

# Policy Gradient

# Reinforcement learning



- An **agent** interacts with an **environment** (e.g. game of Breakout)
- In each time step  $t$ ,
  - the agent receives **observations** (e.g. pixels) which give it information about the **state**  $s_t$  (e.g. positions of the ball and paddle)
  - the agent picks an **action**  $a_t$  (e.g. keystrokes) which affects the state
- The agent periodically receives a **reward**  $r(s_t, a_t)$ , which depends on the state and action (e.g. points)
- The agent wants to learn a **policy**  $\pi_\theta(a_t | s_t)$ 
  - Distribution over actions depending on the current state and parameters  $\theta$

# Markov Decision Processes

- The environment is represented as a **Markov decision process**  $\mathcal{M}$ .
- Markov assumption: all relevant information is encapsulated in the current state; i.e. the policy, reward, and transitions are all independent of past states given the current state
- Components of an MDP:
  - initial state distribution  $p(\mathbf{s}_0)$
  - policy  $\pi_\theta(\mathbf{a}_t | \mathbf{s}_t)$
  - transition distribution  $p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$
  - reward function  $r(\mathbf{s}_t, \mathbf{a}_t)$
- Assume a **fully observable** environment, i.e.  $\mathbf{s}_t$  can be observed directly
- **Rollout**, or **trajectory**  $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)$
- Probability of a rollout

$$p(\tau) = p(\mathbf{s}_0) \pi_\theta(\mathbf{a}_0 | \mathbf{s}_0) p(\mathbf{s}_1 | \mathbf{s}_0, \mathbf{a}_0) \cdots p(\mathbf{s}_T | \mathbf{s}_{T-1}, \mathbf{a}_{T-1}) \pi_\theta(\mathbf{a}_T | \mathbf{s}_T)$$

# Markov Decision Processes

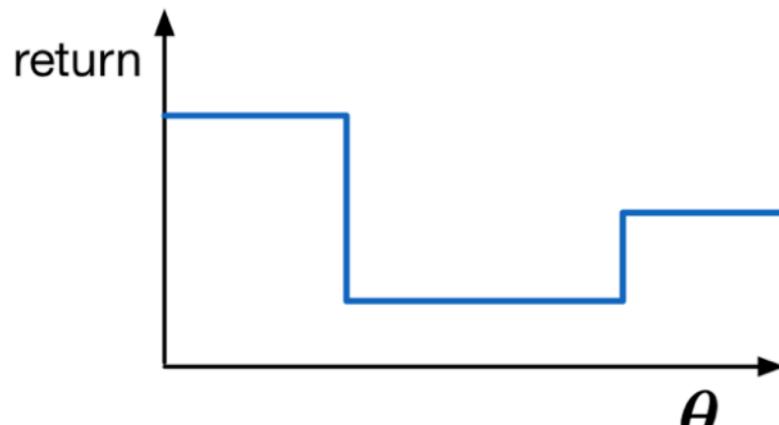
Continuous control in simulation, e.g. teaching an ant to walk



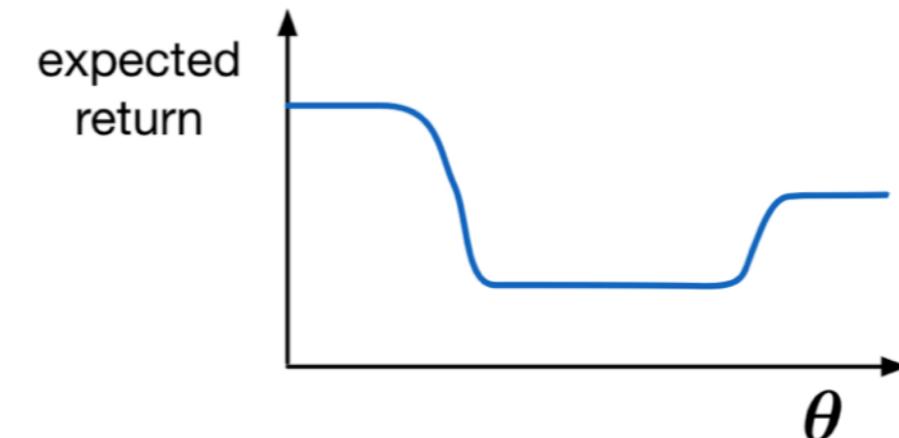
- State: positions, angles, and velocities of the joints
- Actions: apply forces to the joints
- Reward: distance from starting point
- Policy: output of an ordinary MLP, using the state as input
- More environments: <https://gym.openai.com/envs/#mujoco>

