Fast Fixed-Time Nonsingular Terminal Sliding Mode Control and Its Application to Chaos Suppression in Power System

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***Abstract*—This brief presents a novel control scheme to achieve fast fixed-time system stabilization. Based on fixed-time stability theory, a novel fixed-time stable system is presented. Using the proposed fixed-time stable system, a fast fixed-time nonsingular terminal sliding mode control method is derived. Our control scheme achieves system stabilization within bounded time independent of the initial condition and has an advantage in convergence rate over the existing result of the fixed-time stable control method. The proposed control strategy is applied to suppress chaotic oscillation in power systems, and its effectiveness as well as superiority is verified through numerical simulation. The proposed control strategy can be applied to address the control and synchronization problem for other complex systems.**

***Index Terms*—Chaos suppression, fixed-time stability, nonsingular terminal sliding mode control, power system.**

I. Introduction

POWER SYSTEM is a complex nonlinear dynamical system, and when the power system operates near its stability boundary, parameter variations [1], time delay [2], and external disturbances [3] can induce chaos. Chaotic oscillation is an undesirable phenomenon for power systems, and it may result in voltage collapse [4], [5] and even catastrophic blackouts [6]. Therefore, it is necessary to study chaos control method to avoid voltage collapse in power systems. Different control schemes have been proposed for chaos suppression and voltage stabilization, such as the parameter perturbation method [7], washout filter-aided feedback [8], feedback linearization [9], ANFIS-based control [10], adaptive control using LaSalle's invariance principle [11], etc. However, all aforementioned control methods can achieve only asymptotic stability, i.e., the convergence time cannot be assigned in advance. From the viewpoint of power system operation, oscillations are acceptable if they can be damped within a limited time. Moreover, they ignore the fact that voltage instability always accompanies with frequency oscillation caused by active power imbalance. In power systems, energy storage device and static var compensator (SVC) can provide flexible active and reactive power compensation, respectively. Therefore, designing the SVC controller and energy storage device controller to achieve system stabilization within a limited time is significant for chaos suppression in power systems.

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Finite-time control can achieve system stabilization within a prescribed time, and it has been applied in many fields (for instance, [12] and [13]). However, finite-time control cannot guarantee the system convergence within bounded time independent of the initial condition, which prohibits its application into practical systems if the initial condition is unknown in advance. The fixed-time stability introduced by Polyakov [14] can overcome this drawback. Due to this attractive property, fixed-time stability has found applications in uniform exact differentiator design [15] and consensus for multiagent systems [16]. However, there are few results about fixed-time stable control. Zuo [17] proposed a nonsingular fixed-time terminal sliding mode controller for a class of second-order nonlinear systems. However, the convergence time of the nonsingular fixed-time terminal sliding mode controller is not an optimal one; moreover, the method used to eliminate singularity is complicated.

Motivated by the above discussion, this brief proposes a fast fixed-time nonsingular terminal sliding mode control method and applies it to design the energy storage device controller and SVC controller for chaos suppression in power systems. As far as we know, no results on the application of the fixed-time stable controller into chaos and power system stability control are available up to now. The main advantage of the proposed controller is that it can guarantee finite-time system stabilization independent of the initial state and ensure fast convergence both far away from and at a close range of the control objective. This feature can reduce the loss caused by chaotic oscillation and avoid voltage collapse. In order to overcome the singularity problem of the terminal sliding mode control, a saturation function is introduced to limit the amplitude of the control input. In comparison with the existing results on fixed-time nonsingular terminal sliding mode control [17], the proposed controller applies a simpler method to overcome the singularity problem and achieves faster convergence.

II. Main Results

Consider the following second-order system:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = f(y) + g(y)u \end{cases} \tag{1}$$

where $[y_1, y_2]^T \in R^2$ is the state vector, $f(y), g(y) \neq 0$ are smooth real vector fields, and u is the control input. The control objective is to achieve fixed-time system stabilization.

Before controller design, a definition for fixed-time stability and a novel fixed-time stable system are given.

Definition 1 [14]: Consider a system of differential equation

$$\dot{x}(t) = f(x(t)), \quad x(0) = x_0. \tag{2}$$

The origin of (2) is said to be a fixed-time stable equilibrium point if it is globally finite-time stable with bounded settling-time function $T(x_0)$, i.e., $\exists T_{\max} > 0$ such that $T(x_0) < T_{\max}$.

