Forelesning B

Logikk og Bevisteknikk

Pensum

- **Pensumhefte, appendiks A**

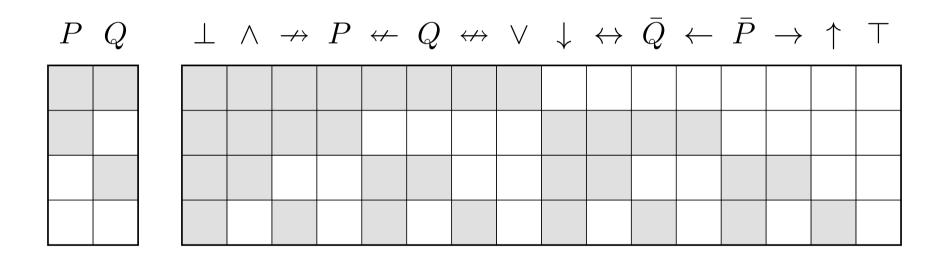# To nivåer

- **Setningslogikk:**
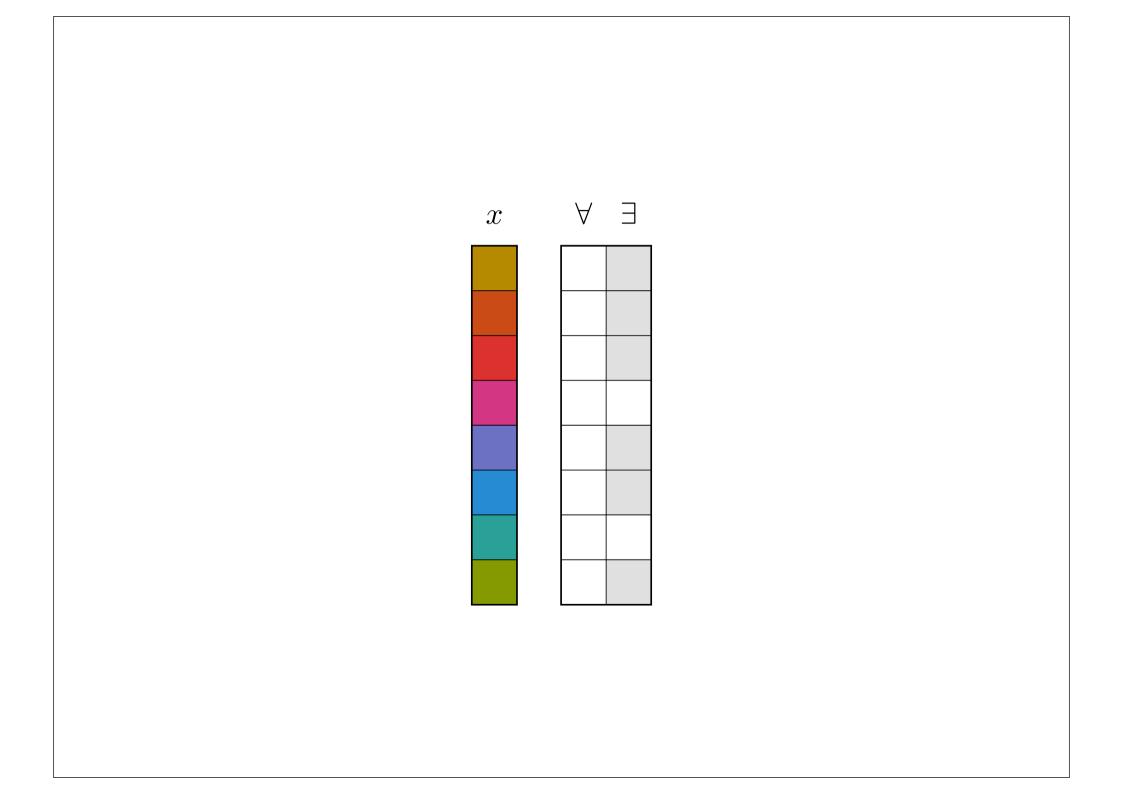
  «The logic of 'and', 'or' and 'not'.»
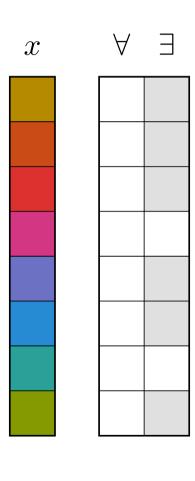
- **Predikatslogikk:**

