

Forelesning

3

Splitt & hersk



Spionproblemet



Spionproblemet

- En forsamling folk
- Noen kjenner andre
- Det er ikke nødvendigvis gjensidig
- En spion kjenner alle, men ingen kjenner spionen
- Har forsamlingen en spion?

Callback corner

«[Von Neumann's] manuscript, written in ink, is 23 pages long; the first page still shows traces of the penciled phrase "TOP SECRET," which was subsequently erased. (In 1945, work on computers was classified, due to its connections with military problems.)»

Donald Knuth, «Von Neumann's First Computer Program»
<http://dl.acm.org/citation.cfm?doid=356580.356581>

Merge Sort

⑤

(g) We now formulate a set of instructions to effect this 4-way decision between $(\alpha)-(s)$. We state again the contents of the short tanks already assigned:

$T_1, N^{m_{(20)}}$ $T_2, N^{m_{(20)}}$ $T_3, N^{m_{(20)}}$ $T_4, N^{m_{(20)}}$

$T_5, N^{m_{(20)}}$ $T_6, N^{m_{(20)}}$ $T_7, N^{m_{(20)}}$ $T_8, N^{m_{(20)}}$

$T_9, N^{l_{(20)}}$ $T_{10}, N^{l_{(20)}}$ $T_{11}, N^{l_{(20)}}$ $T_{12}, N^{l_{(20)}}$

Now let the instructions occupy the (long tank) words $1_{11}, 2_{11}, \dots$:

$1_{11} = T_1 - \bar{T}_1$ $0) N^{m_1 - m_{(20)}}$ for $m_1 \geq m$
 $1_{12} = T_2 - \bar{T}_2$ $0) N^{l_1 - l_{(20)}}$ for $m_1 \geq m$
 $1_{13} = T_3 - \bar{T}_3$ $0) N^{l_1 - m_{(20)}}$ for $m_1 \geq m$
 $1_{14} = T_4 - \bar{T}_4$ $0) N^{l_1 - m_{(20)}}$ for $m_1 \geq m$

MERGE(A, p, q, r)

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1 $n_1 = q - p + 1$

MERGE(A, p, q, r)

$$1 \quad n_1 = q - p + 1$$

$$2 \quad n_2 = r - q$$

MERGE(A, p, q, r)

1 $n_1 = q - p + 1$

2 $n_2 = r - q$

3 let $L[1..n_1 + 1]$ and $R[1..n_2 + 1]$ be new arrays

MERGE(A, p, q, r)

1 $n_1 = q - p + 1$

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4 **for** $i = 1$ **to** n_1

MERGE(A, p, q, r)

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2 $n_2 = r - q$

3 let $L[1..n_1 + 1]$ and $R[1..n_2 + 1]$ be new arrays

4 **for** $i = 1$ **to** n_1

5 $L[i] = A[p + i - 1]$

MERGE(A, p, q, r)

1 $n_1 = q - p + 1$

2 $n_2 = r - q$

3 let $L[1..n_1 + 1]$ and $R[1..n_2 + 1]$ be new arrays

4 **for** $i = 1$ **to** n_1

5 $L[i] = A[p + i - 1]$

6 **for** $j = 1$ **to** n_2

MERGE(A, p, q, r)

1 $n_1 = q - p + 1$

2 $n_2 = r - q$

3 let $L[1..n_1 + 1]$ and $R[1..n_2 + 1]$ be new arrays

4 **for** $i = 1$ **to** n_1

5 $L[i] = A[p + i - 1]$

6 **for** $j = 1$ **to** n_2

7 $R[j] = A[q + j]$

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5 $L[i] = A[p + i - 1]$

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7 $R[j] = A[q + j]$

8 $L[n_1 + 1] = \infty$

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 - 13 **if** $L[i] \leq R[j]$

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```

	A	L	R
p	2	1	i
	4	2	j
	5	3	1
	7	4	2
	1	5	3
	2	7	3
	3	4	6
	6	∞	4
	7	5	∞
r	8	6	5

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k, p	2	1	i
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r	6	8	5

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```

	A	L	R
p	1 2	1 2	1 2
i		4	j
k	5 3 7 1 2 3 6	2 4 5 7 ∞ 5	1 2 3 4 5 ∞ 5
r	6	7 8	

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p	1 2	1 2	1 2
i		4	j
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	A	L	R
p	1 2	1 2	1 2
i		4	j
k	2 3 7 1 2 3 6	2 3 5 4 ∞ 5	1 2 3 4 5 ∞
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	A	L	R
p	1 2	1 2	1 2
k	2 3 7 1 2 3 6	4 5 7 4 5 6 ∞	3 3 4 3 5 4 ∞
r	6	7 ∞	5

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MERGE( $A, p, q, r$ )
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	A	L	R	
p	1 2 2 7 1 2 3	1 2 4 5 ∞	2 2 3 4 5 ∞	1 2 3 4 5
k		i	j	
r	6 7 8			

```

MERGE( $A, p, q, r$ )
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	A	L	R
p	1 2 2 7 1 2 3	1 2 4 5 ∞	2 2 3 4 5 ∞
k		i	j
r	6 7 8		1 2 3 4 5

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	A	L	R	
p	1 2 2 3 4 1 2 3 6	1 2 3 5 7 ∞ 6 7 8	2 4 5 3 4 5 ∞	1 2 3 3 4 4 ∞
k		i	j	
r				

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	A	L	R	
p	1 2 2 3 4 1 2 3 6	1 2 4 5 7 ∞ 5	2 2 5 3 4 5 ∞	1 2 3 3 6 4
k		i	j	
r	8			

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MERGE( $A, p, q, r$ )
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	A	L	R
p	1	1	2
	2	2	2
	2	3	3
	3	4	4
	1	5	3
	2	7	6
	3	4	4
	6	∞	5
k	5	5	5
	6	6	6
	7	7	7
	8	8	5
r	6		

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MERGE( $A, p, q, r$ )
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	A	L	R
p	1 2 2 3 4 1 2 3 6	1 2 4 5 7 ∞ 5	1 2 2 3 6 ∞
i		2 4 5 7 4 ∞	
k	1 2 3 6	6 7 8	
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	A	L	R
p	1 2 2 3 3 4 2 3 6	1 2 4 5 7 ∞ 5	1 2 2 3 3 4 6 ∞
i		2 4 5 7 4 ∞	1 2 3 3 4 6
k	1 2 2 3 3 4 2 3 6	1 2 4 5 7 ∞ 5	1 2 2 3 3 4 6 ∞
r	6	8	5

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	A	L	R
p	1 2 2 3 4 5 6 7	1 2 4 5 7 ∞ 5 8	1 2 2 3 4 5 6 5
k		i	j
r			

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	A	L	R
p	1	1	2
	2	2	4
	2	3	5
	3	4	7
	4	5	∞
k	2	6	4
	3	7	3
r	6	8	5

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	2	3	3
	3	4	4
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	5	7	6
	6	∞	4
	7	5	5
	8		5
k	2	6	
	3	7	
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p	1	1	2
	2	2	2
	2	3	3
	3	4	4
	4	5	5
	5	∞	6
	6		∞
i		5	
j		4	6
k	6		
	7		
r	8		

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MERGE( $A, p, q, r$ )
1  copy into  $L$  and  $R$ 
2  for  $k = p$  to  $r$ 
3      if  $L[i] \leq R[j]$ 
4           $A[k] = L[i]$ 
5           $i = i + 1$ 
6      else  $A[k] = R[j]$ 
7           $j = j + 1$ 

```

	A	L	R
p	1	2	1
	2	4	2
	2	5	3
	3	7	4
	4	∞	5
k	5	6	
	3	7	
r	6	8	

```

MERGE( $A, p, q, r$ )
1  copy into  $L$  and  $R$ 
2  for  $k = p$  to  $r$ 
3      if  $L[i] \leq R[j]$ 
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7           $j = j + 1$ 

```

	A	L	R	
p	1 2 2 3 4 5 6	1 2 4 5 7 ∞ 5	2 4 2 3 4 6 4	1 2 3 3 6 4 5
i		i	j	
k	3	7		
r	6	8		

```

MERGE( $A, p, q, r$ )
1  copy into  $L$  and  $R$ 
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```

	A	L	R	
p	1 2 2 3 3 4 5 6	1 2 4 5 7 ∞ 6 5	2 4 3 3 4 5 6 4	1 2 3 3 6 4 5
i		i	j	
k	3	7		
r	6	8		

```

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```

	A	L	R	
p	1 2 2 3 3 4 5 6	1 2 4 5 7 ∞ 5	2 4 5 3 4 6 ∞	1 2 3 3 4
i		i	j	
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```

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```

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	2	5	3
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	5		
	6	7	
k	6		
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```

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```

	A	L	R
p	1	2	1
	2	4	2
	2	5	3
	3	7	4
	4	∞	5
	5		
	6		
	6		
k, r	6	8	

i

```

MERGE( $A, p, q, r$ )
1  copy into  $L$  and  $R$ 
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```

	A	L	R
p	1	2	1
	2	4	2
	2	5	3
	3	7	4
	4	∞	5
	5		
	6	7	
	6		
k, r	6	8	

```

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```

	A	L	R
p	1	2	1
	2	4	2
	2	5	3
	3	7	4
	4	∞	5
	5	5	∞
	6	6	4
	7	7	3
k, r	7	8	5

```

MERGE( $A, p, q, r$ )
1  copy into  $L$  and  $R$ 
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```

	A	L	R
p	1	1	2
	2	2	2
	2	3	3
	3	4	3
	4	7	6
	5	∞	4
	6	5	6
	7	7	4
k, r	8	∞	5

```

MERGE( $A, p, q, r$ )
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```

	A	L	R	
p	1 2 2 3 3 4 5 6 7	1 2 4 5 7 ∞ 5	2 4 5 3 4 6 5 ∞	1 2 3 3 4 4 5 5
		i	j	
r	7	8		

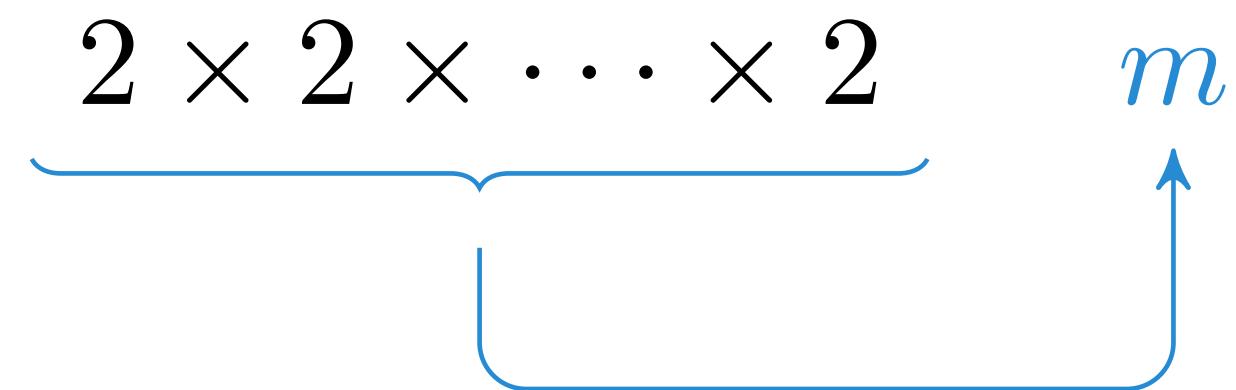
Max-subarray:
Neste callback corner

Pensum

- Kap. 4. Divide-and-conquer:
Innledning, 4.1 og 4.3–4.5
- Kap. 7. Quicksort
- Oppgaver 2.3–5 og 4.5–3 med løsning
(binærsøk)

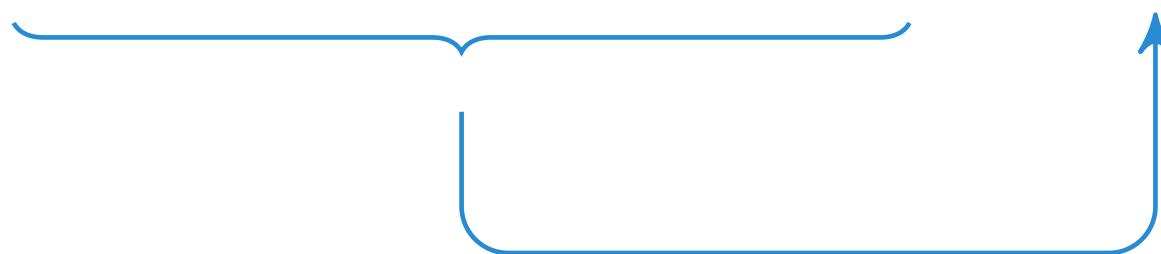
Potenser og logaritmer

$$2 \times 2 \times \cdots \times 2$$

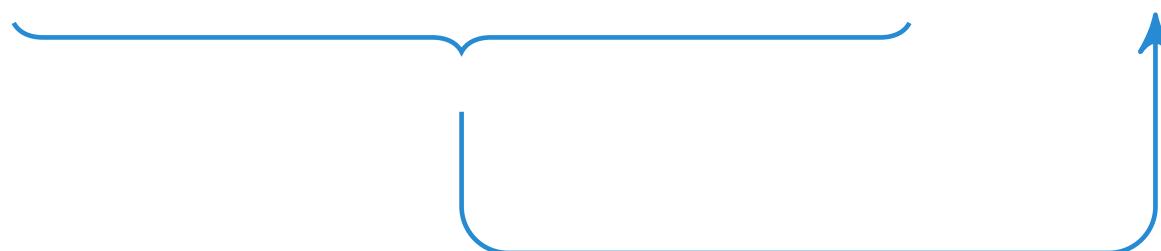
$$2 \times 2 \times \cdots \times 2$$


A blue brace is positioned under the multiplication expression $2 \times 2 \times \cdots \times 2$. The brace is a horizontal line with a vertical line extending downwards from its center, ending in an arrowhead pointing towards the right. To the right of the arrowhead, the letter m is written in blue.

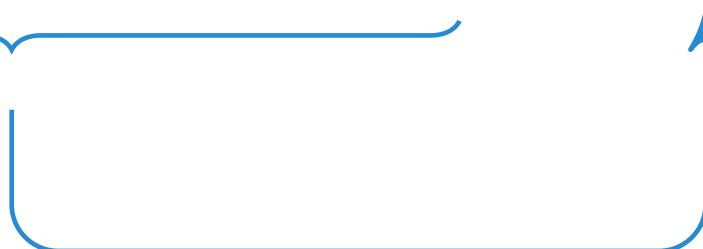
$$\text{length}(2 \times 2 \times \cdots \times 2) = m$$



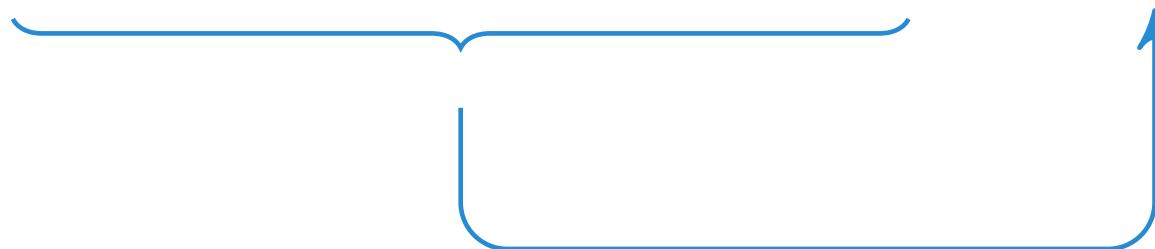
length($2 \times 2 \times \cdots \times 2$) = m



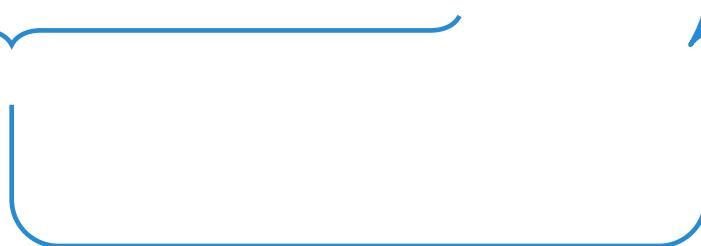
$$\lg(\underbrace{2 \times 2 \times \cdots \times 2}_\text{m}) = m$$



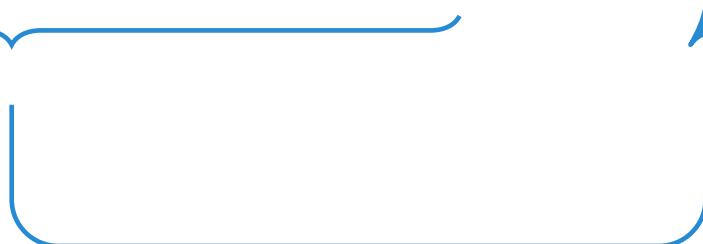
$$\log_2(2 \times 2 \times \cdots \times 2) = m$$



$$\log_3(\underbrace{3 \times 3 \times \cdots \times 3}_\text{m}) = m$$



$$\log_k(\underbrace{k \times k \times \cdots \times k}_\text{m}) = m$$



$$\underbrace{\frac{n}{2 \times 2 \times \cdots \times 2}}_i = \frac{n}{2^i} = 1 \iff i = \lg n$$

$$(n = 2^{\lg n})$$

$$\begin{array}{r} & & & & 1 \\ & & & & 2 \\ + & & & & 2 \times 2 \\ + & & & & 2 \times 2 \times 2 \\ + & & 2 \times 2 \times 2 \times 2 \\ \hline = & 2 \times 2 \times 2 \times 2 \times 2 & - & 1 \\ \hline \hline \end{array}$$

$$\begin{array}{r} & & 2 \\ & + & 2 \times 2 \\ & + & 2 \times 2 \times 2 \\ & + & 2 \times 2 \times 2 \times 2 \\ \hline = & 2 \times 2 \times 2 \times 2 \times 2 - 2 \\ \hline \hline \end{array}$$

$$+ \quad \quad \quad 2^2$$

$$+ \quad \quad \quad 2 \times \cdots \times 2$$

$$+ \quad \quad \quad 2 \times 2 \times \cdots \times 2$$

=

$$+ \quad \quad \quad 2 \times 2$$

$$+ \quad \quad \quad 2 \times \cdots \times 2$$

$$= \quad \quad \quad 2 \times 2 \times \cdots \times 2 - 2$$

$$+ \quad \quad \quad 2 \times 2 \times \cdots \times 2$$

$$=$$

$$\begin{aligned} & + 2 \times 2 \\ & + 2 \times \cdots \times 2 \\ & = 2 \times 2 \times \cdots \times 2 - 2 \\ & + 2 \times 2 \times \cdots \times 2 \\ \hline & = 2 \times 2 \times 2 \times \cdots \times 2 - 2 \\ \hline \hline \end{aligned}$$

$$+ \quad \quad \quad 2^2$$

$$+ \quad \quad \quad 2 \times 2$$



$$+ \quad \quad \quad 2 \times \cdots \times 2$$

$$+ \quad \quad \quad 2 \times 2 \times \cdots \times 2$$

$$= \quad 2 \times 2 \times 2 \times \cdots \times 2 - 2$$



Litt grunnlag

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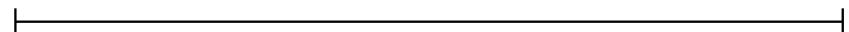
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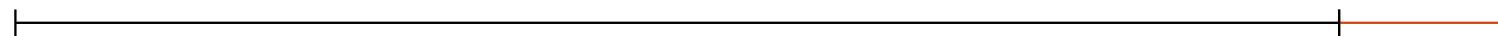
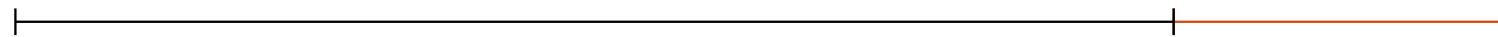
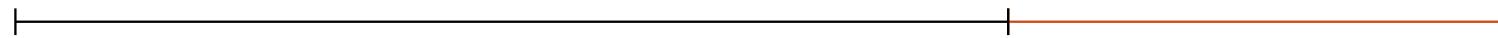
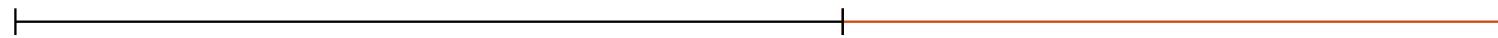
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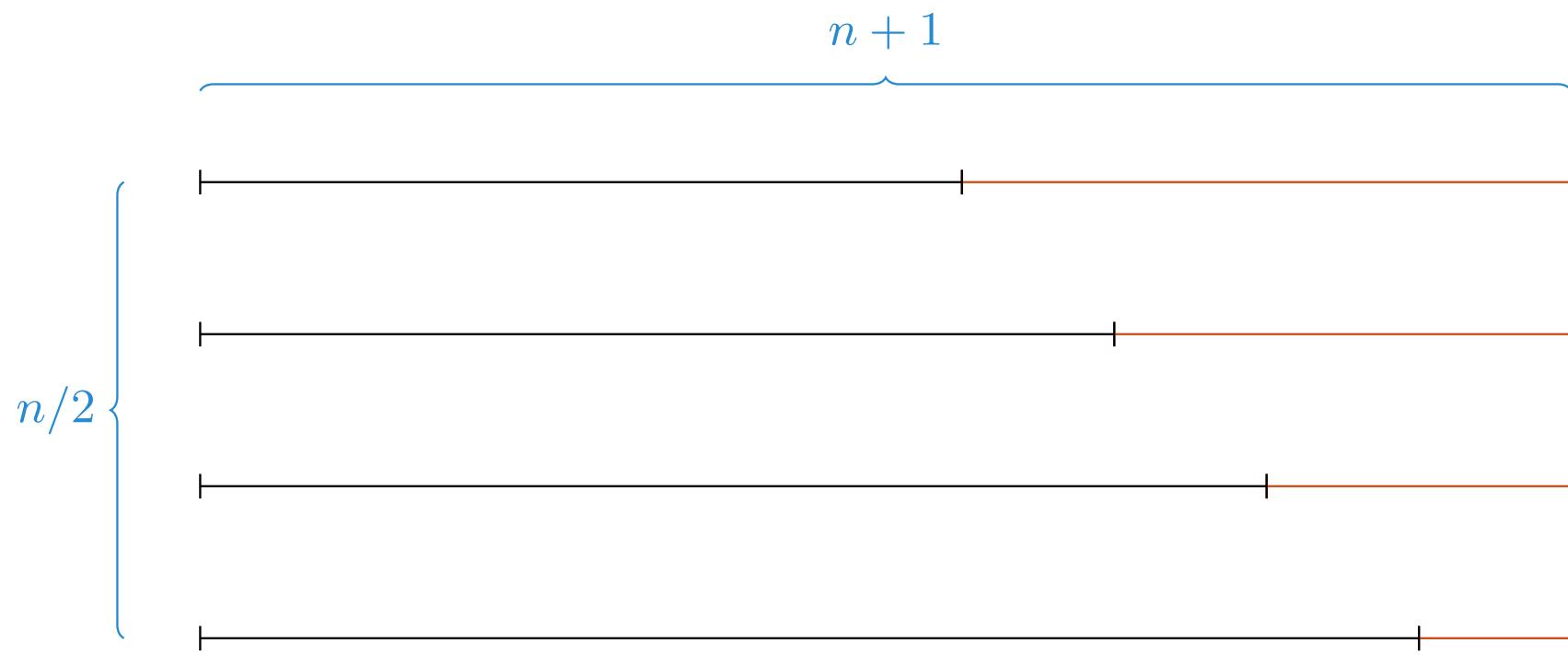
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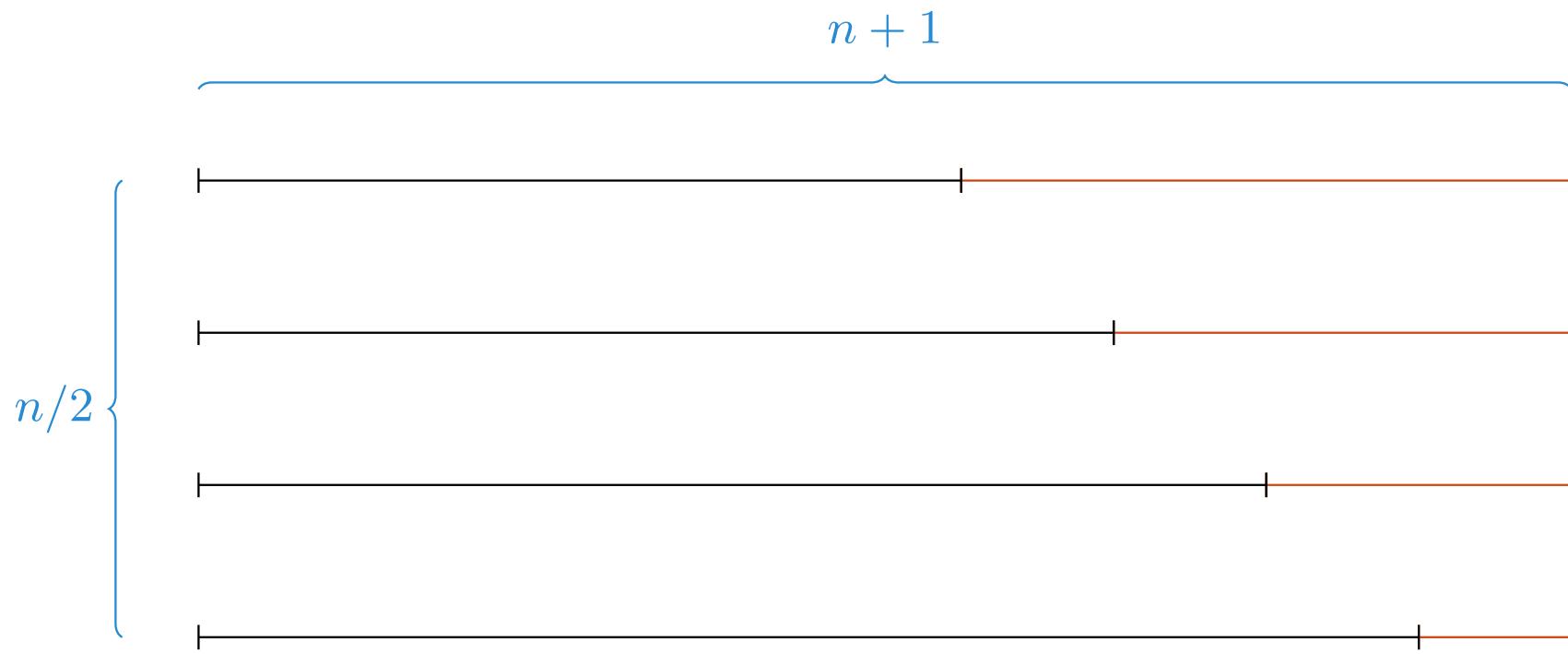
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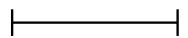


