Advanced Macroeconomics

13. Model-based Forecasting

The Halle Economic Projection Model (HEPM)

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The State of Macro

(Blanchard 2008)

"That the predictions were wildly incorrect, and that the doctrine on which they were based was fundamentally flawed, are now simple matters of fact. The task which faces contemporary students of the business cycle is that of sorting through the wreckage, determining what features of ... can be salvaged and put to good use, and which others must be discarded."

Lucas and Sargent (1978) about the Keynesian Revolution

Animal Spirits and Behavioral Macro

(Akerlof 2002, Akerlof and Shiller 2009)

"New Classical macroeconomics was based on the competitive, general-equilibrium model. But it differed in being much more zealous in insisting that all decisions – consumption and labor supply by households, output, employment and pricing decisions by producers, and the wage bargains between both workers and firms – be consistent with maximizing behavior. ... But the behavioral assumptions were so primitive that the model faced extreme difficulty in accounting for at least six macroeconomic phenomena."

Akerlof (2002) in *Behavioral Macroeconomics*

The Pretense-of-Knowledge Syndrome

(Caballero 2010)

"The natural next step ... is to incorporate massive uncertainty. This step may also harbor some of the answers on how to deal with quantitative statements in a highly uncertain environment. As a starting point, we probably need to relax the artificial micro-foundation constraints imposed just for the sake of being able to generate 'structural' general equilibrium simulations. When closing a *quantitative* model, there should be an explicit correspondence between the knowledge we assume in such closure and the state of our knowledge about such closure. This means replacing artificial structural equations for looser ones, or even for reduced-form data-relationships if this all that we really know."

Three Core New-Keynesian Equations

(*inter alia* Woodford 2003, Galì 2008, Blanchard 2008)

- Aggregate demand relation, in which output is determined by demand, and demand depends in turn on anticipations of both future output and future real interest rates
- Phillips-curve like relation, in which inflation depends on both output and anticipations of future inflation
- Monetary policy relation, which embodies the proposition that monetary policy can be used to affect the current real interest rate
- Anticipations and the uncertainty about the current state of the economy play a major role

The Halle Economic Projection Model (HEPM)

Overview

- Macroeconomic projection model (Carabenciov et al. 2008) which can be used for forecasting purposes, simulation of different policy scenarios, and identification of unobserved variables
- Combines core equations of New-Keynesian standard DSGE models with empirically useful ad-hoc equations
- Implicit microfoundation of core equations facilitates structural interpretation of economic fluctuations and of adjustment processes to shocks
- Ad-hoc modifications and additional equations improve statistical fit and forecasting performance (compared to pure DSGE models)

Model Specification

- Multi-country framework: U.S. and Euro area (Germany, France and Italy, $\approx$ 70 % of the euro area)
- Basic relations: aggregate demand (IS) curve, Phillips curve (PC), monetary policy rule (MP)
- Observable variables: real GDP, CPI inflation, short-term interest rate, unemployment rate, U.S. dollar exchange rate

Model Specification – The Long Run

- Country-specific exogenous stochastic processes for $i \in \{US, DE, FR, IT\}$
- Potential output

$$\begin{aligned} \overline{Y}_{i,t} &= \overline{Y}_{i,t-1} + g^{\overline{Y}}_{i,t}/4 + \varepsilon^{\overline{Y}}_{i,t} \\ g^{\overline{Y}}_{i,t} &= \tau_i g_i^{\overline{Y}^{ss}} + (1-\tau_i) g^{\overline{Y}}_{i,t-1} + \varepsilon^{g^{\overline{Y}}}_{i,t} \end{aligned}$$

$$g^{\overline{Y}^{ss}}_{US} = 2.5,\ g^{\overline{Y}^{ss}}_{DE} = 1.4,\ g^{\overline{Y}^{ss}}_{FR} = 1.3$$

- Equilibrium real interest rate $\overline{R}$

$$\overline{R}_{i,t} = \rho_i \overline{R}^{ss}_i + (1-\rho_i)\overline{R}_{i,t-1} + \varepsilon^{\overline{R}}_{i,t}$$

whith $0 \leq \rho_i \leq 1$

Model Specification – IS Curve

$$
\begin{aligned}
y_{i,t} &= \beta_{i,1} y_{i,t-1} + \beta_{i,2} E_t y_{i,t+1} - \beta_{i,3} r_{i,t-1} \\
&\quad + \beta_{i,4} \sum_{l=1}^{L} \omega_{i,l,4} z_{i,l,t-1} + \beta_{i,5} \sum_{l=1}^{L} \omega_{i,l,5} y_{l,t-1} + \varepsilon_{i,t}^{y}
\end{aligned}
$$

- $i \in \{US, DE, FR, IT\}$
- $y_{i,t}$ output gap, $r_{i,t-1}$ real interest rate gap
- Output gaps of trading partners ($y_{l,t-1}$) and effective real exchange rate gap ($z_{i,l,t-1}$) reflect nature of open economy
- Hybrid form, i.e. lags and leads enter the equation ($y_{i,t-1}$ habit persistence, Fuhrer 2000, e.g.)

Model Specification – Phillips Curve

$$\begin{aligned}\pi_{i,t} &= \lambda_{i,1} E_t \pi^a_{i,t+4} + (1-\lambda_{i,1})\pi^a_{i,t-1} + \lambda_{i,2} y_{i,t-1} \\ &\quad + \lambda_{i,3} \sum_{l=1}^{L} \omega_{i,k,3} \Delta Z_{i,l,t} - \varepsilon^{\pi}_{i,t}\end{aligned}$$

- $i \in \{US, DE, FR\}$
- Relates inflation ($\pi_{i,t} = 400\times$ log difference of CPI) to its future ($E_t \pi^a_{i,t+4}$) and past values ($\pi^a_{i,t-1}$), lagged output gap ($y_{i,t-1}$), change in the effective exchange rate ($\Delta Z_{i,l,t}$)
- Output gap links the real side of the economy to inflation

Model Specification – Monetary Policy

$$
\begin{aligned}
I_{i,t} &= (1-\gamma_{i,1})[\overline{R}_{i,t} + \pi^{a}_{i,t+3} + \gamma_{i,2}(\pi^{a}_{i,t+3} - \pi^{tar}_{i}) + \gamma_{i,4} y_{i,t}] \\
&\quad + \gamma_{i,1} I_{i,t-1} + \varepsilon^{I}_{i,t}
\end{aligned}
$$

$$
\begin{aligned}
\pi^{tar}_{EU} &= 1.8 \text{ (\textit{posterior mode}) } (1.9 \textit{ prior mean}) \\
\pi^{tar}_{US} &= 2.6 \text{ (\textit{posterior mode}) } (2.5 \textit{ prior mean})
\end{aligned}
$$

- $i \in \{US, EU\}$
- Central bank reacts to movements of output gap ($y_{i,t}$) and to deviations of the expected inflation rate from its target ($\pi^{a}_{i,t+3} - \pi^{tar}_{i}$)
- Interest rate smoothing ($I_{i,t-1}$)

Model Specification – Real Interest Rate Parity

$$4 * \left(Z^{e}_{EU,t+1} - Z_{EU,t}\right) = \left(R_{EU,t} - R_{US,t}\right) - \left(\overline{R}_{EU,t} - \overline{R}_{US,t}\right) + \varepsilon^{Z-Z^{e}}_{EU,t}$$

- $Z_{EU,t}$ real exchange rate, $R_{EU,t}$ and $R_{US,t}$ real interest rates
- Difference between equilibrium rates of Euro area and U.S. $(\overline{R}_{EU,t} - \overline{R}_{US,t})$ is equilibrium risk premium
- Z^{e} not fully model-consistent to allow for persistence in the exchange rate: $Z^{e}_{EU,t+1} = \phi_{EU} E_t Z_{EU,t+1} + (1 - \phi_{EU}) Z_{EU,t-1}$
- When $\phi_{EU} = 1$ expectations are fully rational ($\widehat{\phi}_{EU} = 0.88$)

Model Specification – Labor Market

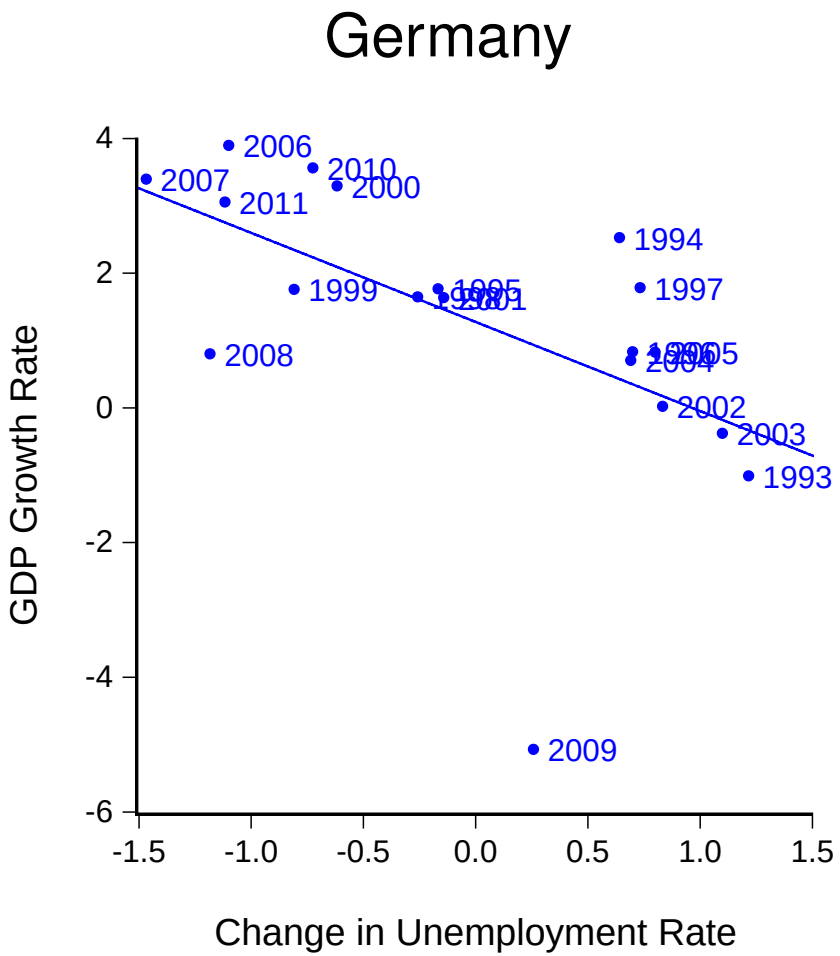
- For $i \in \{US, DE, FR, IT\}$
- NAIRU

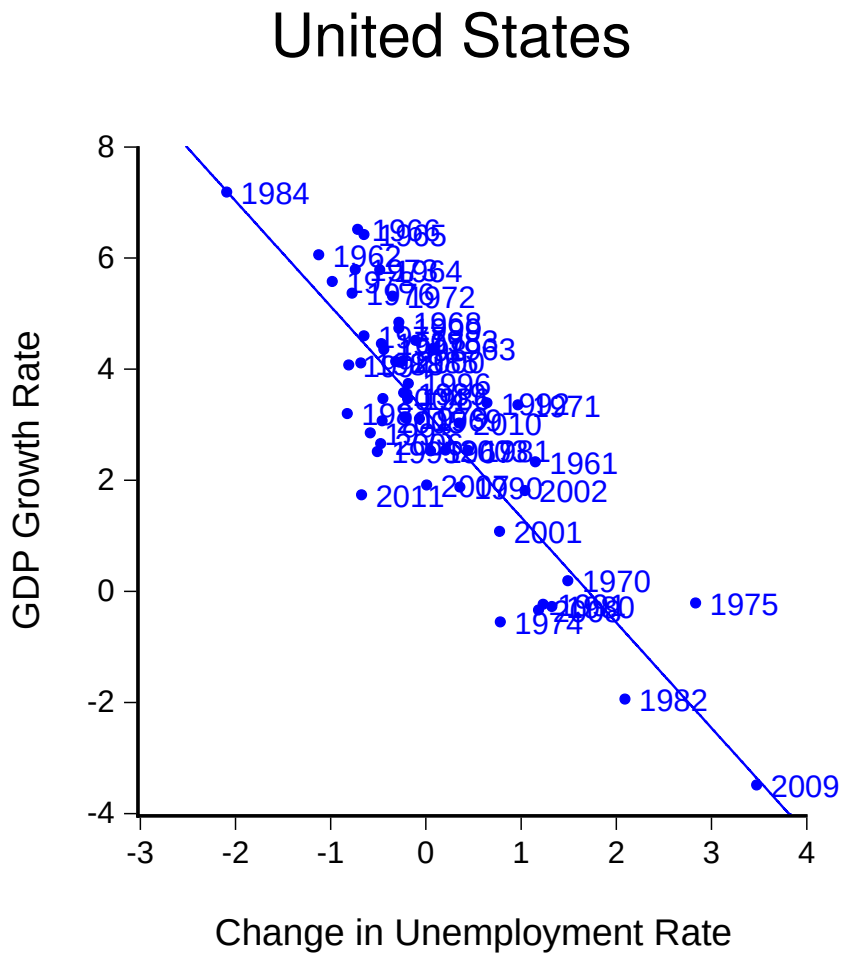
$$\begin{aligned} \overline{U}_{i,t} &= \overline{U}_{i,t-1} + g^{\overline{U}}_{i,t} + \varepsilon^{\overline{U}}_{i,t} \\ g^{\overline{U}}_{i,t} &= (1 - \alpha_{i,3}) g^{\overline{U}}_{i,t-1} + \varepsilon^{g^{\overline{U}}}_{i,t} \end{aligned}$$

- Dynamic version of Okun's law

$$u_{i,t} = \alpha_{i,1} u_{i,t-1} + \alpha_{i,2} y_{i,t} + \varepsilon^{u}_{i,t}$$

Okun's Law – Empirical Evidence





The Halle Economic Projection Model (HEPM)

Model Estimation

- Bayesian estimation
- Incorporates prior knowledge about central economic relations from previous macro or micro studies
- Prior knowledge is combined with observed data in order to derive an estimate and its distribution
- The researcher explicitly decides what role the data plays (flat prior increases impact of actual data on estimated coefficient)
- Posterior distribution computed using the Metropolis-Hastings algorithm
- Data for U.S., Germany, France, Italy, Euro area (1999Q1-2010Q4)

The Halle Economic Projection Model (HEPM)

Forecasting Performance

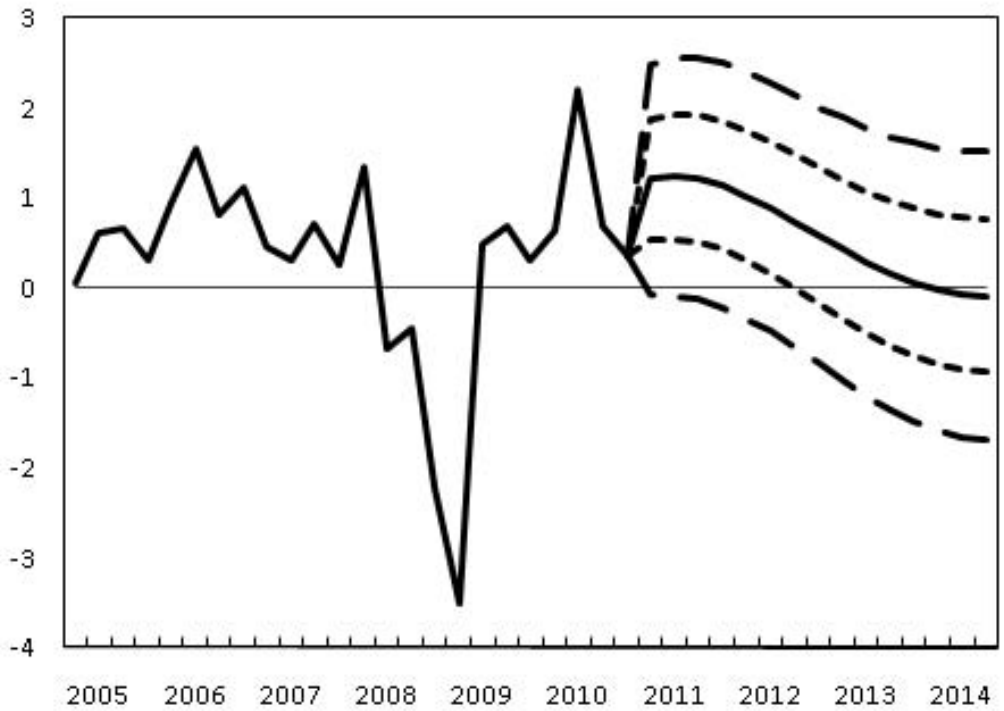
- Out-of-sample relative RMSFEs (h denotes forecast horizon).
- HEPM forecasts do well compared to competitors (RMSFE ≥ 1).

	ARIMA				VAR				Factor			
h	1	2	4	8	1	2	4	8	1	2	4	8
Y_{DE}	1.42	1.39	1.55	1.64	1.51	1.07	**0.87**	1.08	1.43	1.64	2.03	1.93
u_{DE}	**0.90**	1.01	1.41	1.64	1.48	1.79	2.39	5.94	1.86	2.19	3.24	5.15
π^a_{DE}	1.21	1.28	1.06	**0.77**	1.35	1.43	1.16	**0.63**	1.19	1.20	1.09	**0.79**
Y_{US}	**0.91**	**0.92**	1.10	1.08	1.23	1.20	1.23	**0.82**	**0.79**	**0.85**	**0.98**	1.01
u_{US}	1.25	1.44	1.75	1.16	**0.74**	**0.77**	**0.95**	**0.80**	**0.88**	**0.96**	1.18	1.08
π^a_{US}	1.04	1.25	1.15	1.10	1.49	1.46	1.06	**0.90**	1.33	1.27	1.31	1.09

Relative RMSEs for alternative models (ARIMA, VAR and factor model); a value smaller than one indicates that the respective benchmark model performs better than the HEPM

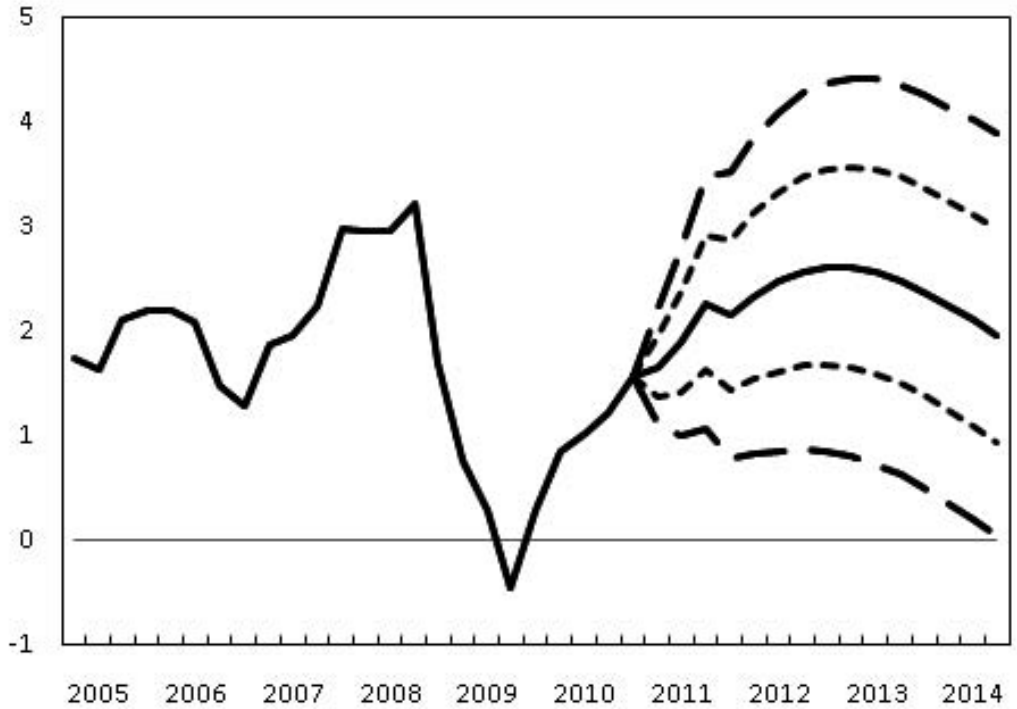
Forecast Results I

- Germany (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- GDP growth (%, Quarter-on-quarter)



