Probability Problems

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1. The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying joint probability table gives the proportions of individuals in the various ethnic group–blood group combinations.

		Blood Group			
		O	**A**	**B**	**AB**
Ethnic Group	**1**	0.082	0.106	0.008	0.004
	2	0.135	0.141	0.018	0.006
	3	0.215	0.200	0.065	0.020

Suppose that an individual is randomly selected from the population, and define events by $A = \{type\ A\ selected\}$, $B = \{type\ B\ selected\}$, and $C = \{ethnic\ group\ 3\ selected\}$.

(a) Calculate $P(A)$, $P(C)$, and $P(A \cap C)$

(b) Calculate $P(A|C)$ and explain in context what this probability represents.

(c) If the selected individual does not have type B blood, what is the probability that he or she is from group 1?

$A = \{\text{type A selected}\}$

$B = \{\text{type B selected}\}$

$C = \{\text{ethnic group 3 selected}\}$

a) $P(A) = 0.106 + 0.141 + 0.2 = 0.447$

$P(C) = 0.215 + 0.2 + 0.065 + 0.02 = 0.5$

$P(A \cap C) = 0.2$

b) $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{0.2}{0.5} = 0.4$

c) $\neg B = \{\text{type other than B}\}$ $P(B) = 0.091$

$G1 = \{\text{ethnic group 1 selected}\}$

$P(G1 | \neg B) = \dfrac{P(G1 \cap \neg B)}{P(\neg B)} = \dfrac{0.192}{1 - P(B)} = \dfrac{0.192}{0.909} \approx 0.21$

2. At a certain gas station, 40% of the customers user regular gas (A_1), 35% use plus gas (A_2), and 25% use premium gas (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested?

$A_1 = \{$use regular gas$\}$ $\quad B = \{$fill the tank$\}$

$A_2 = \{$use plus gas$\}$

$A_3 = \{$use premium gas$\}$

$P(A_1) = 0.4,\ P(A_2) = 0.35,\ P(A_3) = 0.25$

$P(B|A_1) = 0.3,\ P(B|A_2) = 0.6,\ P(B|A_3) = 0.5$

$$P(A_1|B) = \frac{P(B \cap A_1)}{P(B)} = \frac{P(B|A_1) \cdot P(A_1)}{\sum_{i=1}^{3} P(B|A_i) \cdot P(A_i)}$$

$$= \frac{0.3 \times 0.4}{0.3 \times 0.4 + 0.6 \times 0.35 + 0.5 \times 0.25} = \frac{0.12}{0.455} = 0.26$$

3. An employee of a company is traveling to either England, Italy, or Spain. The employee can travel to only one country. There is a 50% chance the employee will go to England and a 20% chance to Italy. Assume the chances of contracting COVID to be proportional to the prevalence of the disease in each country. For example, the chances of contracting COVID in England is 1.2×10^{-3}, in Italy 1.5×10^{-3} and in Spain is 1.6×10^{-3}.

(a) What are the chances that the employee will contract COVID while travelling?

(b) Assume that the employee has traveled to Europe and contracted COVID, what is the probability that he/she traveled to England?

$E = \{$Travel to England$\}$

$I = \{$Travel to Italy$\}$

$S = \{$Travel to Spain$\}$

$C = \{$Contact Covid$\}$

$P(E) = 0.5,\quad P(C|E) = 1.2 \times 10^{-3}$

$P(I) = 0.2,\quad P(C|I) = 1.5 \times 10^{-3}$

$P(S) = 0.3,\quad P(C|S) = 1.6 \times 10^{-3}$

a) $P(C) = P(C|E) \times P(E) + P(C|I) \times P(I) + P(C|S) \times P(S)$

$= 10^{-3}(1.2 \times 0.5 + 1.5 \times 0.2 + 1.6 \times 0.3) = 1.38 \times 10^{-3}$

b) $$P(E|C) = \frac{P(C|E) \times P(E)}{P(C)} = \frac{1.2 \times 10^{-3} \times 0.5}{1.38 \times 10^{-3}} = 0.43$$

4. Two production lines produce the same part. Line 1 produces 1,000 parts per week of which 100 are defective. Line 2 produces 2,000 parts per week of which 150 are defective. If you choose a part randomly from the stock and is found defective. What is the probability it was produced by line 1?

$L_1 = \{\text{Line 1 Production}\}$

$L_2 = \{\text{Line 2 Production}\}$

$D = \{\text{Defective}\}$

$P(L_1) = 1/3$

$P(L_2) = 2/3$

$P(D|L_1) = 0.1$

$P(D|L_2) = 0.075$

$$P(L_1|D) = \frac{P(D|L_1) \times P(L_1)}{P(D)} = \frac{P(D|L_1)P(L_1)}{\sum_{i=1}^{2} P(D|L_i)P(L_i)} = \frac{0.1 \times 1/3}{0.1 \times 1/3 + 0.075 \times 2/3} = 0.4$$

5. A certain disease affects about 1 out of 10,000 people. There is a test to check whether the person has the disease. The test is quite accurate. In particular, we know that: The probability that the test result is positive (suggesting the person has the disease), given that the person does not have the disease, is only 2 percent and the probability that the test result is negative (suggesting the person does not have the disease), given that the person has the disease, is only 1 percent. A random person gets tested for the disease and the result is positive. What is the probability that the person has the disease?

$+ = \{\text{Positive test}\}$

$- = \{\text{Negative test}\}$

$D = \{\text{Presence of Disease}\}$

$P(+|\neg D) = 0.02$, $P(-|\neg D) = 0.98$

$P(-|D) = 0.01$, $P(+|D) = 0.99$

$P(D) = 10^{-4}$

$$P(D|+) = \frac{P(+|D) \times P(D)}{P(+|D) \times P(D) + P(+|\neg D)P(\neg D)} = \frac{0.99 \times 10^{-4}}{0.99 \times 10^{-4} + 0.02 \times (1 - 10^{-4})} = \frac{0.99 \times 10^{-4}}{0.99 \times 10^{-4} + 0.02 \times 0.9999} = 0.0049$$