  «The logic of relations between individuals x, y, z, ... and the quantifiers 'for all x' and 'there exists and x'.»
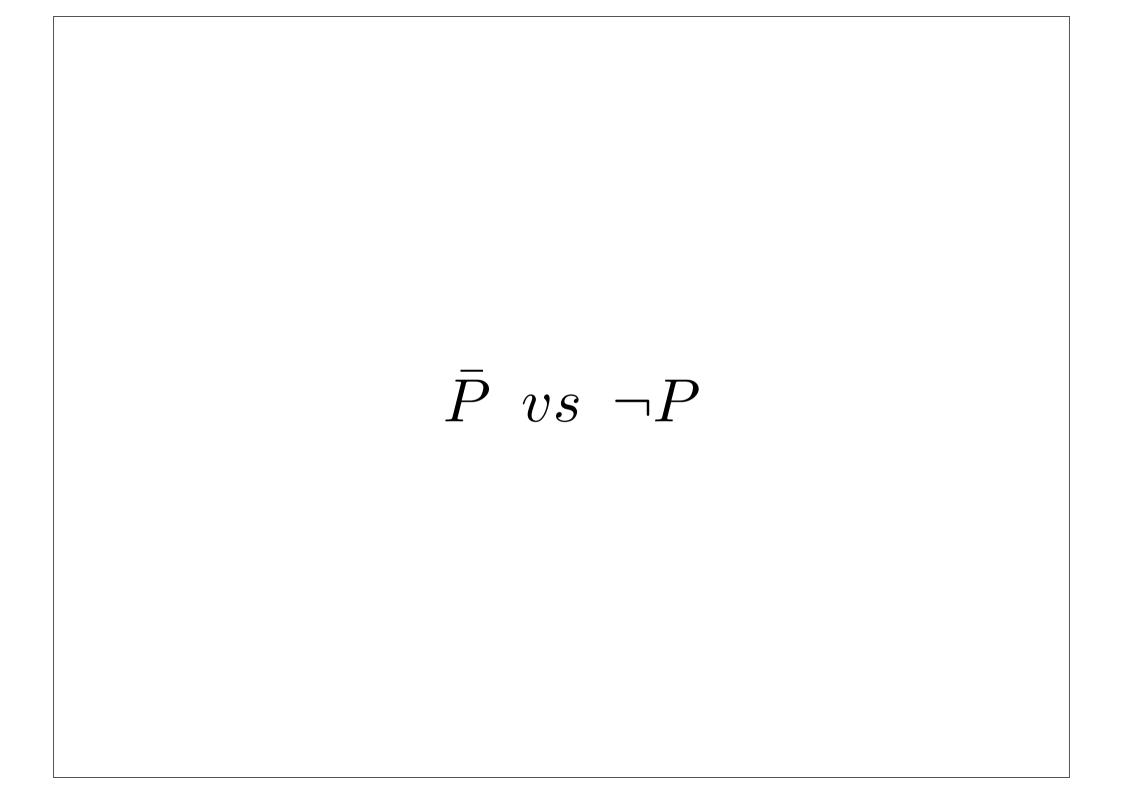
Inferensregler

- Introduksjon og eliminasjon

- Basert på formelle systemer ...

- ... men noe uformelt presentert

P	Q	$\perp$	$\wedge$	$\nrightarrow$	P	$\nleftarrow$	Q	$\nleftrightarrow$	$\vee$	$\downarrow$	$\leftrightarrow$	$\bar{Q}$	$\leftarrow$	$\bar{P}$	$\rightarrow$	$\uparrow$	$\top$
■	■	■	■	■	■	■	■	■	■								
■		■	■	■	■					■	■	■	■				
	■	■	■			■	■			■	■			■	■		
		■		■		■		■		■		■		■		■	

x $\forall$ $\exists$

(Merk: Ikke en sannhetstabell)

$$\bar{P} \ vs \ \neg P$$

$$P \rightarrow Q \quad vs \quad P \Rightarrow Q$$

$\text{I} \Rightarrow$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\text{E} \Rightarrow$