Lemma 1: Consider the following differential equation:

$$\dot{y} = -\alpha y^{\frac{1}{2}+\frac{m}{2n}+\left(\frac{m}{2n}-\frac{1}{2}\right)\mathrm{sign}(|y|-1)} - \beta y^{\frac{p}{q}} \tag{3}$$

where $\alpha > 0$, $\beta > 0$, and m, n, p, q are positive odd integers that satisfy $m > n, p < q$. Then, system (3) is fixed-time stable.

Proof: The differential equation for system (3) can be rewritten as

$$\begin{cases} \dot{y} = -\alpha y^{\frac{m}{n}} - \beta y^{\frac{p}{q}} & |y| > 1 \\ \dot{y} = -\alpha y - \beta y^{\frac{p}{q}} & |y| < 1. \end{cases} \tag{4}$$

Define a new variable as $z = y^{1-(p/q)}$, and the expression for the first equation of system (4) can be rewritten as

$$\dot{z} + \frac{q-p}{q}\alpha z^{\frac{\frac{m}{n}-\frac{p}{q}}{1-\frac{p}{q}}} + \frac{q-p}{q}\beta = 0. \tag{5}$$

Let $\varepsilon = [(m-n)q]/[n(q-p)]$, and we have

$$\dot{z} + \frac{q-p}{q}\alpha z^{1+\varepsilon} + \frac{q-p}{q}\beta = 0. \tag{6}$$

The expression for the second equation of system (4) can be expressed as

$$\dot{z} + \frac{q-p}{q}\alpha z + \frac{q-p}{q}\beta = 0. \tag{7}$$

Solving (6) and (7) for convergence time, the upper bound of convergence time can be estimated as

$$\begin{aligned} \lim_{z_0\to\infty} T(z_0) &= \lim_{z_0\to\infty} \frac{q}{q-p}\left(\int_1^{z_0} \frac{1}{\alpha z^{1+\varepsilon}+\beta}dz + \int_0^1 \frac{1}{\alpha z+\beta}dz\right) \\ &< \lim_{z_0\to\infty} \frac{q}{q-p}\int_1^{z_0} \frac{1}{\alpha z^{1+\varepsilon}}dz + \frac{q}{\alpha(q-p)}\ln\left(1+\frac{\alpha}{\beta}\right) \\ &< \frac{q}{q-p}\left(\frac{1}{\varepsilon\alpha} + \frac{1}{\alpha}\ln\left(1+\frac{\alpha}{\beta}\right)\right) \\ &= \frac{1}{\alpha}\frac{n}{m-n} + \frac{q}{q-p}\frac{1}{\alpha}\ln\left(1+\frac{\alpha}{\beta}\right). \end{aligned} \tag{8}$$

The proof is completed.

Remark 1: In 2015, Zuo [17] proposed a fixed-time stable system $\dot{y} = -\alpha y^{m/n} - \beta y^{p/q}$ and proved that the system can be stabilized within finite time bounded by $T < (1/\alpha)(n/(m-n)) + (1/\beta)(q/(q-p))$. Since the inequality $\ln(1+(\alpha/\beta)) \leq (\alpha/\beta)$ holds, the fixed-time stable system presented in this brief achieves faster convergence than the system proposed by Zuo. The proposed system uses linear term y instead of nonlinear term $y^{m/n}$ in Zuo's approach at the vicinity of the origin, thereby achieving fast convergence both far away from and at a close range of the origin.

Remark 2: Different from finite-time stable systems, fixed-time stable systems can guarantee stabilization within bounded time independent of the initial condition.

Based on Lemma 1, a novel fast fixed-time nonsingular terminal sliding mode control strategy for second-order systems (1) is presented, and the sliding mode surface can be constructed

$$s = y_2 + \alpha_1 y_1^{\frac{1}{2}+\frac{m_1}{2n_1}+\left(\frac{m_1}{2n_1}-\frac{1}{2}\right)\mathrm{sign}(|y_1|-1)} + \beta_1 y_1^{\frac{p_1}{q_1}}. \tag{9}$$

Terminal sliding mode reaching law can be obtained as

$$\dot{s} = -\alpha_2 s^{\frac{1}{2}+\frac{m_2}{2n_2}+\left(\frac{m_2}{2n_2}-\frac{1}{2}\right)\mathrm{sign}(|s|-1)} - \beta_2 s^{\frac{p_2}{q_2}}. \tag{10}$$