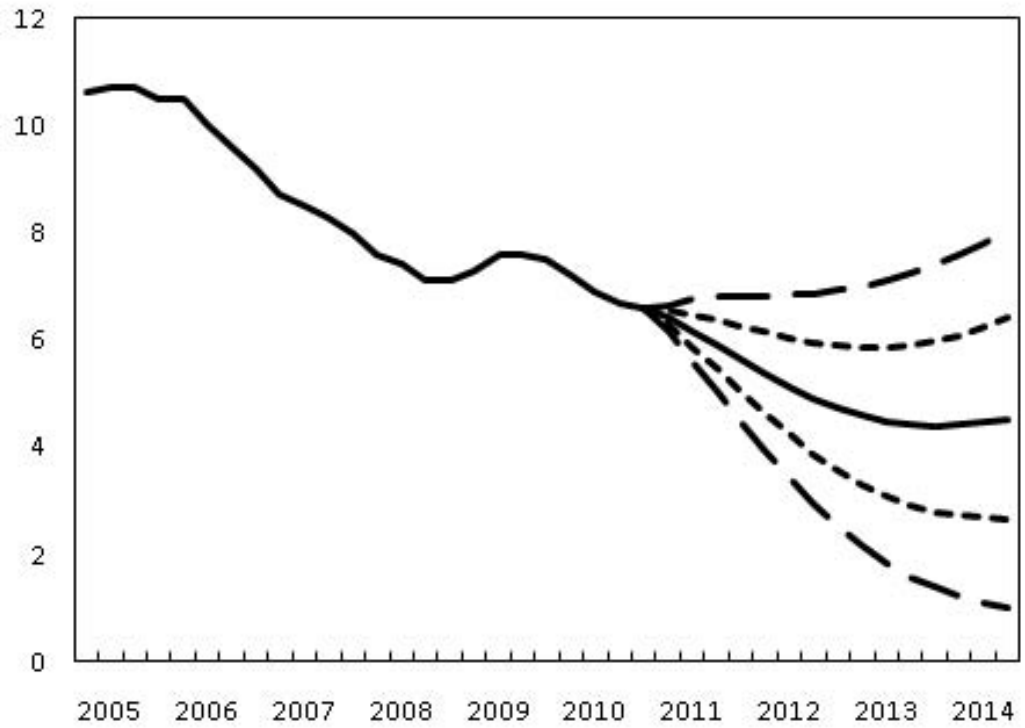
Forecast Results II

- Germany (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- Inflation (%, Year-on-year)



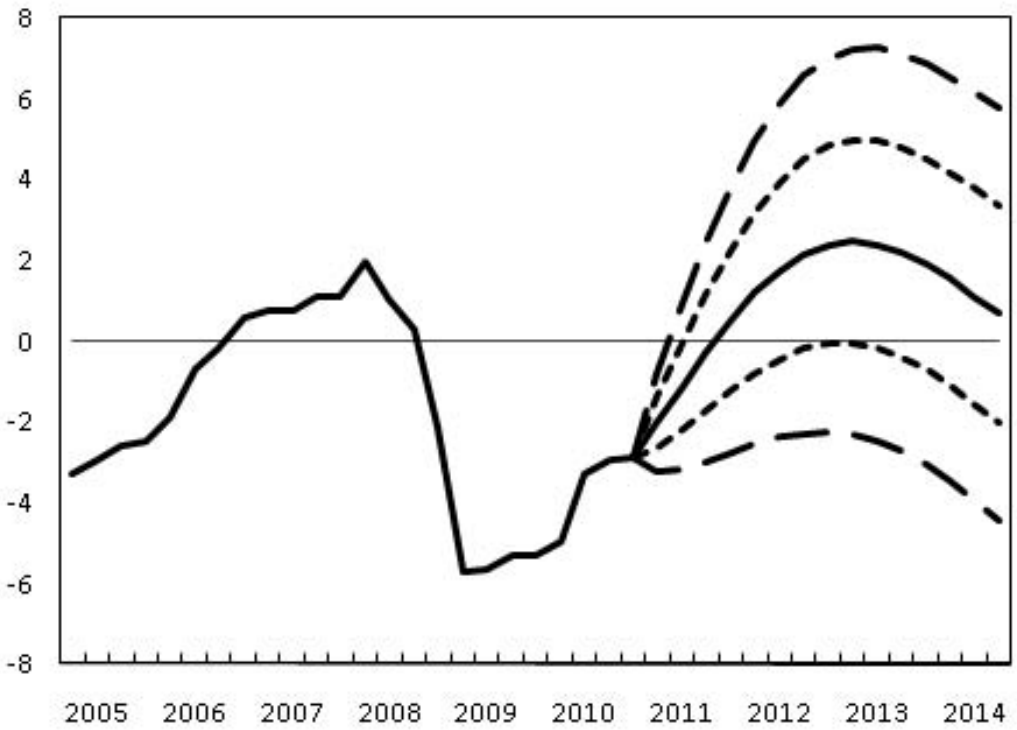
Forecast Results III

- Germany (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- Unemployment rate (%)



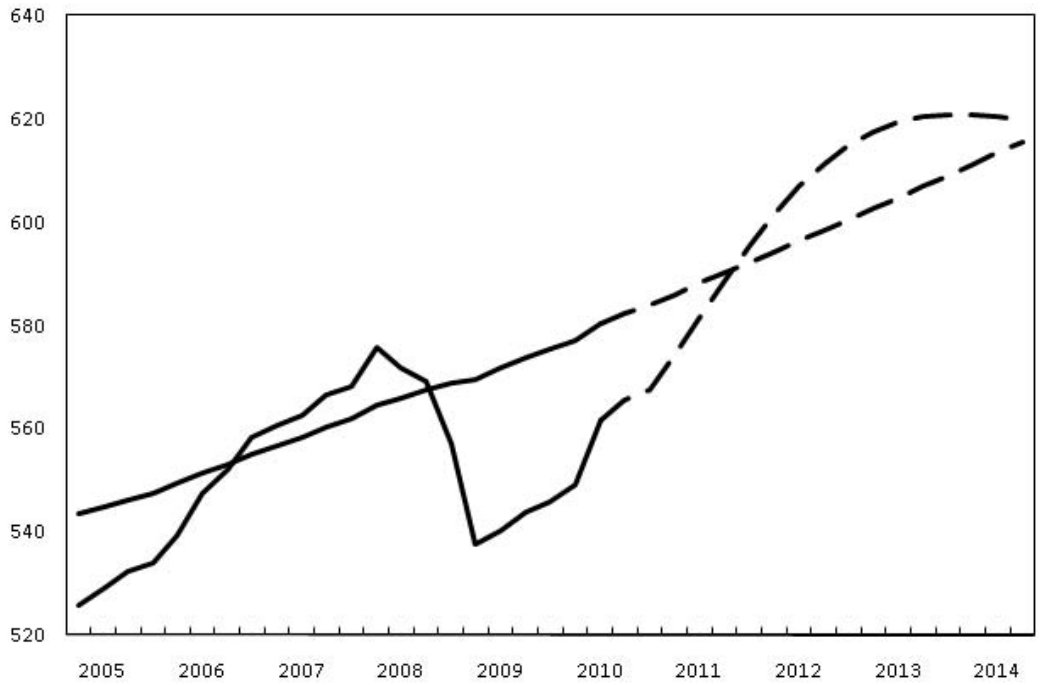
Forecast Results IV

- Germany (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- Output gap (%)



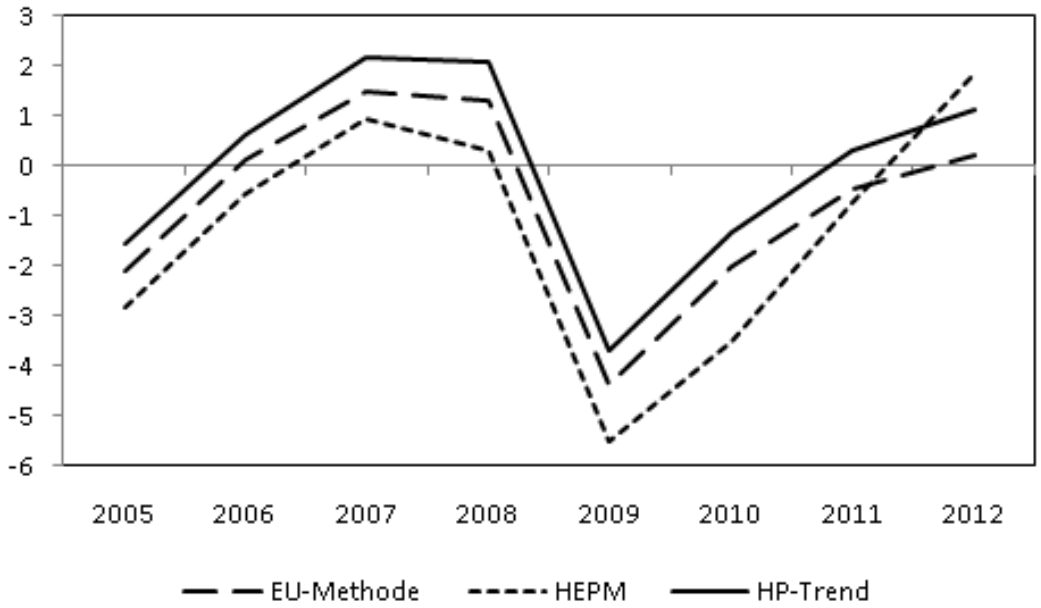
Forecast Results V

- Germany (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- Potential GDP (billion Euros)



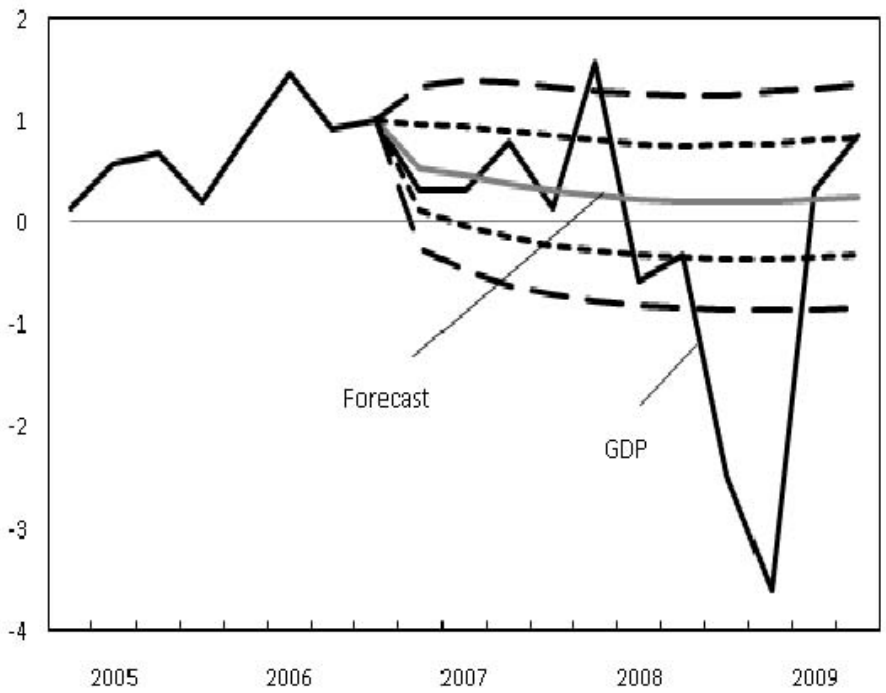
Forecast Results VI

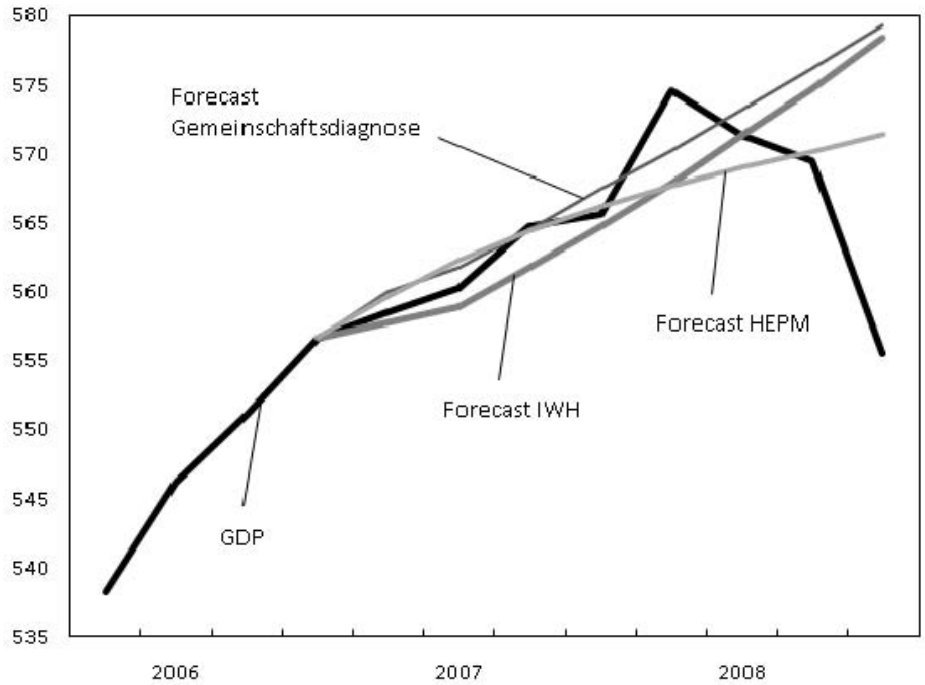
- Germany, Sample 2005Q1-2010Q4, Forecast horizon 2011Q1-2012Q4
- Output Gap (%)



Forecast Results VII

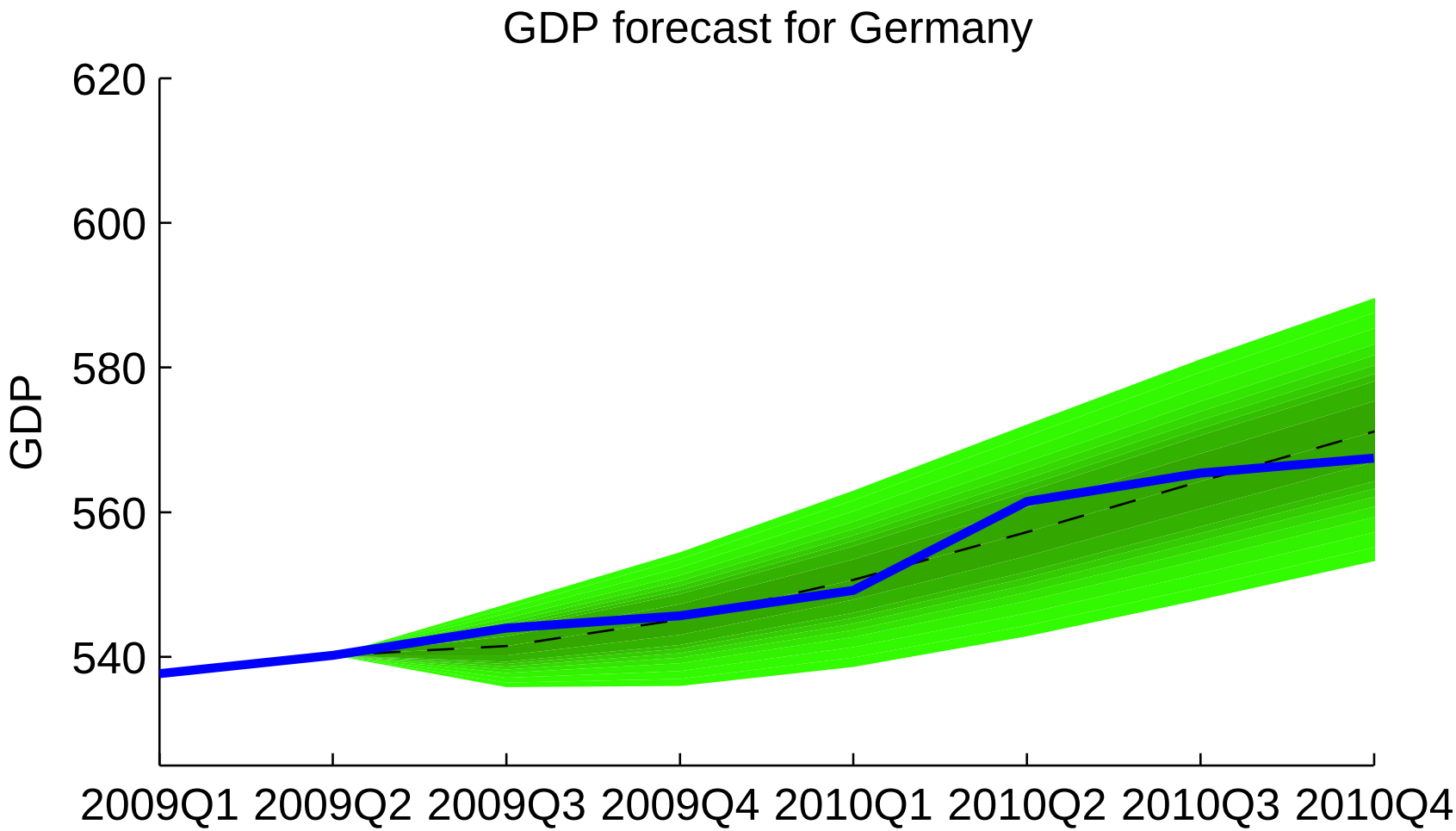
- Germany (70% and 95% confidence intervals), Sample 1999Q1-2006Q4, Forecast horizon 2007Q1-2011Q4
- GDP growth and forecast (left), realized GDP and forecasts (right)





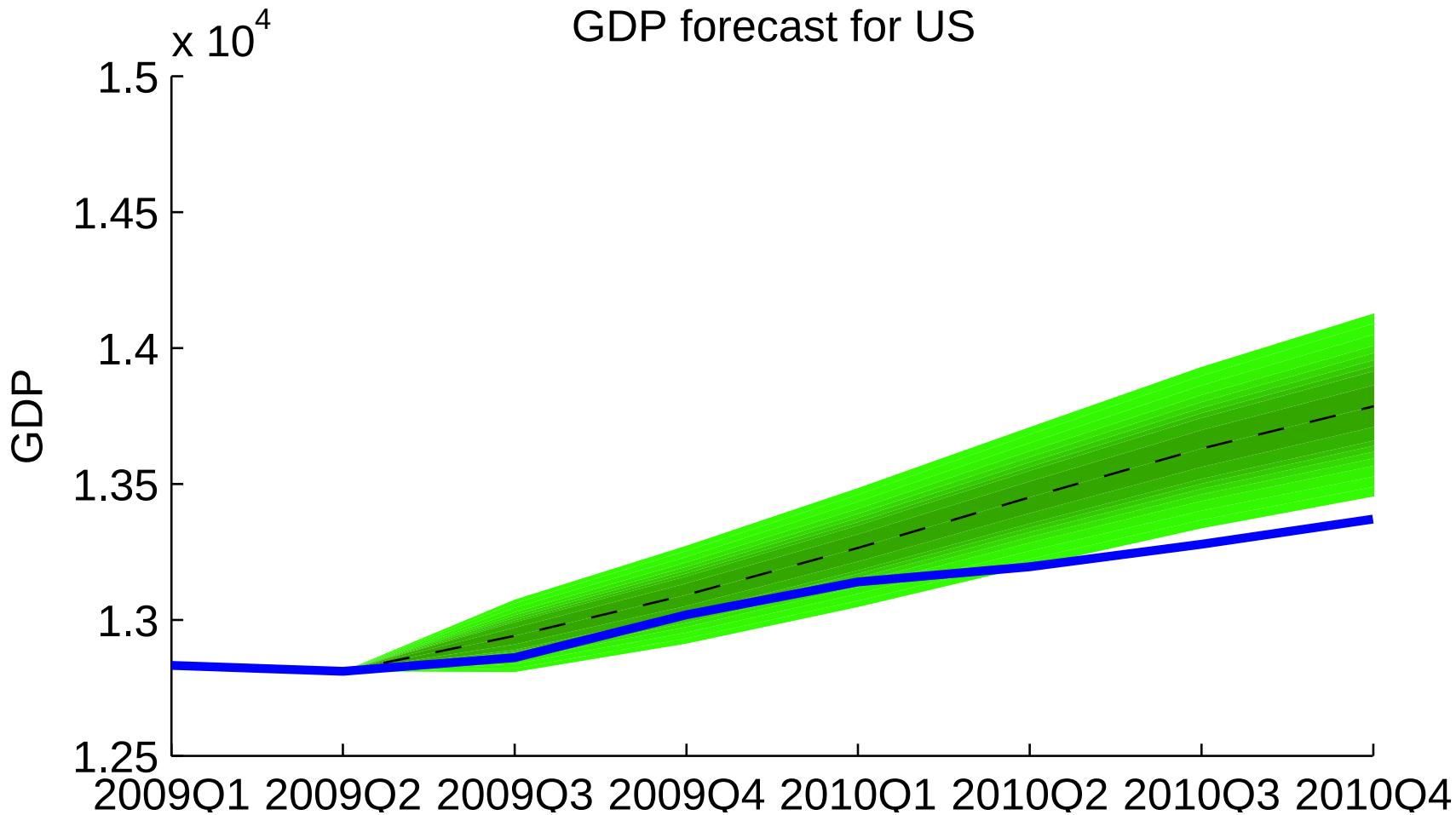
Density Forecasting

- Sample 1999Q1-2009Q2, Forecast horizon 2009Q3-2010Q4



Density Forecasting II

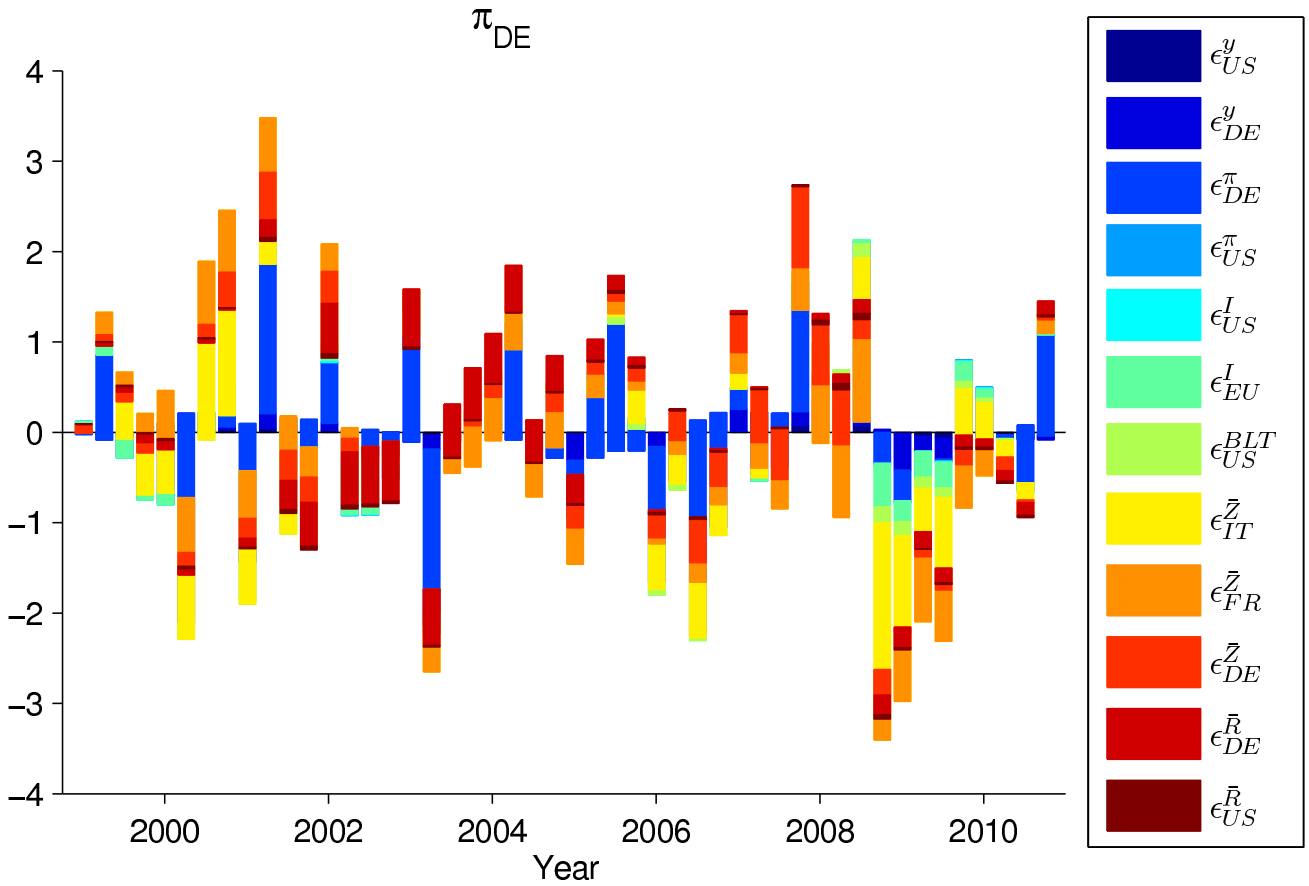
- Sample 1999Q1-2009Q2, Forecast horizon 2009Q3-2010Q4