$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$





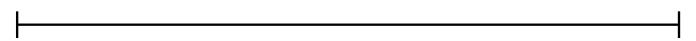
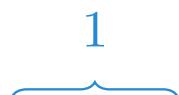
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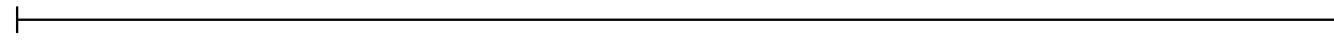
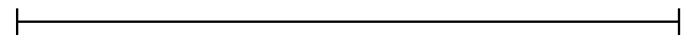
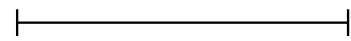
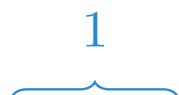
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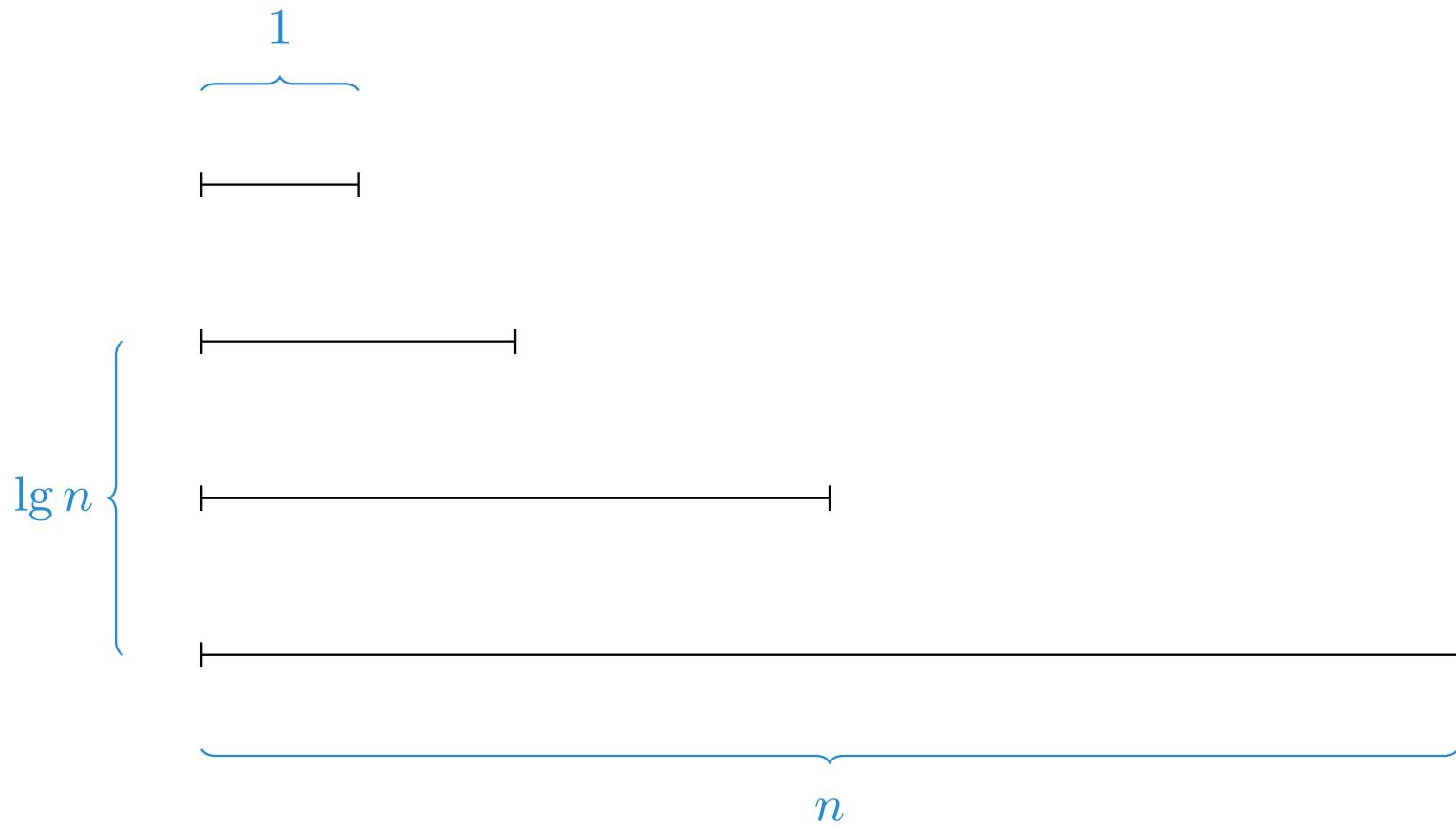


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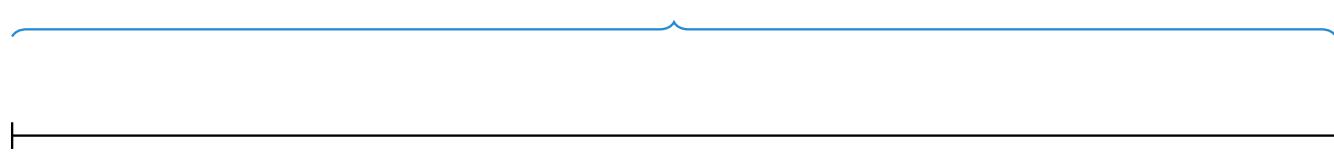


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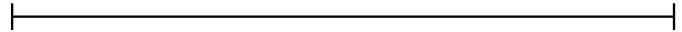
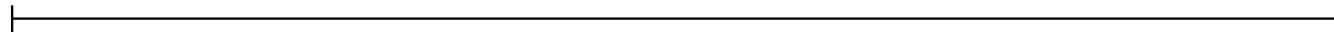




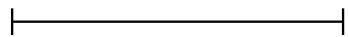
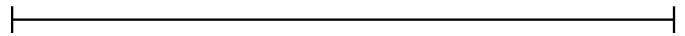
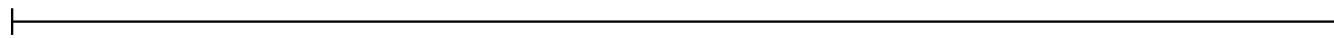
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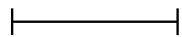
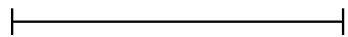
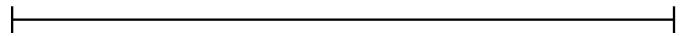
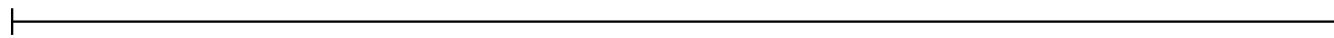
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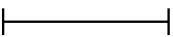
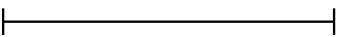
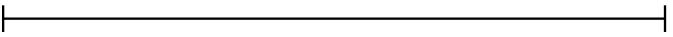
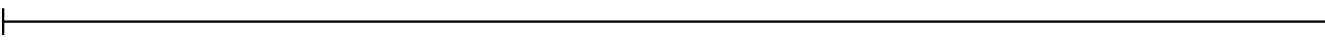
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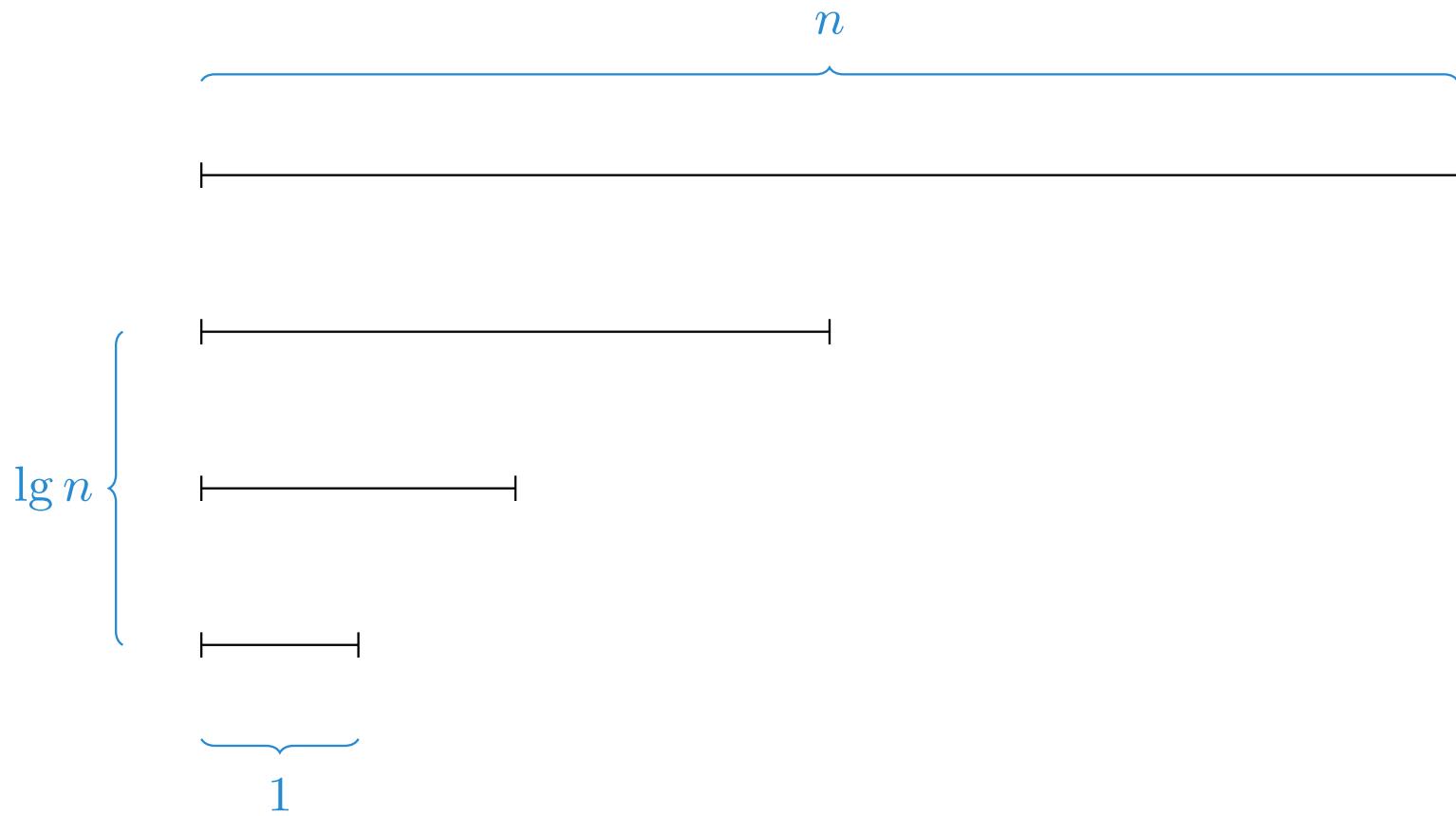
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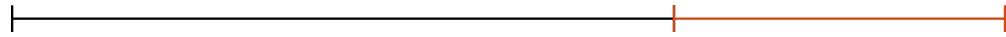


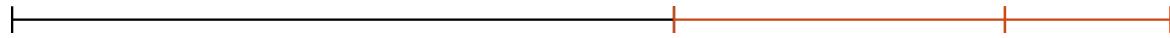
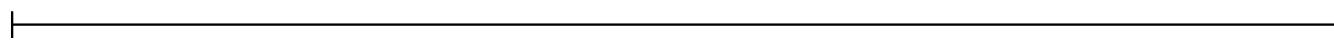
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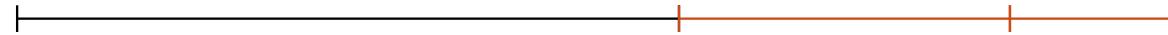
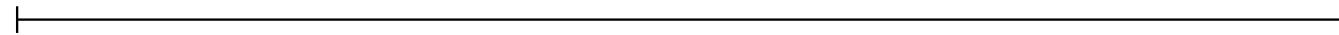
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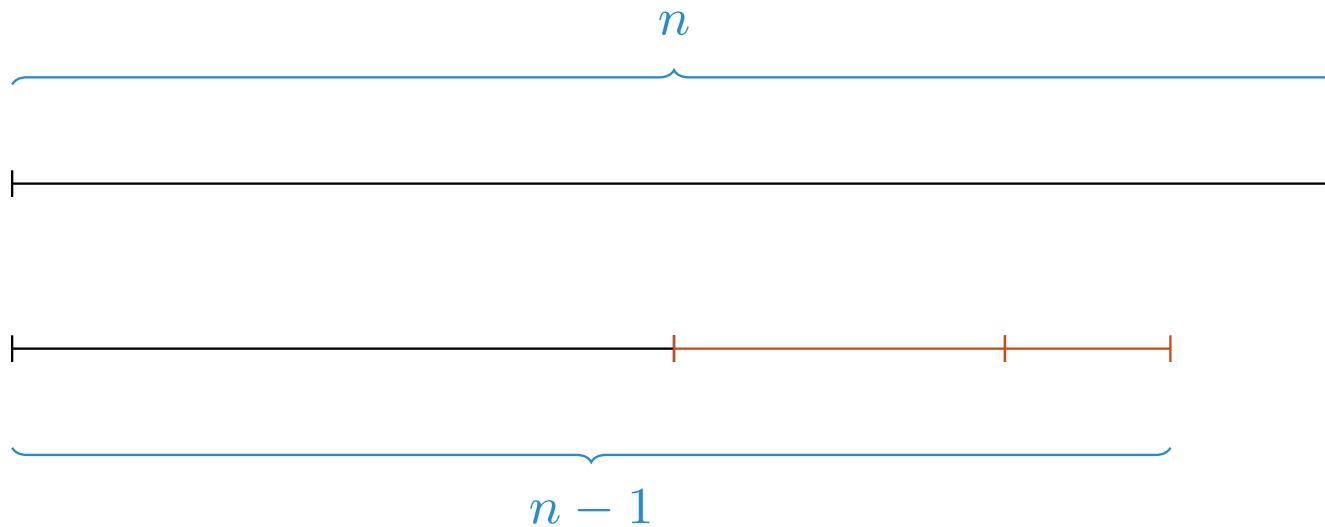
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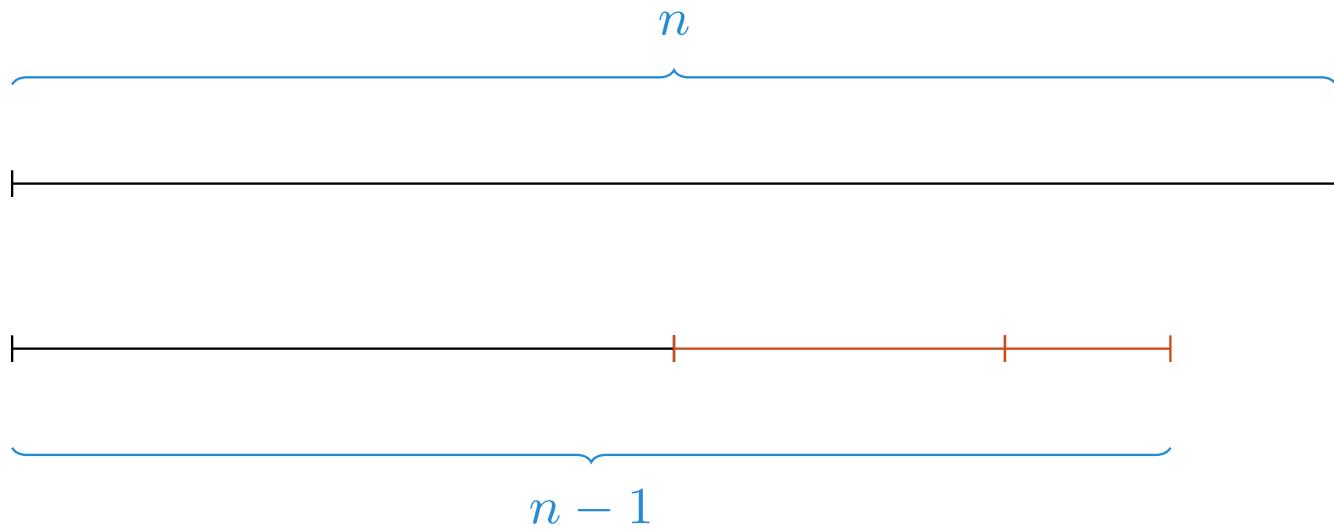




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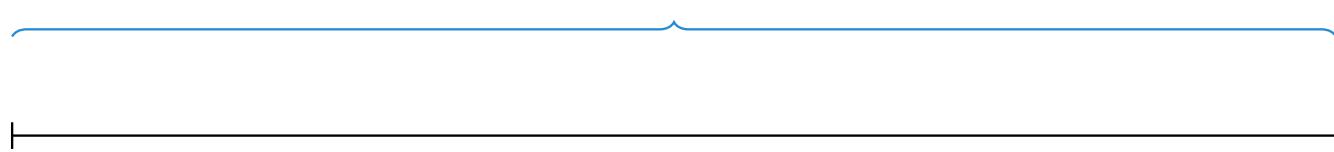


 n $n - 1$

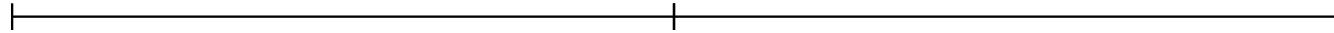
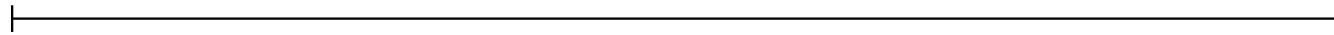


$$\sum_{i=1}^{\lg n} 2^i = 2n - 1$$

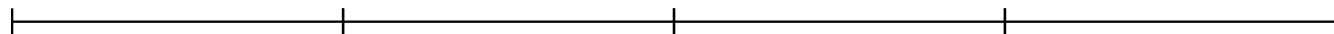
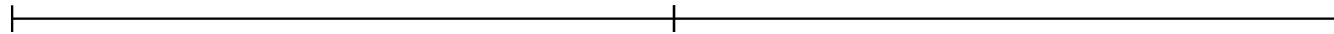
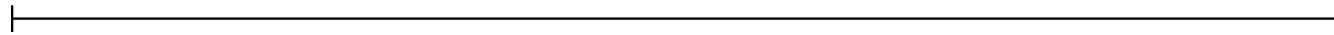
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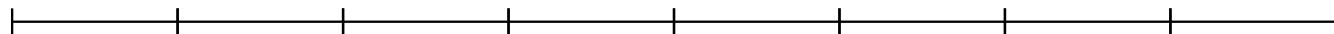
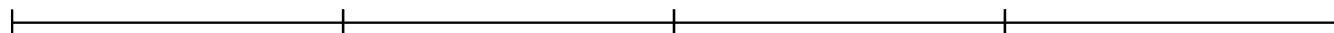
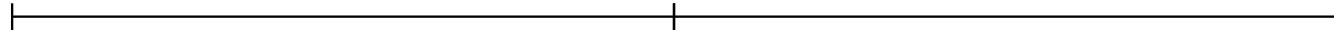
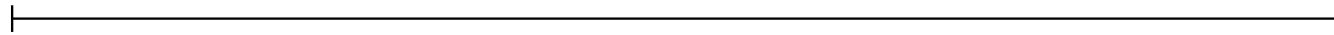
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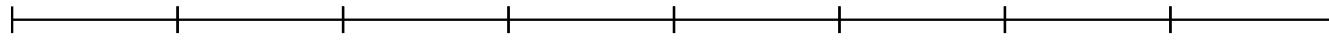
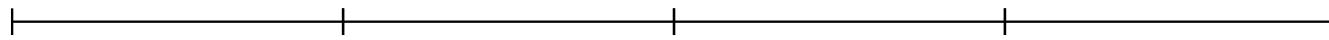
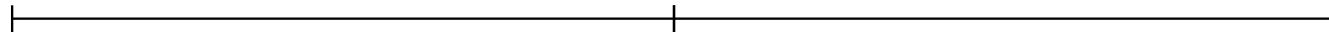
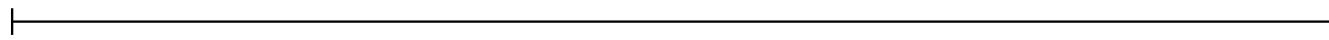
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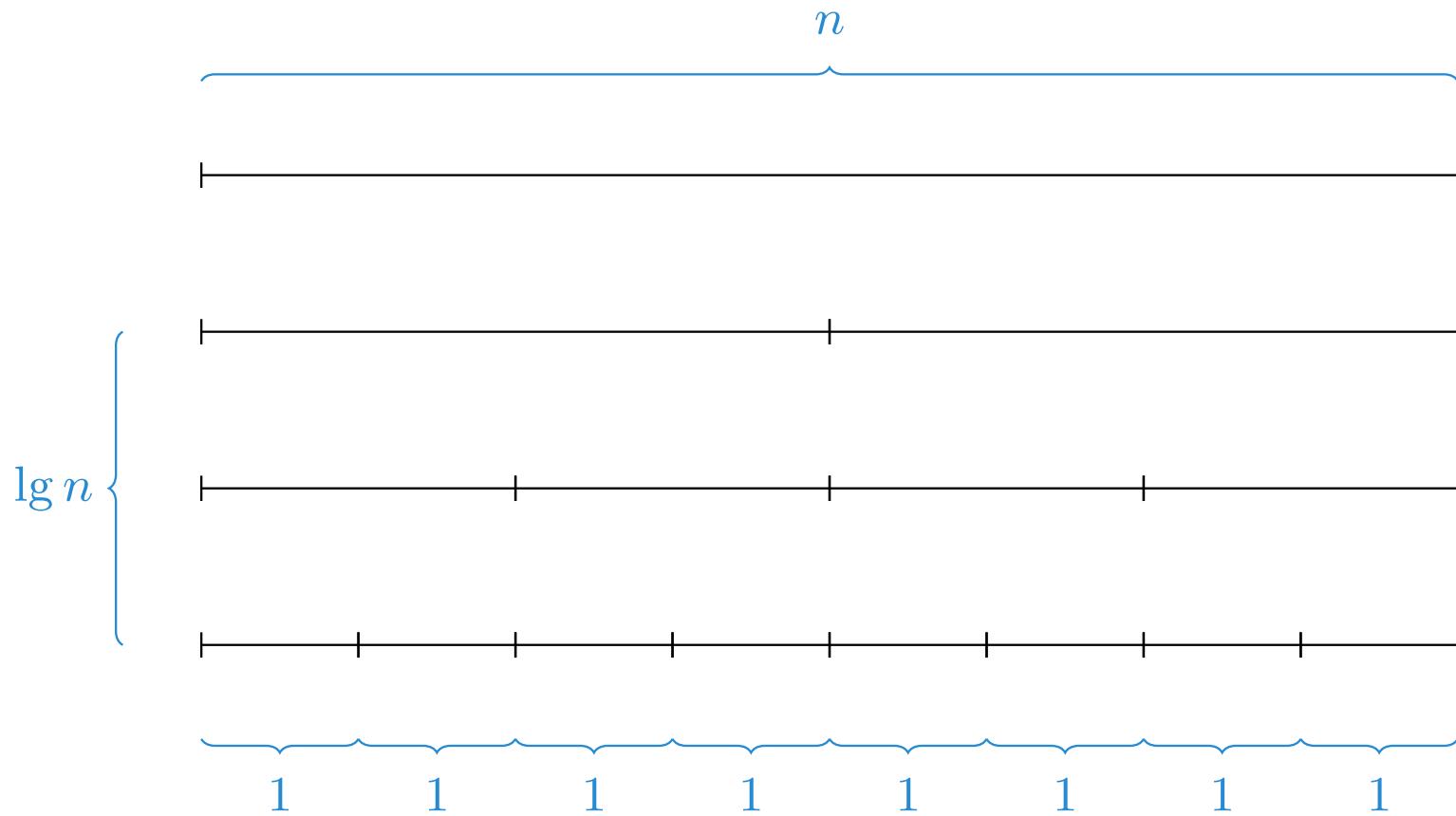
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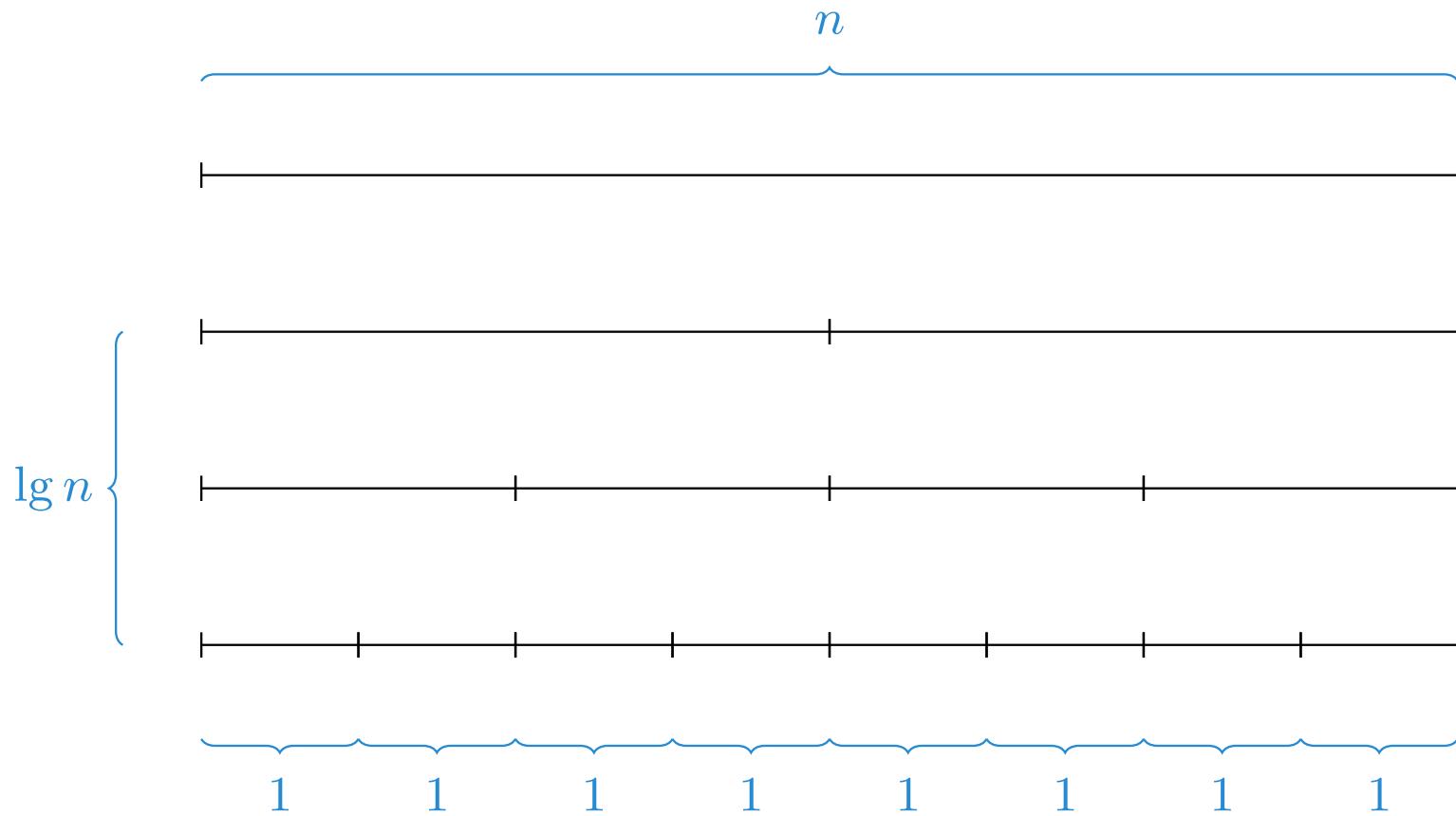
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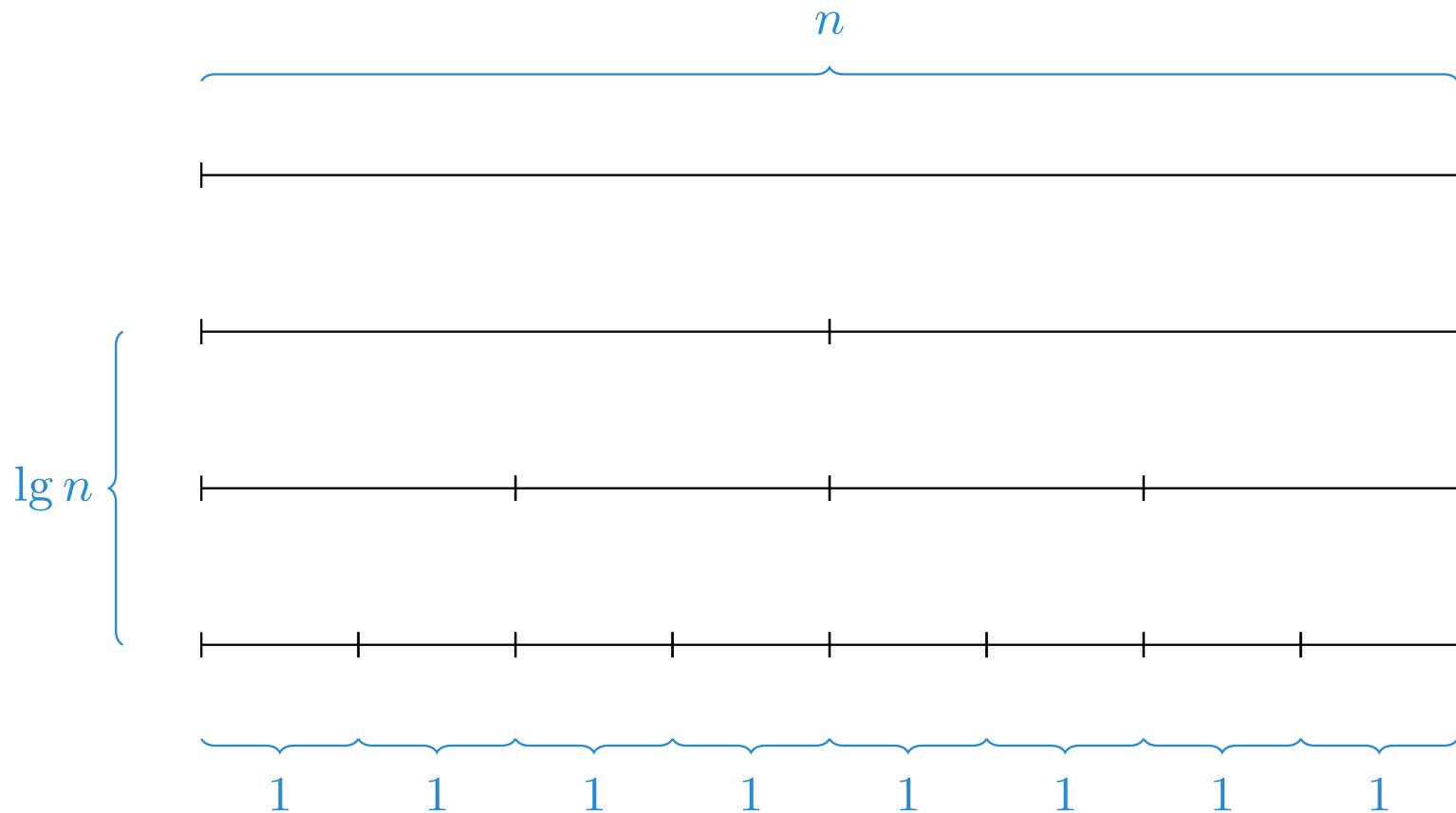
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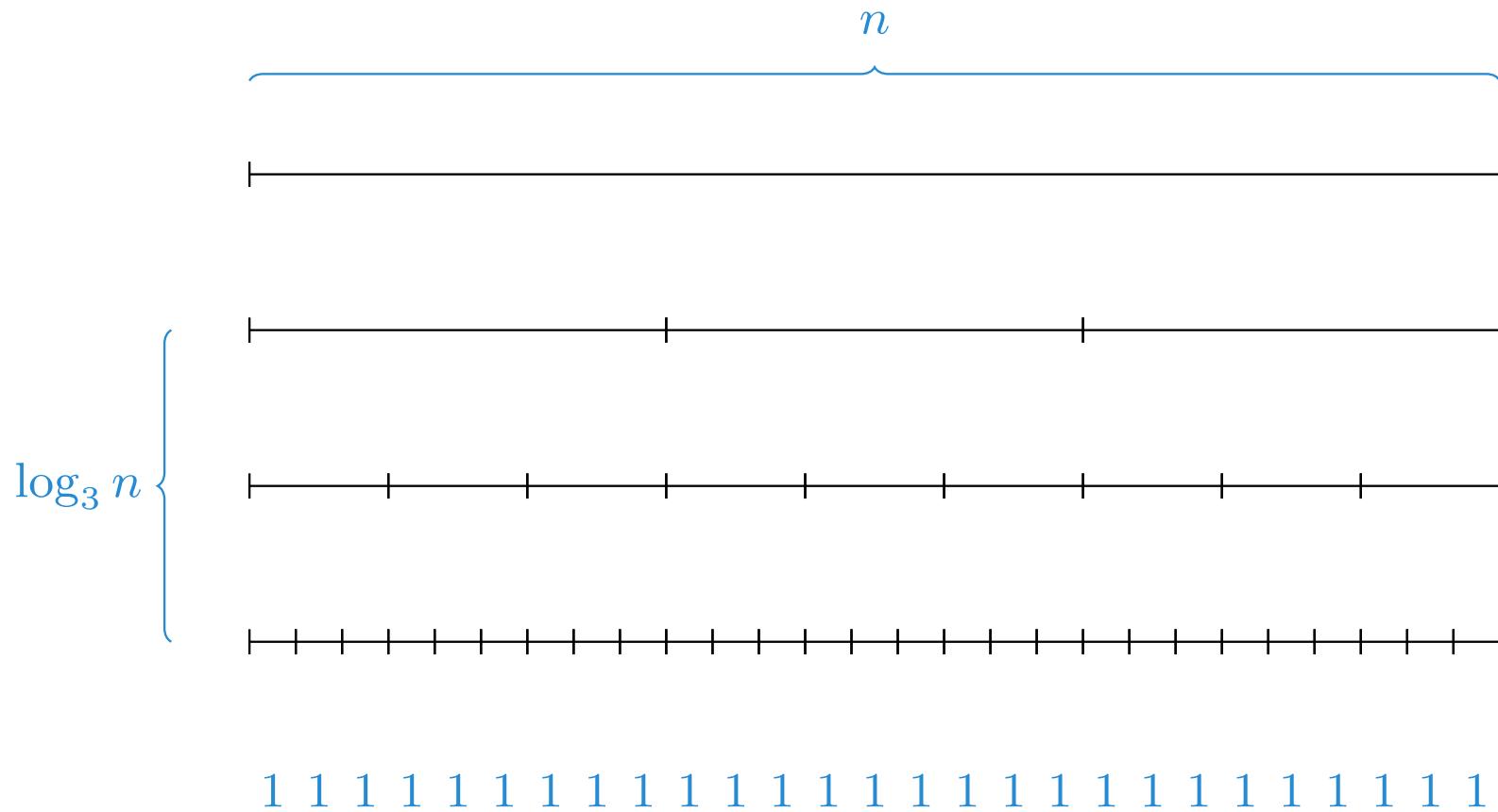