However, the terminal sliding mode has a singularity problem. Here, the saturation function method proposed by Feng *et al.* [21] is adopted to overcome the singularity problem, and according to (10), the control law can be designed as

$$\begin{aligned} u = -\frac{1}{g(y)}\Bigg[& f(y) + \alpha_1\left(\frac{1}{2}+\frac{m_1}{2n_1}+\left(\frac{m_1}{2n_1}-\frac{1}{2}\right)\mathrm{sign}\left(|y_1|-1\right)\right) \\ & \times y_1^{\frac{1}{2}+\frac{m_1}{2n_1}+\left(\frac{m_1}{2n_1}-\frac{1}{2}\right)\mathrm{sign}(|y_1|-1)-1} y_2 \\ & + \mathrm{sat}\left(\beta_1\frac{p_1}{q_1}y_1^{\frac{p_1}{q_1}-1}y_2, h\right) \\ & + \alpha_2 s^{\frac{1}{2}+\frac{m_2}{2n_2}+\left(\frac{m_2}{2n_2}-\frac{1}{2}\right)\mathrm{sign}(|s|-1)} + \beta_2 s^{\frac{p_2}{q_2}}\Bigg]. \end{aligned} \tag{11}$$

In (11), a saturation function is applied to limit the amplitude of singularity term $y_1^{(p_1/q_1)-1}y_2$ in the control input, and the saturation function can be defined as

$$\mathrm{sat}(x,y) = \begin{cases} x, & \text{if } |x| < y \\ y\,\mathrm{sign}(x), & \text{if } |x| \geq y. \end{cases} \tag{12}$$

Theorem 1: For system (1), if the control input is given by (11), then there exist positive constants α_1, β_1, α_2, β_2 and positive odd integers m_1, n_1, m_2, n_2, p_1, q_1, p_2, and q_2, satisfying $m_1 > n_1$, $m_2 > n_2$, $p_1 < q_1$, and $p_2 < q_2$, and $(m_1+n_1)/2$, $(p_1+q_1)/2$, $(m_2+n_2)/2$, and $(p_2+q_2)/2$ being positive odd integers, such that the overall system is fixed-time stable.

Proof: Consider the following Lyapunov candidate function as

$$V_1 = \frac{1}{2}s^2. \tag{13}$$

The time derivative of V_1 can be obtained as

$$\begin{aligned} \dot{V}_1 = s\dot{s} &= -s\left(\alpha_2 s^{\frac{1}{2}+\frac{m_2}{2n_2}+\left(\frac{m_2}{2n_2}-\frac{1}{2}\right)\mathrm{sign}(|s|-1)} + \beta_2 s^{\frac{p_2}{q_2}}\right) \\ &= -\alpha_2 s^{\frac{1}{2}+\frac{m_2}{2n_2}+\left(\frac{m_2}{2n_2}-\frac{1}{2}\right)\mathrm{sign}(|s|-1)+1} - \beta_2 s^{\frac{p_2}{q_2}+1}. \\ &= -\alpha_2(2V_1)^{\frac{\frac{1}{2}+\frac{m_2}{2n_2}+\left(\frac{m_2}{2n_2}-\frac{1}{2}\right)\mathrm{sign}(|s|-1)+1}{2}} - \beta_2(2V_1)^{\frac{\frac{p_2}{q_2}+1}{2}}. \end{aligned} \tag{14}$$

When $|s| \geq 1$, one has

$$\dot{V}_1 = -\alpha_2(2V_1)^{\frac{\frac{m_2}{n_2}+1}{2}} - \beta_2(2V_1)^{\frac{\frac{p_2}{q_2}+1}{2}}. \tag{15}$$

When $|s| < 1$, (14) becomes

$$\dot{V}_1 = -\alpha_2(2V_1) - \beta_2(2V_1)^{\frac{\frac{p_2}{q_2}+1}{2}}. \tag{16}$$

According to Lemma 1, system (1) reaches the sliding surface within a bounded time, and the bound of convergence time can be estimated by

$$T_1 < \frac{1}{2^{\frac{\frac{m_2}{n_2}+1}{2}}\alpha_2}\frac{n_2}{\frac{m_2-n_2}{2}} + \frac{q_2}{q_2-p_2}\frac{1}{2\alpha_2}\ln\left(1+\frac{2\alpha_2}{2^{\frac{\frac{p_2}{q_2}+1}{2}}\beta_2}\right). \quad (17)$$