The Halle Economic Projection Model (HEPM)

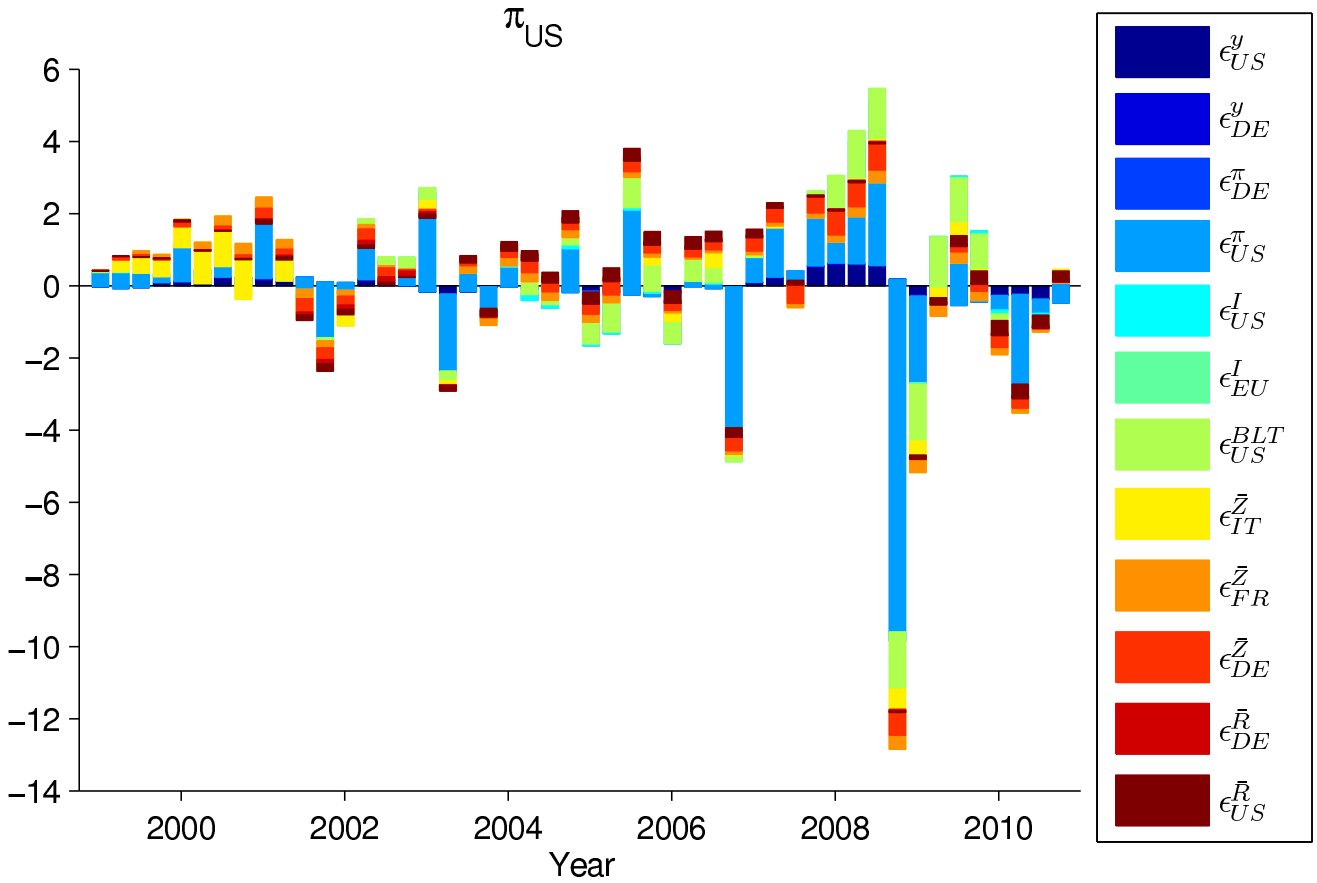
Results I

Germany



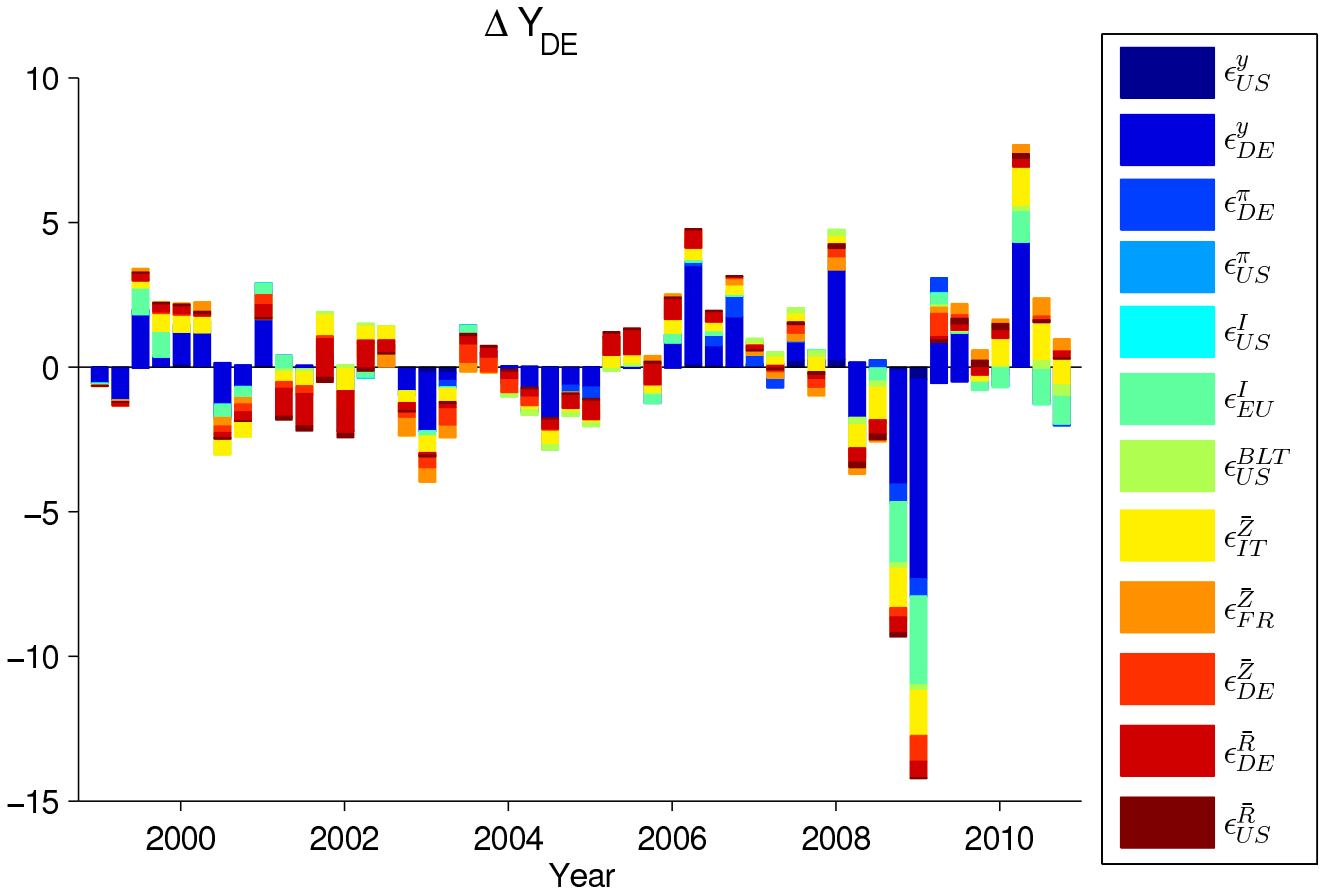
Results II

United States



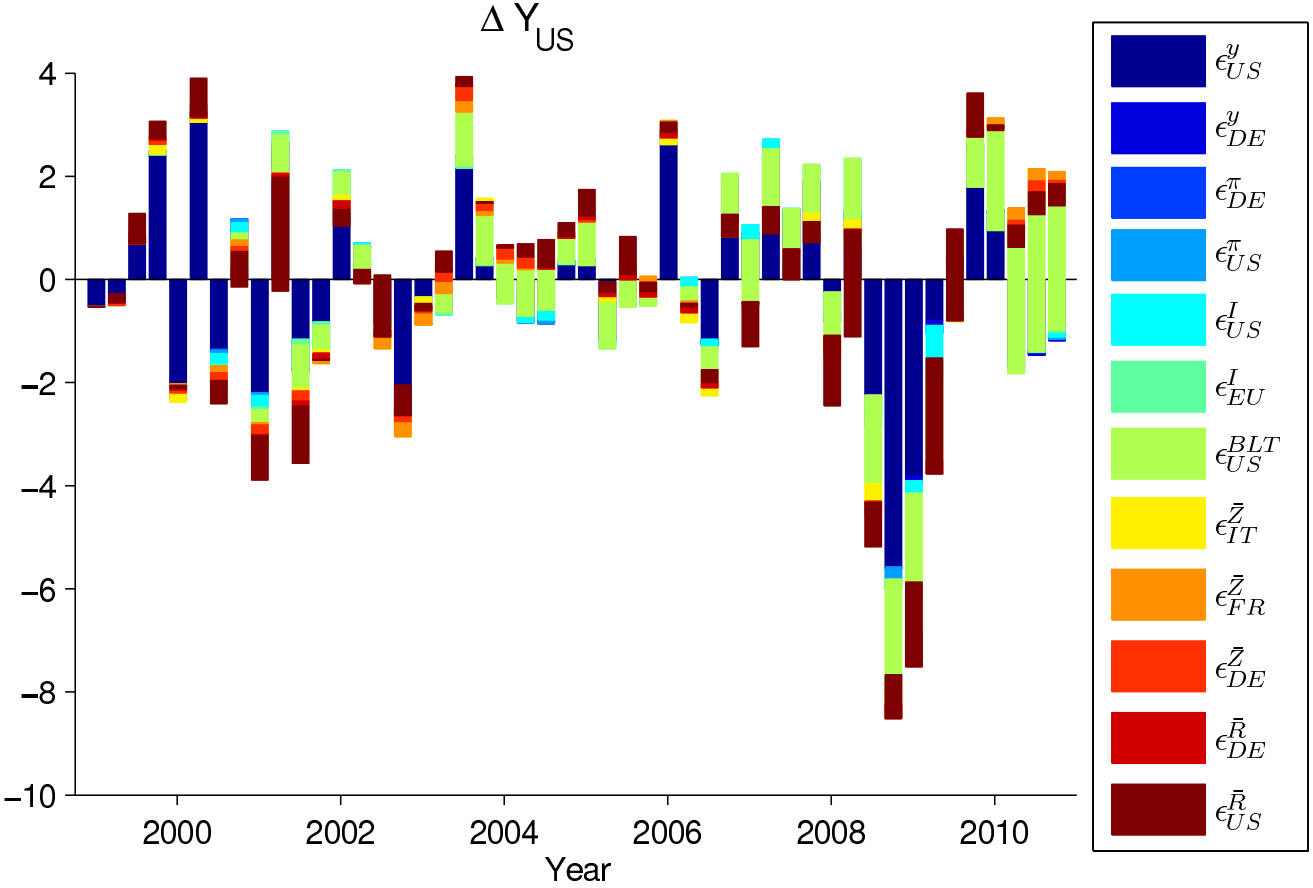
Results III

Germany



Results IV

United States



The Halle Economic Projection Model (HEPM)

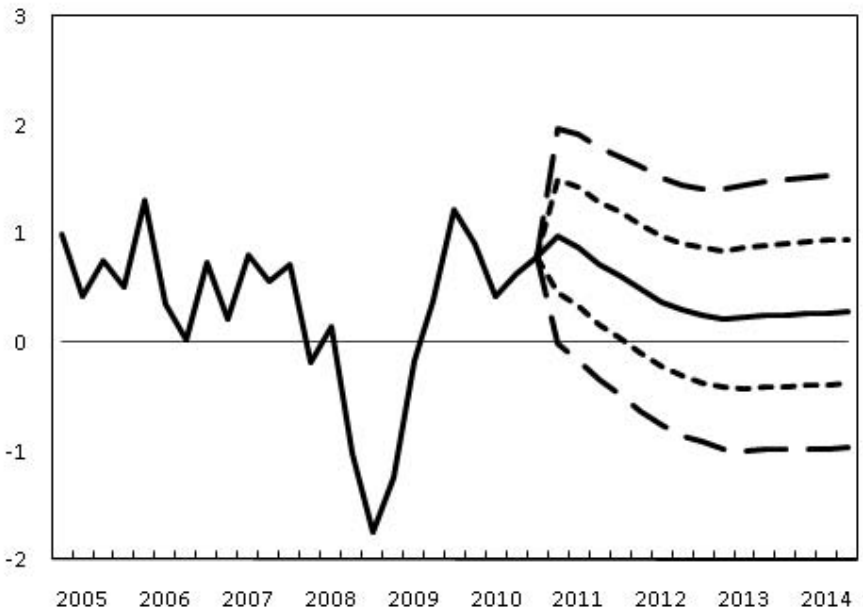
Next Steps

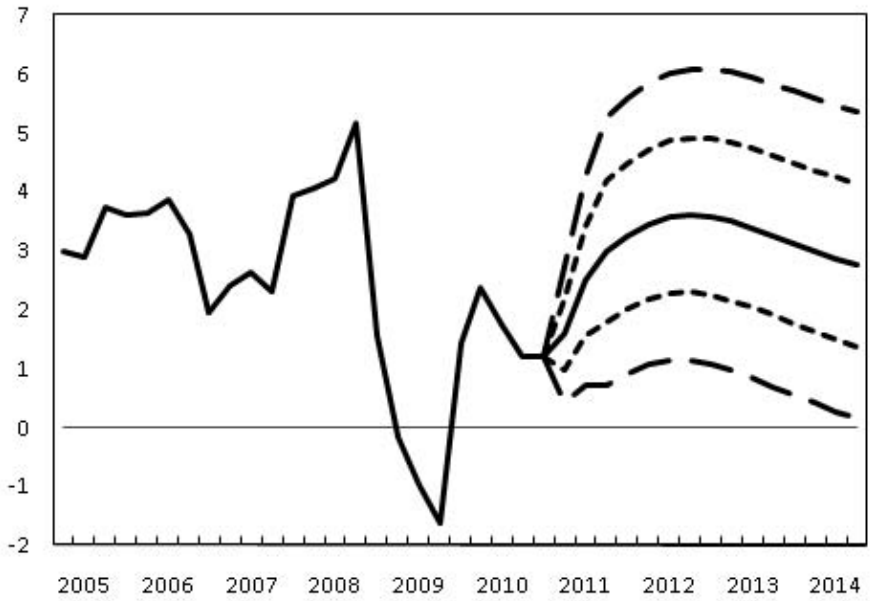
- Macroeconomic projection model to analyze and forecast the interactions between major European economies and the US
- Next steps:
  - Additional European countries
  - Additional variables / equations (government, investment, public debt etc.)
  - Forecast combination
  - Real-time implementation

Appendix

Results

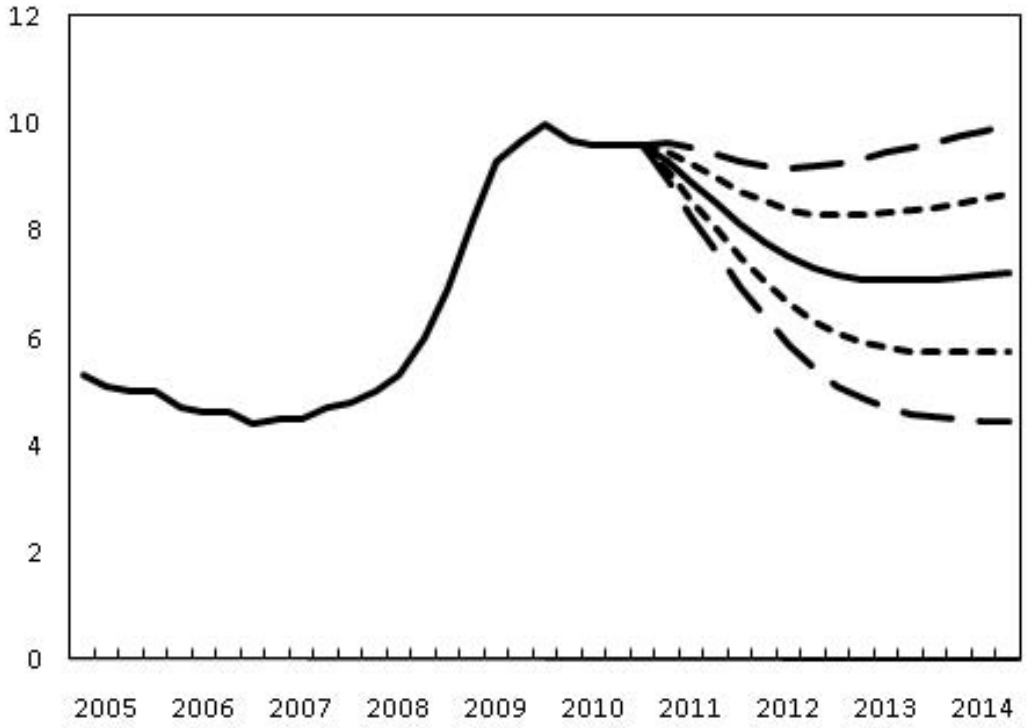
- U.S. (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- GDP growth (%, qoq, left), Inflation (%, yoy, right)