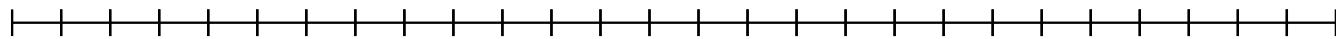
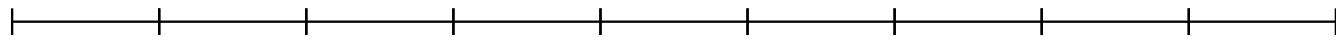
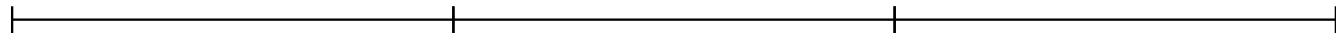
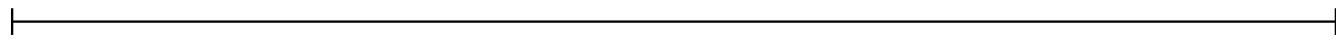
$$\sum_{i=0}^{\lg n} 2^i \cdot \frac{n}{2^i} = n \cdot (\lg n + 1)$$

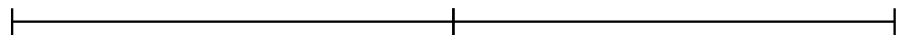
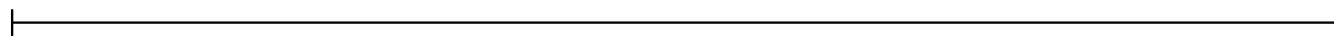


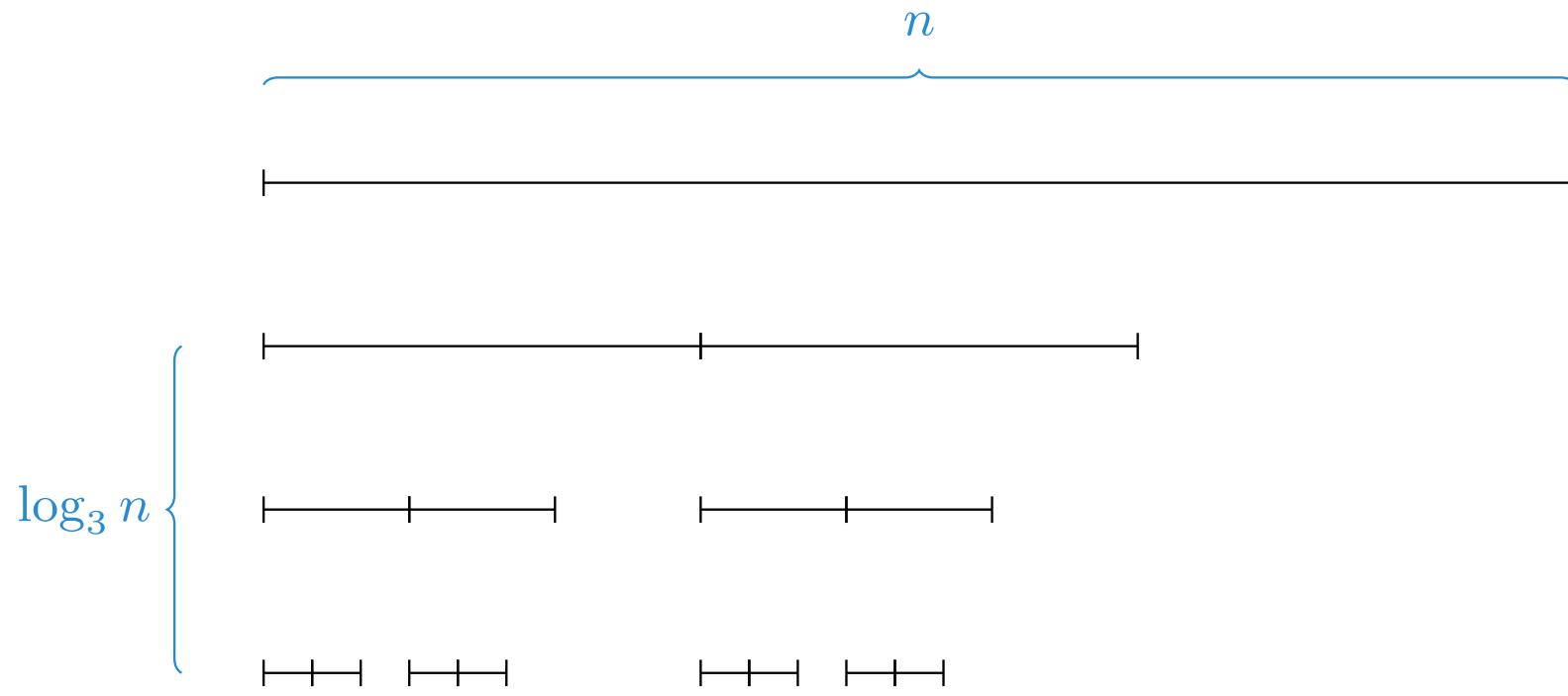
$$\sum_{i=0}^{\lg n} 2^i \cdot \frac{n}{2^i} = n \cdot (\lg n + 1) = n \lg n + n$$

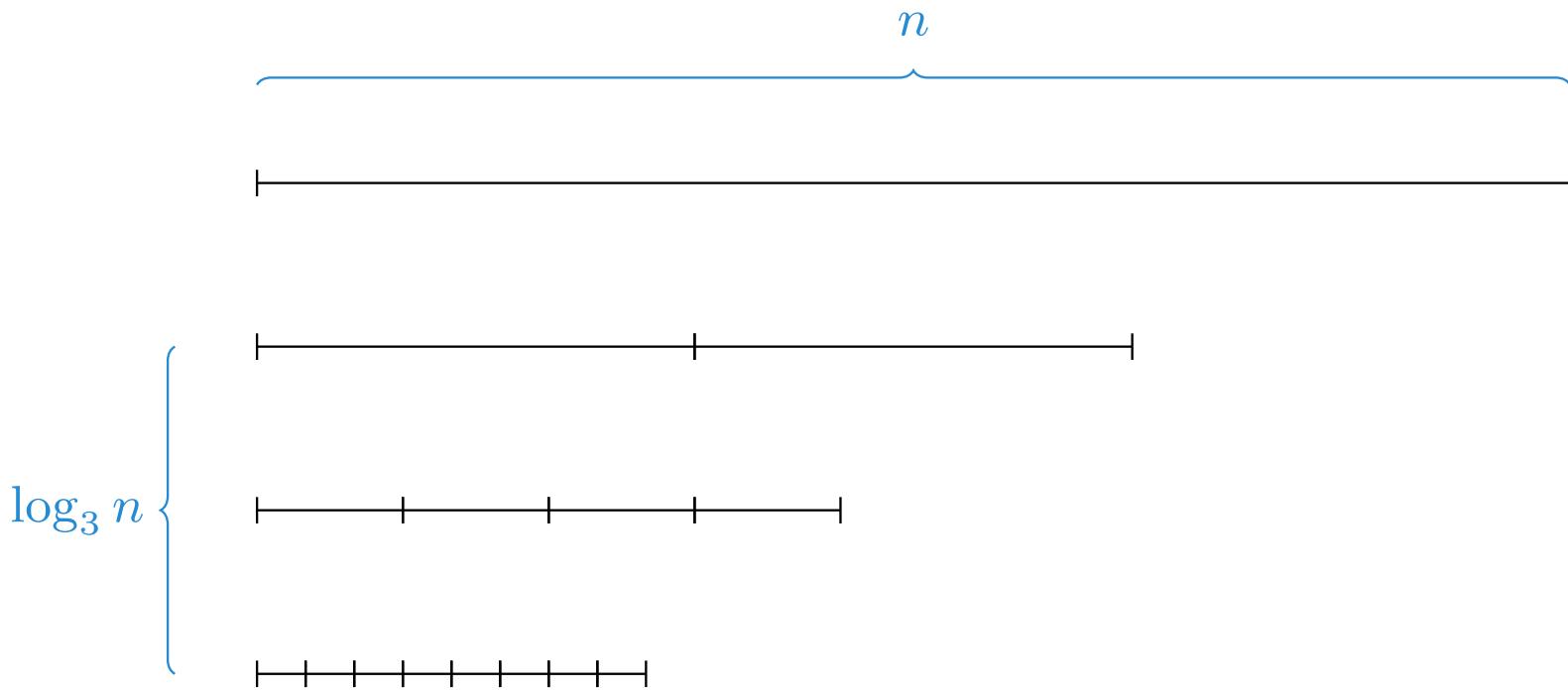


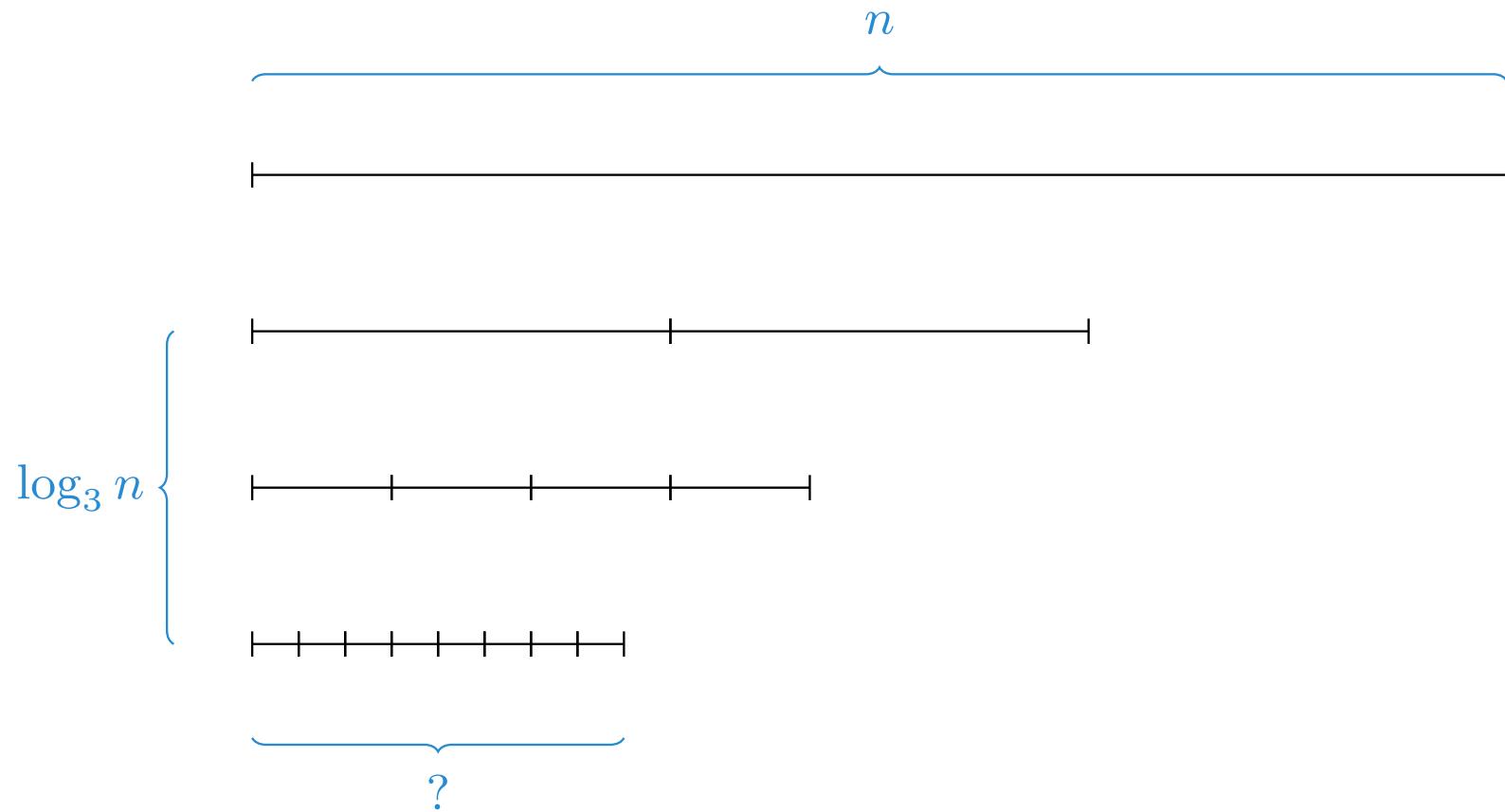
$$\sum_{i=0}^{\log_3 n} 3^i \cdot \frac{n}{3^i} = n \cdot (\log_3 n + 1) = n \log_3 n + n$$