# Markov Decision Processes

- **Return** for a rollout:  $r(\tau) = \sum_{t=0}^T r(s_t, a_t)$ 
  - Note: we're considering a finite **horizon**  $T$ , or number of time steps; we'll consider the infinite horizon case later.
- Goal: maximize the expected return,  $R = \mathbb{E}_{p(\tau)}[r(\tau)]$
- The expectation is over both the environment's dynamics and the policy, but we only have control over the policy.
- The stochastic policy is important, since it makes  $R$  a continuous function of the policy parameters.
  - Reward functions are often discontinuous, as are the dynamics (e.g. collisions)



deterministic policies



stochastic policies

# REINFORCE

- REINFORCE is an elegant algorithm for maximizing the expected return  $R = \mathbb{E}_{p(\tau)} [r(\tau)]$ .
- Intuition: trial and error
  - Sample a rollout  $\tau$ . If you get a high reward, try to make it more likely. If you get a low reward, try to make it less likely.
- Interestingly, this can be seen as stochastic gradient ascent on  $R$ .

# REINFORCE

- Recall the derivative formula for log:

$$\frac{\partial}{\partial \theta} \log p(\tau) = \frac{\frac{\partial}{\partial \theta} p(\tau)}{p(\tau)} \implies \frac{\partial}{\partial \theta} p(\tau) = p(\tau) \frac{\partial}{\partial \theta} \log p(\tau)$$

- Gradient of the expected return:

$$\begin{aligned}\frac{\partial}{\partial \theta} \mathbb{E}_{p(\tau)} [r(\tau)] &= \frac{\partial}{\partial \theta} \sum_{\tau} r(\tau) p(\tau) \\ &= \sum_{\tau} r(\tau) \frac{\partial}{\partial \theta} p(\tau) \\ &= \sum_{\tau} r(\tau) p(\tau) \frac{\partial}{\partial \theta} \log p(\tau) \\ &= \mathbb{E}_{p(\tau)} \left[ r(\tau) \frac{\partial}{\partial \theta} \log p(\tau) \right]\end{aligned}$$

- Compute stochastic estimates of this expectation by sampling rollouts.

# REINFORCE

- For reference:

$$\frac{\partial}{\partial \theta} \mathbb{E}_{p(\tau)} [r(\tau)] = \mathbb{E}_{p(\tau)} \left[ r(\tau) \frac{\partial}{\partial \theta} \log p(\tau) \right]$$

- If you get a large reward, make the rollout more likely. If you get a small reward, make it less likely.
- Unpacking the REINFORCE gradient:

$$\begin{aligned}\frac{\partial}{\partial \theta} \log p(\tau) &= \frac{\partial}{\partial \theta} \log \left[ p(\mathbf{s}_0) \prod_{t=0}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \prod_{t=1}^T p(\mathbf{s}_t | \mathbf{s}_{t-1}, \mathbf{a}_{t-1}) \right] \\ &= \frac{\partial}{\partial \theta} \log \prod_{t=0}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \\ &= \sum_{t=0}^T \frac{\partial}{\partial \theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)\end{aligned}$$

- Hence, it tries to make *all* the actions more likely or less likely, depending on the reward. I.e., it doesn't do **credit assignment**.

# REINFORCE

Repeat forever:

Sample a rollout  $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)$

$$r(\tau) \leftarrow \sum_{k=0}^T r(\mathbf{s}_k, \mathbf{a}_k)$$

For  $t = 0, \dots, T$ :

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha r(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t | \mathbf{s}_t)$$

- Observation: actions should only be reinforced based on future rewards, since they can't possibly influence past rewards.
- You can show that this still gives unbiased gradient estimates.

Repeat forever:

Sample a rollout  $\tau = (\mathbf{s}_0, \mathbf{a}_0, \mathbf{s}_1, \mathbf{a}_1, \dots, \mathbf{s}_T, \mathbf{a}_T)$

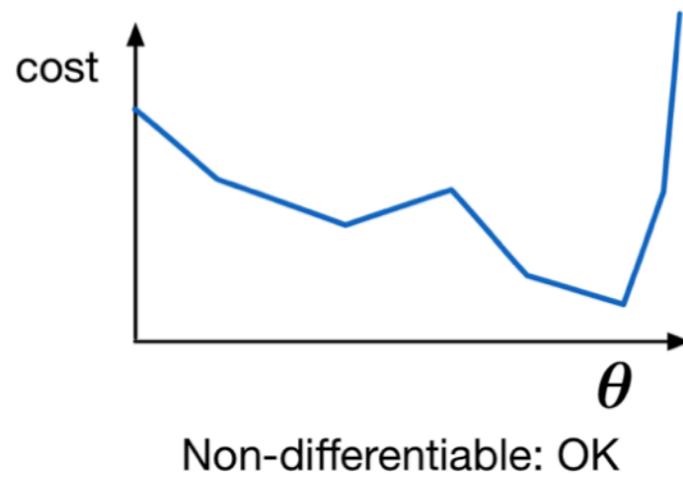
For  $t = 0, \dots, T$ :

