SLE Example

Solve this system of linear equations

$$a + 2b + 3c + 5d = 0$$
$$2a + 4b + 8c + 12d = 6$$
$$3a + 6b + 7c + 13d = -6$$

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & 0 \\ 2 & 4 & 8 & 12 & 6 \\ 3 & 6 & 7 & 13 & -6 \end{array}\right] \xrightarrow{R2 = R2 - 2R1} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & 0 \\ 0 & 0 & 2 & 2 & 6 \\ 3 & 6 & 7 & 13 & -6 \end{array}\right] \xrightarrow{R3 = R3 - 3R1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & 0 \\ 0 & 0 & 2 & 2 & 6 \\ 0 & 0 & -2 & -2 & -6 \end{array}\right] \xrightarrow{R2 = R2/2} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & -2 & -2 & -6 \end{array}\right] \xrightarrow{R3 = R3 + 2R_2}$$

$$\underbrace{\left[\begin{array}{cccc|c} 1 & 2 & 3 & 5 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{\text{REF}} \xrightarrow{R1 = R1 - 3R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & -9 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

Pivot cols

Free cols

<u>X particular</u>

- Set all free variable to zero

$b = 0,\ d = 0$

$c + d = 3 \Rightarrow c = 3$

$a + 2b + 2d = -9 \Rightarrow a = -9$

$$\Rightarrow x_p = \begin{bmatrix} -9 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

<u>X nullspace</u>

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

We have 2 special solutions s_1, s_2 because we have 2 free columns

- Set $b=1$ and $d=0$ for $S1$

$$S1=\begin{bmatrix}-2\\1\\0\\0\end{bmatrix}$$

- Set $b=0$ and $d=1$ for $S2$

$$S2=\begin{bmatrix}-2\\0\\-1\\1\end{bmatrix}$$

$$x_n=\alpha S1+\beta S2=\alpha\begin{bmatrix}-2\\1\\0\\0\end{bmatrix}+\beta\begin{bmatrix}-2\\0\\-1\\1\end{bmatrix},\quad \alpha,\beta\in\mathbb{R}$$

Complete Solution

$$x=x_p+x_n=\begin{bmatrix}-9\\0\\3\\0\end{bmatrix}+\alpha\begin{bmatrix}-2\\1\\0\\0\end{bmatrix}+\beta\begin{bmatrix}-2\\0\\-1\\1\end{bmatrix},\quad \alpha,\beta\in\mathbb{R}$$