x}{4} \times \frac{1}{\sqrt{1-x^2}} + sin^{-1}x \times \frac{1}{4}$$
$$= \frac{x}{4\sqrt{1-x^2}} + \frac{sin^{-1}x}{4}$$

h)

Let f'(x) = y' and now differentiating both sides with respect to x we get,

$$x^2y' + 2xy = y' + ysin(x) + x(y'sin(x) + cos(x) \times y)$$
$$\implies y'(x^2 - 1 - xsin(x)) = -2xy + ysin(x) + xycos(x)$$
$$\therefore y' = f'(x) = \frac{ysin(x) + xycos(x) - 2xy}{x^2 - xsin(x) - 1}$$

or if you have done by separating x from y totally and then differentiating y w.r.t to x, the answer will be,

$$\frac{(2sin(x) + 2xcos(x) - 4x)}{(x^2 - xsin(x) - 1)^2}$$

Ans 3.

a)

We basically have to find $f'(39)$.

Now, taking the value of h=10 in central difference approach we get,

$$
\begin{aligned}
f'(39) &= \frac{f(39+10) - f(39-10)}{2 \times 10} \\
&= \frac{145 - 115}{20} \\
&= \frac{30}{20} \\
&= 1.5
\end{aligned}
$$

$\therefore$ The wind speed is increasing at the rate of 1.5 mph/hr (that basically means accelerating)

b)

Similarly like the last question we now have to find f'(83), and we take h=2.

In this case and use the central difference approximation once again,

$$
\begin{aligned}
f'(83) &= \frac{f(83+2) - f(83-2)}{2 \times 2} \\
&= \frac{95 - 125}{4} \\
&= -7.5
\end{aligned}
$$

This means that the wind speed is decreasing at the rate of 7.5 mph/hr