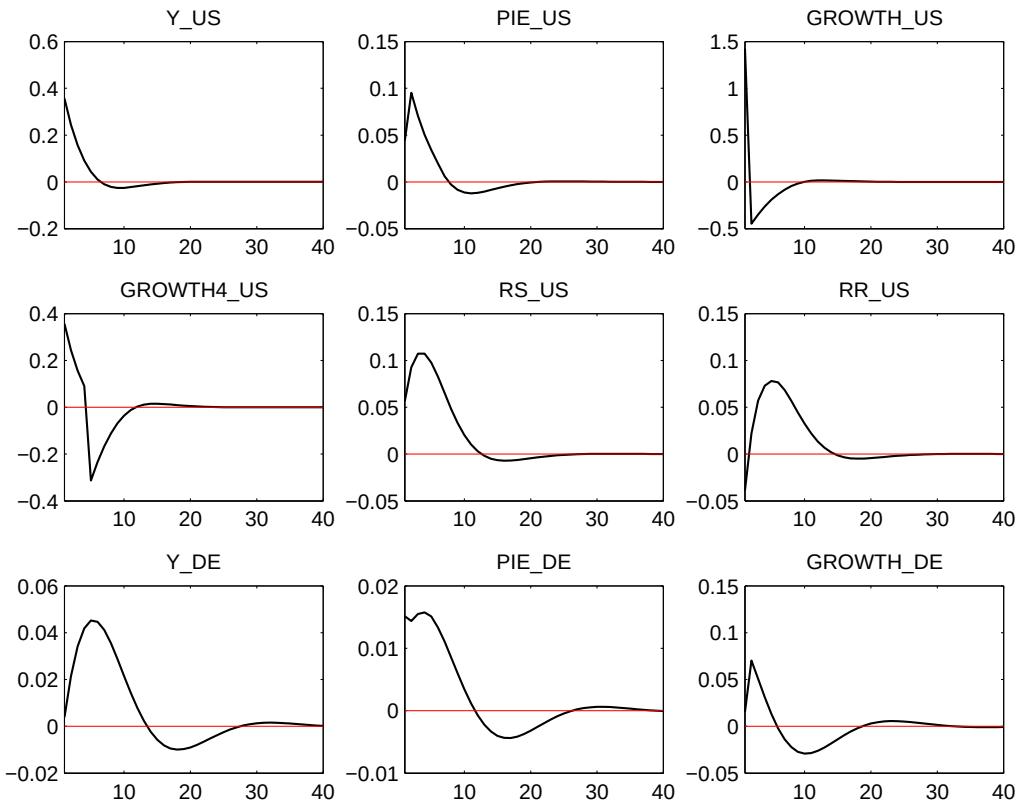
Results

- U.S. (70% and 95% confidence intervals), Sample 1999Q1-2010Q4, Forecast horizon 2011Q1-2014Q4
- Unemployment rate (%)



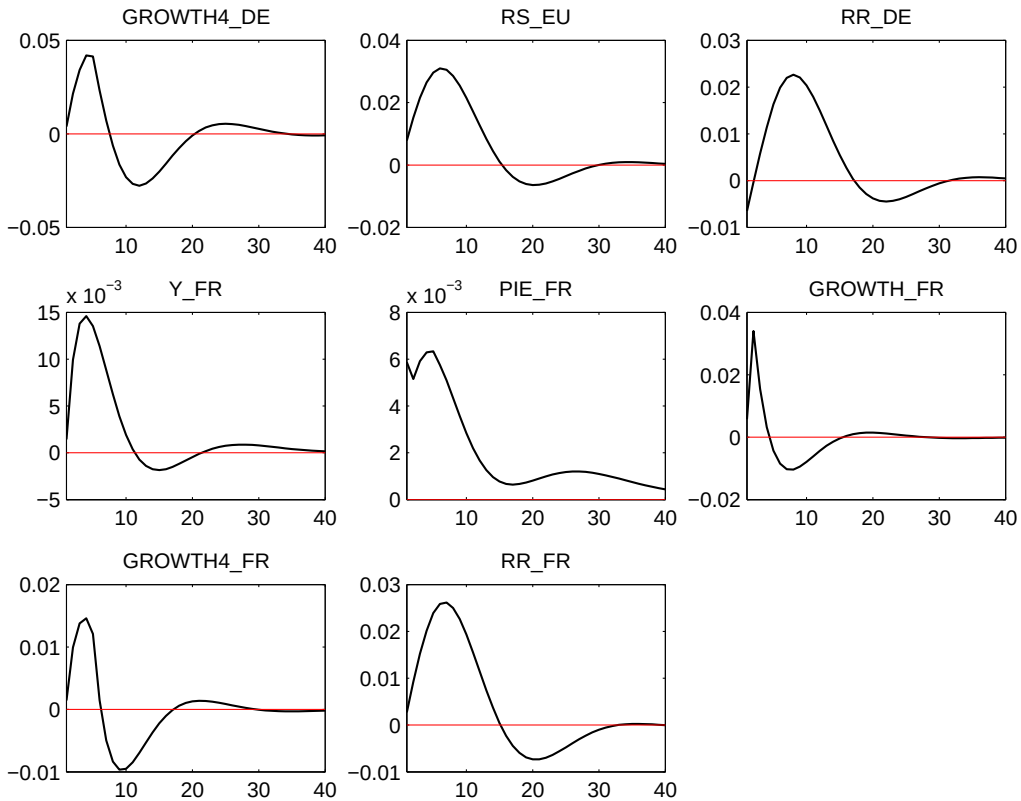
Results

- Demand shock U.S.



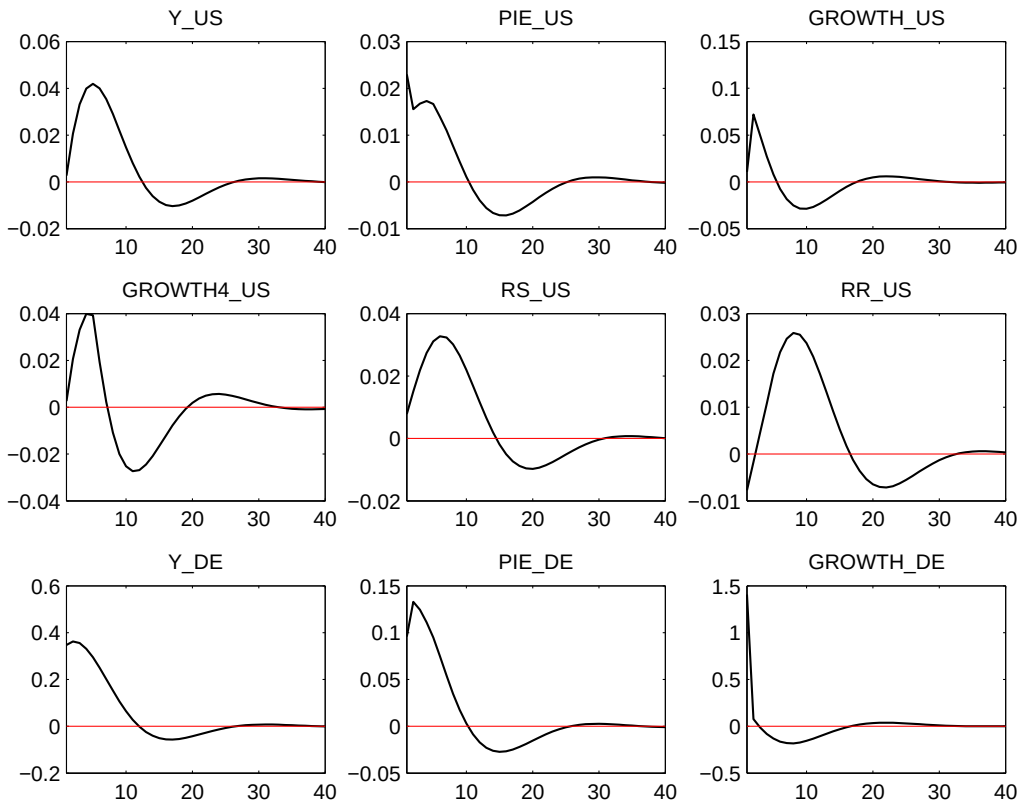
Results

- Demand shock U.S., continued



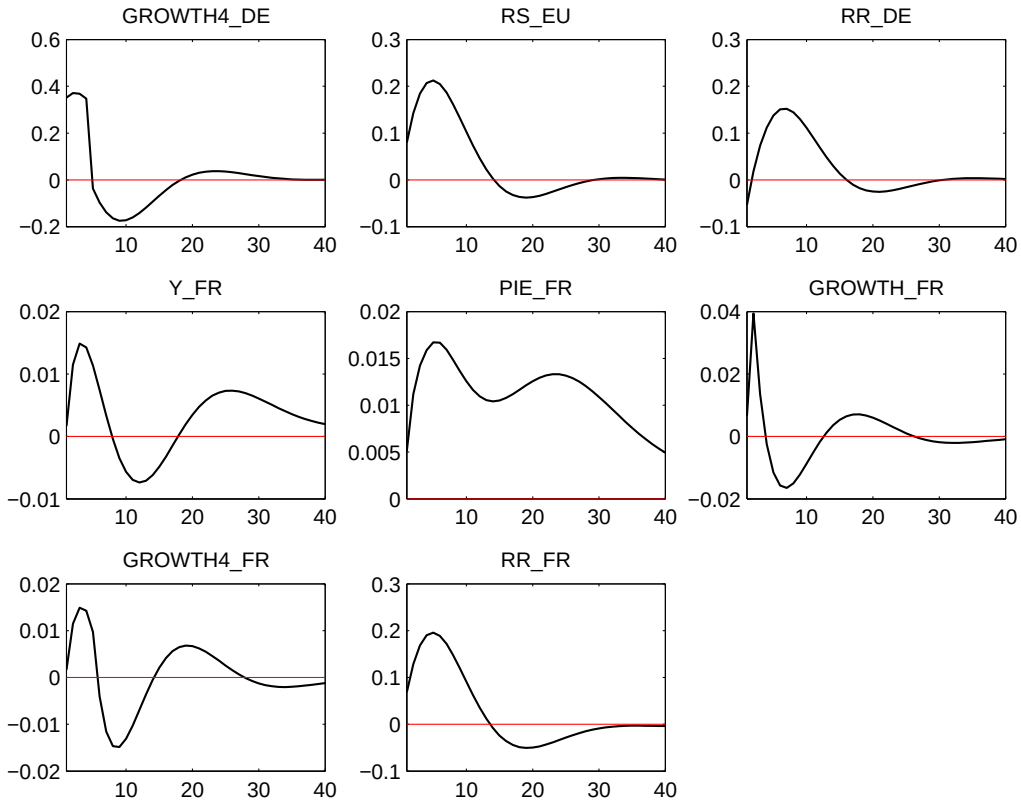
Results

- Demand shock Germany



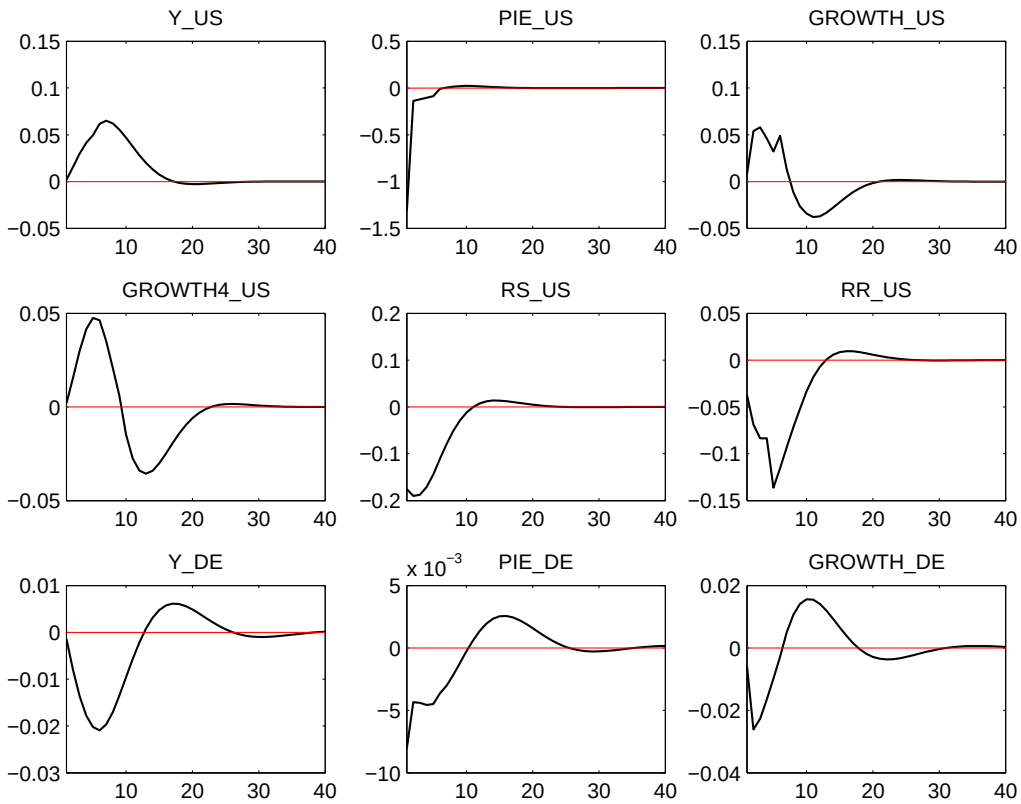
Results

- Demand shock Germany, continued



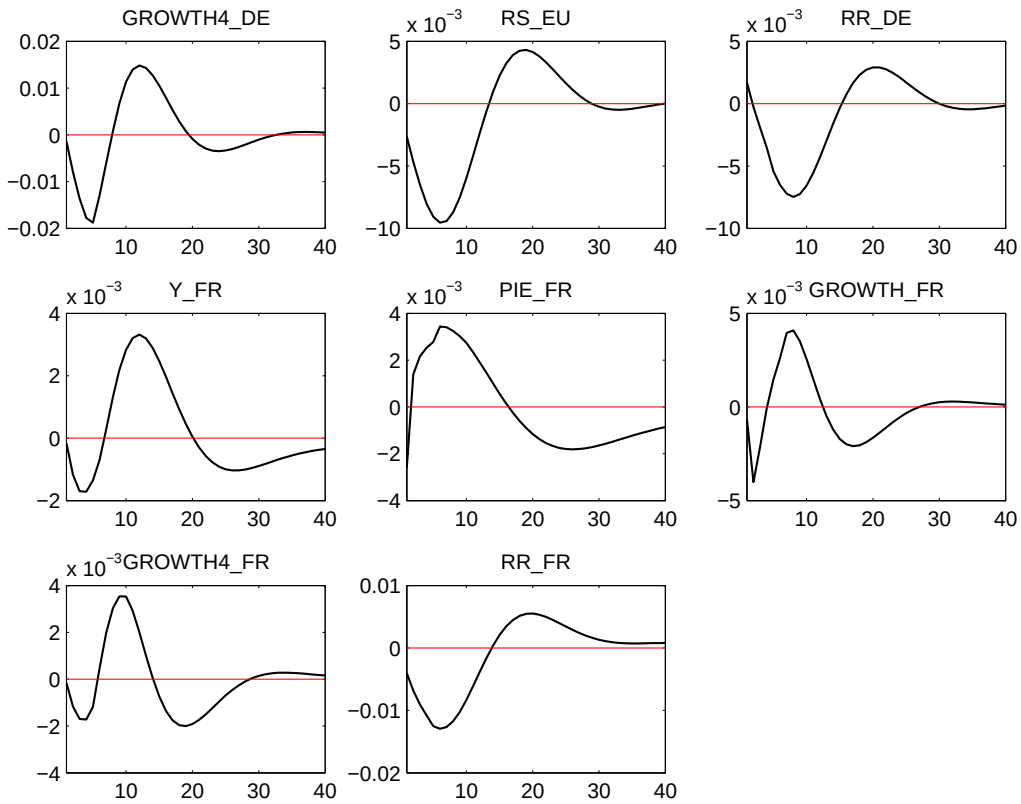
Results

- Cost-push-shock U.S.