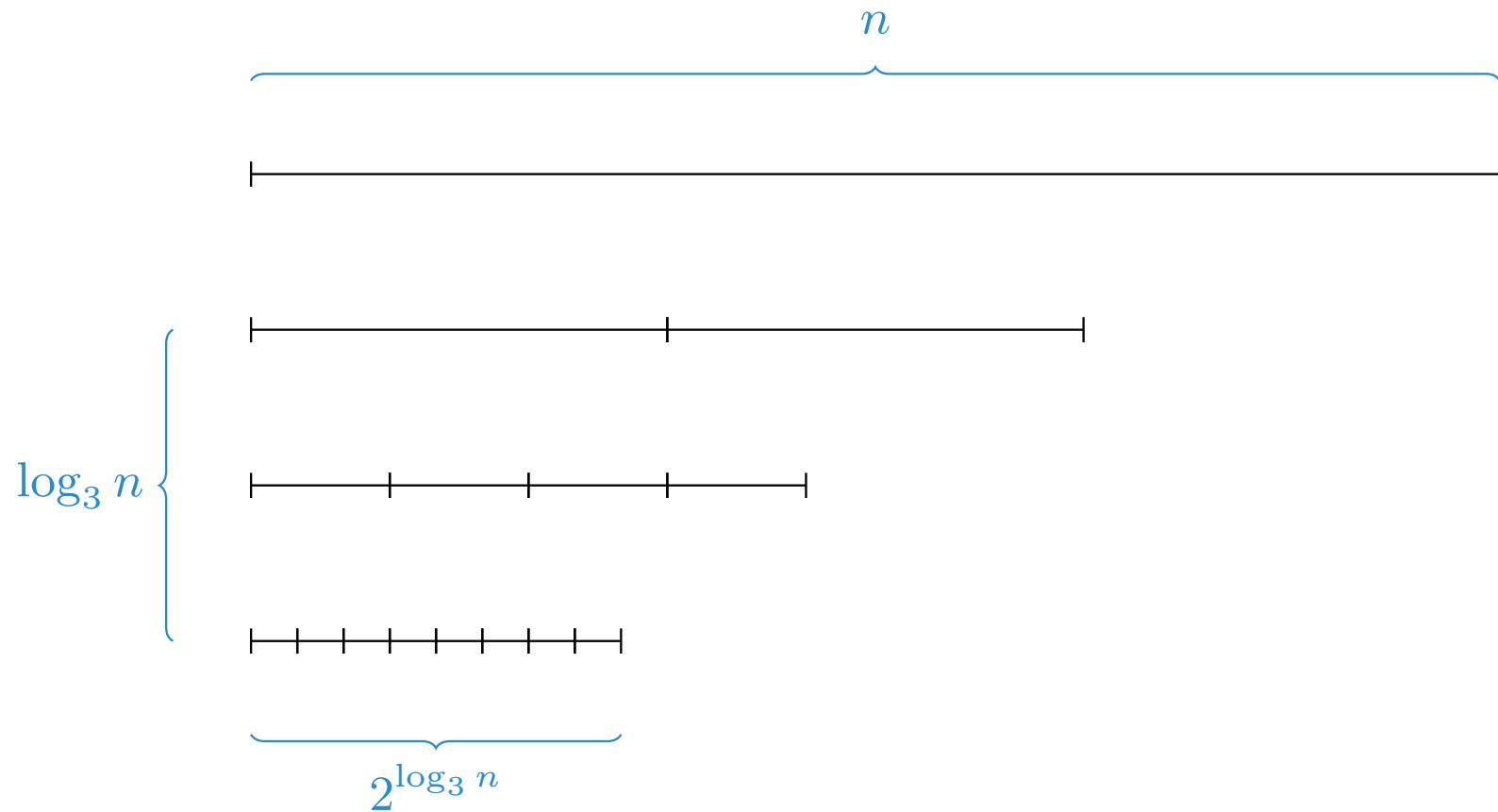


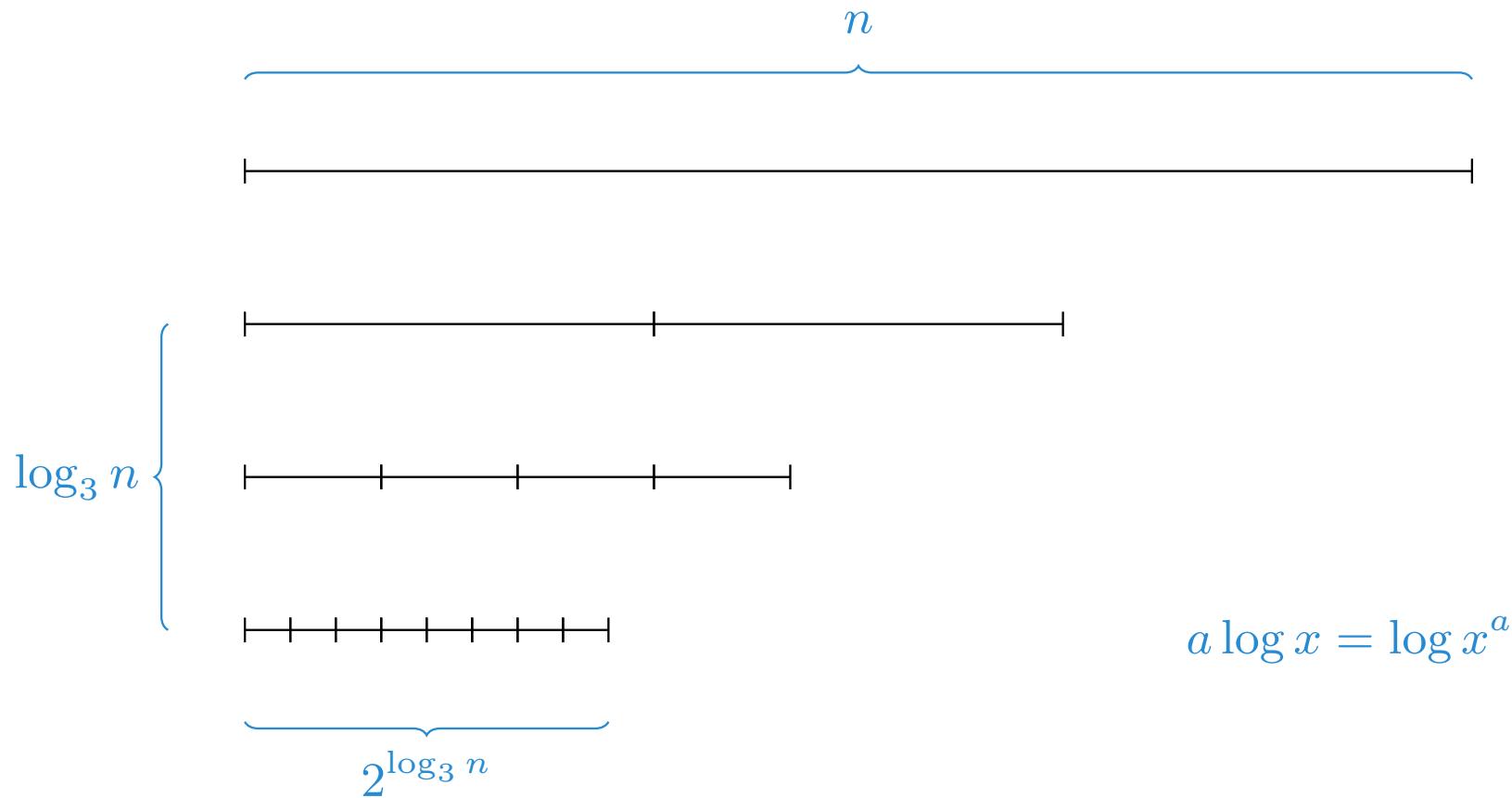


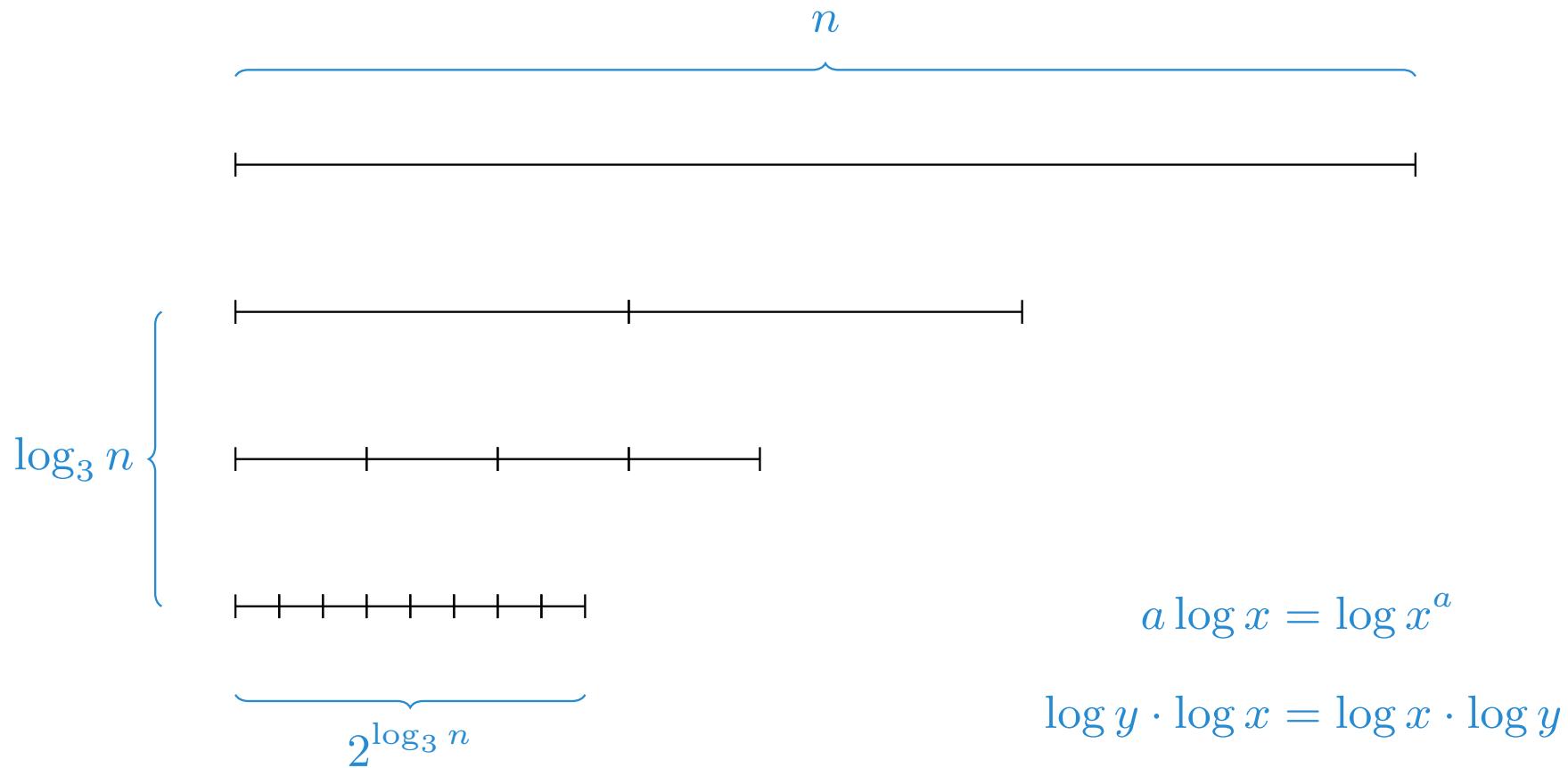


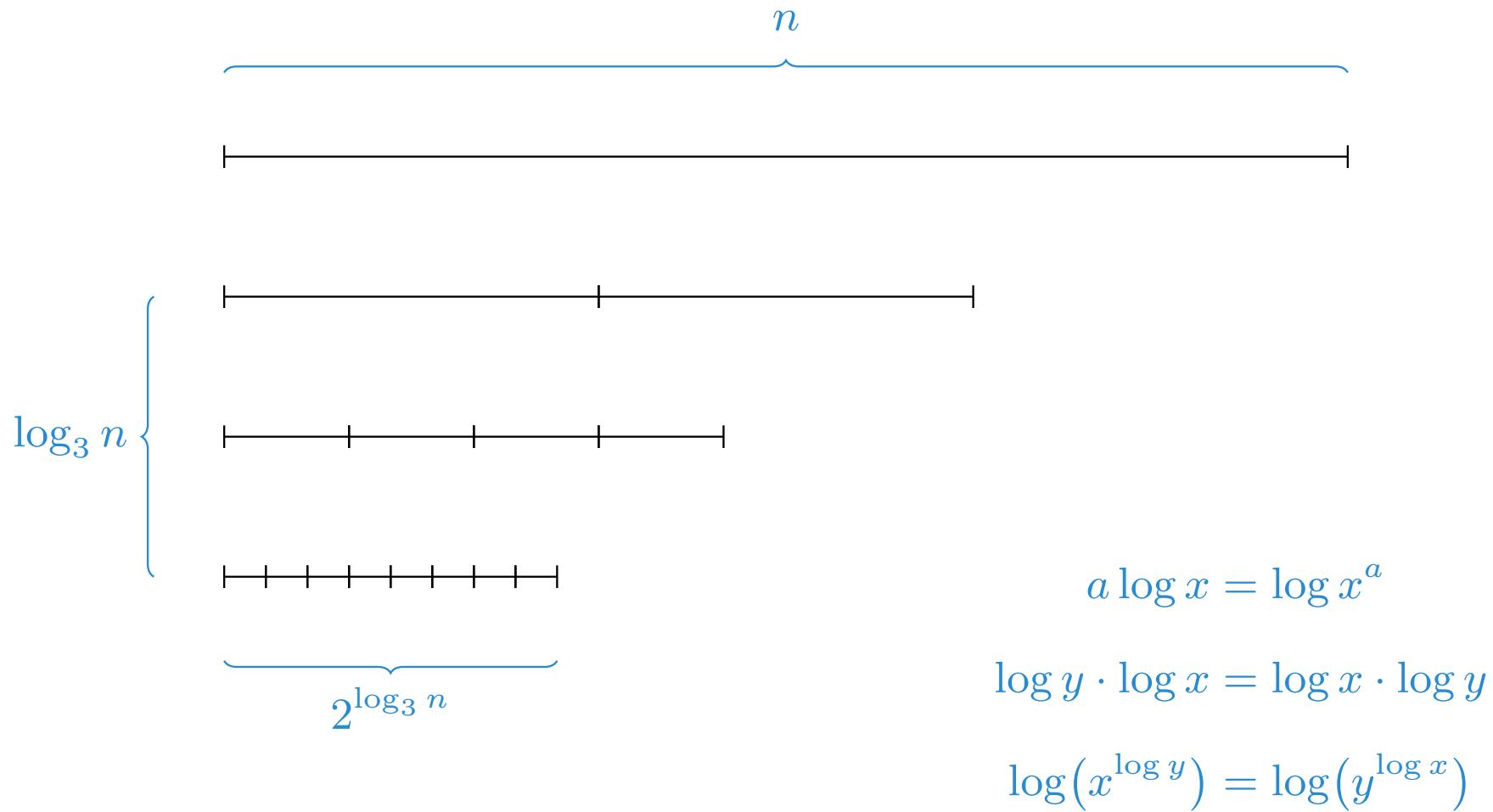


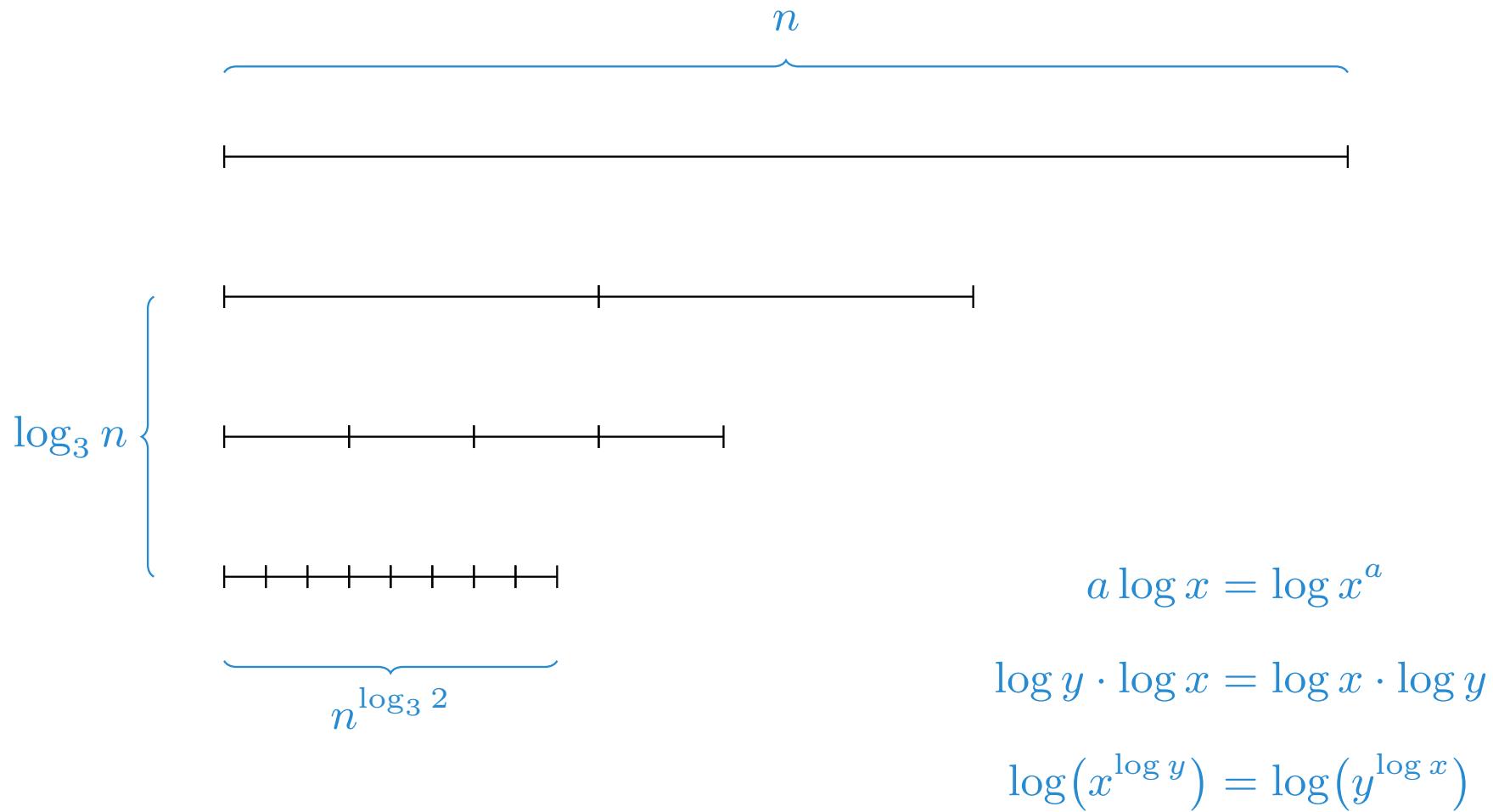


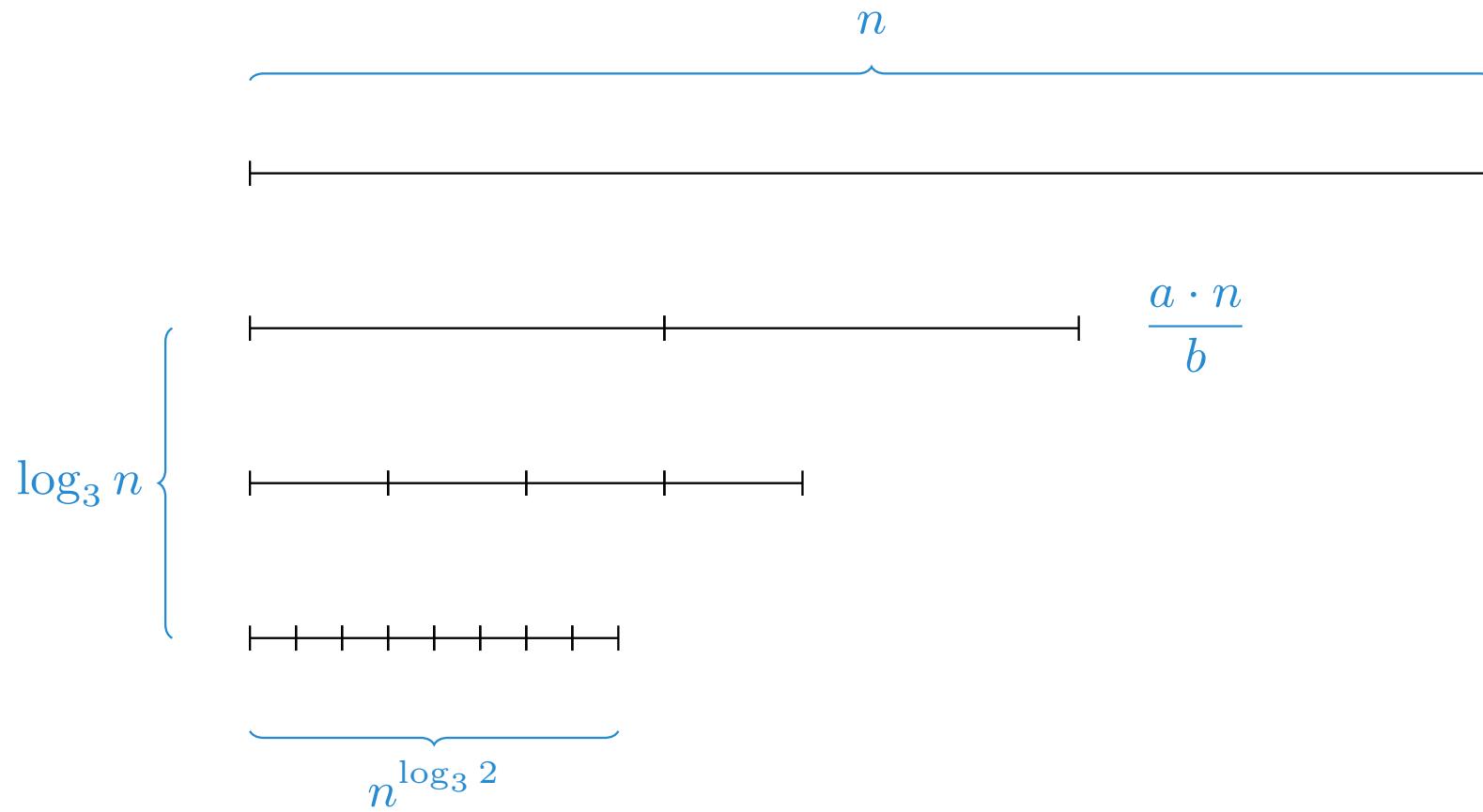


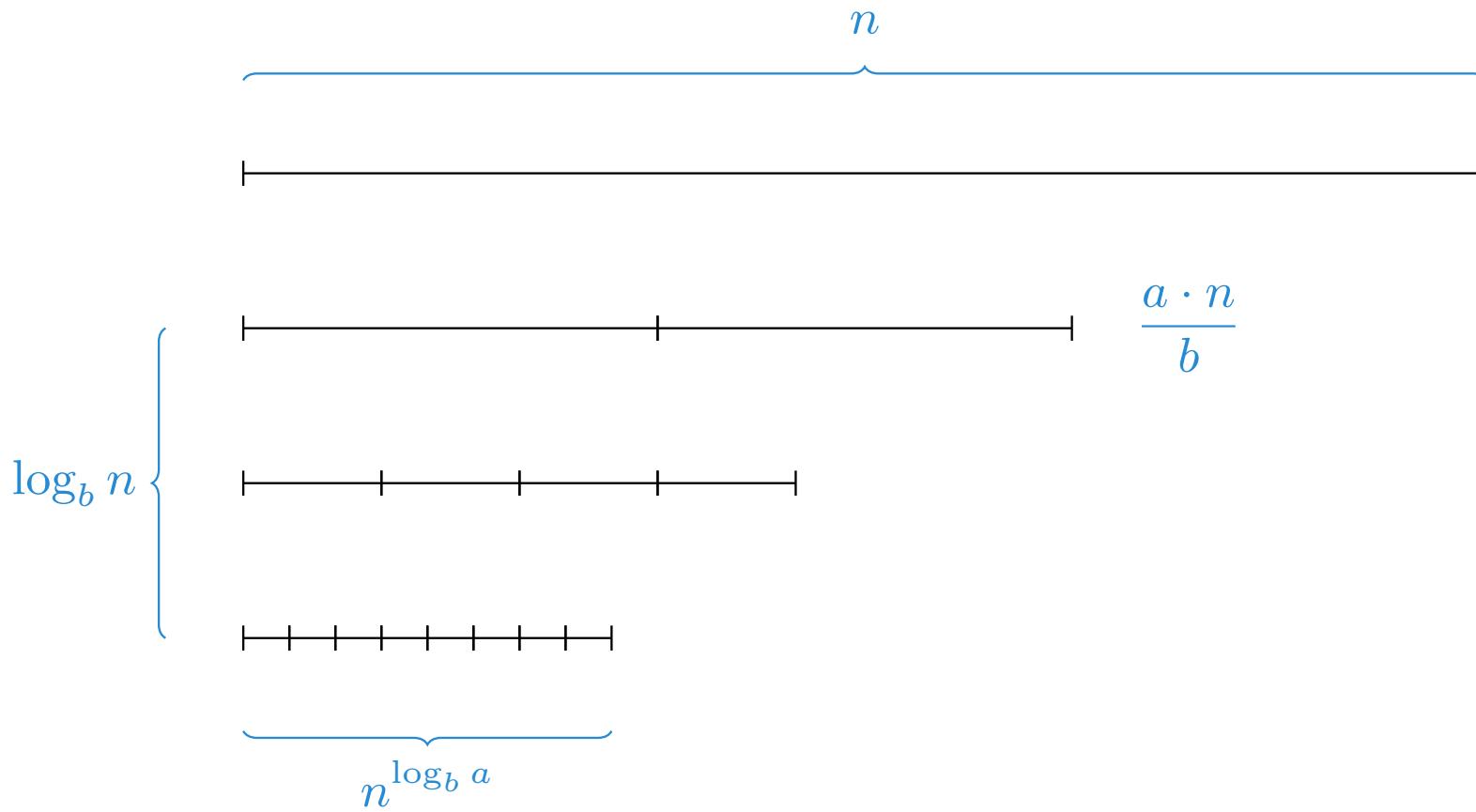


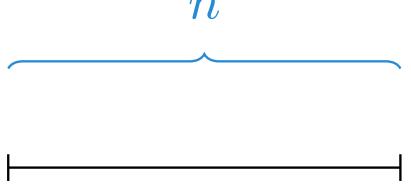


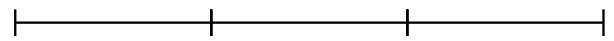


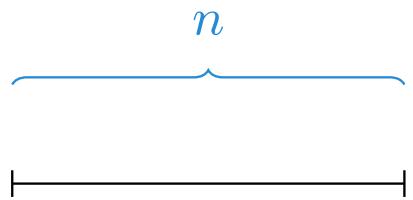






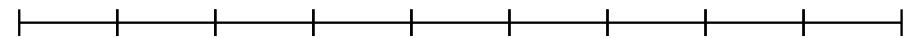
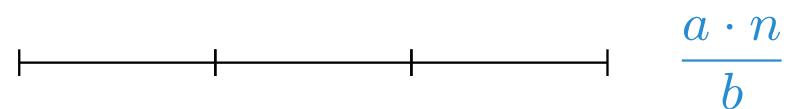
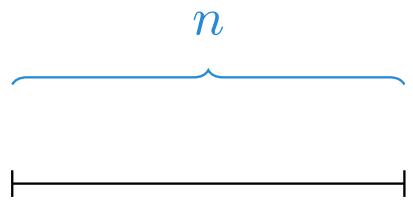
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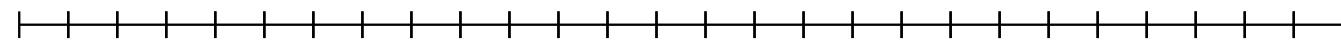
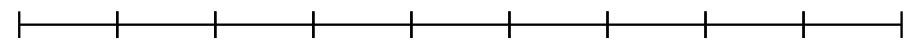
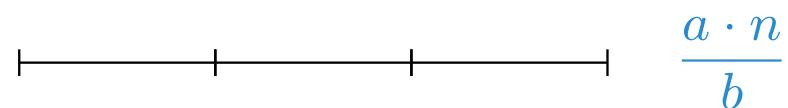
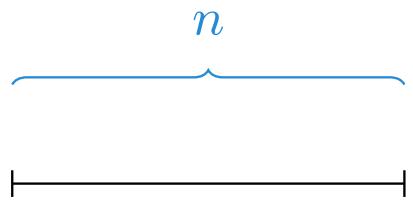
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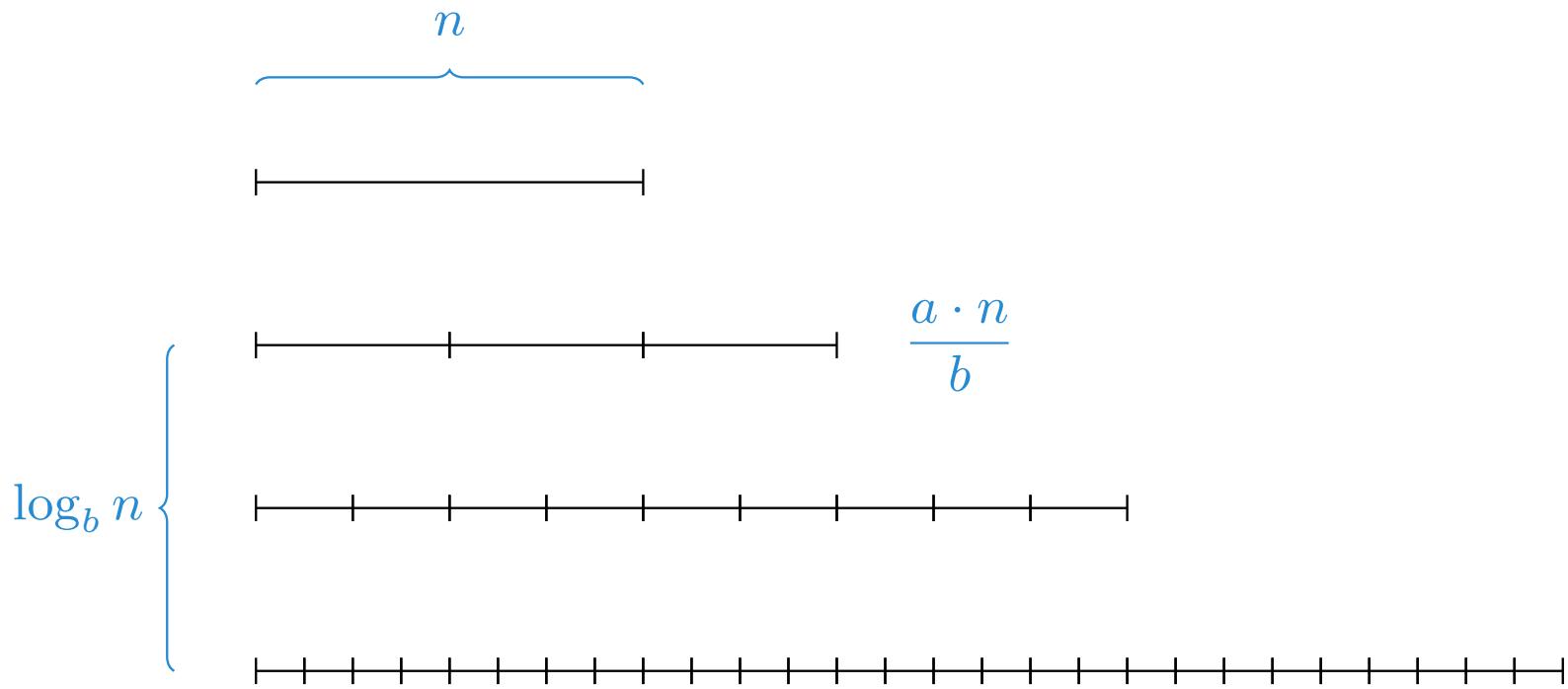


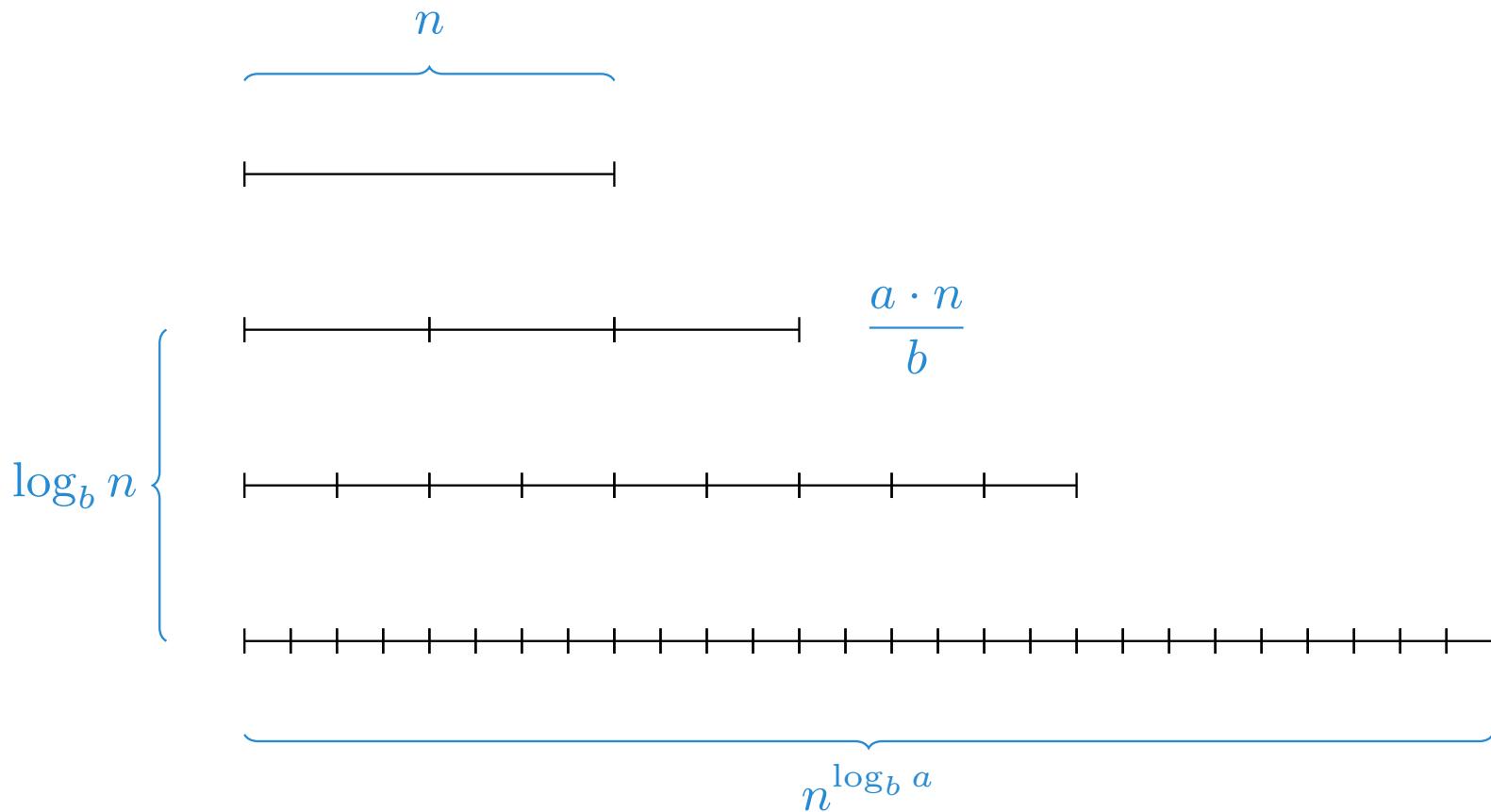
A horizontal line segment with two vertical tick marks dividing it into three equal segments. To its right, the expression $\frac{a \cdot n}{b}$ is written in blue.

