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Problem Set 5
Advanced Macroeconomics
Winter 2025/26

Two-Period Model with Retirement

A household lives for two periods. In the first period, it earns money from lending a capital endowment k_1 to a firm which pays the rental rate r_1. Furthermore, it works hours n for wage w paid by the firm and has to pay a tax rate τ on labor income.

In the second period, the households stops working and gets capital income from lending k_2 to the firm at interest rate r_2 as well as governmental transfers T which equal the tax revenue from the last period. The producer is able to substitute the labor force by using capital k_2 as input factor.

Assume utility function

$$U(c_1, c_2) = u(c_1) - \frac{n^{1+\varphi}}{1+\varphi} + \frac{1}{1+\rho}u(c_2), \quad \text{where } u(c) = \frac{c^{1-\theta}}{1-\theta},$$

and production technology

$$y_1 = A_1 k_1^\alpha n^{1-\alpha} \quad \text{in period 1,}$$
$$y_2 = A_2 k_2 \qquad \text{in period 2.}$$

The exogenously given variables and parameters are as follows: $A_1 = 1.2, A_2 = 1, k_1 = 1$ and $\alpha = 0.3, \varphi = 1.5, \rho = 0.05, \tau = 0.2, \theta = 1.8$.

1. Derive the first order conditions for a utility maximum.

2. Derive the first order conditions for a profit maximum.

3. Summarize the model: exogenous and endogenous variables, equations and parameters.

4. Compute the two period general equilibrium.

5. * Display the results.

6. Do a sensitivity analysis of transfers and labor input with respect to the tax rate.

*Note: the task marked with * follows directly the lecture code and is intended for independent self-study.*