When the system reaches the sliding surface $s = 0$, the ideal sliding motion of the system satisfies the following nonlinear differential equation:

$$\dot{y}_1 = y_2 = -\alpha_1 y_1^{\frac{1}{2}+\frac{m_1}{2n_1}+\left(\frac{m_1}{2n_1}-\frac{1}{2}\right)\mathrm{sign}(|y_1|-1)} - \beta_1 y_1^{\frac{p_1}{q_1}}. \quad (18)$$

Similarly, the state variable y_1 can be stabilized within a finite time bounded by

$$T_2 < \frac{1}{2^{\frac{\frac{m_1}{n_1}+1}{2}}\alpha_1}\frac{n_1}{\frac{m_1-n_1}{2}} + \frac{q_1}{q_1-p_1}\frac{1}{2\alpha_1}\ln\left(1+\frac{2\alpha_1}{2^{\frac{\frac{p_1}{q_1}+1}{2}}\beta_1}\right). \quad (19)$$

When state variable y_1 settles down to the origin, the state variable y_2 also converges to zero. Consequently, the convergence time for system (1) can be estimated as

$$\begin{aligned} T = T_1 + T_2 < {} & \frac{1}{2^{\frac{\frac{m_1}{n_1}+1}{2}}\alpha_1}\frac{n_1}{\frac{m_1-n_1}{2}} \\ & + \frac{q_1}{q_1-p_1}\frac{1}{2\alpha_1}\ln\left(1+\frac{2\alpha_1}{2^{\frac{\frac{p_1}{q_1}+1}{2}}\beta_1}\right) \\ & + \frac{1}{2^{\frac{\frac{m_2}{n_2}+1}{2}}\alpha_2}\frac{n_2}{\frac{m_2-n_2}{2}} + \frac{q_2}{q_2-p_2}\frac{1}{2\alpha_2}\ln\left(1+\frac{2\alpha_2}{2^{\frac{\frac{p_2}{q_2}+1}{2}}\beta_2}\right). \end{aligned} \quad (20)$$

The proof is completed.

Remark 3: In the proof process, the situation where $|\beta_1(p_1/q_1) y_1^{(p_1/q_1)-1}y_2| > h$ holds is ignored. The reasons are given as follows. Define the singularity area as the region where inequality $|\beta_1(p_1/q_1)y_1^{(p_1/q_1)-1}y_2| > h$ holds. According to the first equation of system (1), the solution for state variable y_1 can be obtained

$$y_1(t) = y_1(0) + \int_0^t y_2(\tau)d\tau. \quad (21)$$

If $y_2(t) > 0$ holds, $y_1(t)$ will increase monotonically and leave the singularity area. If $y_2(t) < 0$ holds, $y_1(t)$ will decrease monotonically and also leave the singularity area. Both situations prove that the system lies in the singularity region transiently. Therefore, as pointed out in [21], the existence of singularity region does not influence the results of the stability analysis.

Remark 4: In order to guarantee that $s = 0$ lies outside the singularity area, h can be selected to satisfy

$$\beta_1\frac{p_1}{q_1}y_{1\max}^{\frac{p_1}{q_1}-1}\left(\alpha_1 y_{1\max}^{\frac{1}{2}+\frac{m_1}{2n_1}+(\frac{m_1}{2n_1}-\frac{1}{2})\mathrm{sign}(|y_1|-1)} + \beta_1 y_{1\max}^{\frac{p_1}{q_1}}\right) < h$$

where $|y_1| < y_{1\max}$.

Remark 5: Massive numerical simulations have shown that the time it takes to travel through a singularity area takes up a very small proportion of the total settling time and the upper bound of settling time estimated by (20) has a large margin. Therefore, the upper bound of settling time exists and can be estimated by (20).

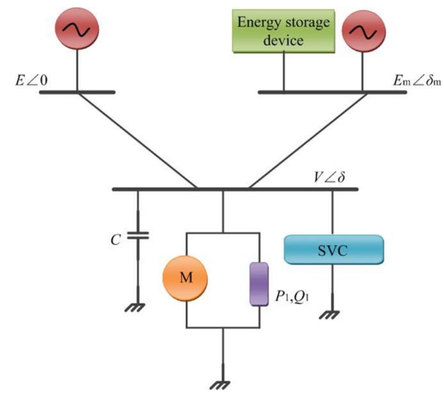

Fig. 1. Studied power system.