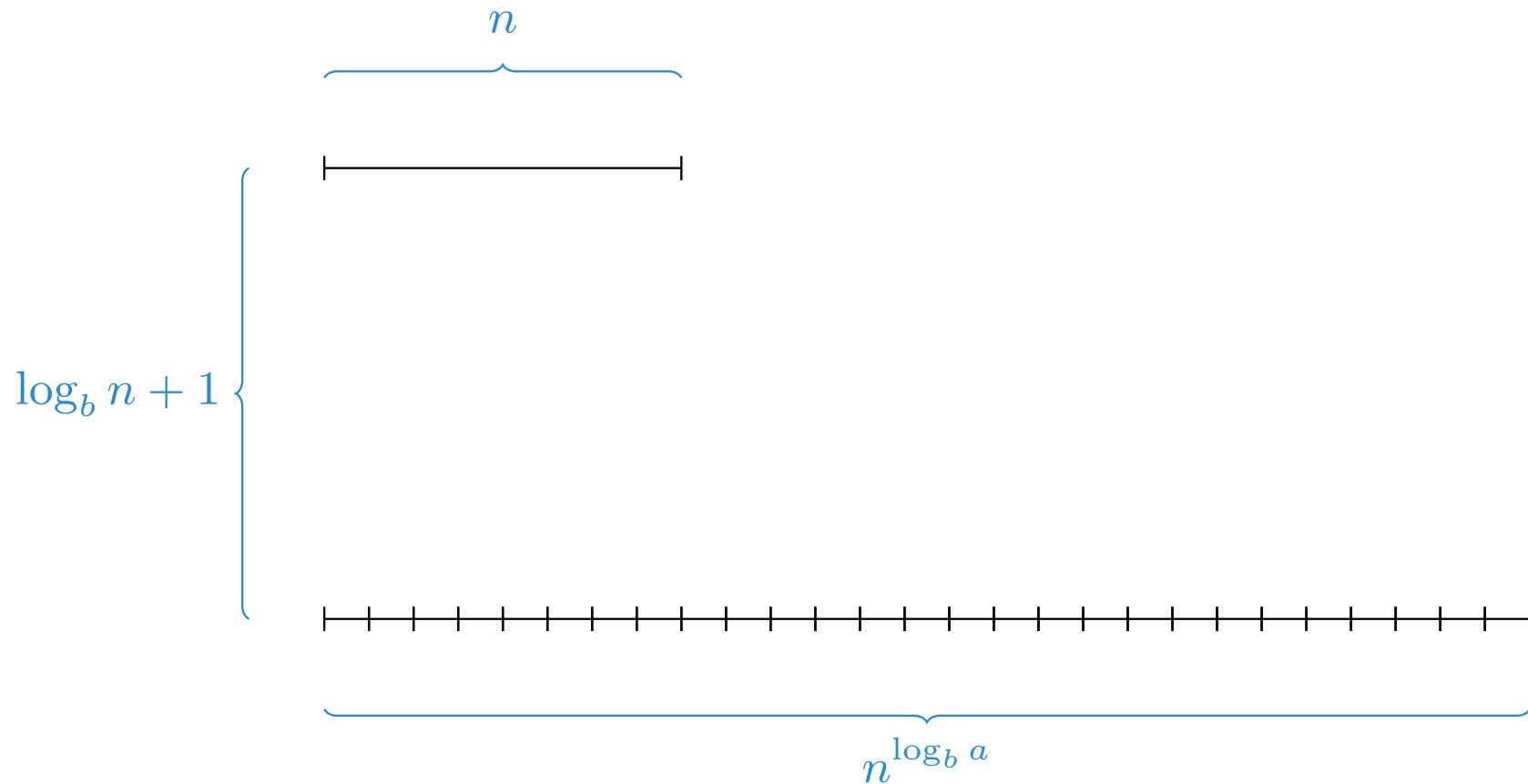
$$\frac{a \cdot n}{b}$$

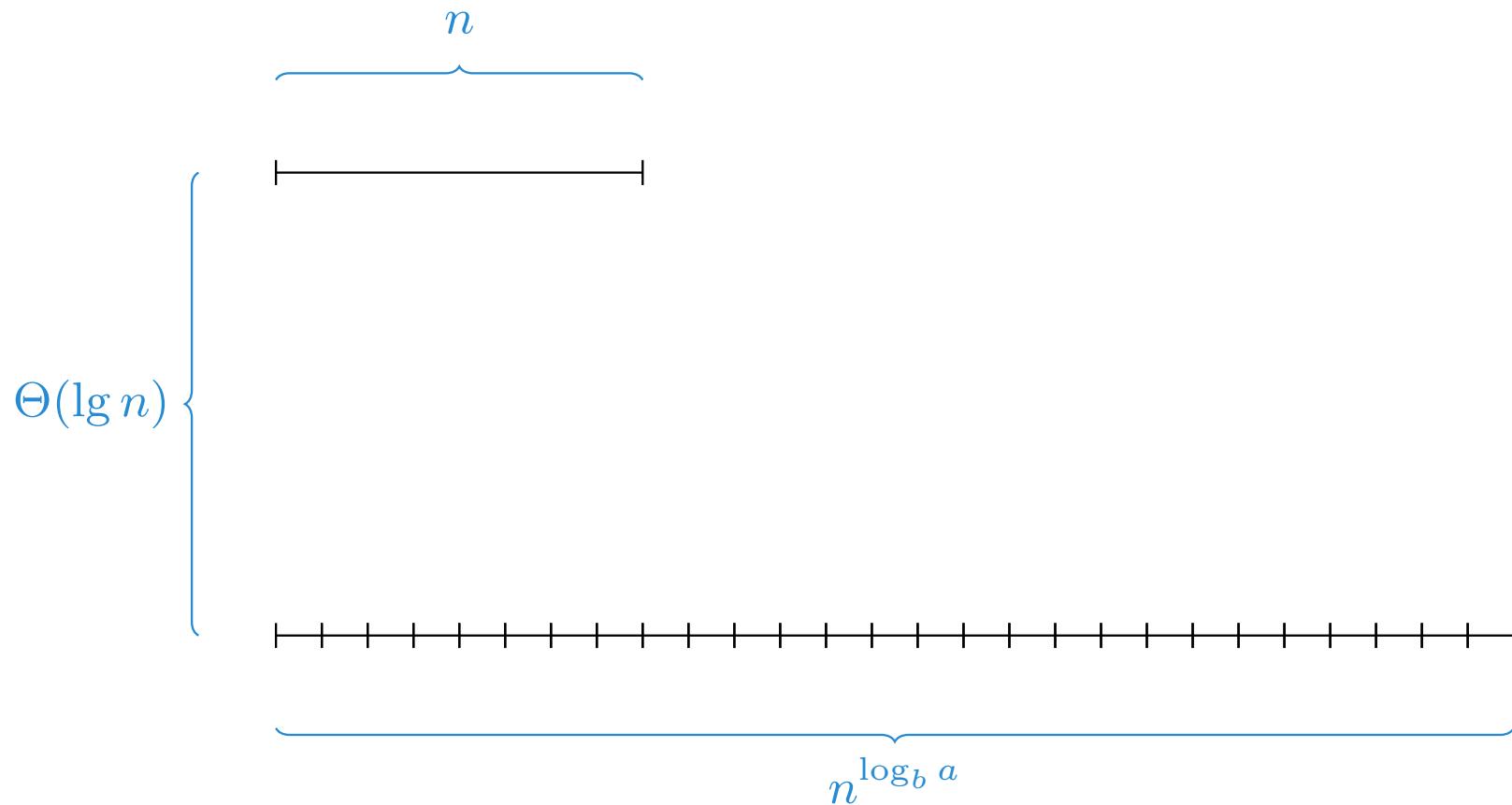


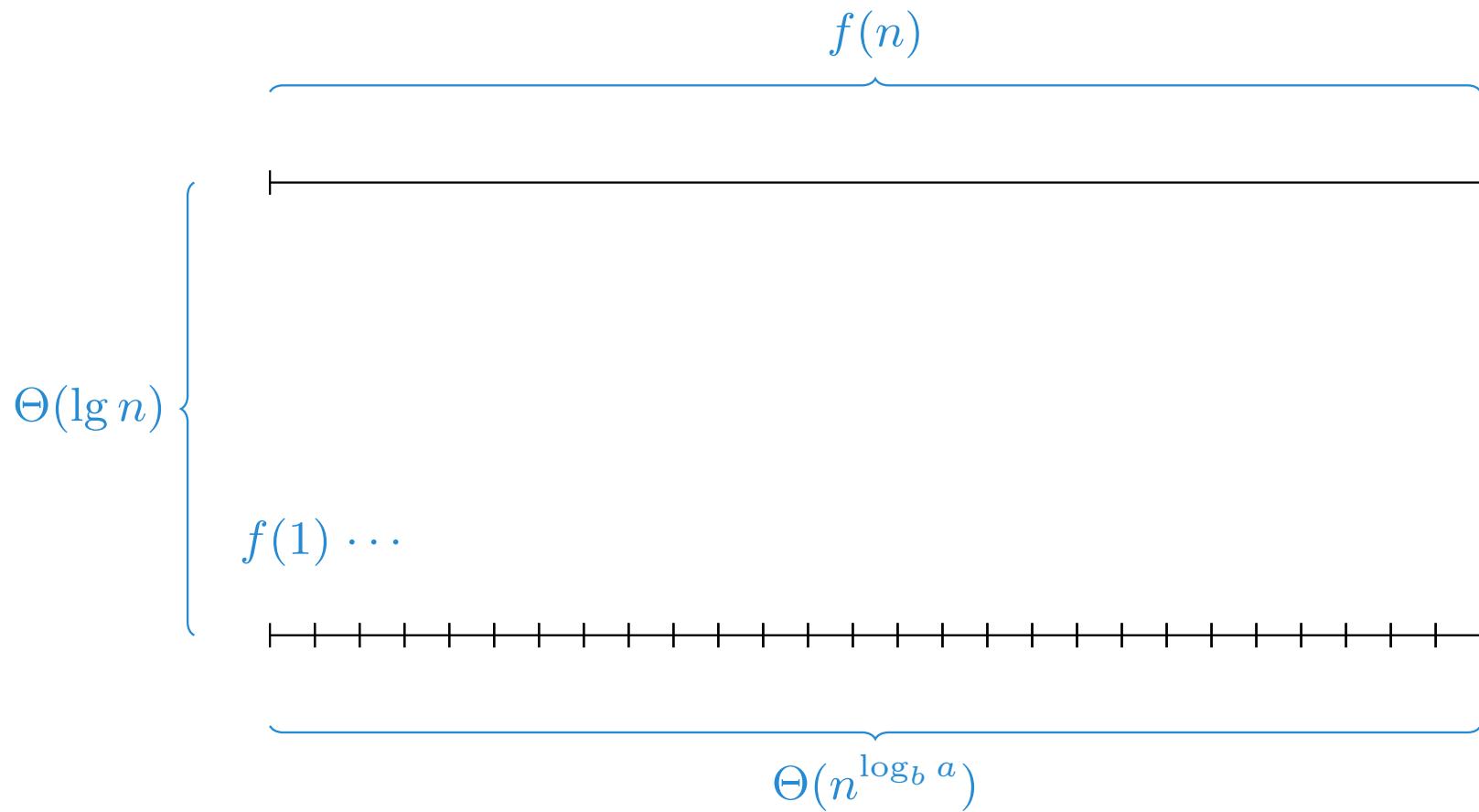


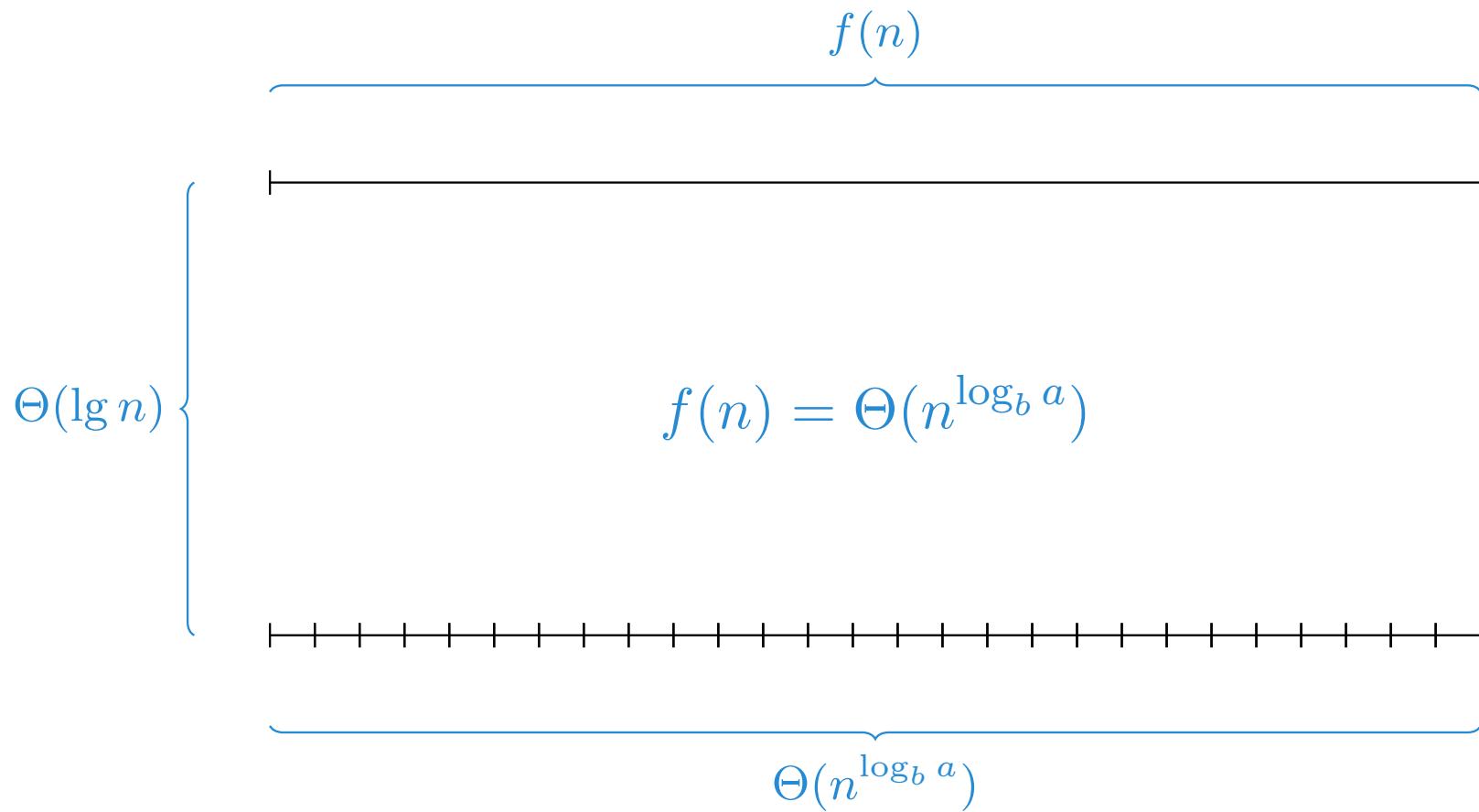


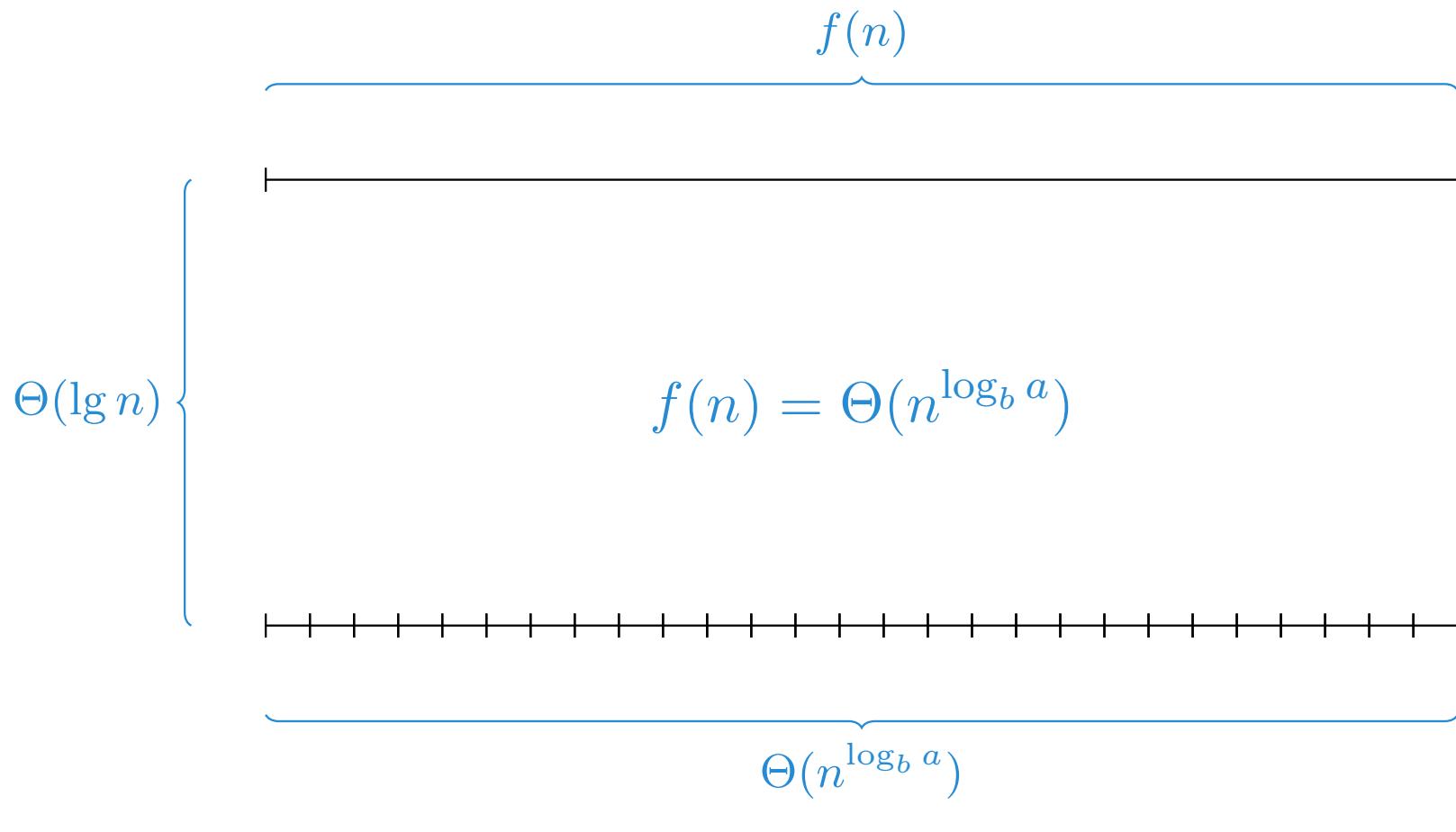




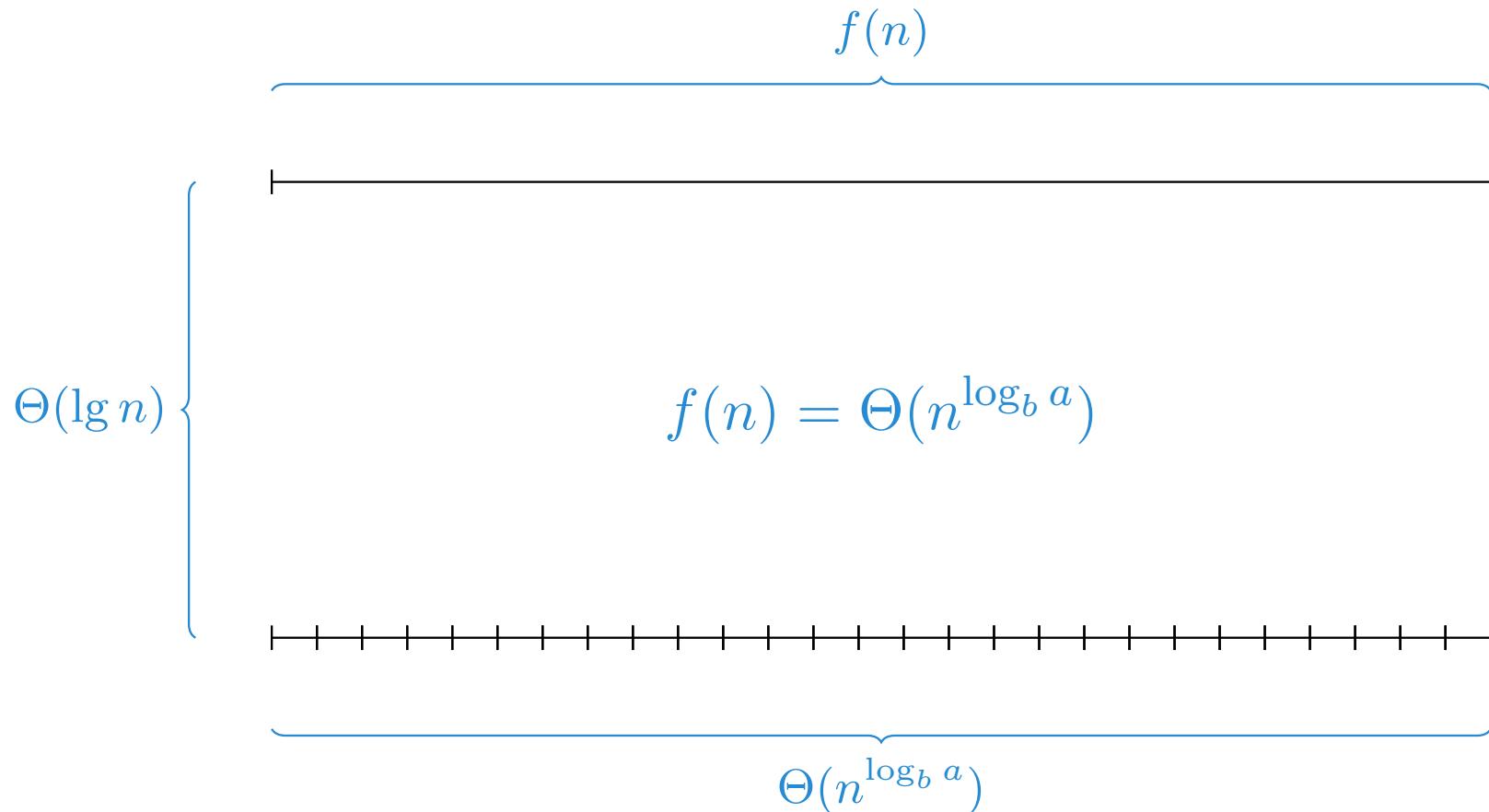




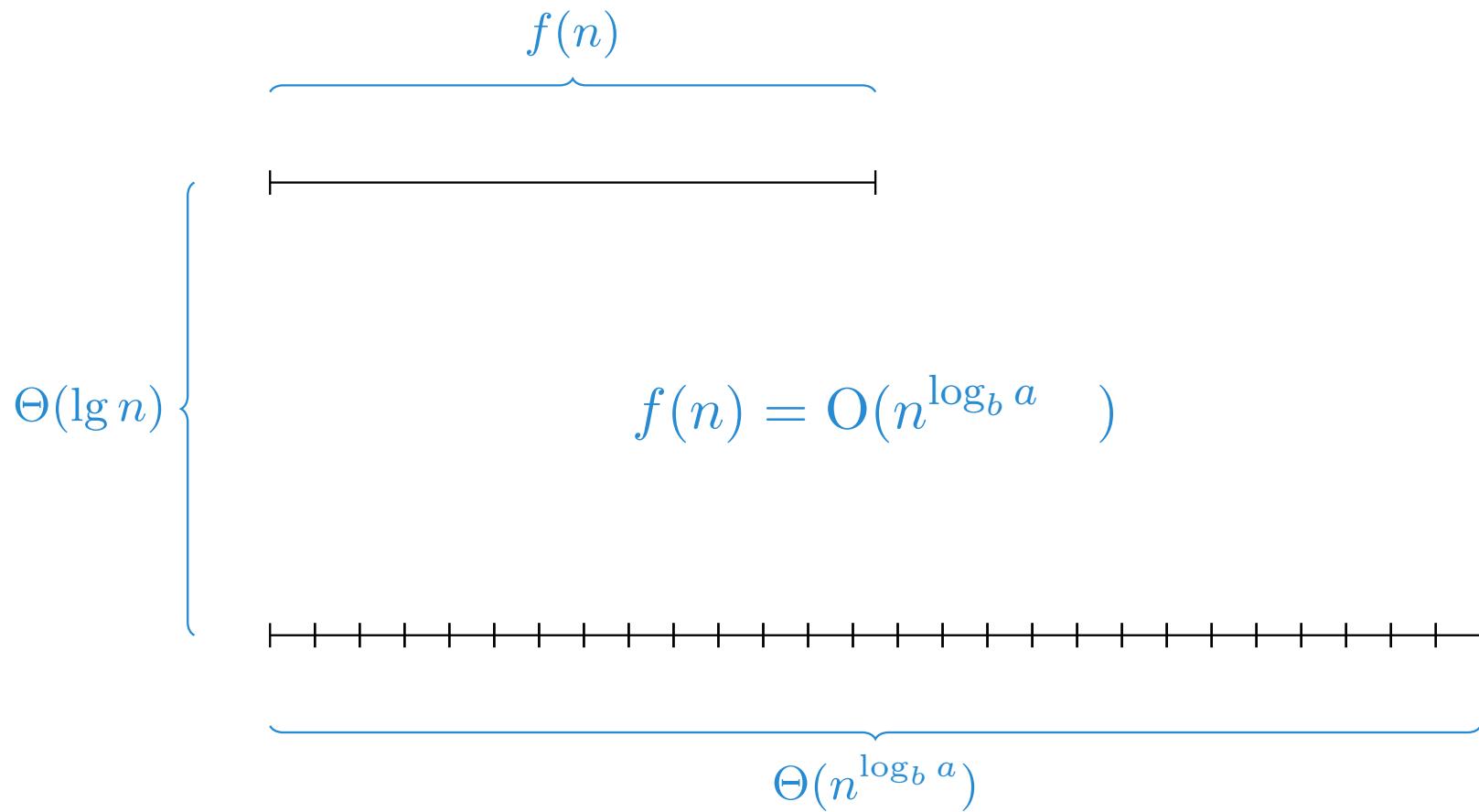




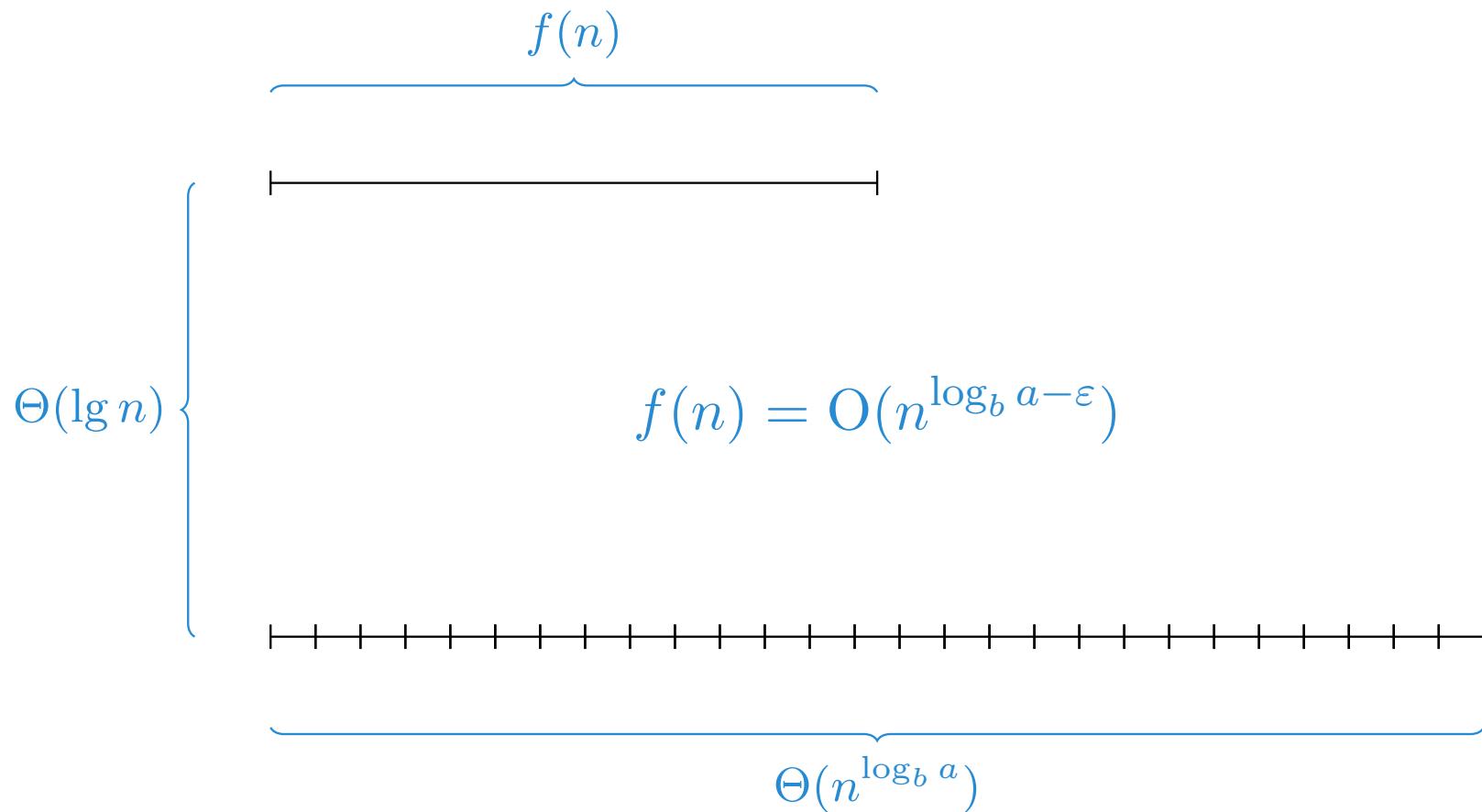
Totalt:



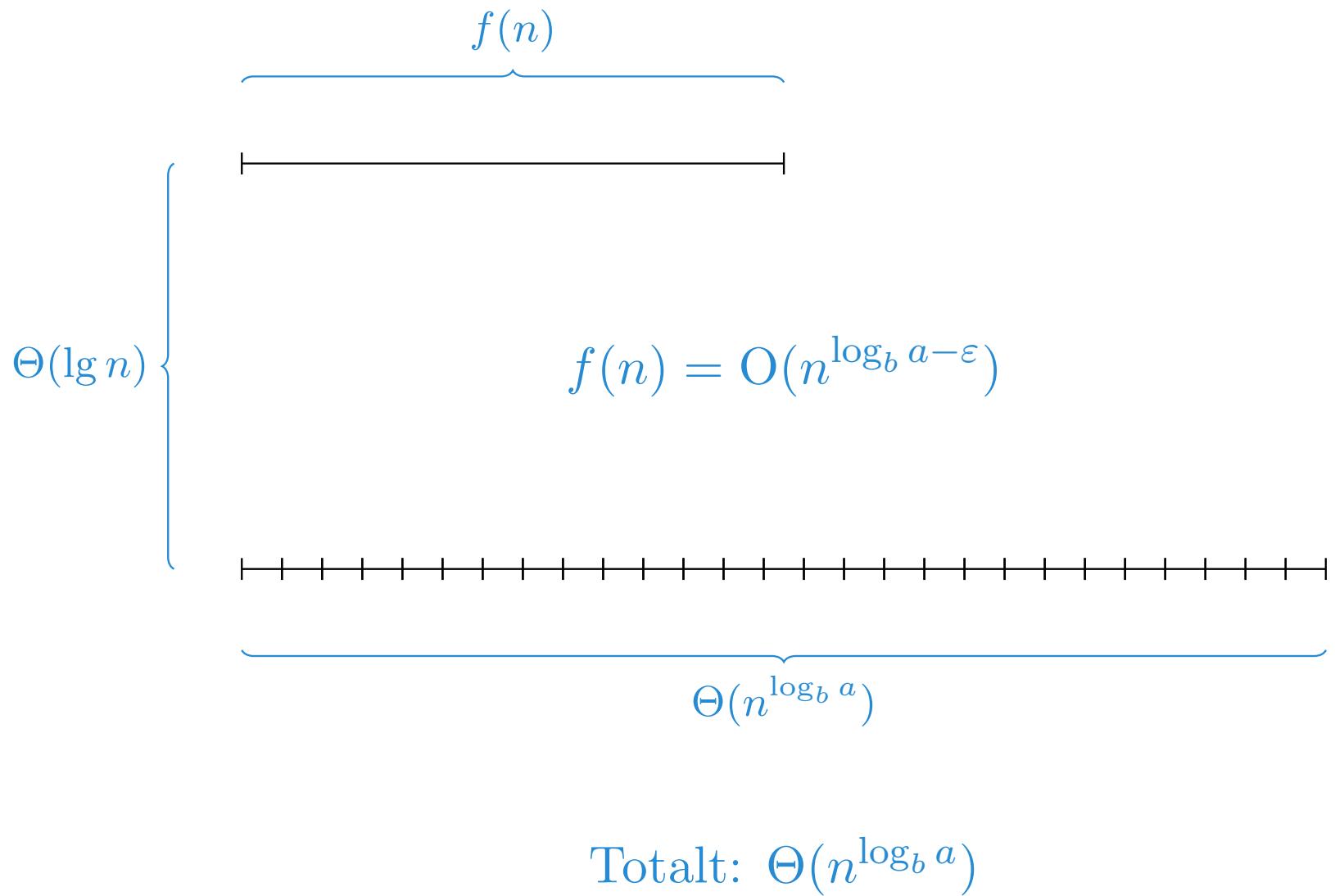
Totalt: $\Theta(n^{\log_b a} \lg n)$

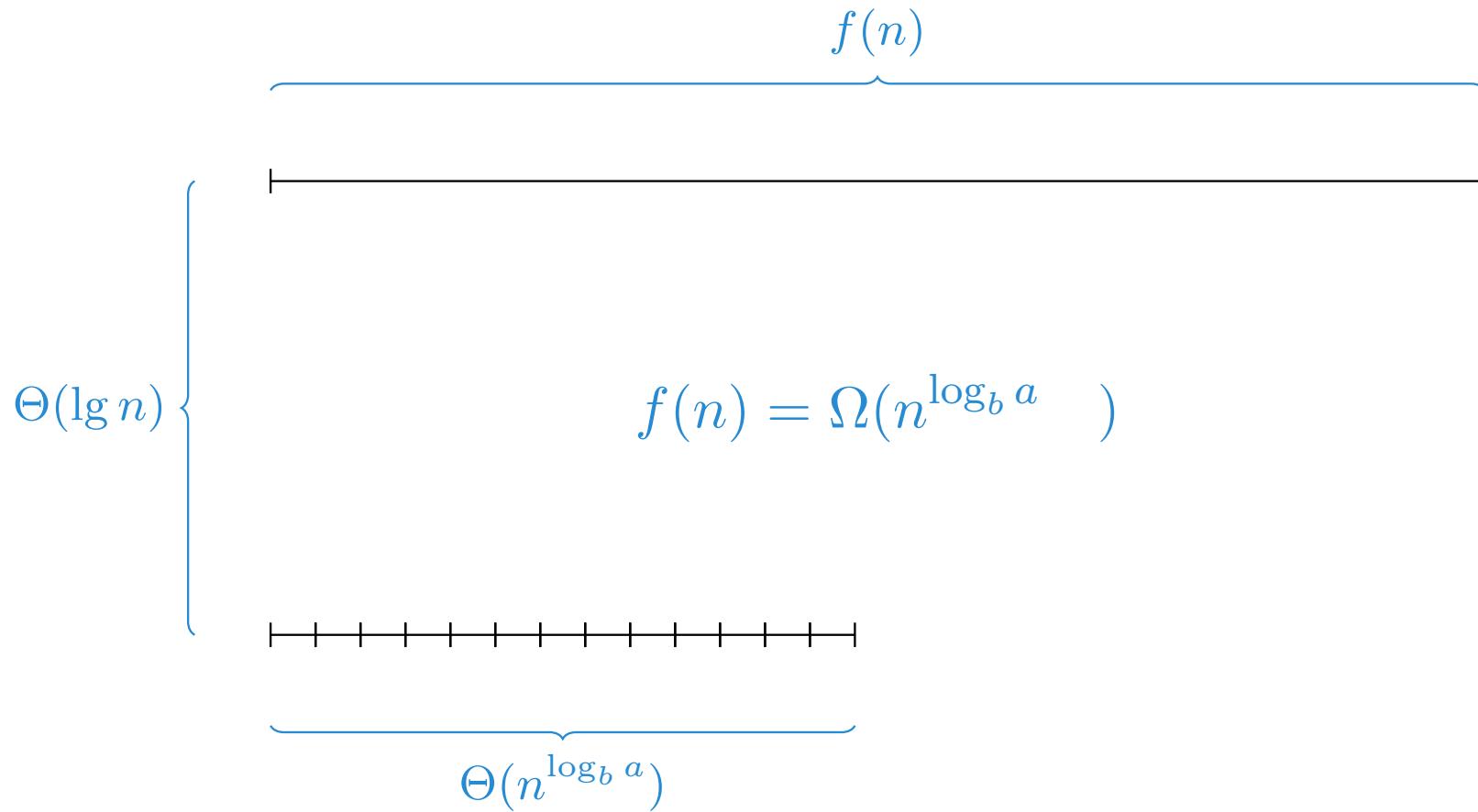


Totalt:

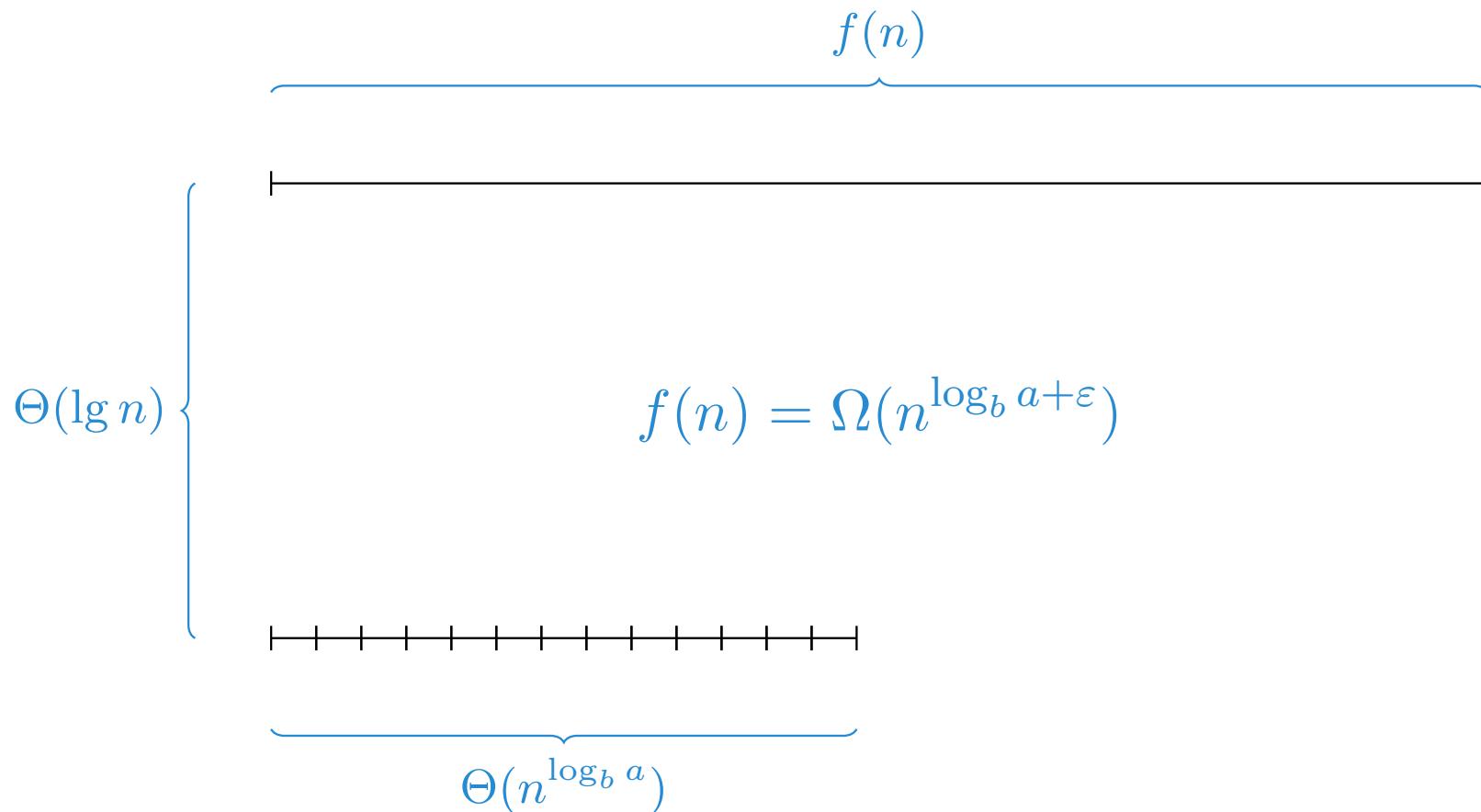


Totalt:

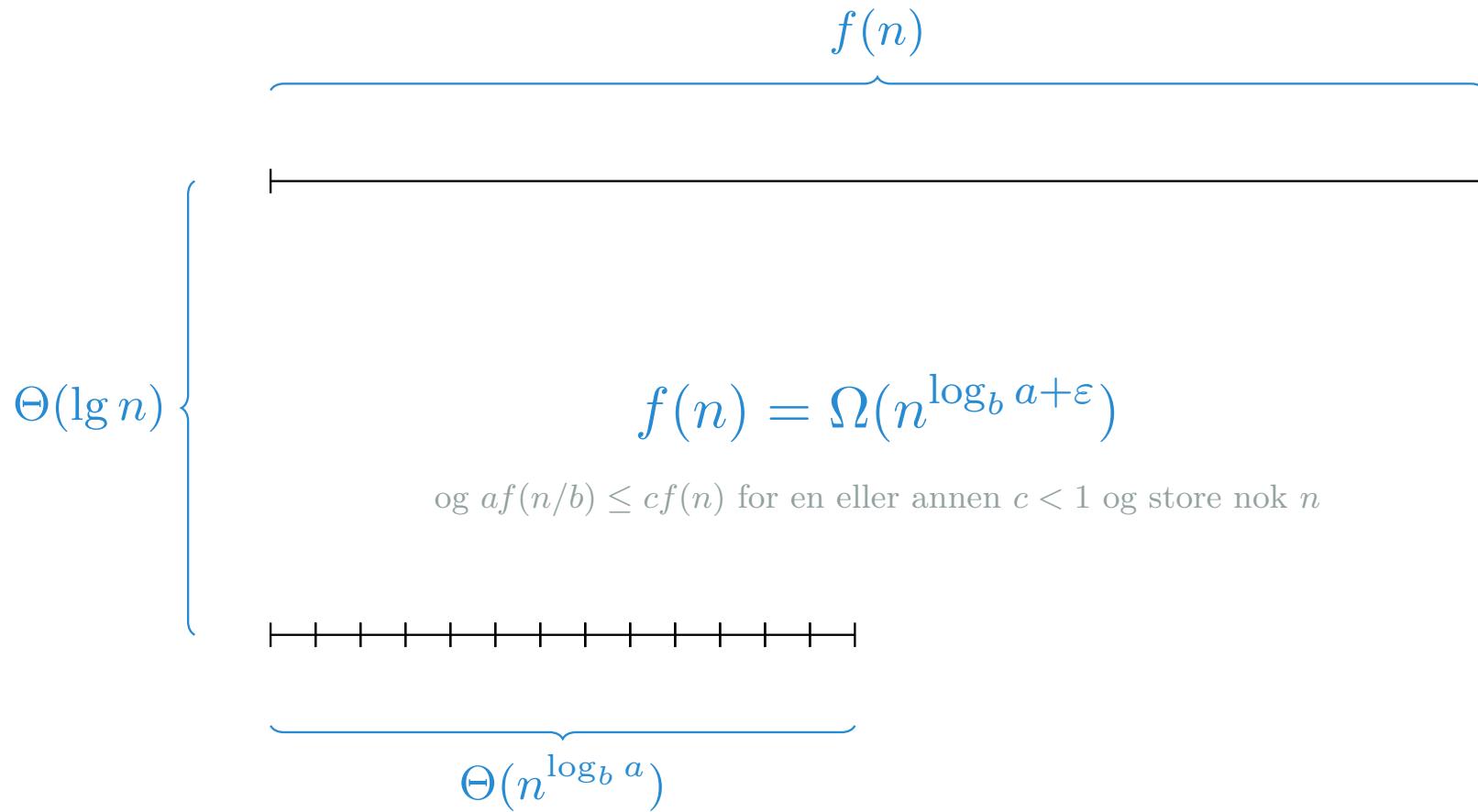




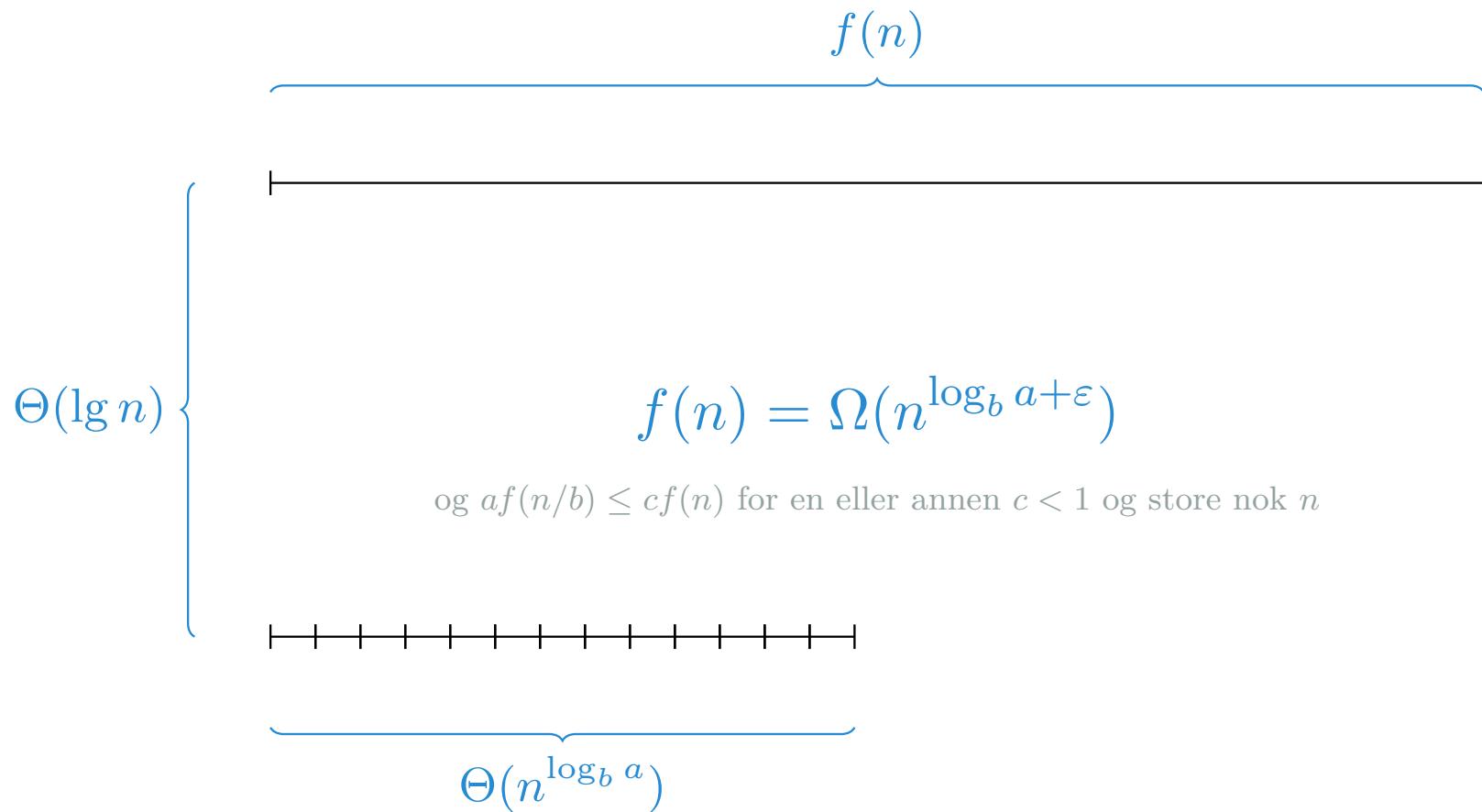
Total:



Total:



Totalt:



Totalt: $\Theta(f(n))$

Rekurrens

En rekurrens er en rekursiv ligning, der en funksjon defineres delvis ved hjelp av seg selv.

$$f(x) = 1 + f(x)$$

$$\boxed{f(x)} = \boxed{1 + f(x)}$$

$$\boxed{f(x)} = \boxed{1 + \boxed{f(x)}}$$

$$f(x) = 1 + 1 + f(x)$$

$$f(x) = 1 + 1 + f(x)$$

$$f(x) = 1 + 1 + 1 + f(x)$$

$$f(x) = 1 + 1 + 1 + f(x)$$

$$f(x) = 1 + 1 + 1 + \dots$$

Dette konvergerer ikke ...
rekursjonen terminerer
aldri. Vi trenger et
grunntilfelle!

$$f(x) = 1 + f(x-1)$$

$$\boxed{f(x)} = \boxed{1 + f(x - 1)}$$

$$f(x) = 1 + f(x - 1)$$

$$f(x) = 1 + [1 + f(x - 2)]$$

$$f(x) = 1 + 1 + f(x - 2)$$

$$f(x) = 1 + \boxed{1 + \boxed{1 + f(x-3)}}$$

$$f(x) = 1 + 1 + 1 + \dots + f(1)$$

$$f(x)$$

$$= 1 + \boxed{1 + \boxed{1 + \cdots + \boxed{1}}}$$

$$f(n) = n + f(n-1)$$

$$f(n) = n + (n-1) + f(n-1)$$

$$f(n) = n + (n-1) + (n-2) + f(n-2)$$

$$f(n) = n + (n - 1) + (n - 2) + \cdots + f(1)$$

$$f(n)=n+(n-1)+(n-2)+\cdots+1$$

Gjentatt innsetting

Ekvivalent med rekursjonstrærne i boka

$$T(n) = 1 + \boxed{T(n-1)}$$

$$\textcolor{blue}{i=0}$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow 1 + \boxed{T(n-i-1)}$$

$$T(n) = 1 + 1 + \boxed{T(n-2)}$$

$$i = 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow 1 + \boxed{T(n-i-1)}$$

$$T(n) = 1 + 1 + 1 + \boxed{T(n-3)}$$

$$i = 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow 1 + \boxed{T(n-i-1)}$$

$$T(n) = 1 + 1 + 1 + 1 + \cdots + \boxed{T(1)}$$

$$\textcolor{blue}{i = n-2}$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow 1 + \boxed{T(n-i-1)}$$

Lineært!

$$T(n) = 1 + 1 + 1 + 1 + \cdots + 1$$

$$i = n - 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n - i)} \rightarrow 1 + \boxed{T(n - i - 1)}$$

$$T(n) = 1 + \boxed{T(n/2^1)}$$

$$i = 0$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow 1 + \boxed{T(n/2^{i+1})}$$

$$T(n) = 1 + 1 + \boxed{T(n/2^2)}$$

$$i = 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow 1 + \boxed{T(n/2^{i+1})}$$

$$T(n) = 1 + 1 + 1 + \boxed{T(n/2^3)}$$

$$i = 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow 1 + \boxed{T(n/2^{i+1})}$$

$$T(n) = 1 + 1 + 1 + 1 + \cdots + \boxed{T(1)}$$

$$i = \lg \textcolor{blue}{n} - 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow 1 + \boxed{T(n/2^{i+1})}$$

Logaritmisk!

$$T(n) = 1 + 1 + 1 + 1 + \dots + 1$$

$$i = \lg n - 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow 1 + \boxed{T(n/2^{i+1})}$$

$$T(n) = n + \boxed{T(n/2^1)}$$

$$i = 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow n/2^i + \boxed{T(n/2^{i+1})}$$

$$T(n) = n + n/2^1 + \boxed{T(n/2^2)}$$

$$i = 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow n/2^i + \boxed{T(n/2^{i+1})}$$

$$T(n) = n + n/2^1 + n/2^2 + \boxed{T(n/2^3)}$$

$$i = 3$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow n/2^i + \boxed{T(n/2^{i+1})}$$

Lineært!

$$T(n) = n + n/2^1 + n/2^2 + n/2^3 + \dots + 1$$

$$i = \lg n$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n/2^i)} \rightarrow n/2^i + \boxed{T(n/2^{i+1})}$$

$$T(n) = n + \boxed{T(n-1)}$$

$$\textcolor{blue}{i=0}$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow (n-i) + \boxed{T(n-i-1)}$$

$$T(n) = n + (n-2) + \boxed{T(n-2)}$$

$$i = 1$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow (n-i) + \boxed{T(n-i-1)}$$

$$T(n) = n + (n-2) + (n-3) + \boxed{T(n-3)}$$

$$i = 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow (n-i) + \boxed{T(n-i-1)}$$

$$T(n) = n + (n-2) + (n-3) + (n-4) + \cdots + \boxed{T(1)}$$

$$\textcolor{blue}{i=n-2}$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n-i)} \rightarrow (n-i) + \boxed{T(n-i-1)}$$

Kvadratisk!

$$T(n) = n + (n - 2) + (n - 4) + \cdots + 1$$

$$i = n - 2$$

$$\boxed{T(1)} \rightarrow 1$$

$$\boxed{T(n - i)} \rightarrow (n - i) + \boxed{T(n - i - 1)}$$

Rekurrenstræ

$T(4)$

$T(4)$

$$T(1) = \boxed{1}$$
$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$

$$T(4)$$

$$T(4) =$$

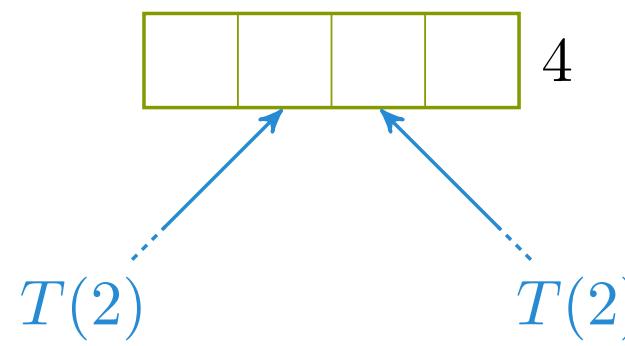
$$T(1) = \boxed{1}$$

$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$

$T(4)$ $T(4) =$

$$T(1) = \boxed{1}$$

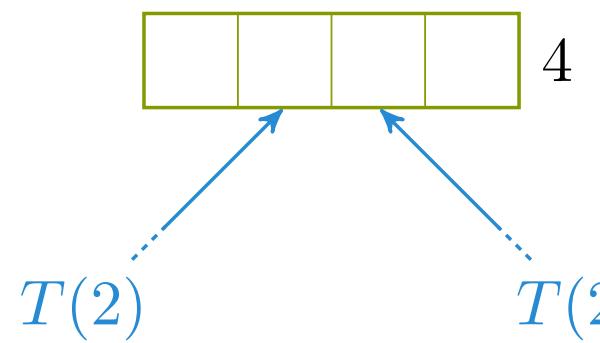
$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$