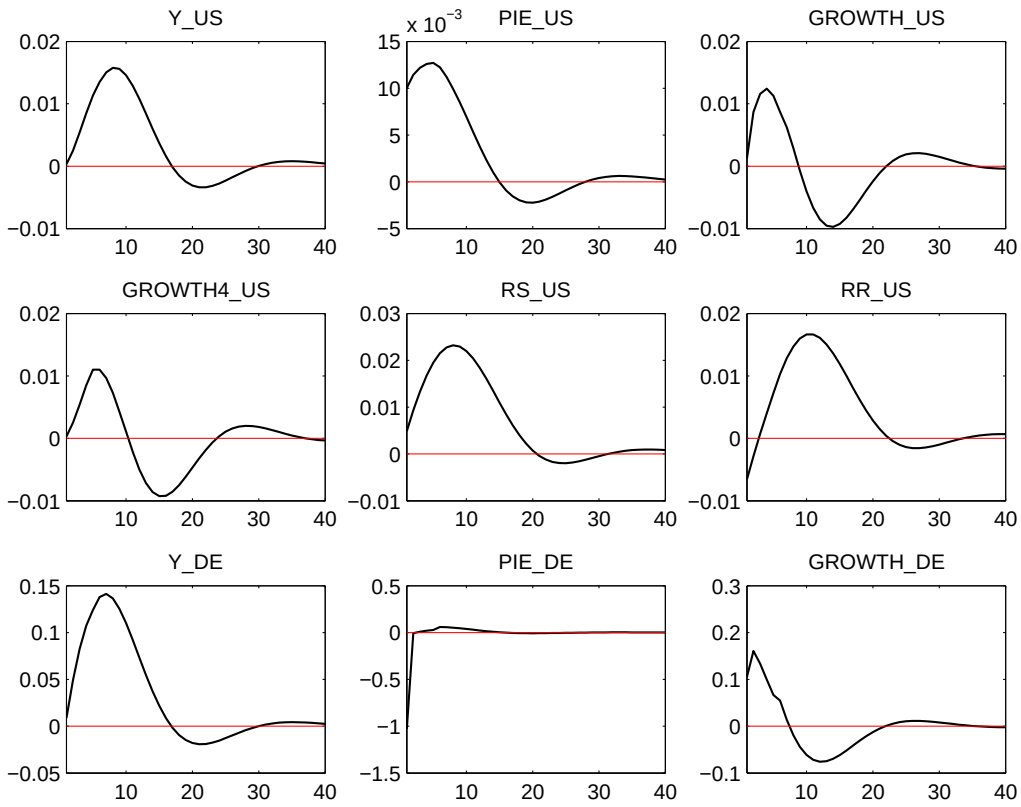
Results

- Cost-push-shock U.S., continued



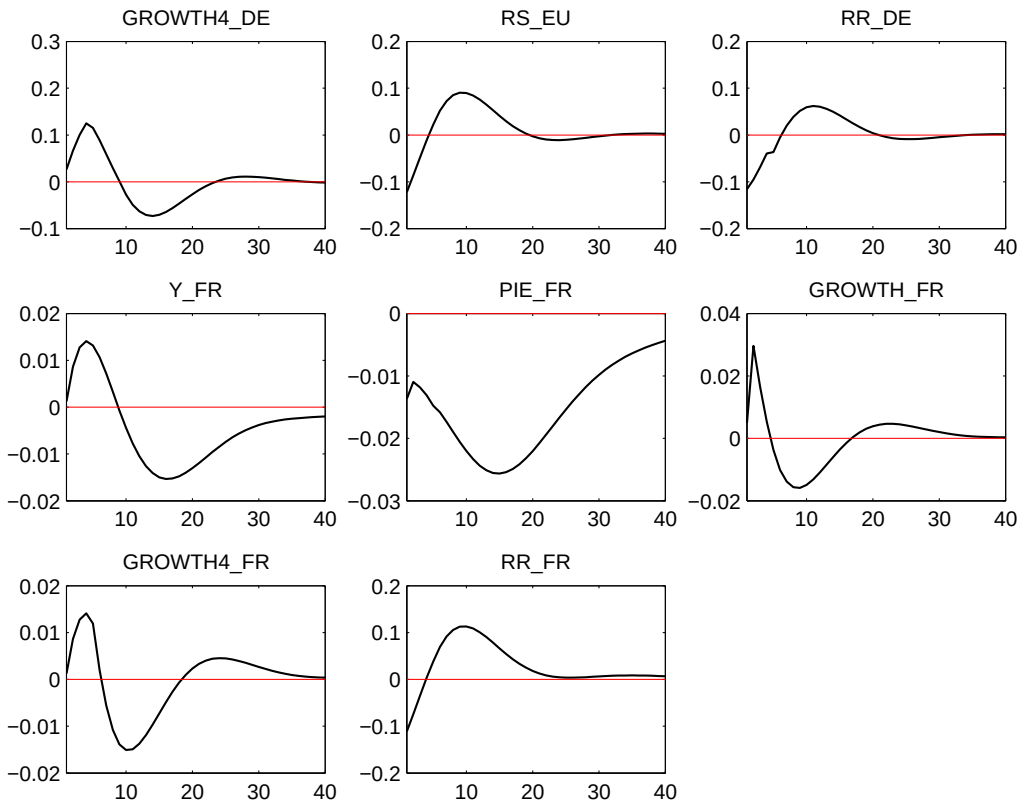
Results

- Cost-push-shock Germany



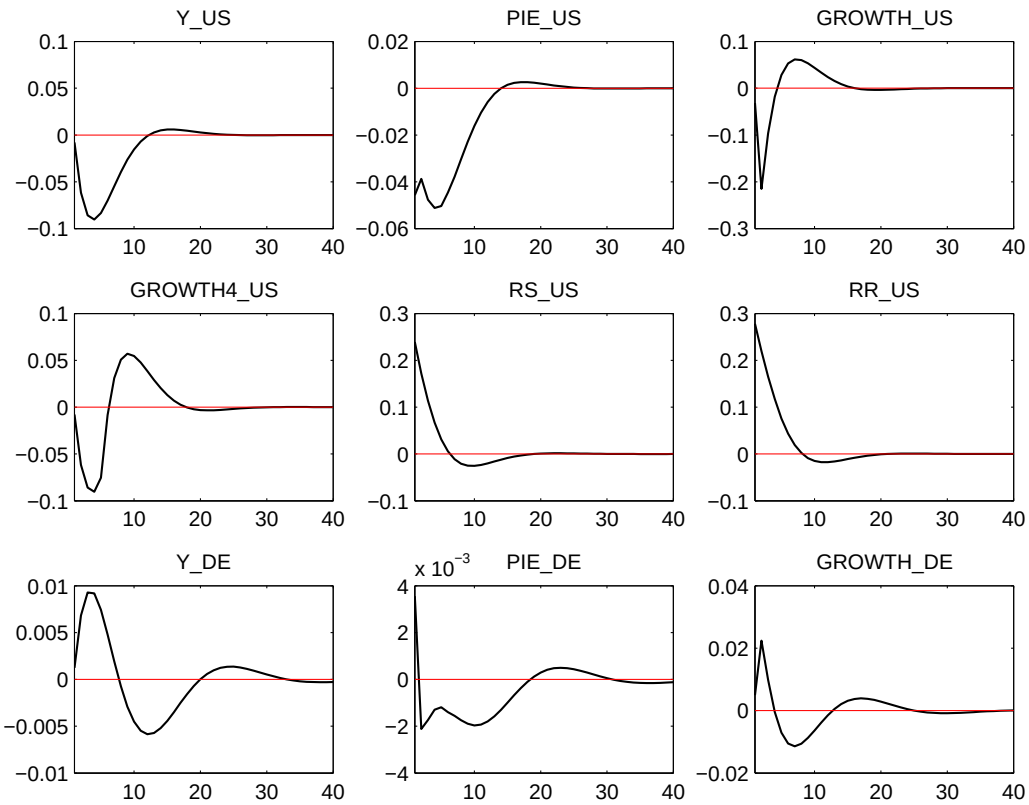
Results

- Cost-push-shock Germany, continued



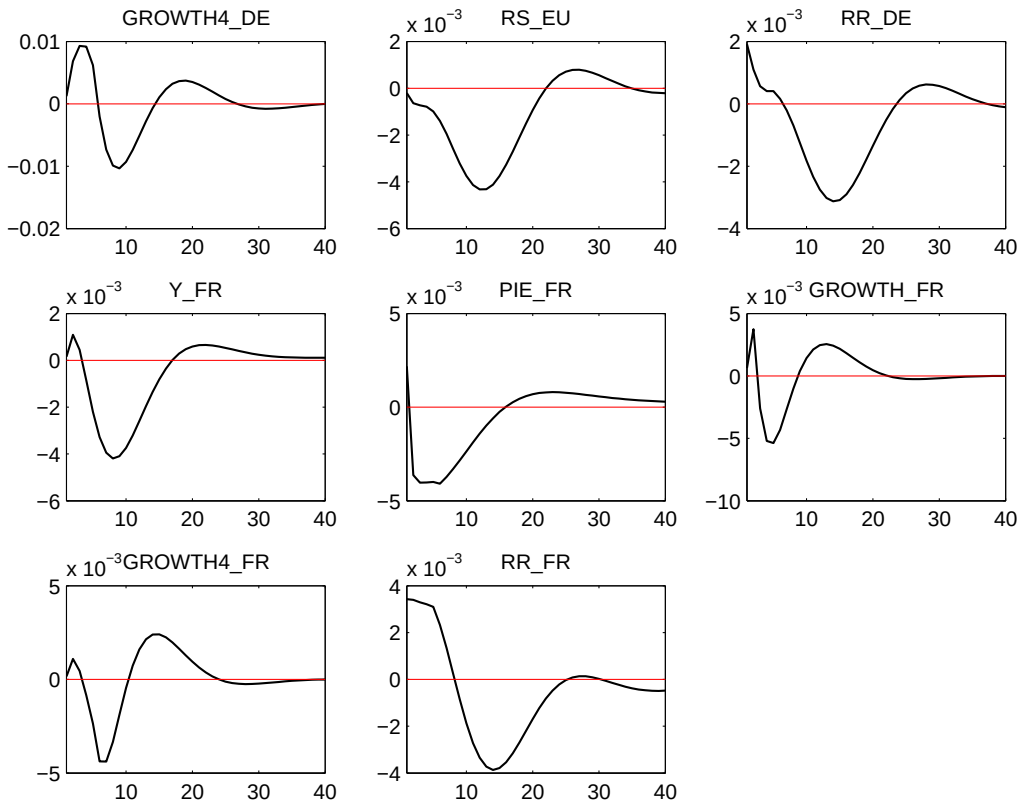
Results

- Monetary policy shock U.S.



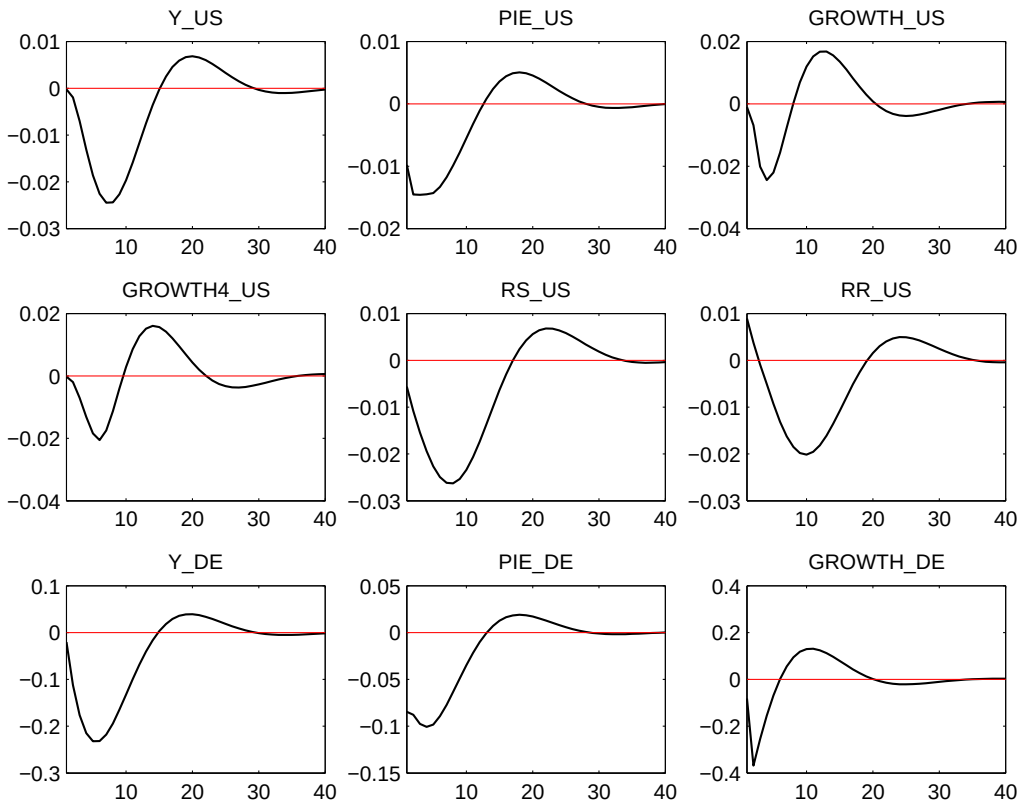
Results

- Monetary policy shock U.S., continued



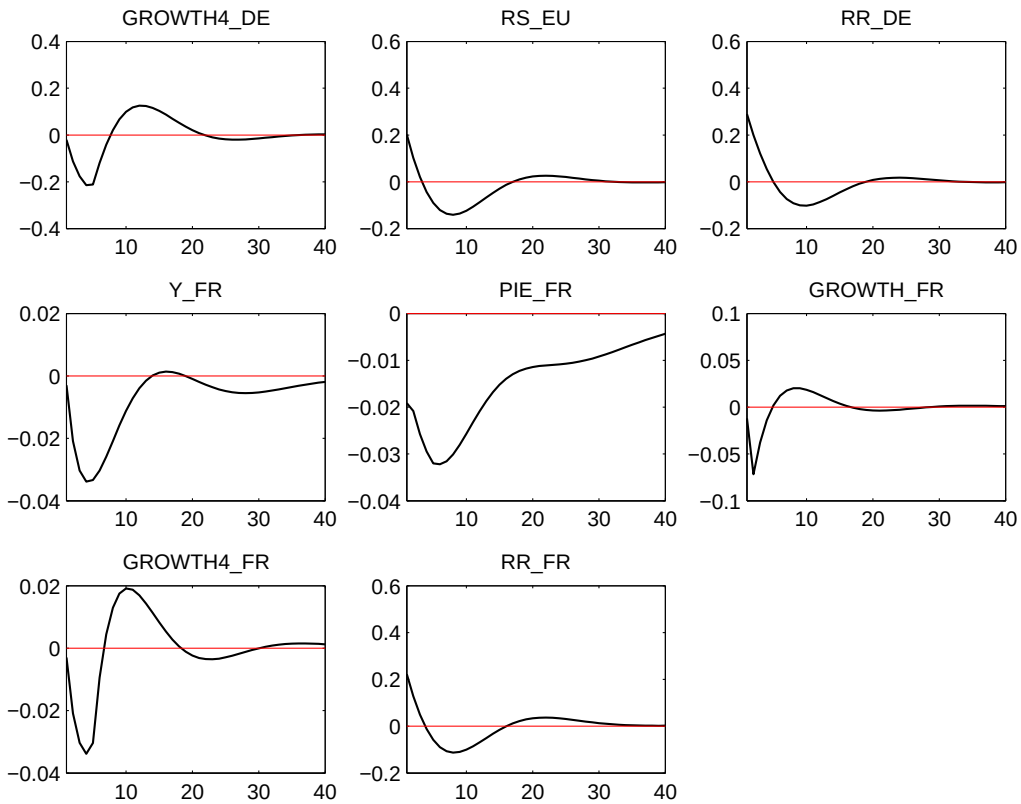
Results

- Monetary policy shock EU



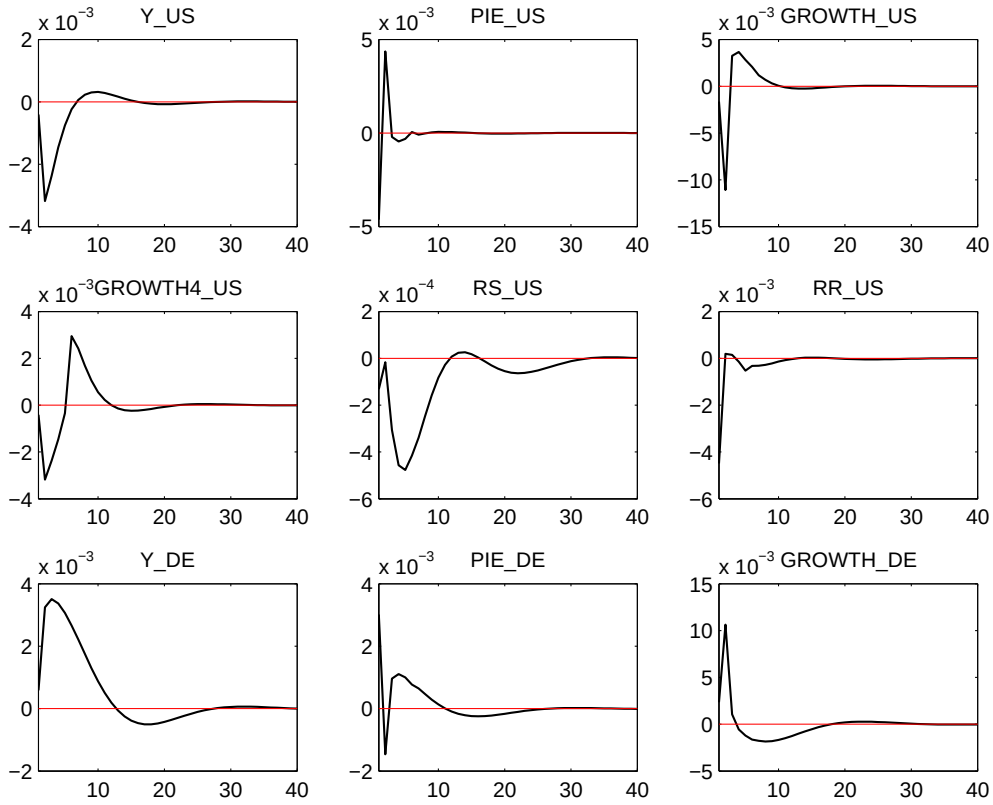
Results

- Monetary policy shock EU, continued



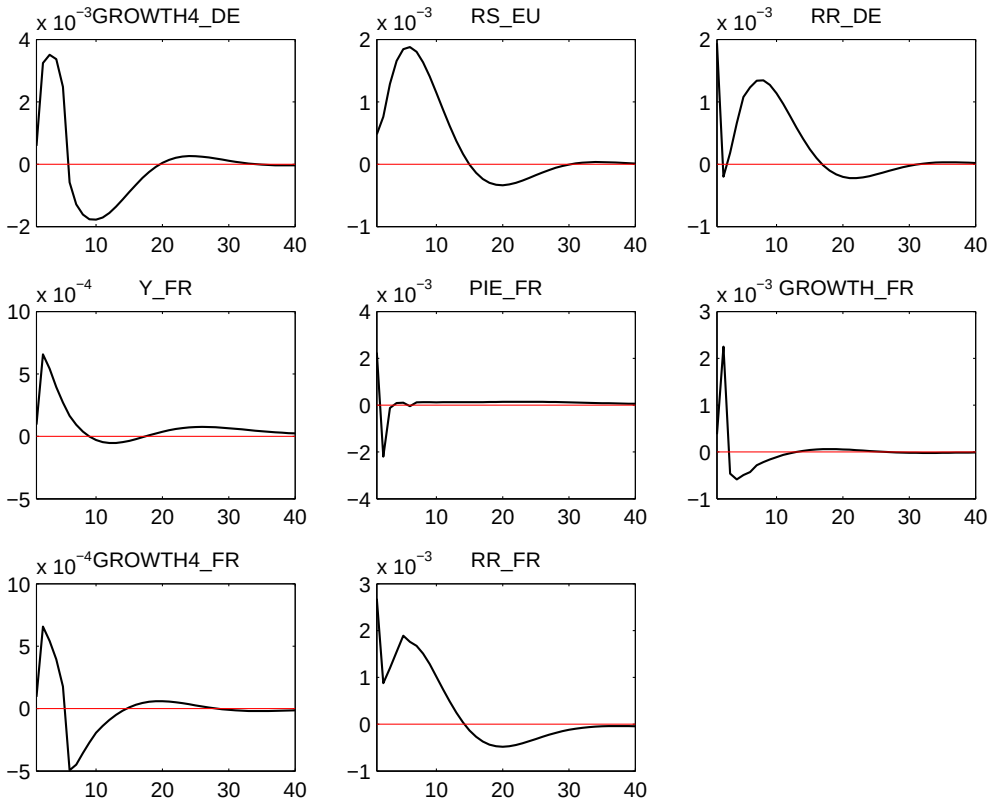
Results

- Real interest rate parity shock



Results

- Real interest rate parity shock, continued



13. Model-based Forecasting

Expectation Formation, Financial Frictions, and Forecasting Performance of Dynamic Stochastic General Equilibrium Models

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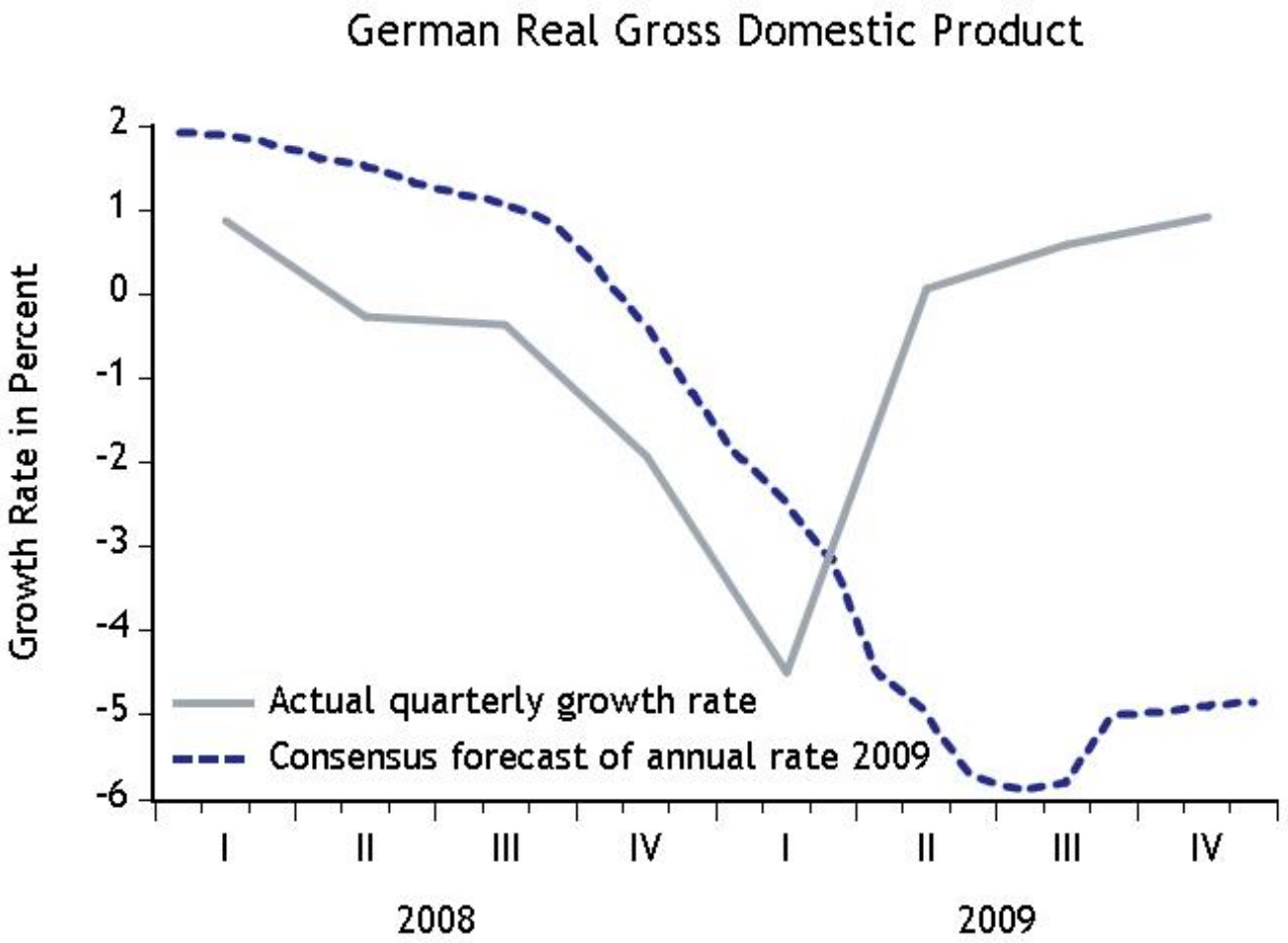
&
Halle Institute for Economic Research (IWH)

Historical Social Research 44(2), 2019, 313-339

Forecasts During the Great Recession

(Consensus Economics, Federal Statistical Office of Germany)

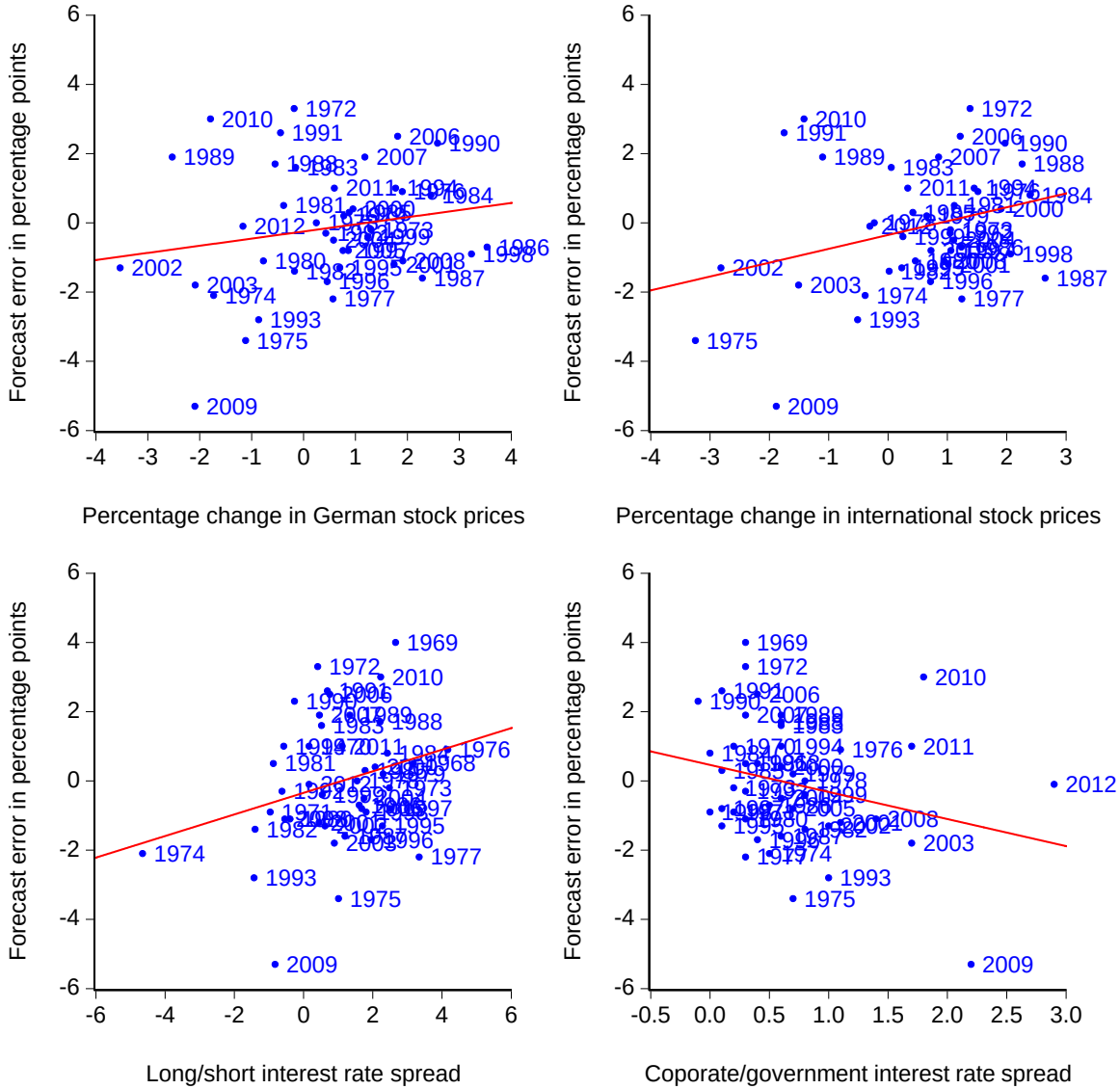


What Helps to Improve Forecast Accuracy?