III. Application to Power System Chaos Suppression

In this section, the proposed control scheme is applied to design SVC and energy storage device controllers for chaos suppression in power systems. The considered power system is a benchmark model for voltage stability study, which is shown in Fig. 1. The power system is composed of two generator buses and one load bus. One generator bus is treated as a slack bus, while the other generator is equipped with an energy storage device, and its dynamics can be described by a swing equation. The load bus includes a dynamic induction motor in parallel with a constant PQ load and an SVC. The mathematical models for the dynamic motor, SVC, and energy storage device controller are taken from [8], [18], and [20], respectively. The dynamics of the controlled power system model can be described as

$$\left\{\begin{aligned} \dot{\delta}_m &= f_1(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) = \omega \\ M\dot{\omega} &= f_2(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) \\ &= -d_m\omega + P_m + E_m Y_m V\sin(\delta - \delta_m - \theta_m) \\ &\quad + E_m^2 Y_m \sin\theta_m - P_{\mathrm{es}} \\ \dot{P}_{\mathrm{es}} &= f_3(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) + \tfrac{K_{\mathrm{es}}}{T_{\mathrm{es}}}u_{\mathrm{es}} \\ &= -\tfrac{1}{T_{\mathrm{es}}}P_{\mathrm{es}} + \tfrac{K_{\mathrm{es}}}{T_{\mathrm{es}}}u_{\mathrm{es}} \\ K_{qw}\dot{\delta} &= f_4(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) \\ &= -K_{qv2}V^2 - K_{qv}V + E_0'Y_0'V\cos(\delta + \theta_0') \\ &\quad + E_m Y_m V\cos(\delta - \delta_m + \theta_m) \\ &\quad - (Y_0'\cos\theta_0' + Y_m\cos\theta_m)V^2 \\ &\quad - Q_0 - Q_1 - Q_{\mathrm{SVC}} \\ TK_{qw}K_{pv}\dot{V} &= f_5(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) \\ &= K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V \\ &\quad + K_{qw}(-E_0'Y_0'V\sin(\delta + \theta_0') \\ &\quad - E_m Y_m V\sin(\delta - \delta_m + \theta_m) \\ &\quad + (Y_0'\sin\theta_0' + Y_m\sin\theta_m)V^2 - P_0 - P_1) \\ &\quad - K_{pw}(E_0'Y_0'V\cos(\delta + \theta_0') \\ &\quad + E_m Y_m V\cos(\delta - \delta_m + \theta_m) \\ &\quad - (Y_0'\cos\theta_0' + Y_m\cos\theta_m)V^2 \\ &\quad - Q_0 - Q_1 - Q_{\mathrm{SVC}}) \\ \dot{B} &= f_6(\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B) + \tfrac{K_{\mathrm{SVC}}}{T_{\mathrm{SVC}}}u_{\mathrm{SVC}} \\ &= \tfrac{1}{T_{\mathrm{SVC}}}(K_{\mathrm{SVC}}u_{\mathrm{SVC}} - B) \end{aligned}\right. \quad (22)$$

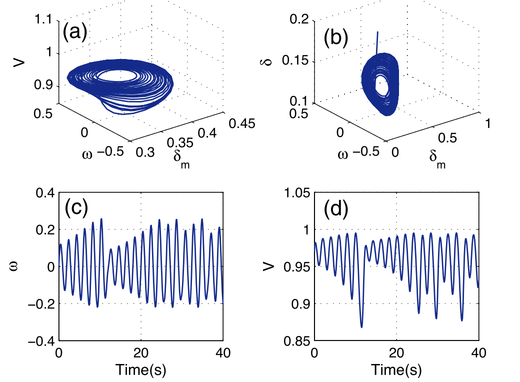

Fig. 2. Chaotic behavior of the power system. (a) Phase portrait projected onto V, δ_m, ω space. (b) Phase portrait projected onto δ, ω, δ_m space. (c) Time domain waveform of ω. (d) Time domain waveform of V.