$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$T(1) = \boxed{1}$$

$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$

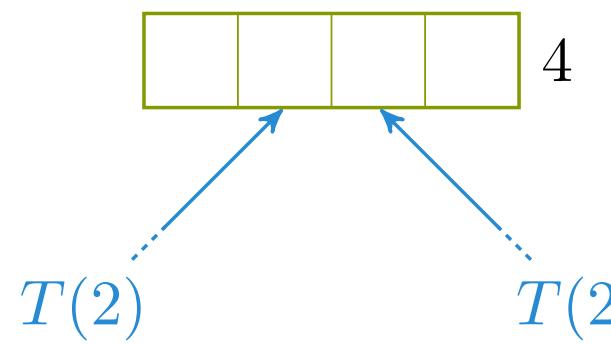


$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

=

$$T(1) = \boxed{1}$$

$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$

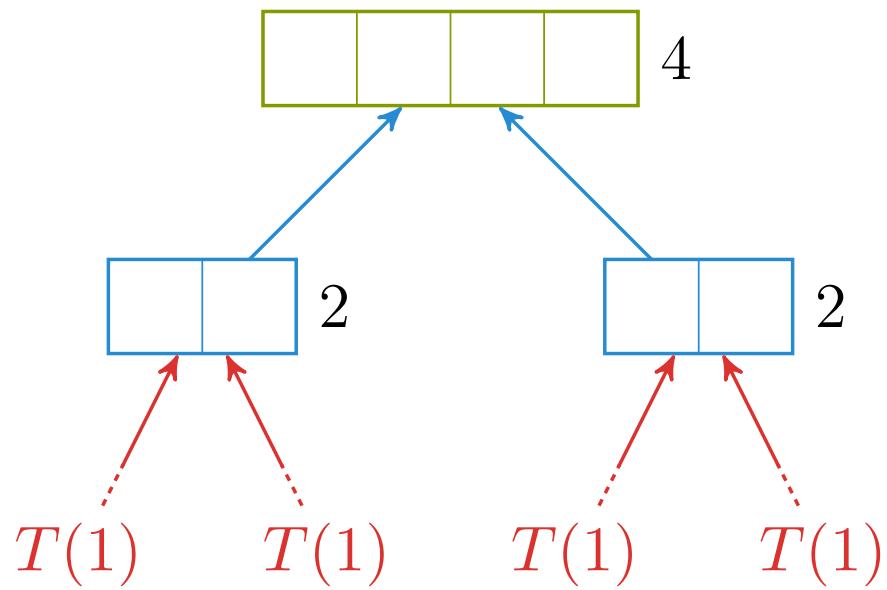


$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

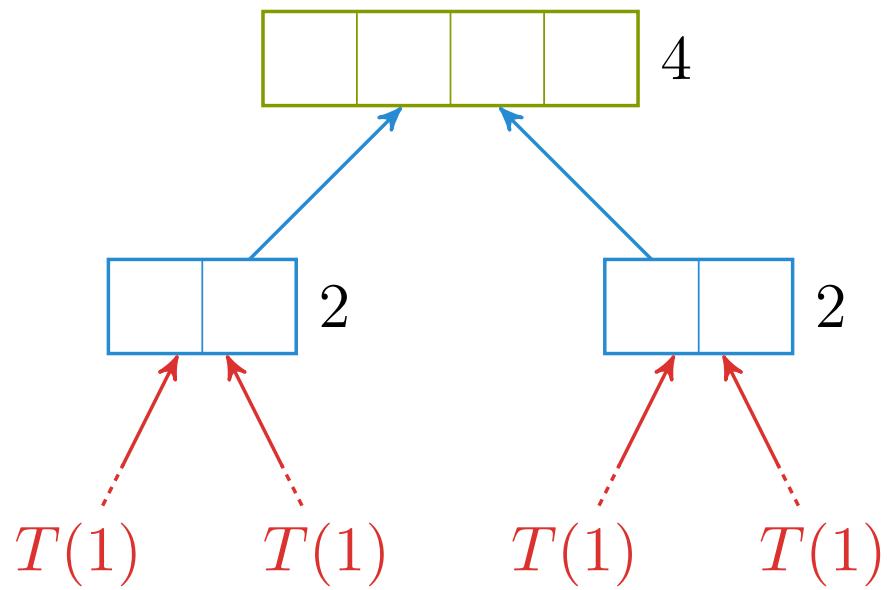
=

$$T(1) = \boxed{1}$$

$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$



$$\begin{aligned}
 T(4) &= \boxed{4 + 2 \cdot T(4/2)} \\
 &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot T(4/4)}} \\
 &\quad \uparrow \\
 &T(1) = \boxed{1} \\
 &T(n) = \boxed{n + 2 \cdot T(n/2)}
 \end{aligned}$$



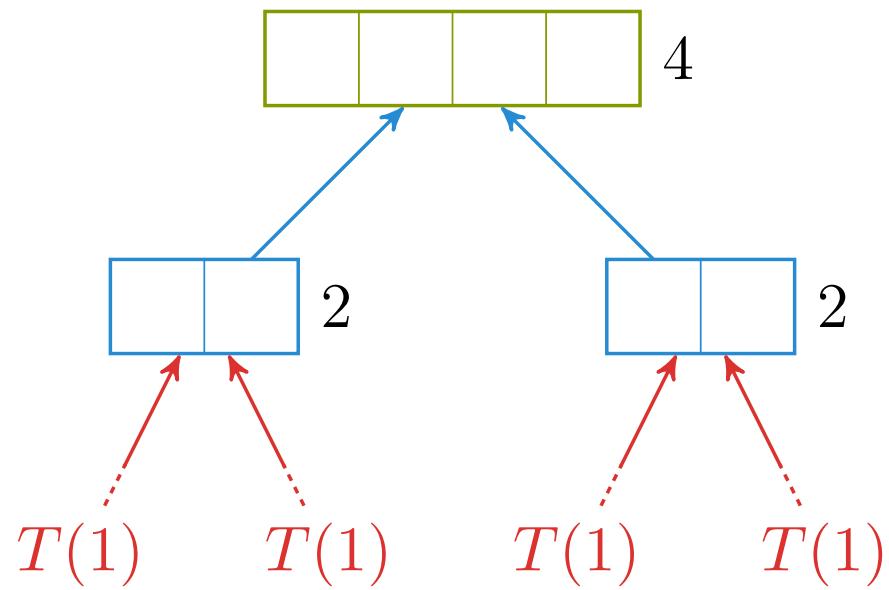
$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot T(4/4)}}$$

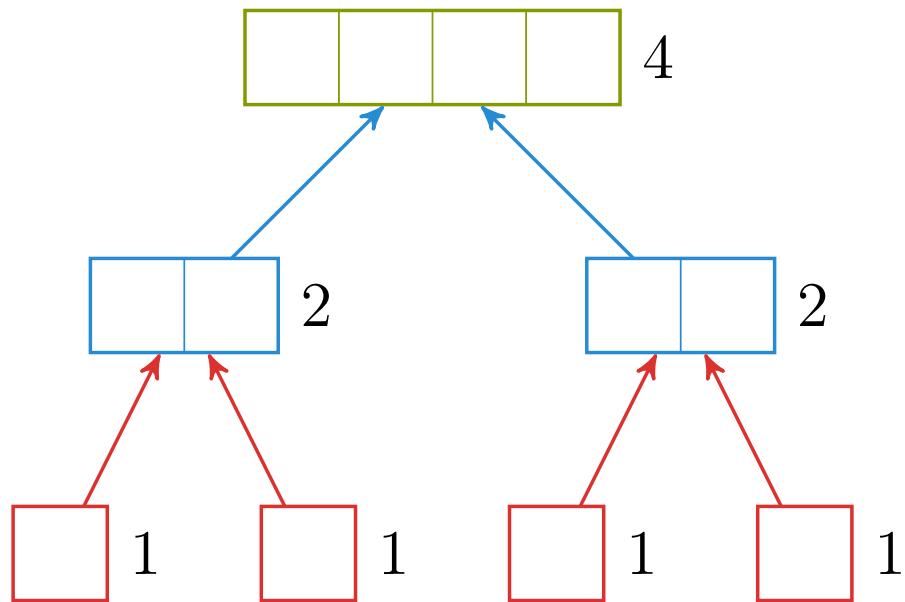
=

$$T(1) = \boxed{1}$$

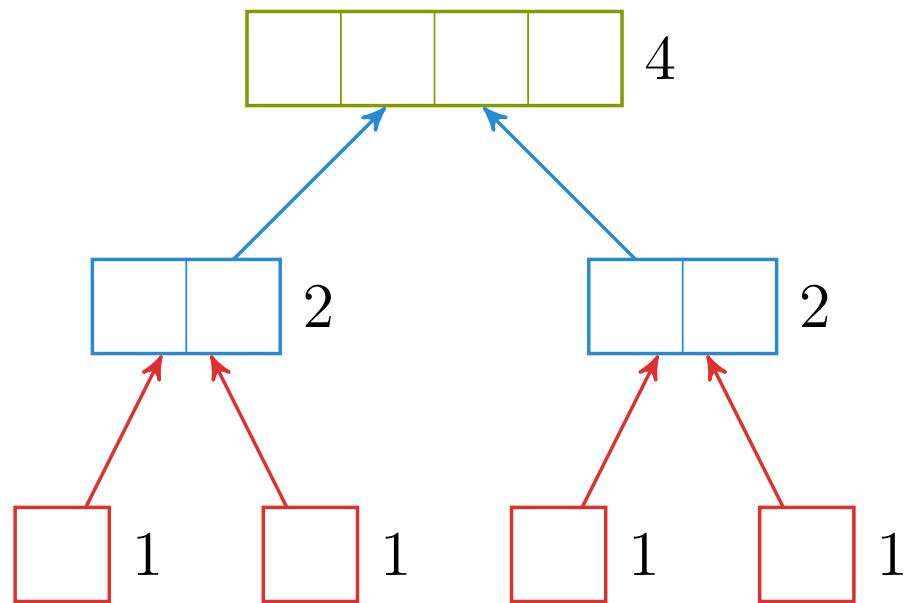
$$T(n) = \boxed{n + 2 \cdot T(n/2)}$$



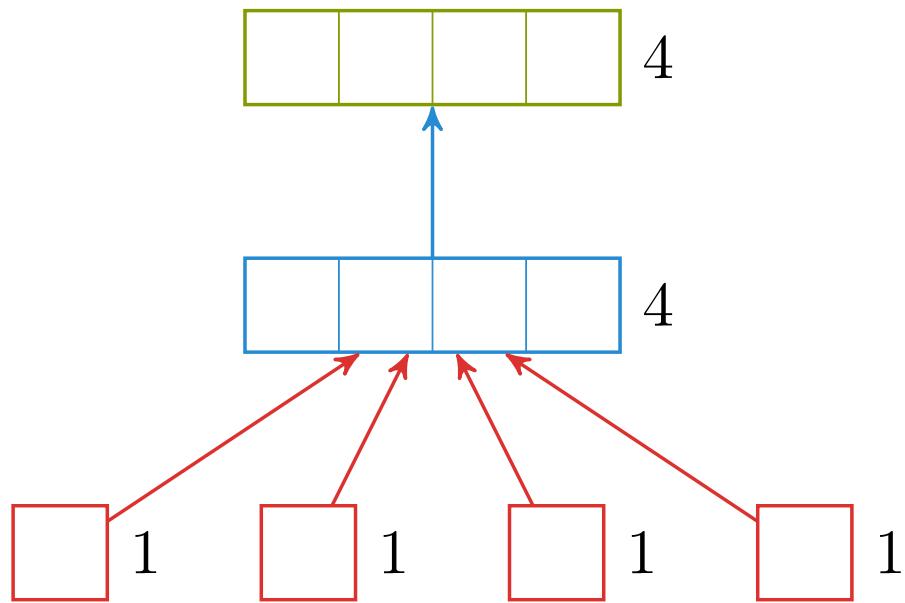
$$\begin{aligned}
 T(4) &= \boxed{4 + 2 \cdot T(4/2)} \\
 &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot T(4/4)}} \\
 &= \\
 T(1) &= \boxed{1} \\
 T(n) &= \boxed{n + 2 \cdot T(n/2)}
 \end{aligned}$$



$$\begin{aligned}
 T(4) &= \boxed{4 + 2 \cdot T(4/2)} \\
 &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot T(4/4)}} \\
 &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot \boxed{1}}} \\
 T(1) &= \boxed{1} \\
 T(n) &= \boxed{n + 2 \cdot T(n/2)}
 \end{aligned}$$



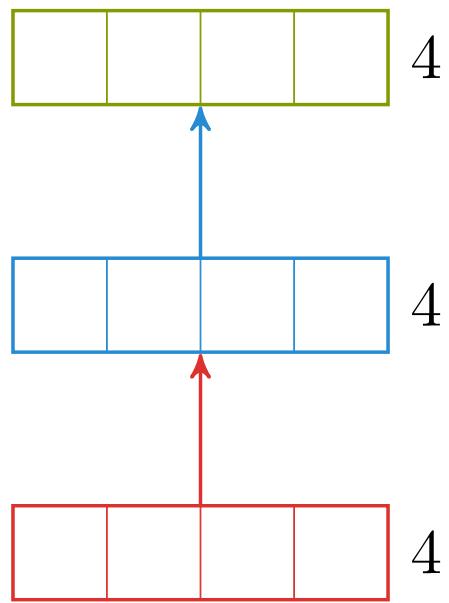
$$\begin{aligned} T(4) &= \boxed{4 + 2 \cdot T(4/2)} \\ &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot T(4/4)}} \\ &= \boxed{4 + 2 \cdot \boxed{4/2 + 2 \cdot \boxed{1}}} \end{aligned}$$



$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + \boxed{4 + 4 \cdot T(4/4)}}$$

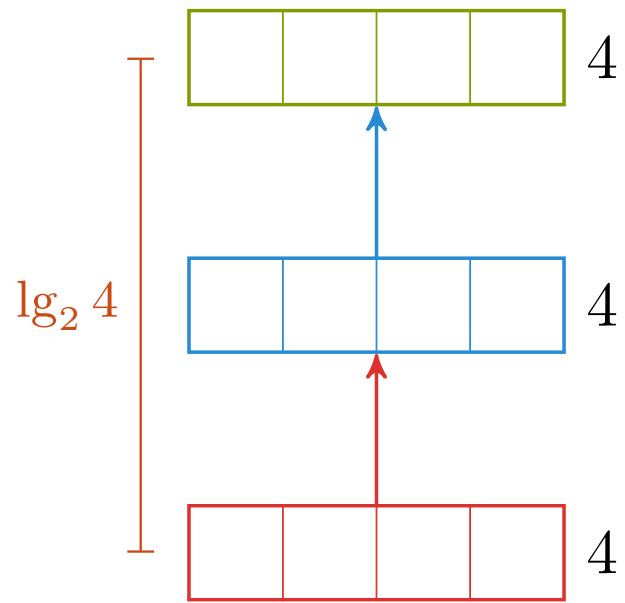
$$= \boxed{4 + \boxed{4 + 4 \cdot \boxed{1}}}$$



$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + \boxed{4 + 4 \cdot T(4/4)}}$$

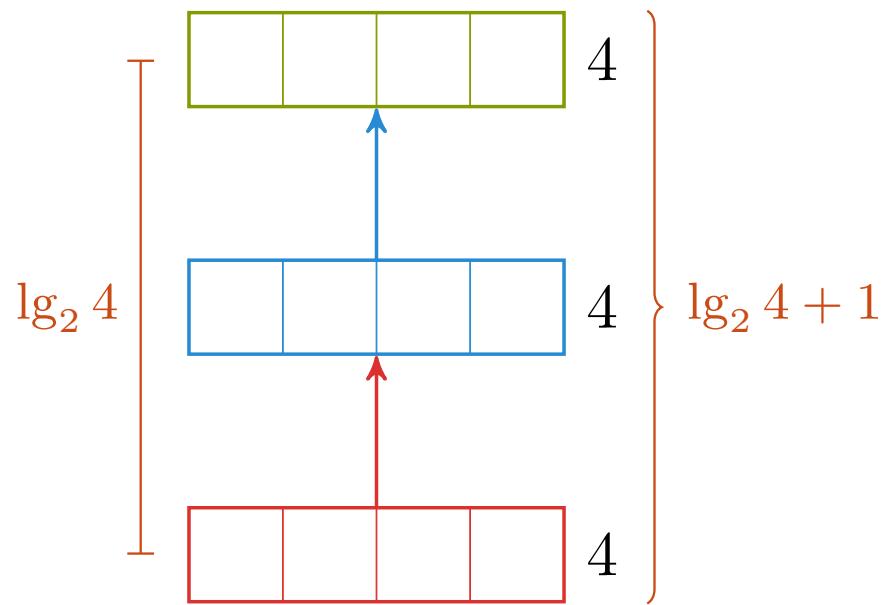
$$= \boxed{4 + \boxed{4 + \boxed{4}}}$$



$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + \boxed{4 + 4 \cdot T(4/4)}}$$

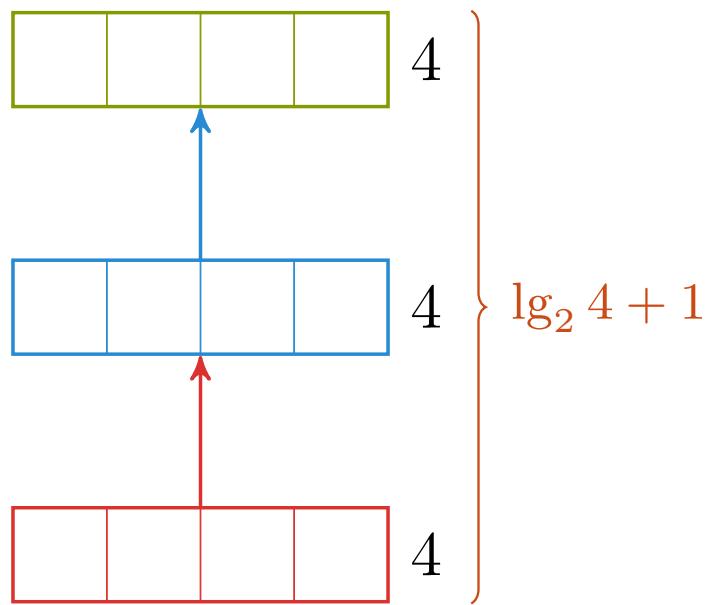
$$= \boxed{4 + \boxed{4 + \boxed{4}}}$$



$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + \boxed{4 + 4 \cdot T(4/4)}}$$

$$= \boxed{4 + \boxed{4 + \boxed{4}}}$$

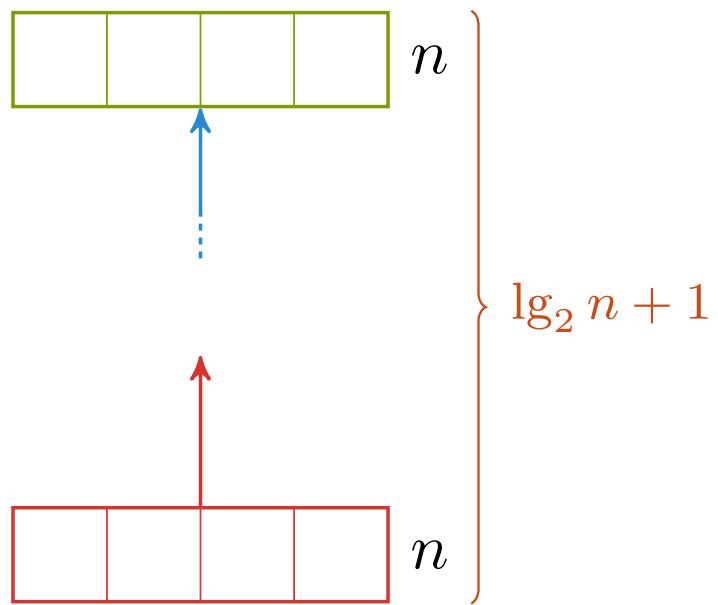


$$T(4) = \boxed{4 + 2 \cdot T(4/2)}$$

$$= \boxed{4 + \boxed{4 + 4 \cdot T(4/4)}}$$

$$= \boxed{4 + \boxed{4 + \boxed{4}}}$$

$\underbrace{\hspace{10em}}_{\lg_2 4 + 1}$



$$\begin{aligned} T(n) &= \boxed{n + 2 \cdot T(n/2)} \\ &\vdots \\ &= \boxed{n + \underbrace{\cdots + \boxed{n}}_{\lg_2 n + 1}} \end{aligned}$$

Substitusjon

(Dvs. induksjon...)

- Vi tar ofte snarveier
- F.eks.: Vi ignorerer avrundingsfeil
- Sjekk løsningen med induksjon!
- Evt. gjett løsning og sjekk den

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$T(1) = 1$$

$$T(n) = T(n - 1) + n$$

$$n > 1$$


$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$T(n) = \frac{n(n+1)}{2}$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) \quad \left| \quad \frac{n(n+1)}{2}$$

$$T(1) = 1$$
$$T(n) = T(n-1) + n$$

$$n = 1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$n = 1$$

$$1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$n = 1$$

$$1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$\frac{1(1+1)}{2} = 1$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$n > 1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$\text{H: } T(n-1) = \frac{(n-1)n}{2}$$

$$\begin{aligned}T(1) &= 1 \\T(n) &= T(n-1) + n\end{aligned}$$

$$n > 1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$\text{H: } T(n-1) = \frac{(n-1)n}{2}$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$n > 1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) =$$

$$\text{H: } T(n-1) = \frac{(n-1)n}{2}$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$$n > 1$$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) = T(n-1) + n$$

$$\text{H: } T(n-1) = \frac{(n-1)n}{2}$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$n > 1$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) = \textcolor{blue}{T}(n-1) + n$$

$$\text{H: } T(n-1) = \frac{(n-1)n}{2}$$

$$\frac{n(n+1)}{2}$$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$n > 1$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) = T(n-1) + n$$

$$= \frac{(n-1)n}{2} + n$$

$$\frac{n(n+1)}{2}$$

H: $T(n-1) = \frac{(n-1)n}{2}$

$$\begin{aligned} T(1) &= 1 \\ T(n) &= T(n-1) + n \end{aligned}$$

$n > 1$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) = T(n-1) + n$$

$$= \frac{(n-1)n}{2} + n$$

$$= \frac{n^2 - n + 2n}{2}$$

H: $T(n-1) = \frac{(n-1)n}{2}$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

$n > 1$

$$T(n)$$

$$\frac{n(n+1)}{2}$$

$$T(n) = T(n-1) + n$$

$$= \frac{(n-1)n}{2} + n$$

$$= \frac{n^2 - n + 2n}{2}$$

$$= \frac{n(n+1)}{2}$$

H: $T(n-1) = \frac{(n-1)n}{2}$

$$T(1) = 1$$

$$T(n) = T(n-1) + n$$

Masterteoremet

$$T(n) = aT(n/b) + f(n)$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon})$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

$$f(n) = \Theta(n^{\log_b a})$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

$$f(n) = \Theta(n^{\log_b a}) \implies T(n) = \Theta(n^{\log_b a} \lg n)$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

$$f(n) = \Theta(n^{\log_b a}) \implies T(n) = \Theta(n^{\log_b a} \lg n)$$

$$f(n) = \Omega(n^{\log_b a + \varepsilon})$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = \mathcal{O}(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

$$f(n) = \Theta(n^{\log_b a}) \implies T(n) = \Theta(n^{\log_b a} \lg n)$$

$$f(n) = \Omega(n^{\log_b a + \varepsilon}) \implies T(n) = \Theta(f(n))$$

$$T(n) = aT(n/b) + f(n)$$

$$f(n) = O(n^{\log_b a - \varepsilon}) \implies T(n) = \Theta(n^{\log_b a})$$

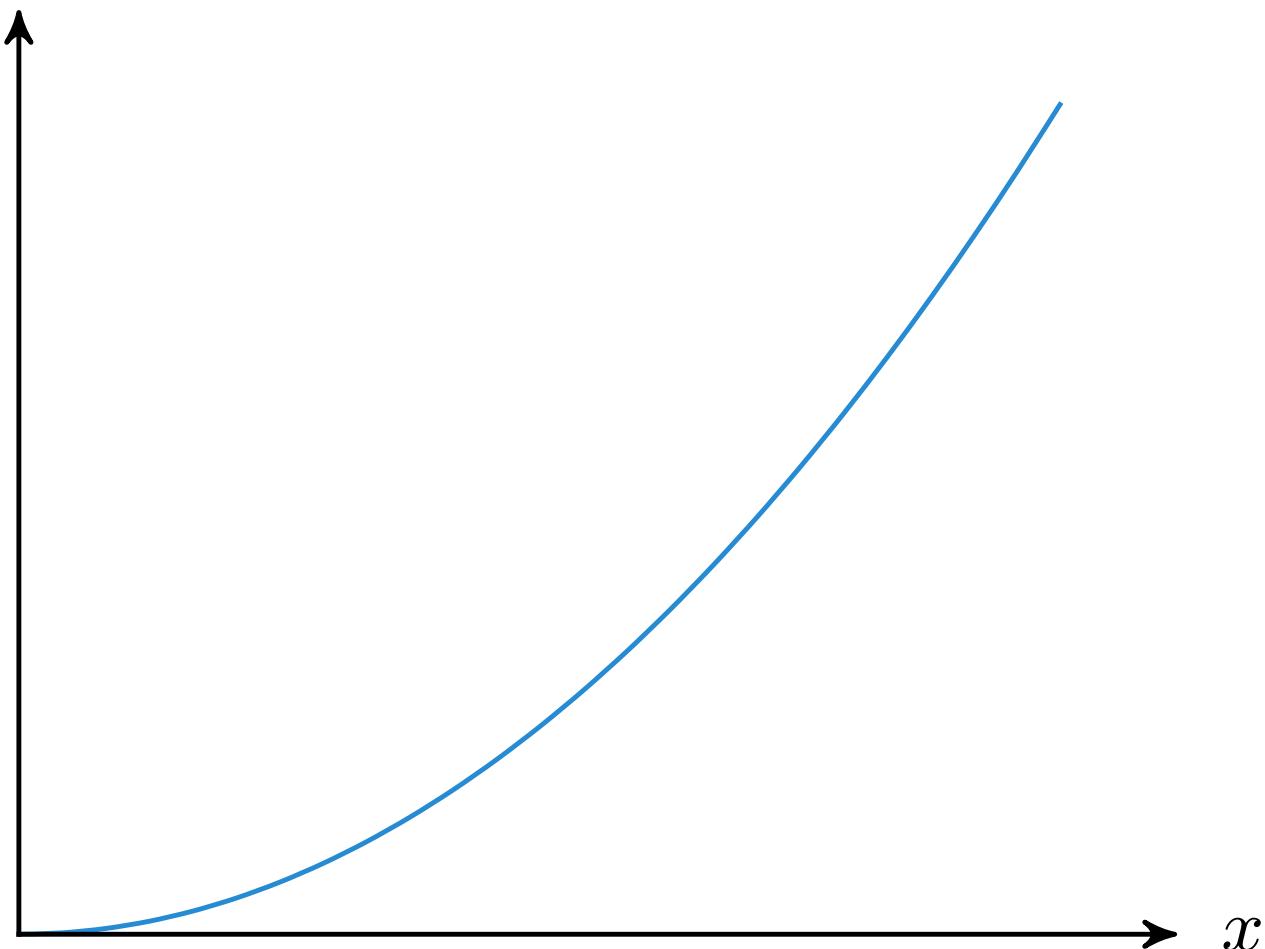
$$f(n) = \Theta(n^{\log_b a}) \implies T(n) = \Theta(n^{\log_b a} \lg n)$$

$$f(n) = \Omega(n^{\log_b a + \varepsilon}) \implies T(n) = \Theta(f(n))^*$$

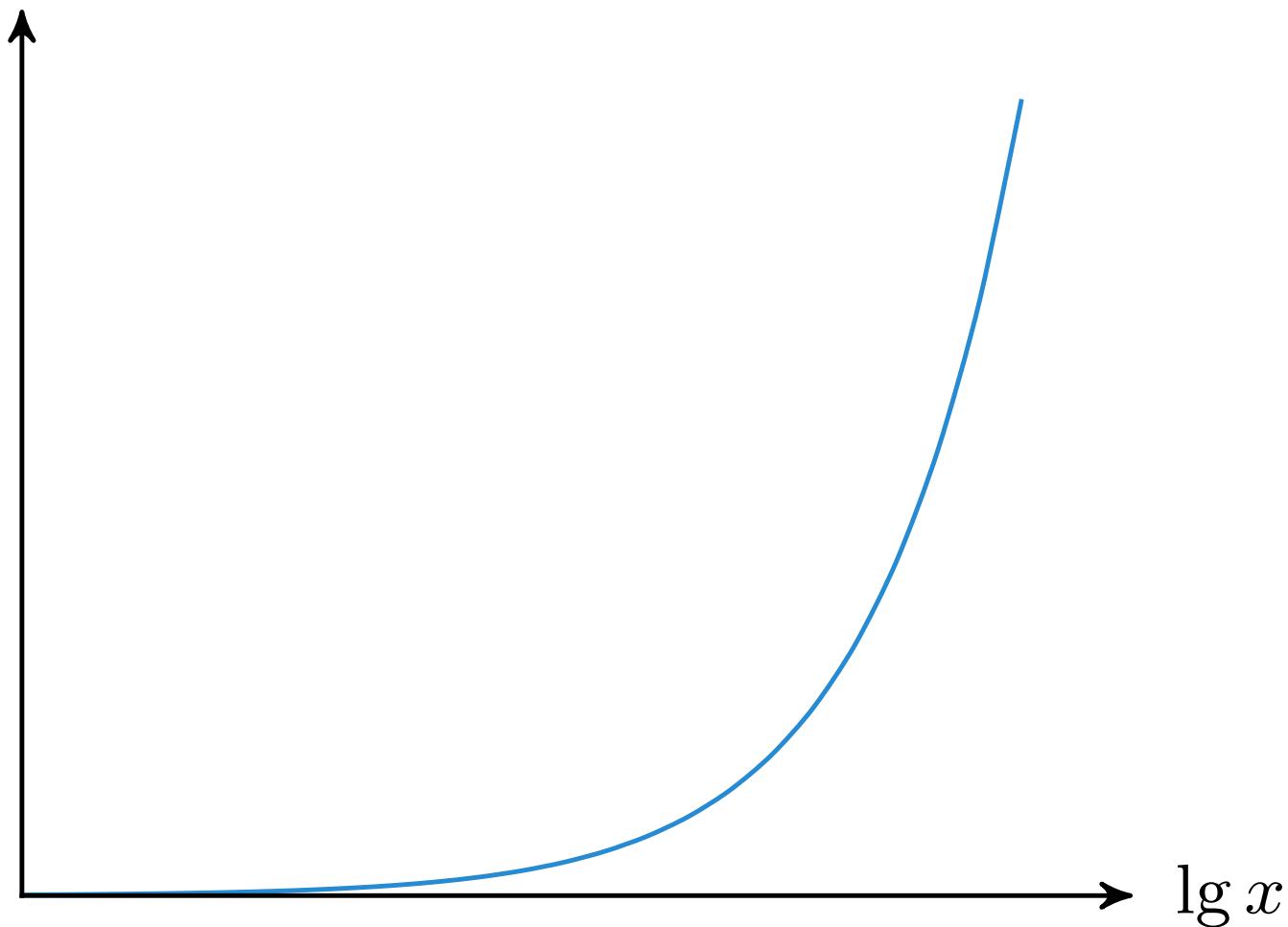
hvis $af(n/b) \leq cf(n)$ for en eller annen $c < 1$ og store nok n