- Macro profession has been blamed for not forecasting the financial crisis 2007/2008
- DSGE models useful for forecasting?
- Rational expectations?
- Financial sector?

Financial Data and Forecast Errors

(Buch and Holtemöller 2014)



Contribution

- How well perform DSGE forecasts in comparison to unrestricted VARs?
- What is the impact of different model features on forecast accuracy?
  - Adaptive learning
  - Financial frictions

Expectation Formation, Financial Frictions, and Forecasting Performance of Dynamic Stochastic General Equilibrium Models

Model Framework

- Baseline: Smets and Wouters 2007 (SW-NR) without price and wage rigidities
- Modifications
  - SW with price rigidities (SW-PR)
  - SW with wage rigidities (SW-WR)
  - SW with price and wage rigidities (SW)
  - SW with price and wage rigidities and financial accelerator (SW-FA)
- Each variant combined with adaptive learning (AL) and rational expectations (RE)

Financial Accelerator Nominal Rigidities

Adaptive Learning vs. Rational Expectations

- Rational expectations (RE): Sims 2002
- Adaptive learning (AL):

$$\xi_t = \mu_t + \mathbf{T}_t \xi_{t-1} + \mathbf{R}_t \varepsilon_t \ , \tag{1}$$

$$y_t = \mathbf{C} \xi_t \tag{2}$$

- $\mathbf{R}_t$ and $\mathbf{T}_t$ depend on structural parameters and beliefs Beliefs
- RE: $\mathbf{T}_t$ and $\mathbf{R}_t$ are time invariant

Expectation Formation, Financial Frictions, and Forecasting Performance of Dynamic Stochastic General Equilibrium Models

Data

- Quarterly seasonally adjusted data:

  GDP, consumption, investment, wages and salaries, total hours worked

  GDP deflator, short-term interest rate, spread between AAA and BAA rated corporate bonds yields

- US
  - FRED (Federal Reserve Bank of St. Louis)
  - 1954-Q3 to 2017-Q3

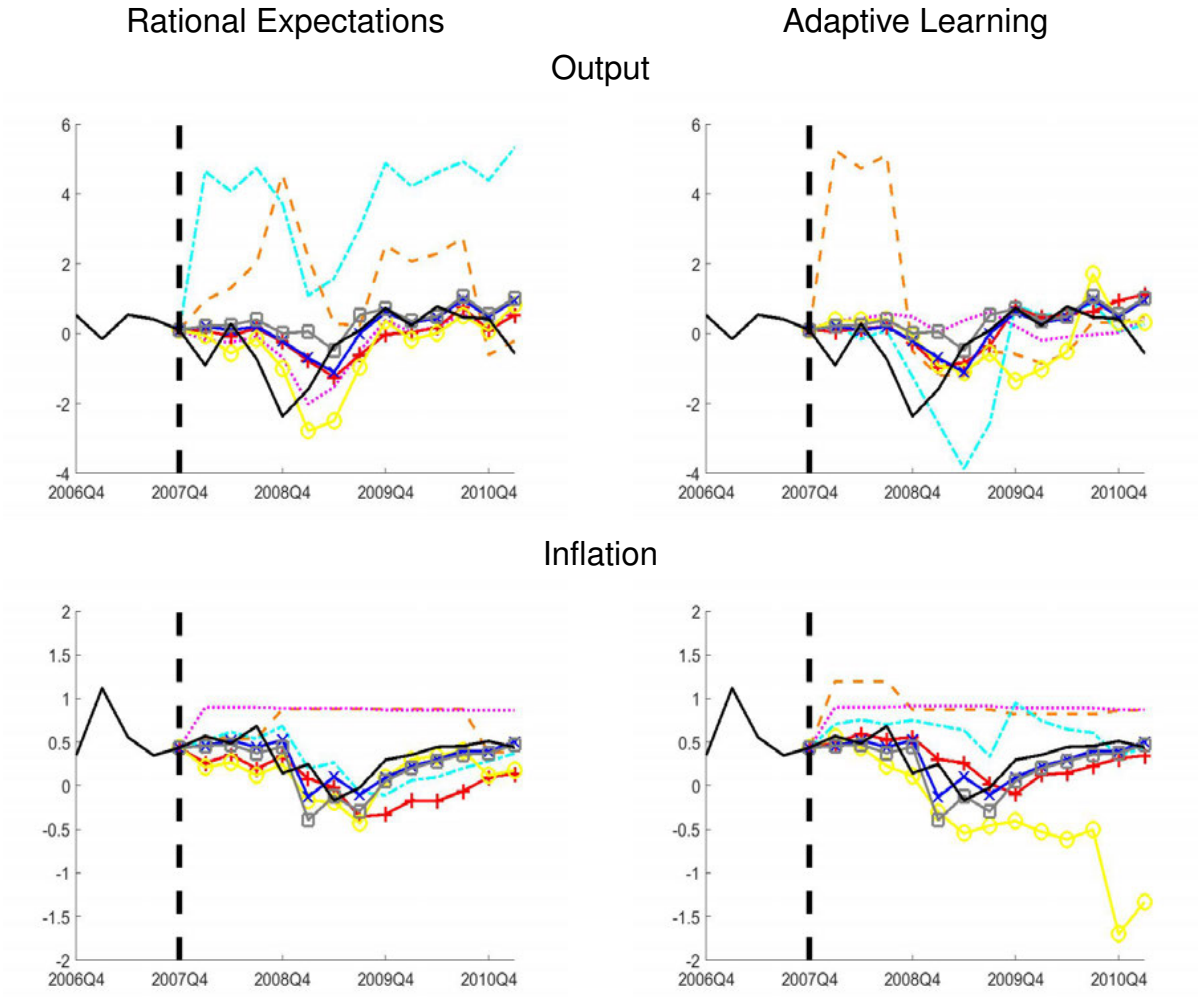
- Euro area and UK
  - Eurostat
  - Corporate bond yields (IBOXX) from Datastream
  - 1999-Q1 to 2017-Q3

Estimation

- We follow Slobodyan and Wouters 2012
- Same priors for all countries
- Prior means of trend parameters set to sample averages (inflation, output growth, interest rate)
- Standard Bayesian techniques
- Pseudo-out-of-sample forecasts
  1. Initial estimation using data until 2006-Q2
  2. Compute forecast errors at mode (point with highest likelihood found by optimization routine)
  3. Expand estimation window by one year
  4. Repeat 2-3 until 2017-Q2

Expectation Formation, Financial Frictions, and Forecasting Performance of Dynamic Stochastic General Equilibrium Models

Forecasts US



Notes: Lines depict actual data (black), forecasts: SW-NR (orange), SW-WR (magenta), SW-PR (cyan), SW-RI (red), SW-FA (yellow) and VAR without external finance premium (blue) and with external finance premium (grey).

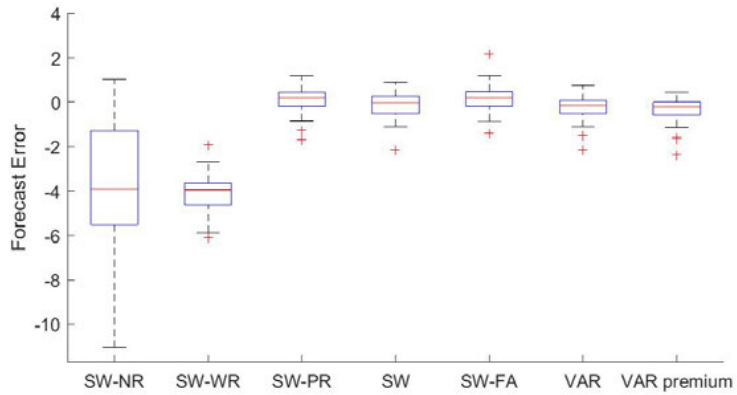
Source: own calculation Box Plots Root Mean Squared Percentage Errors External FInance Premium Short Sample

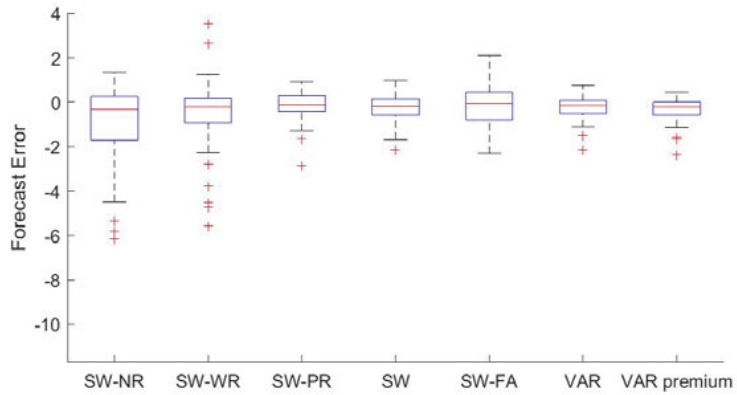
Boxplots for Forecasts US

Rational Expectations

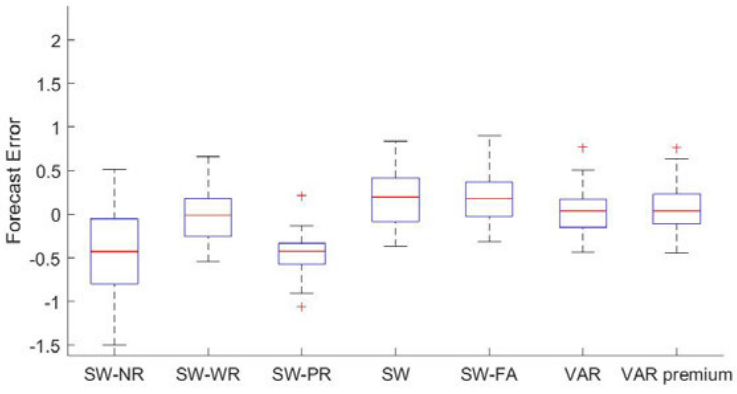
Adaptive Learning

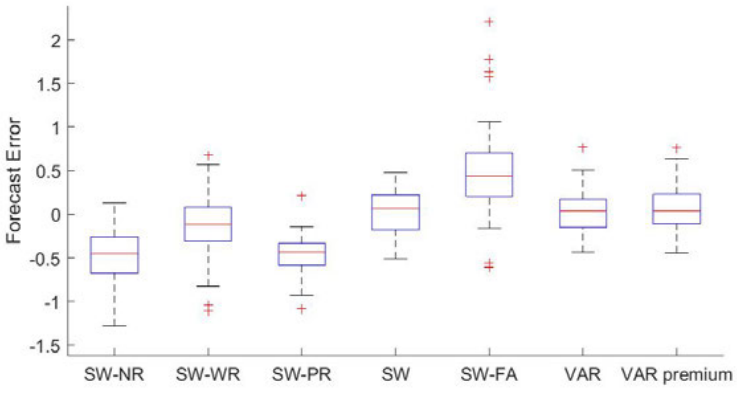
Output





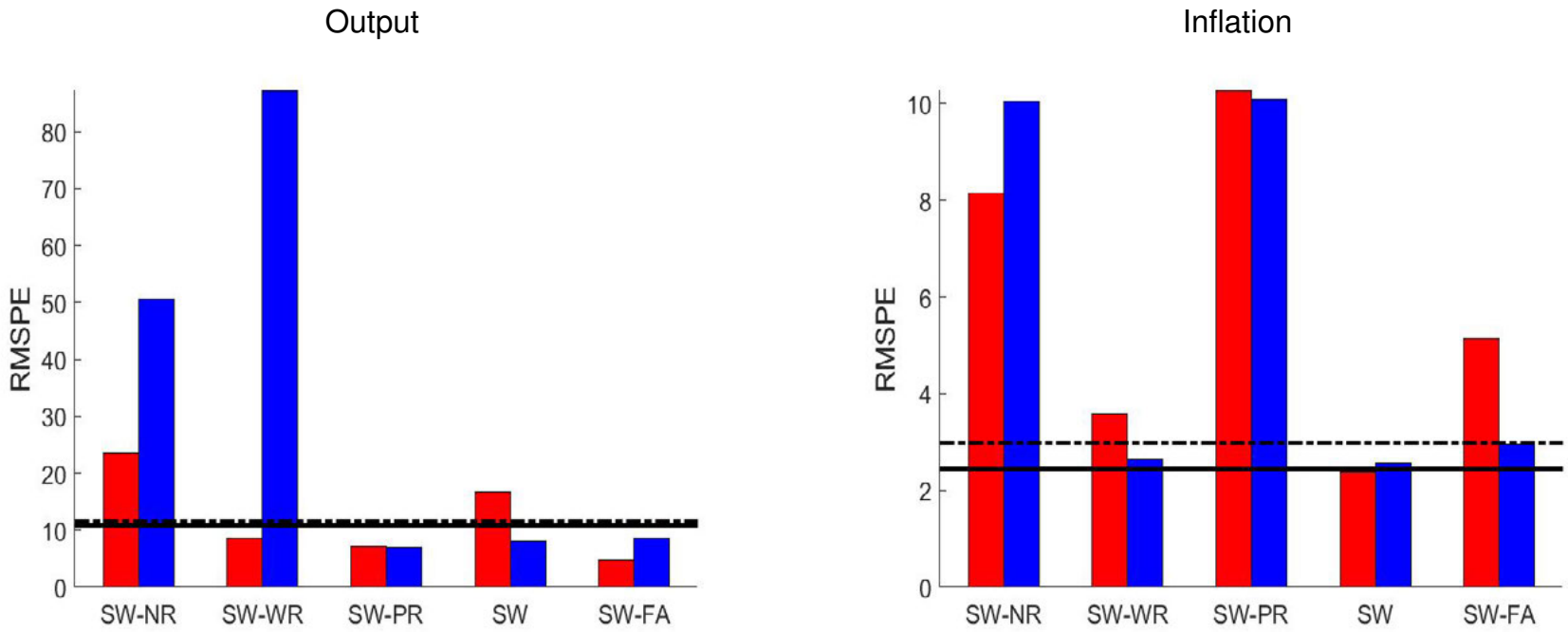
Inflation





Notes: Box plots for one-quarter ahead forecast errors for the different models.
Source: own calculation Back

Root Mean Squared Percentage Errors for Forecasts US



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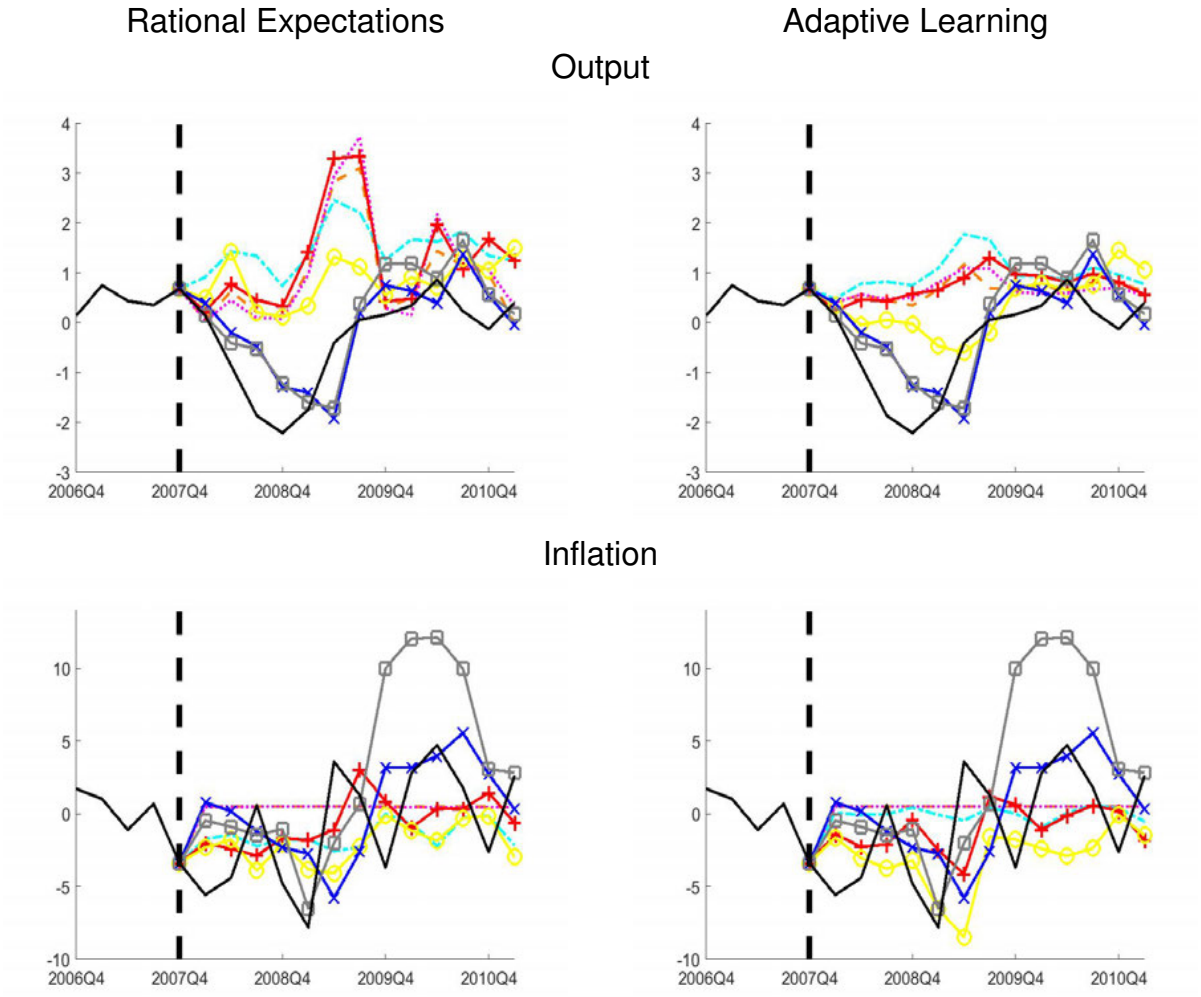
Forecasts UK



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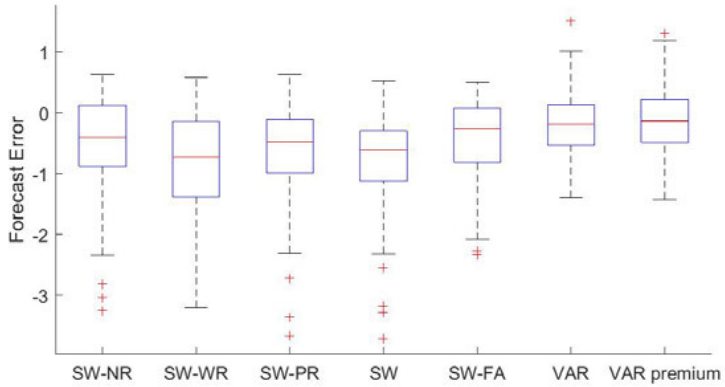
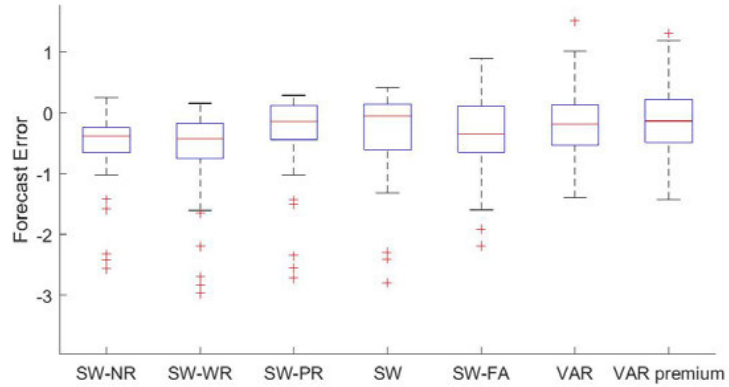
Boxplots for Forecasts UK

