$$Ax = b$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 2 & 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 30 \end{bmatrix}$$

① Particular Solution - $x_{particular}$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 1 & 3 & 15 \\ 1 & 2 & 4 & 21 \\ 2 & 2 & 6 & 30 \end{array}\right] \xrightarrow{R2=R2-R1} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 15 \\ 0 & 1 & 1 & 6 \\ 2 & 2 & 6 & 30 \end{array}\right] \xrightarrow{R3=R3-R1} \left[\begin{array}{ccc|c} 1 & 1. & 3 & 15 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R1=R1-R2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$$x = 9,\ y = 6,\ z = 0$$

$$x_{particular} = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$$

② $x_{nullspace}$

$$\left[\begin{array}{ccc|c} \underline{1} & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

pivot column, free column

$z = 1$

$x + 2 = 0 \rightarrow x = -2$

$y + 1 = 0 \rightarrow y = -1$

Special Solution

$$s = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$x_{nullspace} = c \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

$$x = x_{particular} + x_{nullspace}$$

$$\boxed{x = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix} + c \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix},\ c \in \mathbb{R}}$$

Check, $c = 0 \rightarrow x = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$

$c = 1 \rightarrow x = \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix}$ ✓