Variabelskifte

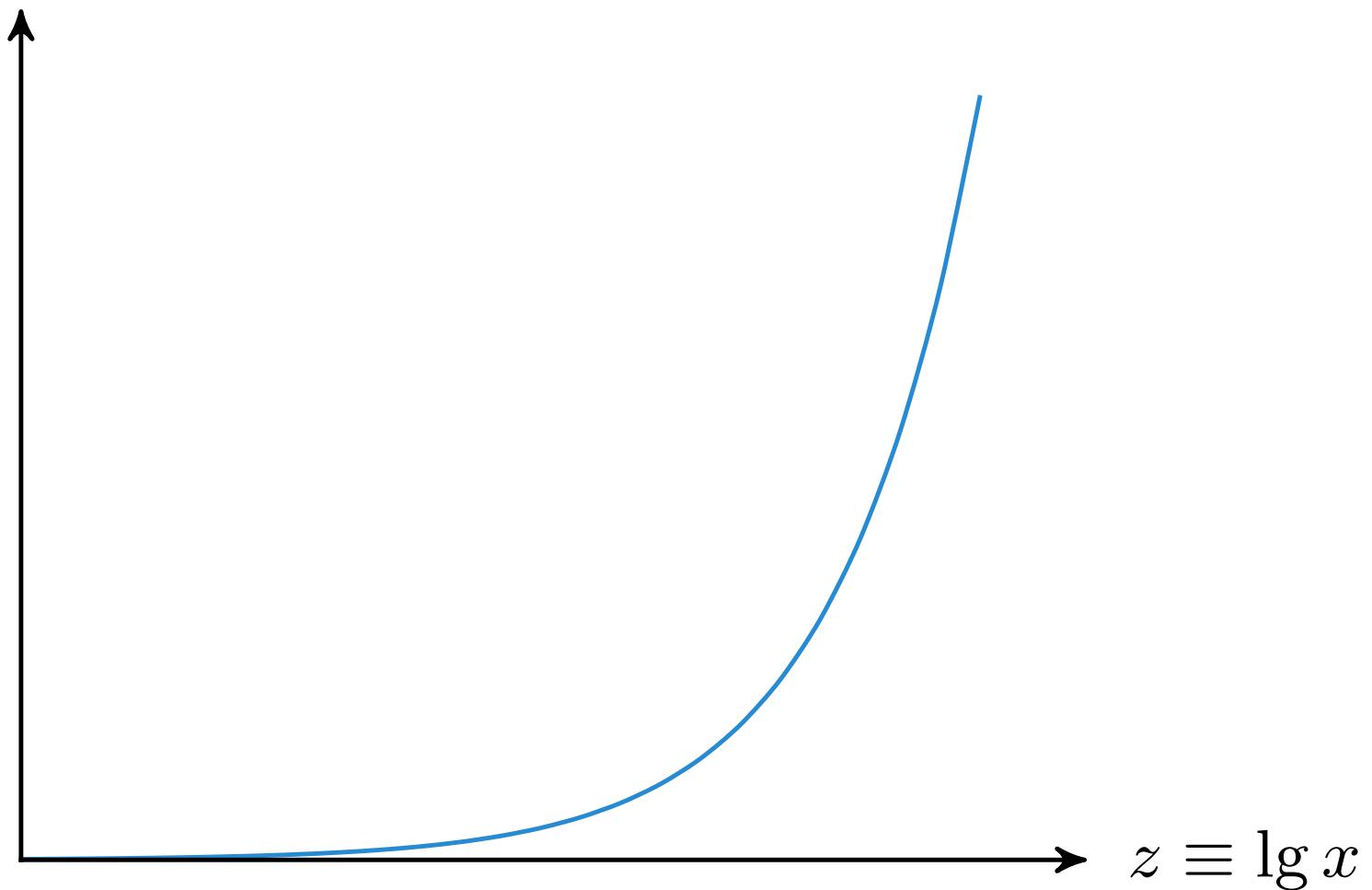
$$f(x) = x^2$$



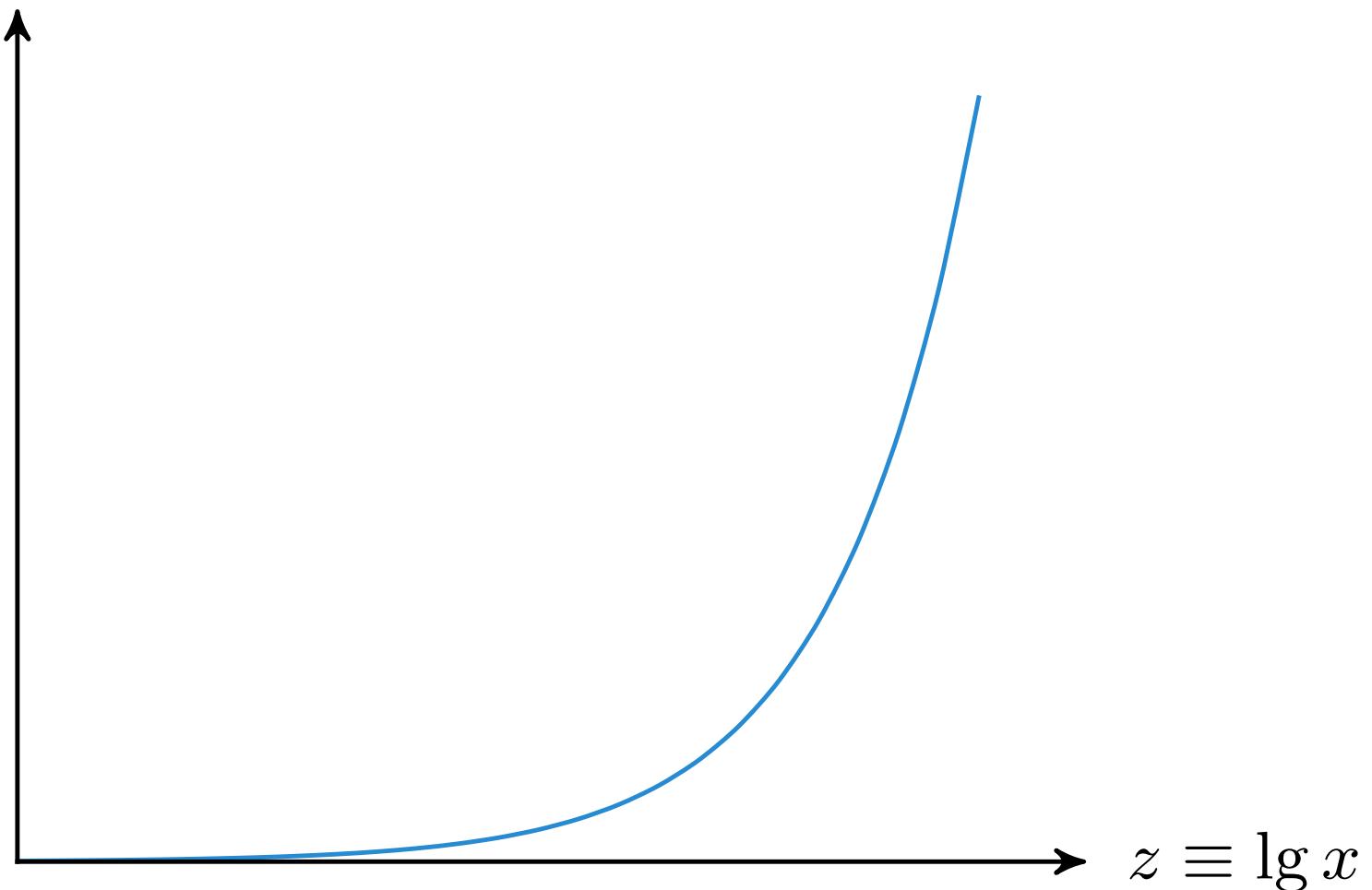
$$f(x) = x^2$$



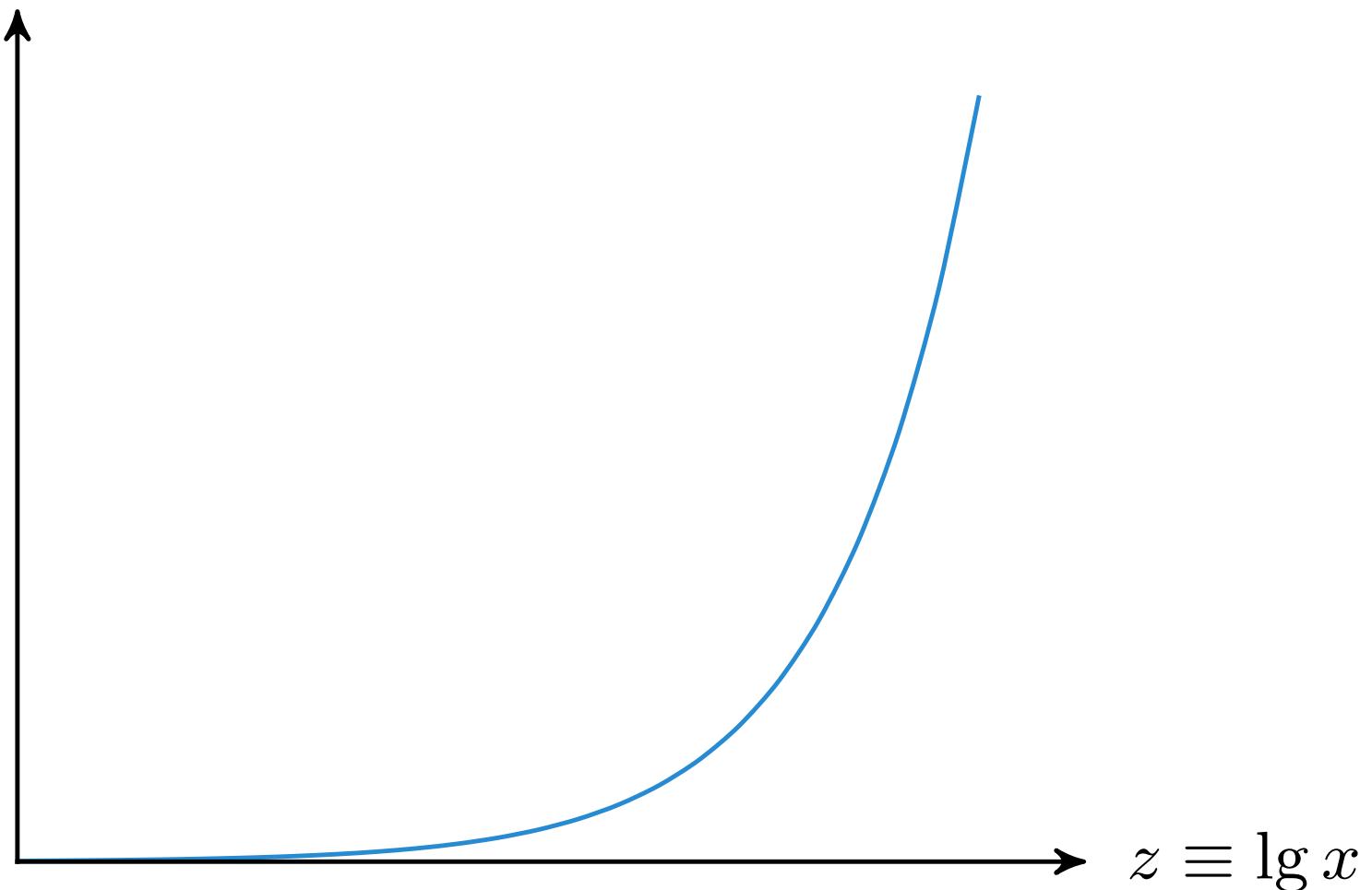
$$f(x) = x^2$$



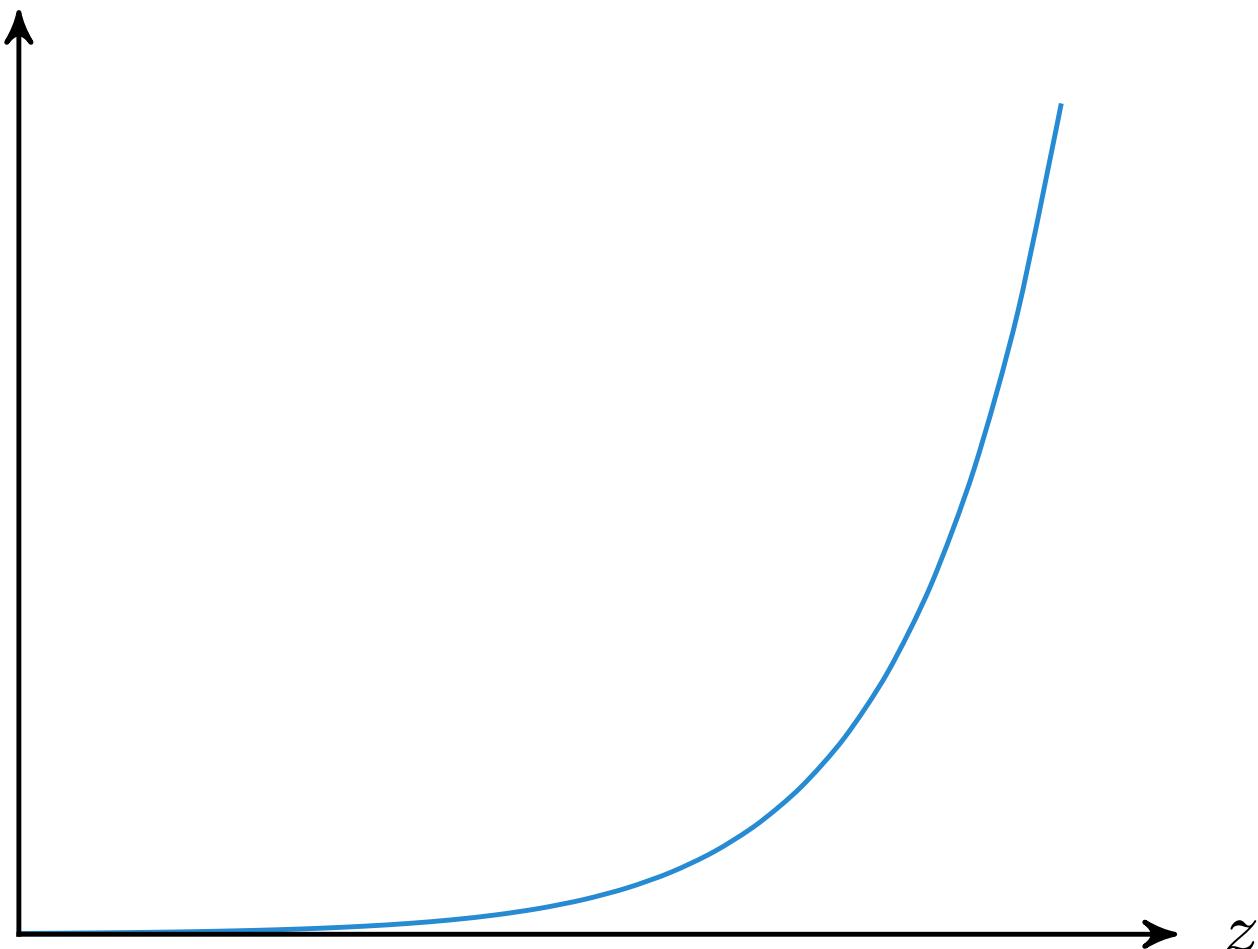
$$f(2^z) = f(x) = x^2$$



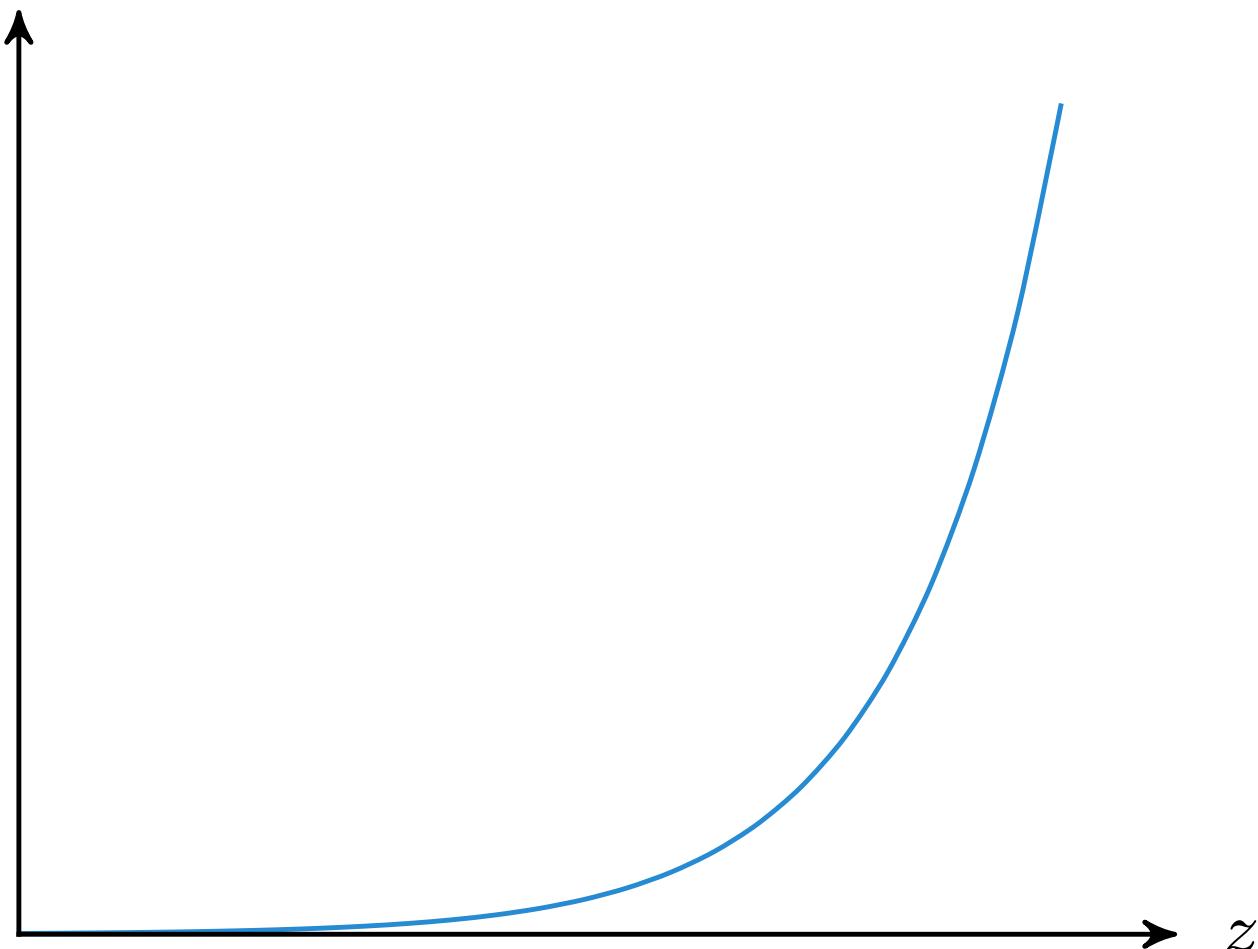
$$g(z) \equiv f(2^z) = f(x) = x^2$$



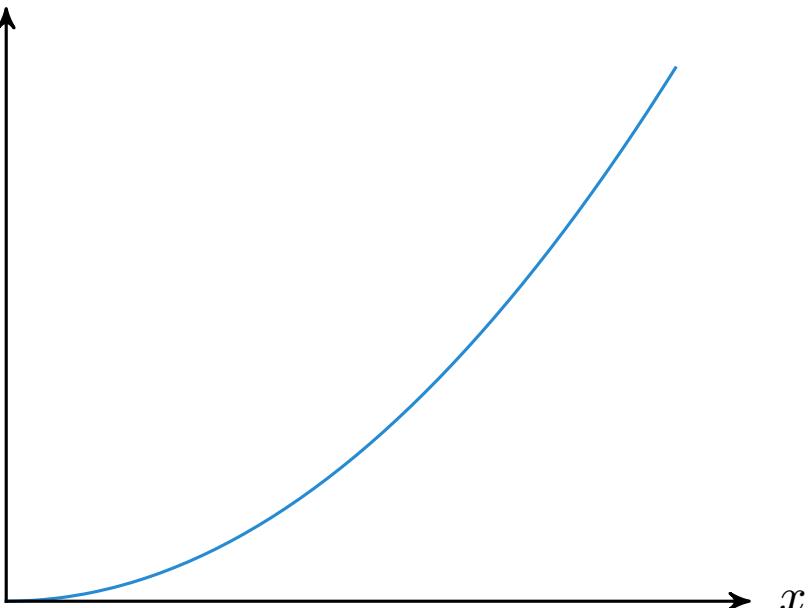
$$g(z) = (2^z)^2$$



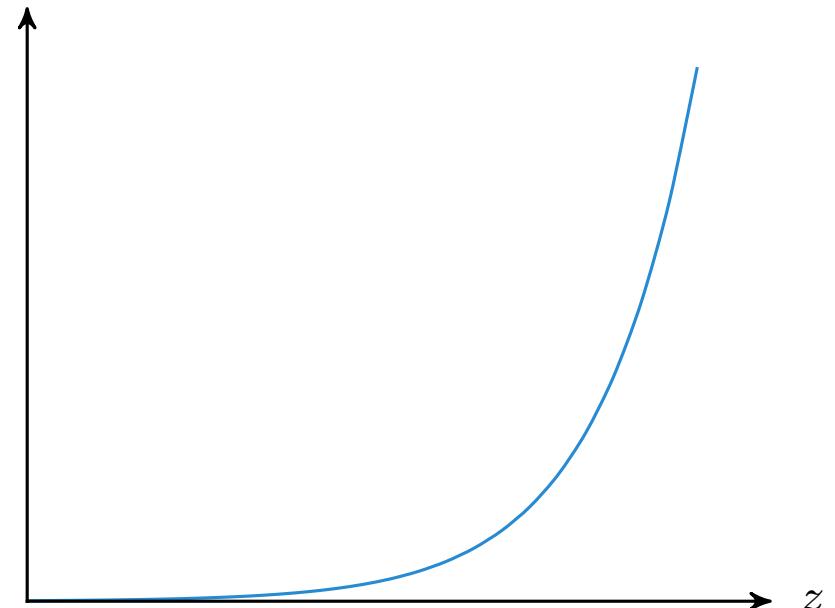
$$g(z) = 2^{2z}$$



$$f(x) = x^2$$



$$g(z) = 2^{2z}$$



Bytt ut variabelen –
gjerne med logaritmen
til den opprinnelige
variabelen – og bytt
funksjonen med en
annen funksjon som
oppveier for
transformasjonen.

$$z \equiv \lg x$$

$$g(z) \equiv f(x)$$

$$T(n) = 2T(\sqrt{n}) + \lg n$$

$$m \equiv \lg n$$

$$T(n) = 2T(\sqrt{n}) + \lg n$$

$$m \equiv \lg n$$

$$T(n) = 2T(\sqrt{n}) + \lg n$$

$$= 2T(\sqrt{2^m}) + m$$

$$m \equiv \lg n$$

$$T(n) = 2T(\sqrt{n}) + \lg n$$

$$= 2T(\sqrt{2^m}) + m = 2T(2^{m/2}) + m$$

$$m \equiv \lg n$$

$$S(\textcolor{red}{m}) \equiv T(2^{\textcolor{red}{m}}) = T(n) = 2T(\sqrt{n}) + \lg n$$

$$= 2T(\sqrt{2^m}) + m = 2T(2^{\textcolor{red}{m}/2}) + m$$

Vi bytter ut n med m og T med S slik at $S(m) = T(n)$, og ender med en enklere, men ekvivalent, rekurrens.

$$m \equiv \lg n$$

$$S(\textcolor{red}{m}) \equiv T(2^{\textcolor{red}{m}}) = T(n) = 2T(\sqrt{n}) + \lg n$$

$$= 2T(\sqrt{2^m}) + m = 2T(2^{\textcolor{red}{m}/2}) + m$$

$$= 2S(\textcolor{red}{m}/2) + m$$

$$S(m)=2S(m/2)+m$$

$$S(m) = 2S(m/2) + m$$

$$= \Theta(m \lg m)$$

$$m \equiv \lg n$$

$$S(m) = 2S(m/2) + m$$

$$= \Theta(m \lg m)$$

$$m \equiv \lg n$$

$$S(m) = 2S(m/2) + m$$

$$= \Theta(m \lg m)$$

$$= \Theta(\lg n \lg \lg n)$$

$$m \equiv \lg n$$

$$T(n) \equiv S(m) = 2S(m/2) + m$$

$$= \Theta(m \lg m)$$

$$= \Theta(\lg n \lg \lg n)$$

Vi løser den enklere rekurrensen, og bytter så m ned til n og får så svaret vi er ute etter.

$$m \equiv \lg n$$

$$T(n) \equiv S(m) = 2S(m/2) + m$$

$$= \Theta(m \lg m)$$

$$= \Theta(\lg n \lg \lg n)$$



D&C



D&C



D&C