Rational Expectations

Adaptive Learning

Output





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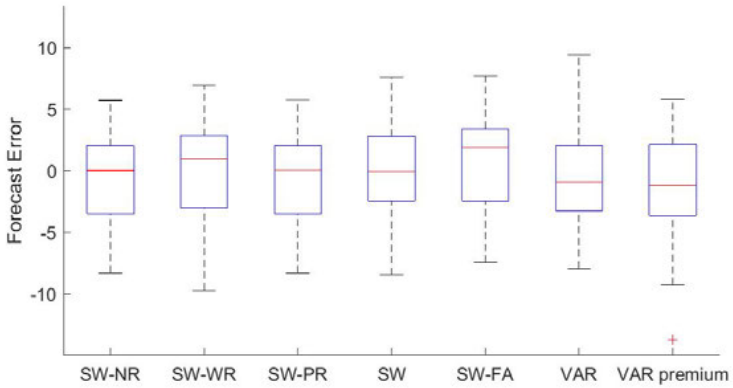

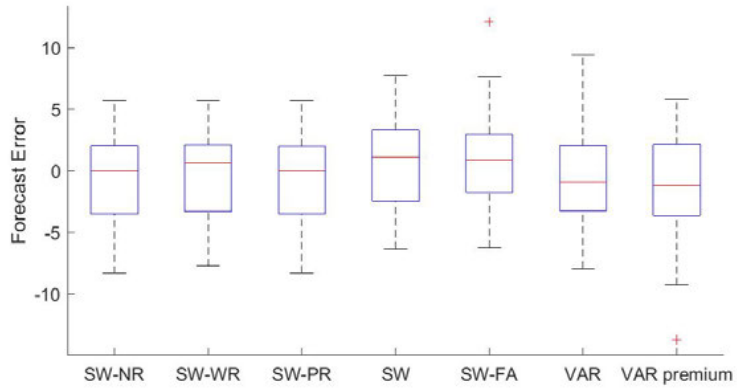

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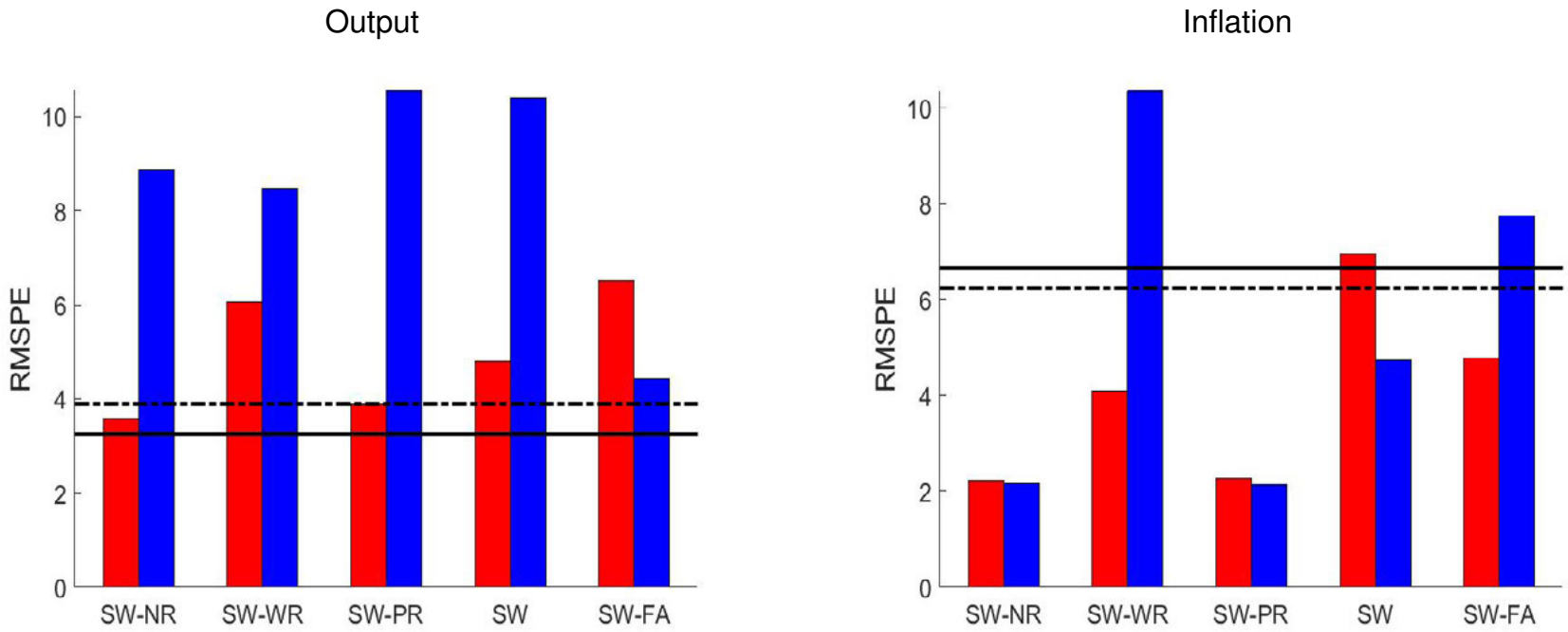
Root Mean Squared Percentage Errors for Forecasts UK



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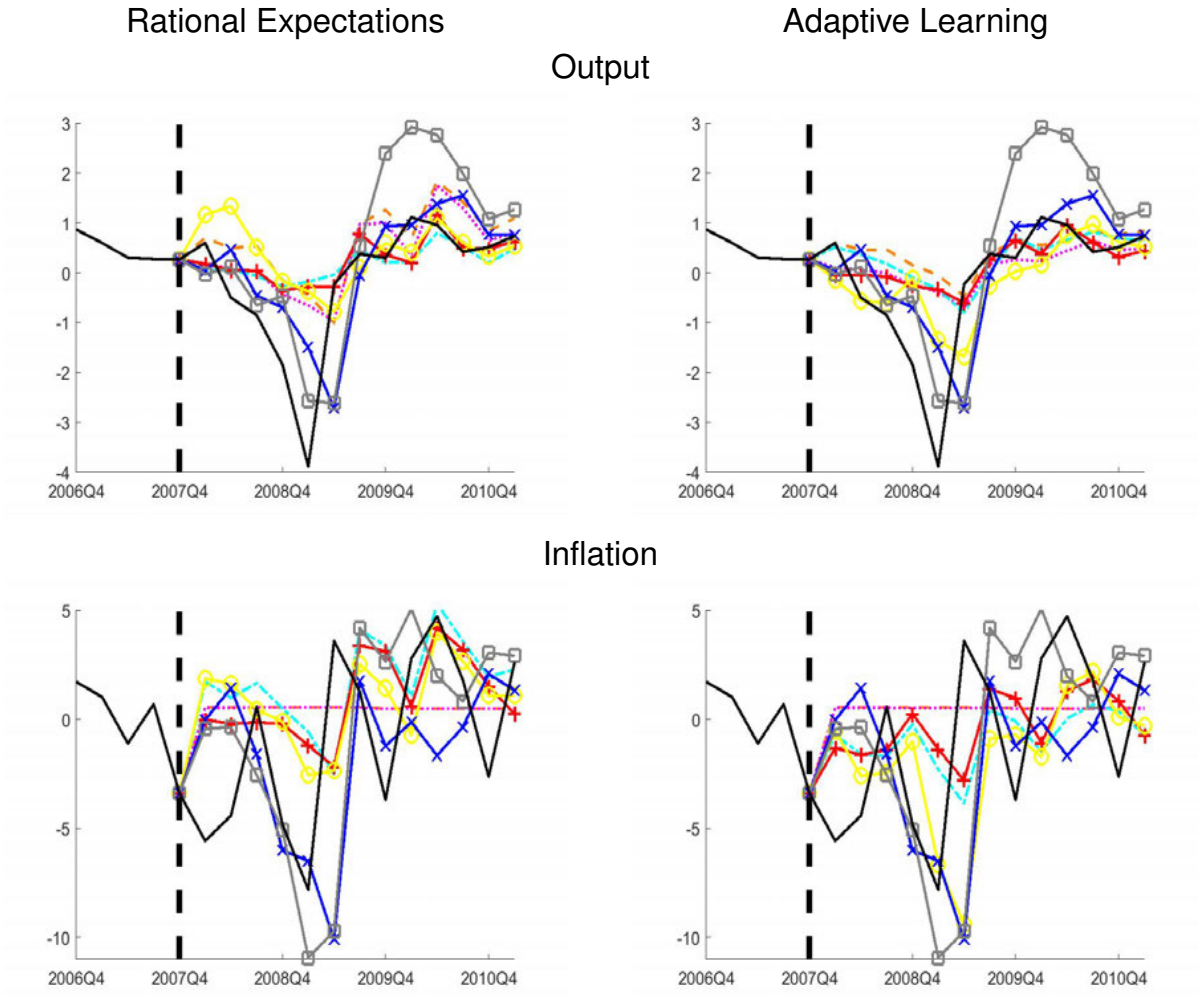
Forecasts Euro area



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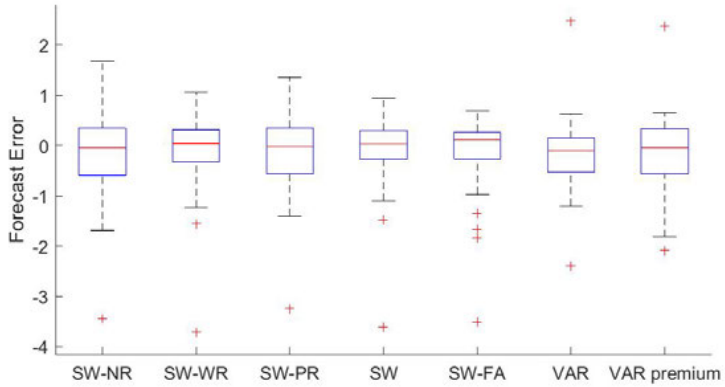
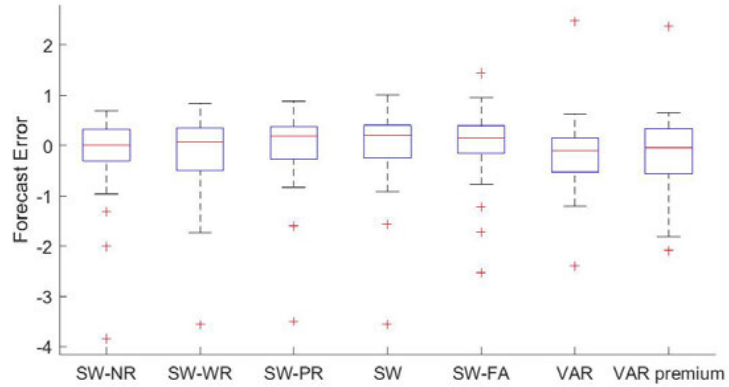
Boxplots for Forecasts Euro area

Rational Expectations

Adaptive Learning

Output





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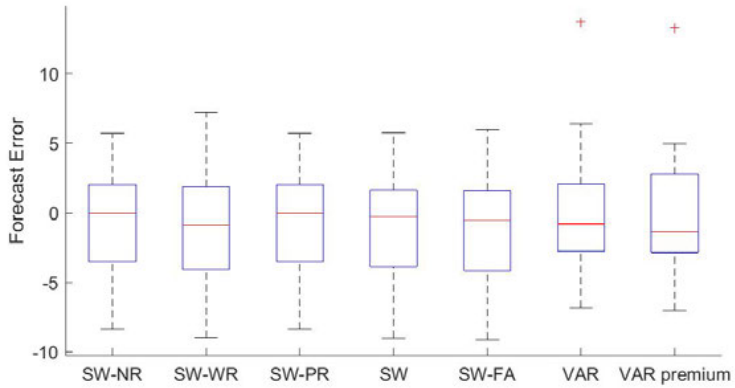

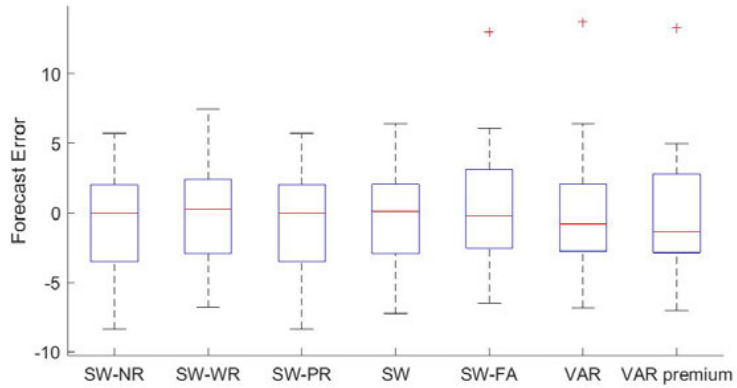

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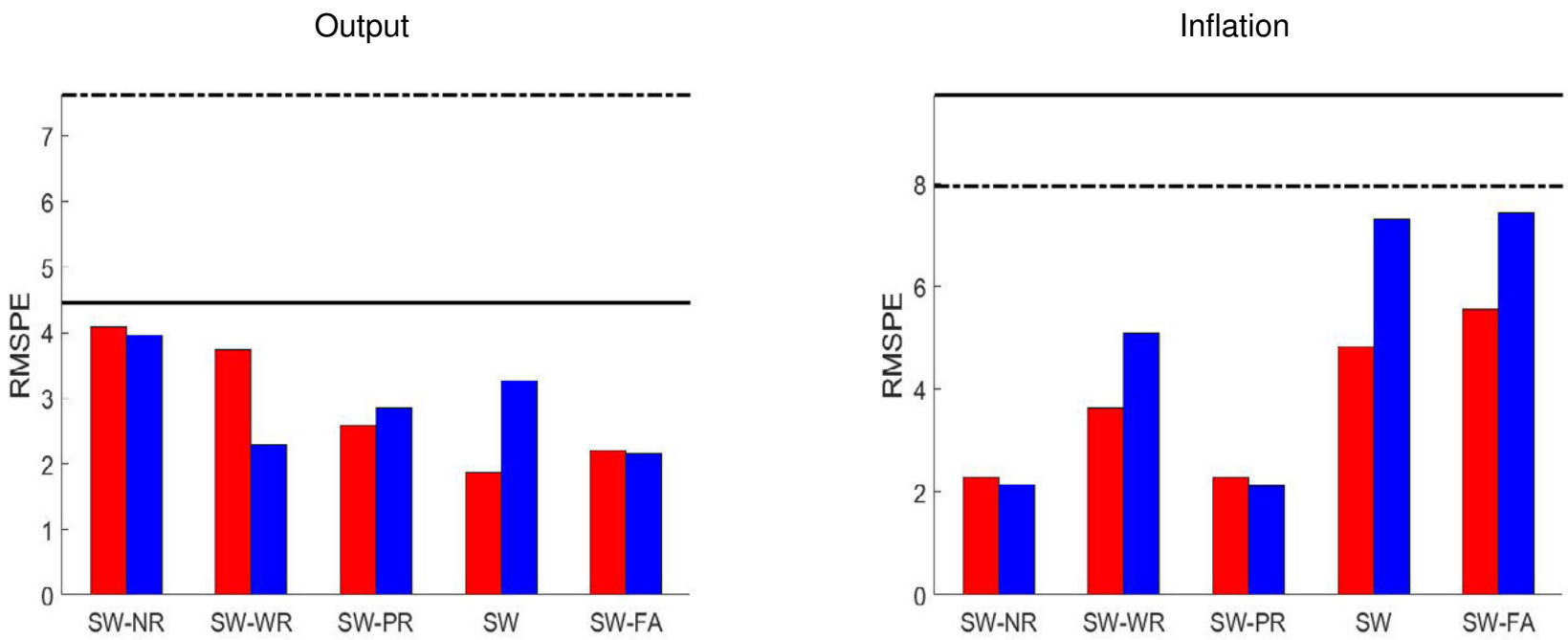
Root Mean Squared Percentage Errors for Forecasts Euro area



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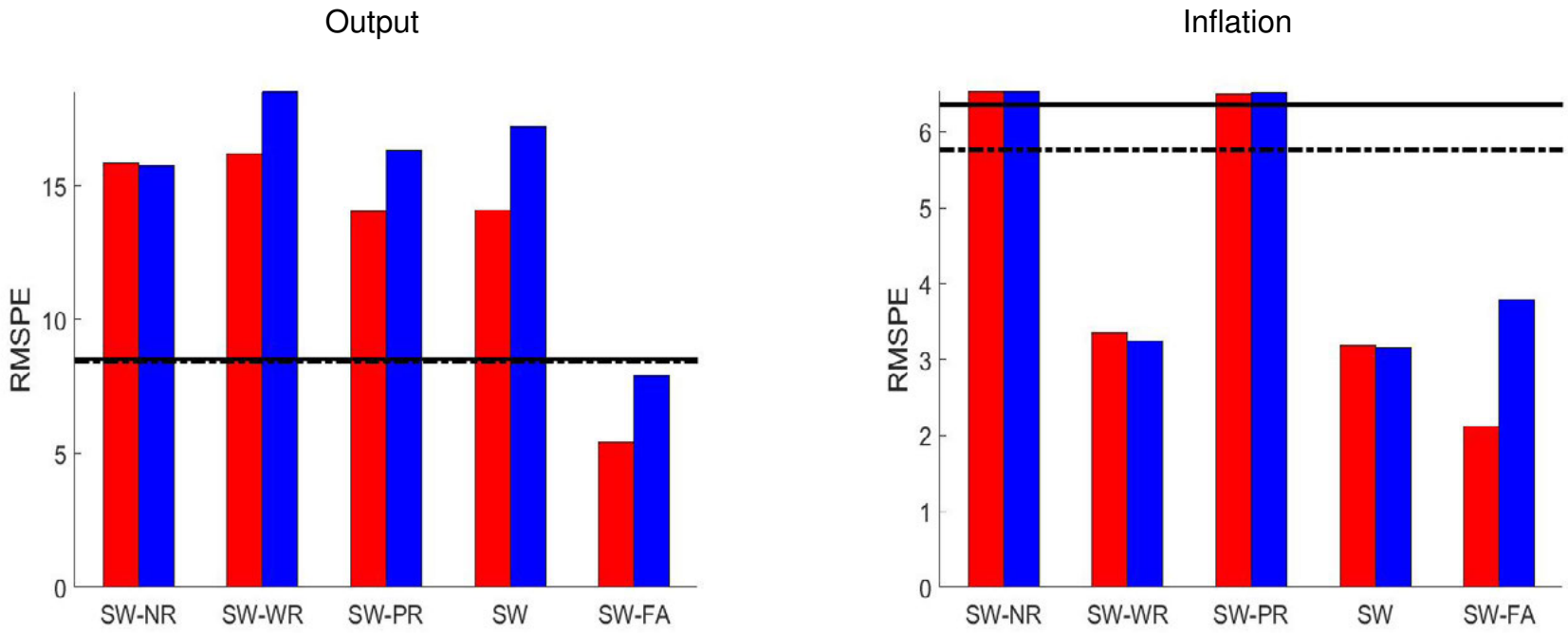
Root Mean Squared Percentage Errors for Forecasts US Short Sample



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Expectation Formation, Financial Frictions, and Forecasting Performance of Dynamic Stochastic General Equilibrium Models

Conclusions

- Additional frictions do not systemically improve forecast performance
- Including financial frictions helps to fit but not necessarily to predict the Great Recession
- Sample selection important for forecast accuracy
- Adaptive learning does not improve overall forecast performance

Thank you for your attention!

References I

Calvo, Guillermo A (1983). "Staggered prices in a utility-maximizing framework." In: *Journal of monetary Economics* 12.3, pp. 383–398.

Sims, Christopher A (2002). "Solving linear rational expectations models." In: *Computational economics* 20.1, pp. 1–20.

Slobodyan, Sergey and Raf Wouters (2012). "Learning in a medium-scale DSGE model with expectations based on small forecasting models." In: *American Economic Journal: Macroeconomics* 4.2, pp. 65–101.

Smets, Frank and Rafael Wouters (2007). "Shocks and frictions in US business cycles: A Bayesian DSGE approach." In: *The American Economic Review* 97.3, pp. 586–606.