where δ_m and ω are the state variables that denote generator angle and frequency deviation, respectively, M refers to the generator inertia, d_m is the damping coefficient, and P_m is the mechanical power; Y_m and θ_m are the admittance and impedance angles of the transmission line, E_m stands for the magnitude of the generator voltage, and δ and V are the phase angle and magnitude of the load voltage; P_1 and Q_1 are the real and reactive power demands of constant PQ load; P_0 and Q_0 are the constant real and reactive power for the induction motor; and E_0', Y_0', and θ_0' are the Thevenin equivalent circuit values with respect to E_0, Y_0, and θ_0 (for detailed derivation, see [5]). $K_{pw}, K_{pv}, K_{qw}, K_{qv}, K_{qv2}$ are constants associated with the induction motor. T_{SVC} is the time constant of the SVC, K_{SVC} represents the gain for the SVC, u_{SVC} is the control input of the SVC, B is the susceptance of the SVC, T_{es} is the time constant of the energy storage device, K_{es} represents the gain for the energy storage device, u_{es} is the control input, P_{es} denotes the injected active power provided by the energy storage device, and Q_{SVC} is the injected reactive power provided by the SVC and has the form of $Q_{\mathrm{SVC}} = BV^2$. The parameter values for the controlled power system are listed in the Appendix.

Before the controllers are carried out, i.e., $P_{\mathrm{es}} = 0$ and $Q_{\mathrm{SVC}} = 0$, the power system shows chaotic behavior, which is displayed in Fig. 2. The existence of the chaotic attractor can be verified by calculating Lyapunov exponents using Wolf algorithm [19], and the computation result is

$$(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0.3088, -0.0073, -4.1459, -66.1876).$$

As can be seen from Fig. 2, chaotic oscillation is accompanied by voltage instability and frequency oscillation. If no control action is taken, chaotic oscillation will result in voltage collapse through chaotic blue-sky bifurcation [5]. Therefore, it is necessary to study control method to suppress chaotic oscillation as soon as possible to avoid voltage collapse.

To ensure excellent power supply quality, frequency deviation should be eliminated, and system voltage should be stabilized to its nominal value. Therefore, the controller needs to be designed to suppress chaos and stabilize the system to its desired operating point ($V_d = 1$ and $\omega_d = 0$). The error of state variables ω and V can be given as $y_1 = \omega - \omega_d$ and $y_2 = V - V_d$, and the controlled power system (22) can be rewritten as

$$\begin{cases} \dot{x} = f(x) + g_1(x)u_{\mathrm{es}} + g_2(x)u_{\mathrm{SVC}} \\ y_1 = h_1(x) \\ y_2 = h_2(x) \end{cases} \tag{23}$$

where

$$x = [\delta_m, \omega, P_{\mathrm{es}}, \delta, V, B]^T, g_1(x) = [0, 0, K_{\mathrm{es}}/T_{\mathrm{es}}, 0, 0, 0]^T$$

$$g_2(x) = [0, 0, 0, 0, 0, K_{\mathrm{SVC}}/T_{\mathrm{SVC}}]^T$$

$$f(x) = [f_1, f_2/M, f_3, f_4/K_{qw}, f_5/TK_{qw}K_{pv}, f_6]^T.$$

Note that error variables y_1 and y_2 have a relative degree of two with respect to control inputs u_{es} and u_{SVC}. Consequently, we introduce the coordinate transformation $z_1 = h_1(x)$, $z_2 = L_f h_1(x)$, $z_3 = h_2(x)$, and $z_4 = L_f h_2(x)$, and system (23) becomes

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = L_f^2 h_1(x) + L_{g_1} L_f h_1(x) u_{\mathrm{es}} + L_{g_2} L_f h_1(x) u_{\mathrm{SVC}} \\ \dot{z}_3 = z_4 \\ \dot{z}_4 = L_f^2 h_2(x) + L_{g_2} L_f h_2(x) u_{\mathrm{SVC}}. \end{cases} \tag{24}$$

After the coordinate transformation

$$\varphi_1 = z_1 - L_{g_2} L_f h_1(x) / L_{g_2} L_f h_2(x) z_3$$
$$\varphi_2 = z_2 - L_{g_2} L_f h_1(x) / L_{g_2} L_f h_2(x) z_4, \quad \varphi_3 = z_3, \varphi_4 = z_4$$

system (24) is transformed into two second-order systems, which have the same form as (1).