$$P \quad \text{H}$$

$$\vdots$$

$$Q$$

$$\frac{}{P \Rightarrow Q} \quad \text{H̶}$$

I⇒   E⇒

$$P \qquad \text{H}$$

$$\vdots$$

$$Q$$
$$\frac{}{P \Rightarrow Q} \quad \cancel{\text{H}}$$

$$\text{I}\Rightarrow$$

$$\frac{P \Rightarrow Q, \ P}{Q}$$

$$\text{E}\Rightarrow$$

$$P \qquad \text{H}$$

$$\vdots$$

$$Q$$

$$\frac{}{P \Rightarrow Q} \quad \cancel{\text{H}}$$

$$\frac{P \Rightarrow Q, \; P}{Q}$$

$$\text{I} \Rightarrow \qquad\qquad\qquad \text{E} \Rightarrow$$

$$P \Rightarrow Q \quad \equiv \quad \neg Q \Rightarrow \neg P$$

$$\neg Q \qquad \text{H}$$

$$\vdots$$

$$\neg P \qquad \cancel{\text{H}}$$

$$\overline{}$$

$$P \Rightarrow Q$$

$$\neg Q \qquad \text{H}$$

$$\vdots$$

$$\frac{\neg P}{P \Rightarrow Q} \qquad \cancel{\text{H}}$$

$$\frac{P \Rightarrow Q, \ \neg Q}{\neg P}$$

$$\neg Q \quad \text{H}$$

$$\vdots$$

$$\frac{\neg P}{P \Rightarrow Q} \quad \text{H}\!\!\!/$$

$$\frac{P \Rightarrow Q, \ \neg Q}{\neg P}$$

$I \Leftrightarrow$

$E \Leftrightarrow$

$$\frac{P \Rightarrow Q, \; Q \Rightarrow P}{P \Leftrightarrow Q}$$

I⇔

E⇔

$$\frac{P \Rightarrow Q, \ Q \Rightarrow P}{P \Leftrightarrow Q}$$

$$\frac{P \Leftrightarrow Q}{P \Rightarrow Q, \ Q \Rightarrow P}$$
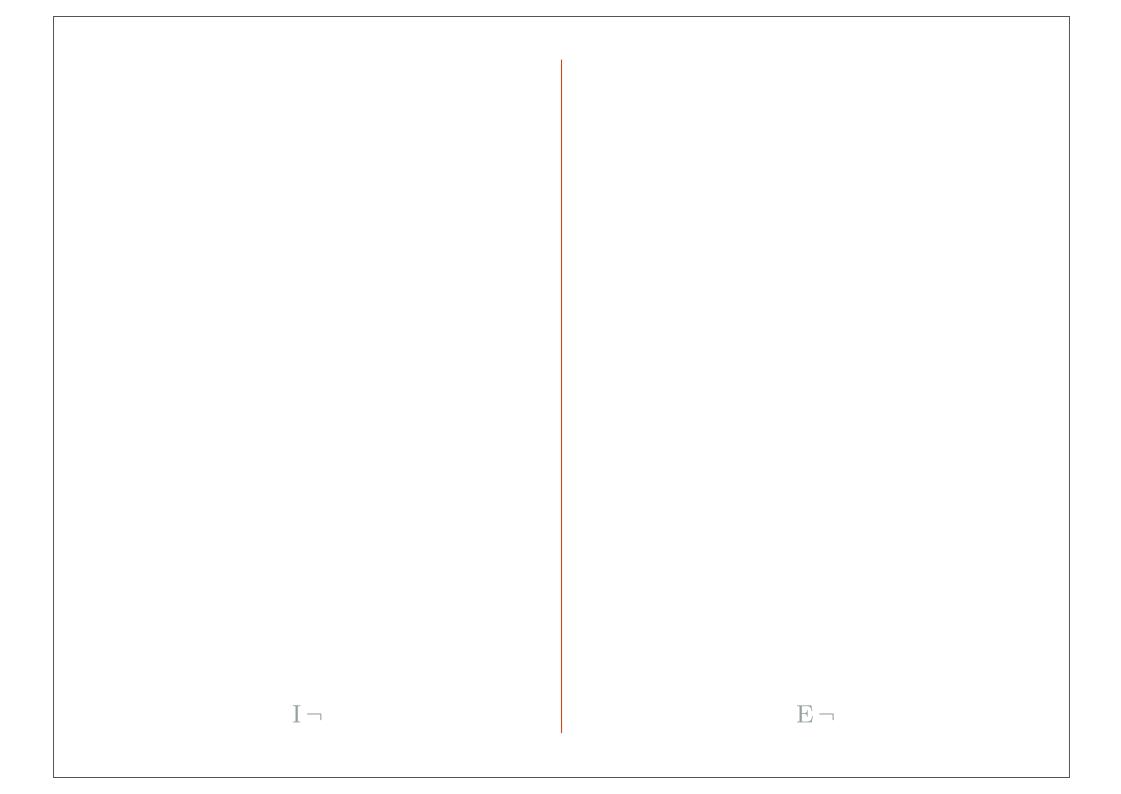
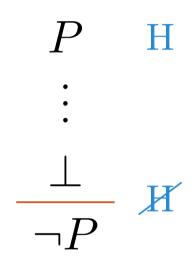$$\text{I} \Leftrightarrow \qquad\qquad\qquad \text{E} \Leftrightarrow$$