$$r_t(\tau) \leftarrow \sum_{k=t}^T r(\mathbf{s}_k, \mathbf{a}_k)$$

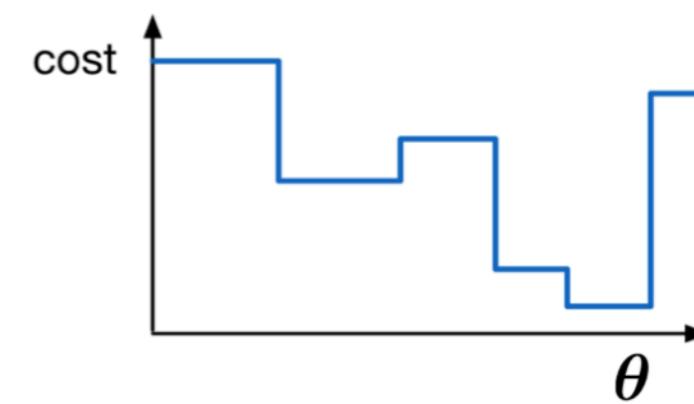
$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha r_t(\tau) \frac{\partial}{\partial \boldsymbol{\theta}} \log \pi_{\boldsymbol{\theta}}(\mathbf{a}_t | \mathbf{s}_t)$$

# Optimizing Discontinuous Objectives

- Edge case of RL: handwritten digit classification, but maximizing accuracy (or minimizing 0–1 loss)
- Gradient descent completely fails if the cost function is discontinuous:



Non-differentiable: OK



Discontinuous: not OK

- Original solution: use a surrogate loss function, e.g. logistic-cross-entropy
- RL formulation: in each episode, the agent is shown an image, guesses a digit class, and receives a reward of 1 if it's right or 0 if it's wrong
- We'd never actually do it this way, but it will give us an interesting comparison with backprop

# Optimizing Discontinuous Objectives

- RL formulation
  - one time step
  - state  $\mathbf{x}$ : an image
  - action  $\mathbf{a}$ : a digit class
  - reward  $r(\mathbf{x}, \mathbf{a})$ : 1 if correct, 0 if wrong
  - policy  $\pi(\mathbf{a} | \mathbf{x})$ : a distribution over categories
    - Compute using an MLP with softmax outputs – this is a **policy network**

# Optimizing Discontinuous Objectives

- Let  $z_k$  denote the logits,  $y_k$  denote the softmax output,  $t$  the integer target, and  $t_k$  the target one-hot representation.
- To apply REINFORCE, we sample  $\mathbf{a} \sim \pi_{\theta}(\cdot | \mathbf{x})$  and apply:

$$\begin{aligned}\theta &\leftarrow \theta + \alpha r(\mathbf{a}, t) \frac{\partial}{\partial \theta} \log \pi_{\theta}(\mathbf{a} | \mathbf{x}) \\&= \theta + \alpha r(\mathbf{a}, t) \frac{\partial}{\partial \theta} \log y_a \\&= \theta + \alpha r(\mathbf{a}, t) \sum_k (a_k - y_k) \frac{\partial}{\partial \theta} z_k\end{aligned}$$

- Compare with the logistic regression SGD update:

$$\begin{aligned}\theta &\leftarrow \theta + \alpha \frac{\partial}{\partial \theta} \log y_t \\&\leftarrow \theta + \alpha \sum_k (t_k - y_k) \frac{\partial}{\partial \theta} z_k\end{aligned}$$

# Discussion

- Why policy gradient?
  - Can handle discontinuous cost functions
  - Don't need an explicit model of the environment, i.e. rewards and dynamics are treated as black boxes
    - Policy gradient is an example of **model-free reinforcement learning**, since the agent doesn't try to fit a model of the environment
    - Almost everyone thinks model-based approaches are needed for AI, but nobody has a clue how to get it to work

# Policy Gradient vs. Q-Learning

- Policy gradient and Q-learning use two very different choices of representation: policies and value functions
- Advantage of both methods: don't need to model the environment
- Pros/cons of policy gradient
  - Pro: unbiased estimate of gradient of expected return
  - Pro: can handle a large space of actions (since you only need to sample one)
  - Con: high variance updates (implies poor sample efficiency)
  - Con: doesn't do credit assignment
- Pros/cons of Q-learning
  - Pro: lower variance updates, more sample efficient
  - Pro: does credit assignment
  - Con: biased updates since Q function is approximate (drinks its own Kool-Aid)
  - Con: hard to handle many actions (since you need to take the max)