

Example : Compute the nullspace of matrix A.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix}$$

① Apply elimination to transform A into RREF.

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 3 & 1 & 4 \end{bmatrix} \xrightarrow{R_3 = R_3 - 3R_1} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & -2 & -2 \end{bmatrix} \xrightarrow{R_2 = -1 \cdot R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 + 2R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 = R_1 - R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{RREF}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

pivot columns free column

② Set free variables to 1 and then find values of pivot variables.

$$z = 1, x = -1, y = -1$$

③ Special solution $s = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

④ Nullspace \rightarrow linear combinations of special solutions

$$Z = c \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, c \in \mathbb{R}$$