Nominal Rigidities

- Calvo 1983 pricing
- First-order condition for optimal price and wage setting

$$\hat{\pi}_t = \begin{cases} \Pi_1^{-1}\left(\beta\gamma^{1-\sigma}\,\hat{\pi}_{t+1} + \iota^p\,\hat{\pi}_{t-1} + \Pi_2\hat{mc}_t\right) + \epsilon_t^p & \text{if } \xi^p > 0, \\ \epsilon_t^p & \text{else.} \end{cases} \tag{3}$$

$$\hat{w}_t = \begin{cases} \Pi_1^{w\,-1}\left\{\hat{w}_{t-1} + \left(\Pi_1^w - 1\right)\hat{w}_{t+1} + \iota^w\hat{\pi}_{t-1} - \Pi_3^w\hat{\pi}_t\right\} + \ldots \\ \frac{\Pi_2^w}{1+\left(\Lambda^w-1\right)\zeta^w}\left(\hat{w}_t^* - \hat{w}_t\right) + \epsilon_t^w & \text{if } \xi^w > 0, \\ \hat{w}_t^* & \text{else.} \end{cases} \tag{4}$$

- $\Pi_1 = 1 + \beta\gamma^{1-\sigma}\,\iota^p$, $\Pi_2 = \frac{\left(1-\xi^p\right)\left(1-\beta\gamma^{1-\sigma}\,\xi^p\right)}{\xi^p\left(1+\left(\Theta^p-1\right)\zeta^p\right)}$
- $\Pi_1^w = 1 + \beta\gamma^{1-\sigma}$, $\Pi_2^w = \frac{\left(1-\xi^w\right)\hat{\pi}_{t+1}\left(1-\beta\gamma^{1-\sigma}\,\xi^w\right)}{\left(1+\beta\gamma^{1-\sigma}\right)\xi^w}$, $\Pi_3^w = \left(1 + \beta\gamma^{1-\sigma}\,\iota^w\right)$,
  $\hat{w}_t^* = \hat{l}_t\,\sigma^l + \hat{c}_t\,\frac{\gamma}{\gamma-h} - \hat{c}_{t-1}\,\frac{h}{\gamma-h}$

Back

Financial Accelerator

- Resource constraint:

$$\hat{y}_t = \hat{c}_t \frac{\bar{c}}{\bar{y}} + \hat{i}_t \frac{\bar{i}}{\bar{y}} + g_t + \hat{u}_t \frac{\bar{r}^k \bar{k}}{\bar{y}} + \frac{\bar{k}}{\bar{y}} \bar{f} \left(1 - \frac{\bar{r}}{\bar{f}}\right) \left(1 - \frac{1}{l\bar{e}v}\right) (f_t + \hat{p}^k_{t-1} + \hat{k}_t). \qquad (5)$$

- Optimal price of capital:

$$\hat{p}^k_t = -\left(\hat{f}_t + b_t\right) + \frac{\bar{r}^k}{\bar{r}^k + (1-\delta)} r^k_{t+1} + \frac{1-\delta}{r^k + (1-\delta)} \hat{p}^k_{t+1}$$

- External finance costs:

$$\hat{f}_{t+1} = \hat{r}_t - \hat{\pi}_{t+1} + \omega \left(\hat{p}^k_{t+1} + \hat{k}_{t+1} - \hat{n}_t\right)$$

- Law of motion for net worth:

$$\frac{1}{v\,f} \hat{n}_t = l\bar{e}v \hat{f}_t - \omega \left(l\bar{e}v - 1\right) \left(\hat{p}^k_{t-1} + \hat{k}_t\right) - \left(l\bar{e}v - 1\right) \left(\hat{r}_{t-1} - \hat{\pi}_t\right) + \left[\omega(lev - 1) + 1\right] \hat{n}_{t-1}$$

- Leverage ratio:

$$\hat{lev}_t = \hat{p}^k_t + \hat{k}_t - \hat{n}_{t-1}$$

Back

Forecast Models

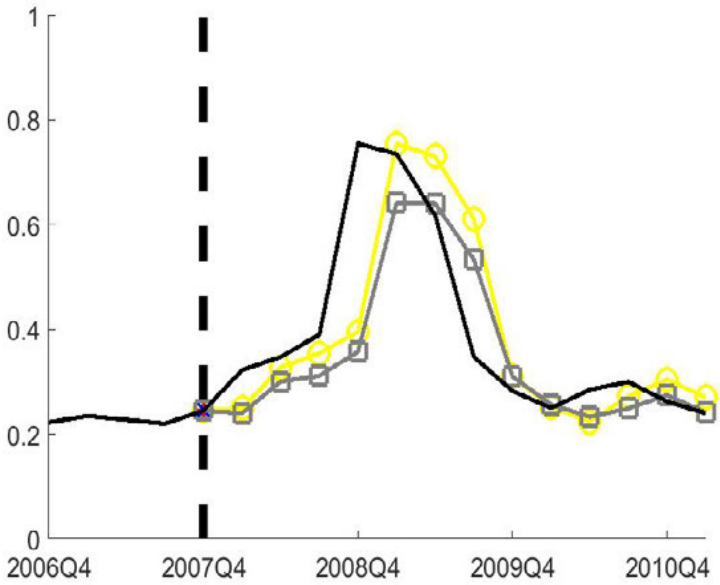
- $\mathbf{y}_t^f$ all variables appearing with a lead are forecasted using AR(1) model ($\mathbf{X}_{t-1}$ regressors)
- β_t coefficients are updated to improve forecasts using the Kalman Filter
- noise $\mathbf{u}_t$ is normally distributed

$$
\begin{aligned}
\mathbf{y}_t^f &= \mathbf{X}_{t-1}^\top \beta_t + \mathbf{u}_t \\
\beta_{t|t} &= \beta_{t|t} + \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}[\Sigma + \mathbf{X}_{t-1}^\top \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}]^{-1}(\mathbf{y}_t^f - \mathbf{X}_{t-1}^\top \beta_{t|t-1}), \\
\beta_{t+1|t} - \bar{\beta} &= \mathbf{F}(\beta_{t|t} - \bar{\beta}), \\
\mathbf{P}_{t|t} &= \mathbf{P}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}[\Sigma + \mathbf{X}_{t-1}^\top \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}]^{-1}\mathbf{X}_{t-1}^\top \mathbf{P}_{t|t-1}, \\
\mathbf{P}_{t+1|t} &= \mathbf{F}\mathbf{P}_{t|t}\mathbf{F}^\top + \mathbf{V},
\end{aligned}
$$

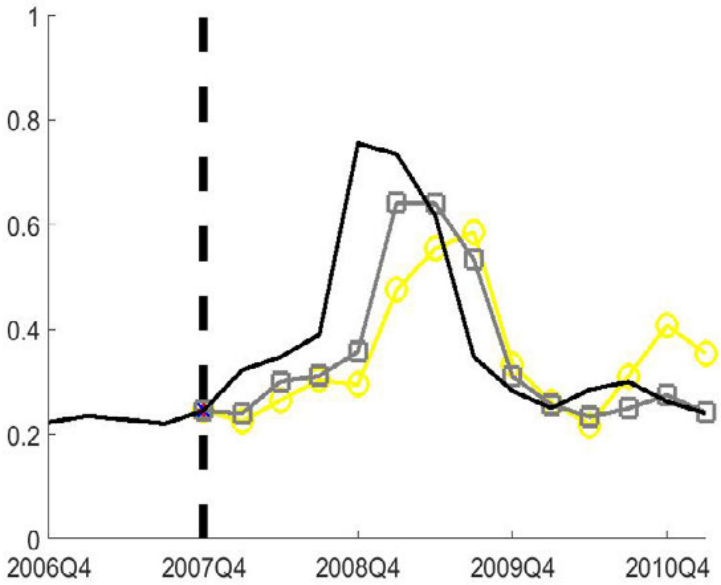
Back

Forecasts US Short Sample External Finance Premium

Rational Expectations



Adaptive Learning

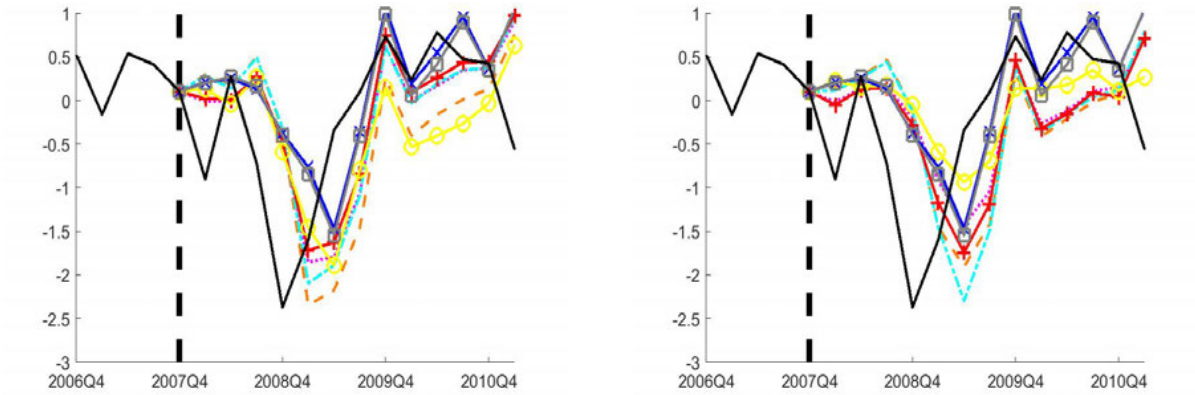


Notes: Lines depict actual data (black), forecasts: SW-FA (yellow) and VAR with external finance premium (grey).
Source: own calculation Back

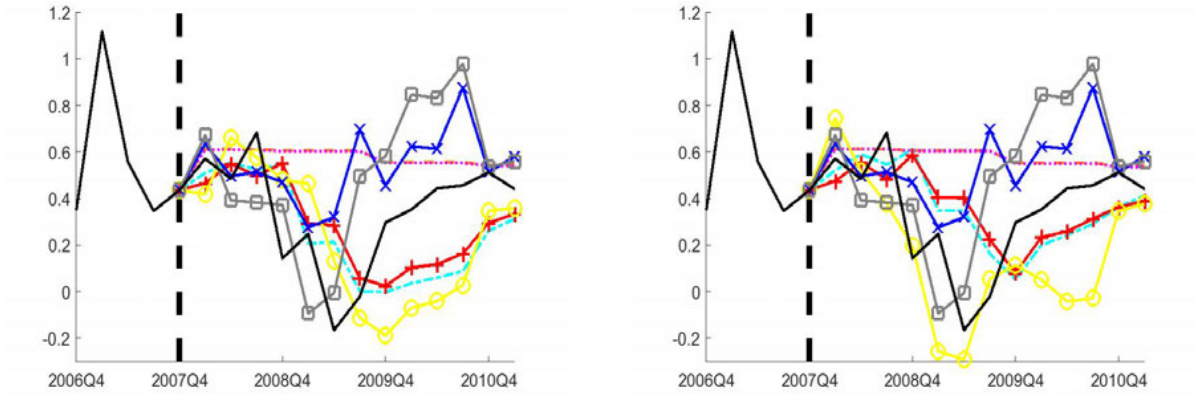
Forecasts US Short Sample

Rational Expectations | Adaptive Learning

Output



Inflation



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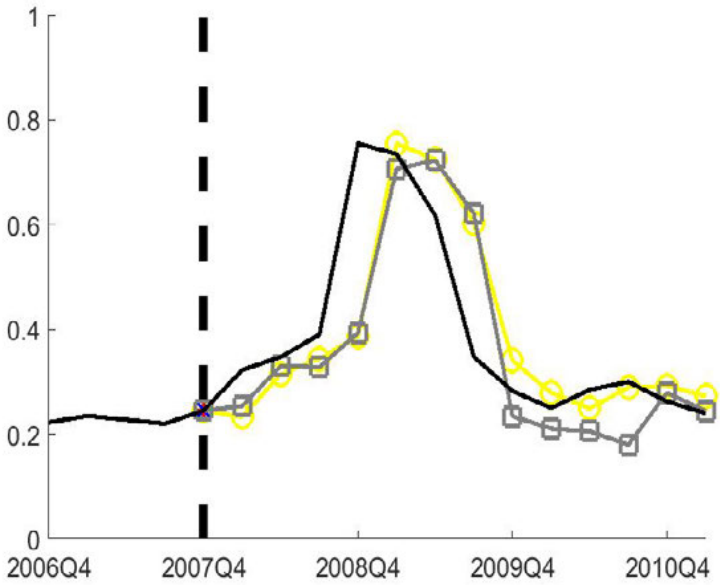
Forecasts US Short Sample External Finance Premium

Rational Expectations



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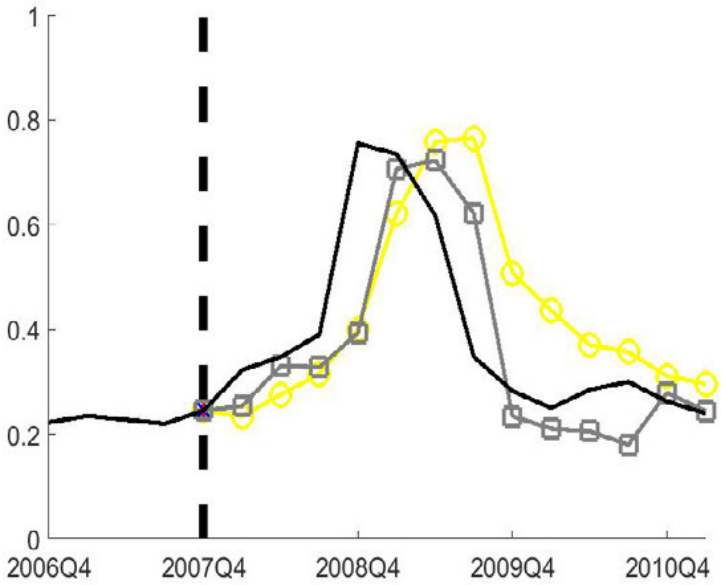

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Boxplots for Forecasts US Short Sample

Rational Expectations

Adaptive Learning

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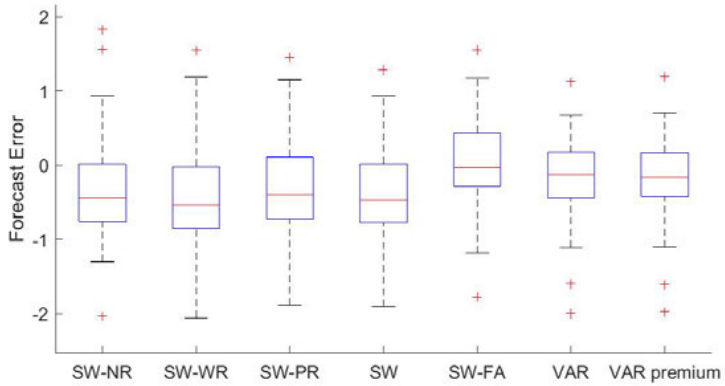

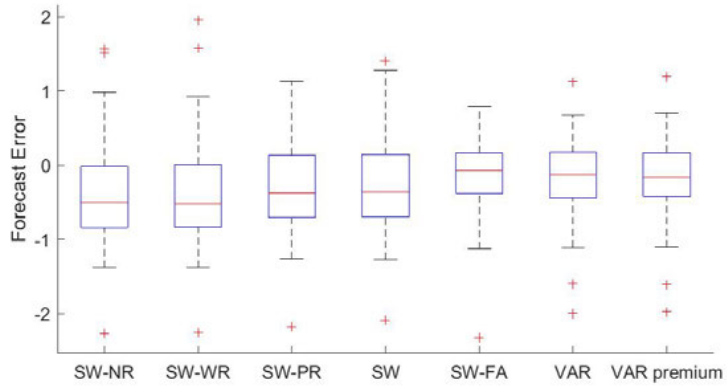

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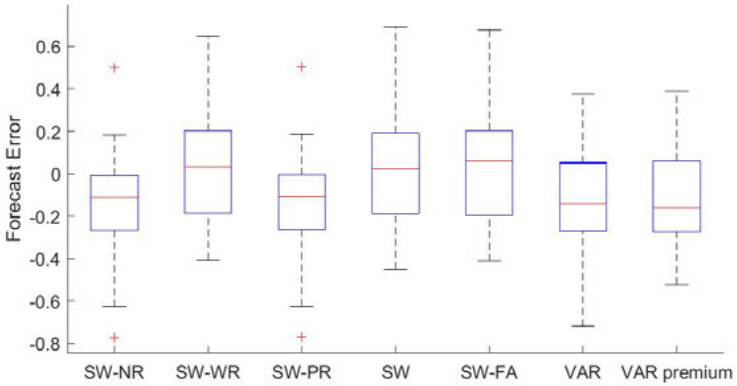

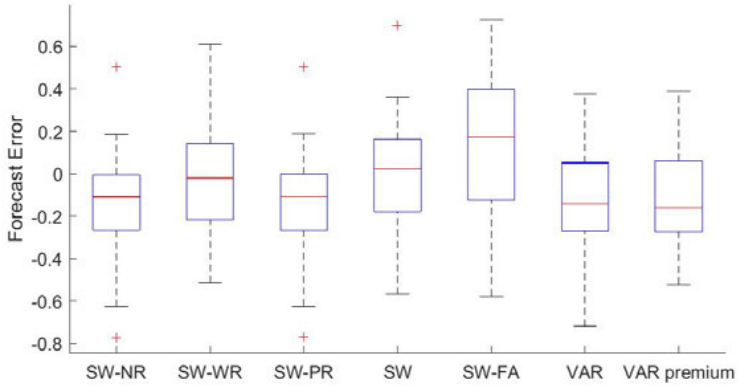

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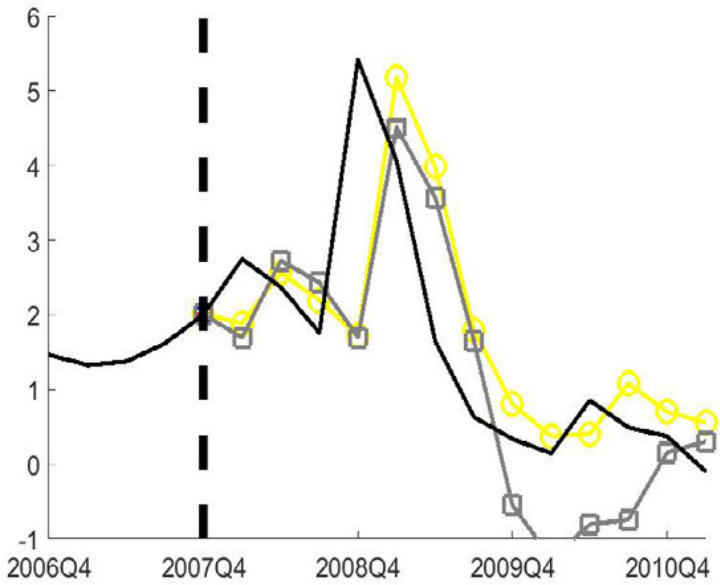
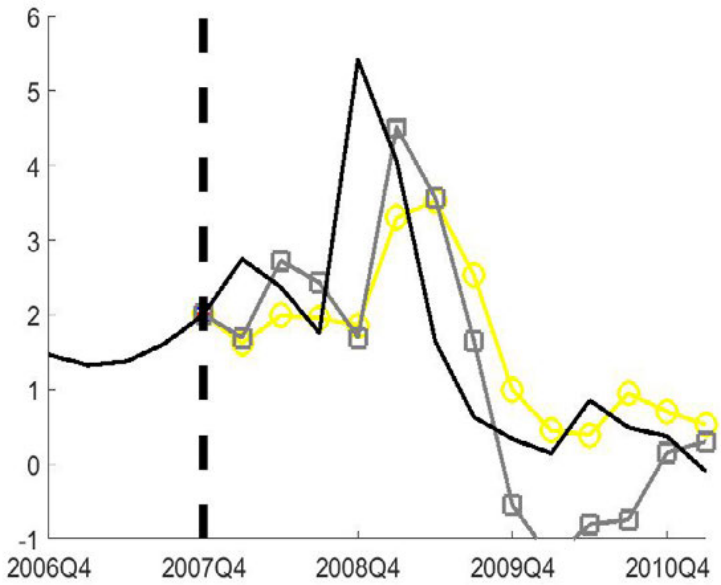
Forecasts UK Sample External Finance Premium

Rational Expectations



Adaptive Learning



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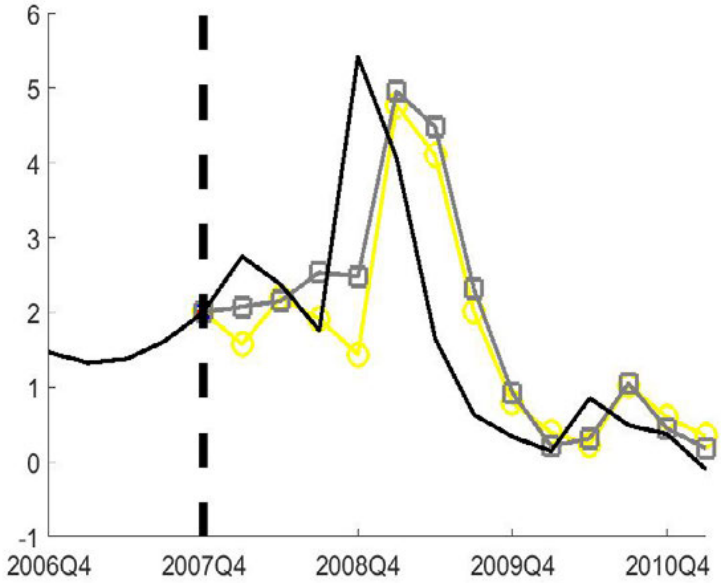
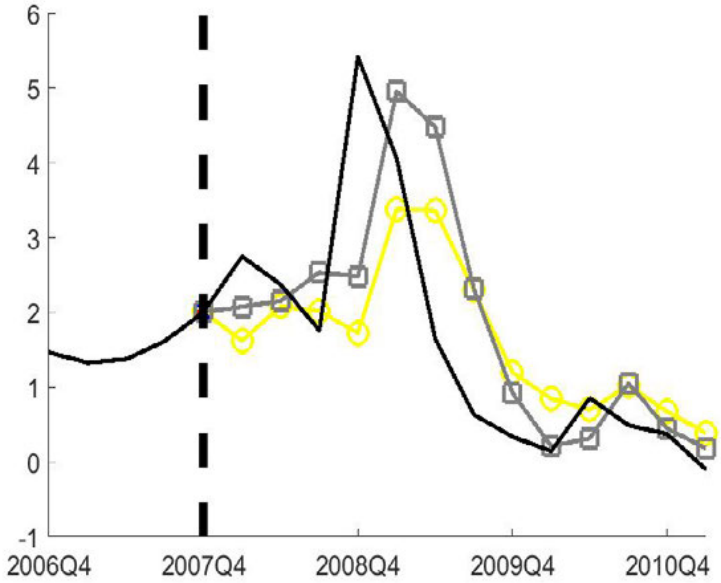
Forecasts Euro Area Sample External Finance Premium

Rational Expectations



Adaptive Learning



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