Remark 6: It is easy to see that the convergence of $\varphi_1 \sim \varphi_4$ to zero also implies that the original states $z_1 \sim z_4$ converge to zero. This means that the coordinate transformation is physically meaningful.

The proposed control method is applied to suppress chaotic oscillation in studied power systems. The parameters for the proposed controller are listed in the Appendix. An immediate control action is activated to suppress chaos in three-bus power systems. The results given in Fig. 3 show that chaotic oscillation has been eliminated effectively and state variables settle down to their desired operating points, respectively, within finite time. Next, a comparative study between the proposed controller and Zuo's approach [17] is given to show the superiority of the proposed controller. Both controllers are implemented to stabilize second-order system (1). Fig. 4 compares the convergence times of the proposed control method and Zuo's approach under different initial conditions. As Fig. 4 clearly demonstrates, the convergence time of both controllers is upper bounded by a constant, and it takes less time for the proposed controller to stabilize the system than Zuo's approach. Simulation results verify the effectiveness and superiority of the proposed control scheme.

IV. Conclusion

This brief has presented a fast fixed-time stable control scheme. A novel fixed-time stable system has been presented, and based on this system, a fast fixed-time nonsingular terminal sliding mode control method has been derived. The proposed control strategy applies a saturation function to overcome the

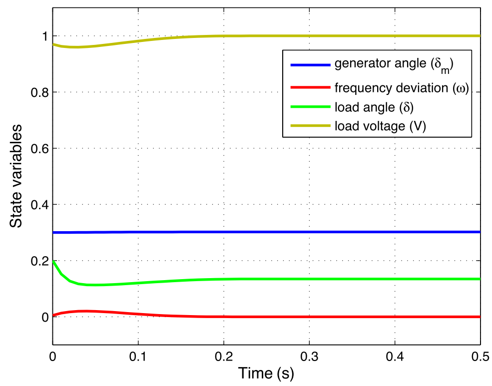

Fig. 3. Time response of the state variables under the proposed controller.

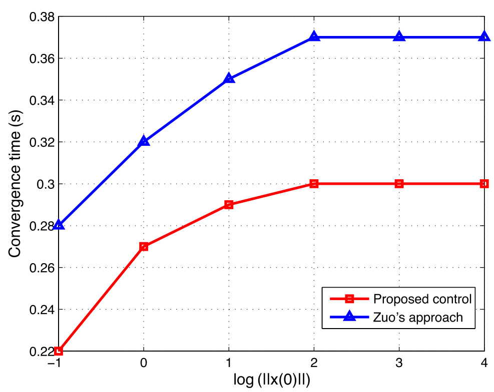

Fig. 4. Convergence time versus the logarithm of norm of the initial condition.

singularity problem and achieves fast convergence by accelerating the convergence speed when the system is closed to the origin. Our control scheme is applied to design the SVC controller and energy storage device controller for chaos suppression in power systems. Simulation results are provided to show the effectiveness and superiority of the proposed control scheme. The proposed control method can be applied to secure communication and network synchronization.

APPENDIX

The three-bus power system parameter values and proposed controller parameter values are given as follows. The parameter values for the network and generator are as follows: $E_m = 1$, $Y_m = 5$, $P_m = 1$, $\theta_m = -5.0$, $E_0' = 2.5$, $Y_0' = 8$, $\theta_0' = -12$, $P_m = 1.0$, $d_m = 0.05$, $M = 0.3$, $K_{\mathrm{es}} = 1$, $T_{\mathrm{es}} = 1$, $K_{\mathrm{SVC}} = 1$, and $T_{\mathrm{SVC}} = 0.01$. The parameter values for load are as follows: $K_{pw} = 0.4$, $K_{qv2} = 2.1$, $K_{qw} = -0.03$, $K_{qv} = -2.8$, $k_{pv} = 0.3$, $T = 8.5$, $P_0 = 0.6$, $P_1 = 0$, $Q_0 = 1.3$, and $Q_1 = 11.377$. The parameter values for the proposed controller are as follows: $\alpha_1 = 10$, $\beta_1 = 5$, $\alpha_2 = 10$, $\beta_2 = 5$, $m_1 = 9$, $n_1 = 5$, $m_2 = 9$, $n_2 = 5$, $h = 100$, $p_1 = 5$, $q_1 = 9$, $p_2 = 5$, and $q_2 = 9$. All values are in per unit, except for angles, which are in degrees.

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