$$P \Leftrightarrow Q \Leftrightarrow R$$

$$\equiv$$

$$P \Rightarrow Q \Rightarrow R \Rightarrow P$$

Negasjon

- Omform til ekvivalent positivt utsagn?

I ¬

E ¬

$$\frac{\begin{array}{c} P \quad \text{H} \\ \vdots \\ \bot \end{array}}{\neg P} \; \text{H}$$

I ¬

E ¬

Reductio ad Absurdum

$$P \qquad \text{H}$$

$$\vdots$$

$$\frac{\bot}{\neg P} \quad \cancel{\text{H}}$$

I¬   E¬

Reductio ad Absurdum

$$P \quad \text{H}$$

$$\vdots$$

$$\frac{\bot}{\neg P} \quad \cancel{\text{H}}$$

$$\text{I} \neg$$

$$\frac{Q, \ \neg Q}{\bot}$$

$$\text{E} \neg$$

I∧   E∧

$$\frac{P,\ Q}{P \wedge Q}$$

I∧

E∧

$$\frac{P, \ Q}{P \land Q}$$

$$\frac{P \land Q}{P, \ Q}$$

I∧

E∧

I∨   E∨

$$\frac{P}{P \lor Q}$$

$$\frac{P}{P \lor Q}$$

$$\frac{P \lor Q, \quad \overset{\displaystyle P}{\vdots} \quad \overset{\displaystyle Q}{\vdots}}{R}$$

$$\frac{P}{P \lor Q}$$

I$\lor$

$$\frac{P \lor Q, \quad R, \quad R}{R}$$

with P and Q leading by vertical dots to R and R respectively:

$$\begin{array}{cc} P & Q \\ \vdots & \vdots \\ P \lor Q, & R, \quad R \end{array}$$
$$\overline{}$$
$$R$$

E$\lor$

$$P \vee Q \equiv \neg P \Rightarrow Q$$

∀I

∀E

$$\frac{P(a)}{\forall x P(x)}$$

I∀   E∀

$$\frac{P(a)}{\forall x P(x)}$$

vilkårlig

I ∀

E ∀

$$\frac{P(a)}{\forall x P(x)}$$

vilkårlig

$\text{I} \forall$

$$\frac{\forall x P(x)}{P(a)}$$

$\text{E} \forall$

IƎ   EƎ

$$\frac{P(a)}{\exists x P(x)}$$

I∃

E∃

$$\frac{\overset{\text{eksempel}}{P(a)}}{\exists x P(x)}$$

I∃

E∃

$$\frac{P(a)}{\exists x P(x)}$$

eksempel

$$\frac{\exists x P(x)}{P(y)}$$

I∃

E∃

$$\frac{P(a)}{\exists x P(x)}$$

eksempel → $P(a)$

$\text{I}\exists$

$$\frac{\exists x P(x)}{P(y)}$$

dummy → $P(y)$

$\text{E}\exists$

$$\exists x P(x) \equiv \neg \forall x \neg P(x)$$

$$\exists ! x P(x)$$

$$\equiv$$

$$\exists x P(x) \quad \wedge \quad \forall x \forall y \big(P(x) \wedge P(y) \Rightarrow x = y \big)$$