Advanced Macroeconomics

4. Saving and Investment in a Two-Period Model

Two-Period Set-up

- Fisher (1930)
- Period 1: the present
- Period 2: the future
- Perfect foresight
- Households consume, save and invest: in period 2, the capital stock is equal to the initial endowment plus investment from first period

Time Preference

- Assumption: One unit of consumption today yields more utility than one unit of consumption tomorrow
- Households choose consumption such that they are indifferent between consuming one unit more today or β units tomorrow
- β is called discount factor $(0 < \beta < 1)$
- The discount factor depends on the time preference rate $(\rho > 0)$:

$$\beta = \frac{1}{1+\rho}$$

Utility Function and Budget Constraint in the Two-Period Model

- Intertemporal utility function

$$U(c_1, c_2) = u(c_1) + \frac{1}{1+\rho} u(c_2)$$

- Budget constraint period 1

$$k_1 + r_1 k_1 + w_1 n_1 - c_1 = k_2$$

- Budget constraint period 2

$$c_2 = (1 + r_2) k_2 + w_2 n_2$$

- Assume $k_1 = 1$ and $n_1 = n_2 = 1$ and combine the two constraints

$$c_1 + \frac{1}{1+r_2} c_2 = (1 + r_1) + w_1 + \frac{1}{1+r_2} w_2$$

Utility Maximization in the Two-Period Model

- Lagrange function

$$\mathcal{L} = u(c_1) + \frac{1}{1+\rho}u(c_2) - \lambda\left(c_1 + \frac{1}{1+r_2}c_2 - (1+r_1) - w_1 - \frac{1}{1+r_2}w_2\right)$$

- First-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= u'(c_1) - \lambda = 0 &\Rightarrow\quad & u'(c_1) = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_2} &= \frac{1}{1+\rho}u'(c_2) - \frac{\lambda}{1+r_2} = 0 &\Rightarrow\quad & \frac{1}{1+\rho}u'(c_2) = \frac{\lambda}{1+r_2}
\end{aligned}$$

- Euler equation for consumption

$$\frac{1}{1+\rho}\frac{u'(c_2)}{u'(c_1)} = \frac{1}{1+r_2} \quad\Leftrightarrow\quad \frac{u'(c_2)}{u'(c_1)} = \frac{1+\rho}{1+r_2}$$

Euler Equation for Power Utility Function

- Power utility (constant elasticity of intertemporal substitution of consumption $1/\theta$)

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}$$

where

$$u'(c) = c^{-\theta}, \qquad u''(c) = -\theta c^{-\theta-1}, \qquad \frac{-cu''(c)}{u'(c)} = c\frac{\theta c^{-\theta-1}}{c^{-\theta}} = \theta$$

- EULER equation

$$\beta \frac{u'(c_2)}{u'(c_1)} = \beta \left(\frac{c_1}{c_2}\right)^{\theta} = \frac{1}{1+r_2}$$

Profit Maximization

- Firms maximize the present value of profits

$$\mathcal{P} = A_1 F(k_1, n_1) - r_1 k_1 - w_1 n_1 + \frac{1}{1+\rho}(A_2 F(k_2, n_2) - r_2 k_2 - w_2 n_2)$$

- First-order conditions

$$\begin{aligned} r_1 &= A_1 F_k(k_1, n_1) \\ r_2 &= A_2 F_k(k_2, n_2) \\ w_1 &= A_1 F_n(k_1, n_1) \\ w_2 &= A_2 F_n(k_2, n_2) \end{aligned}$$

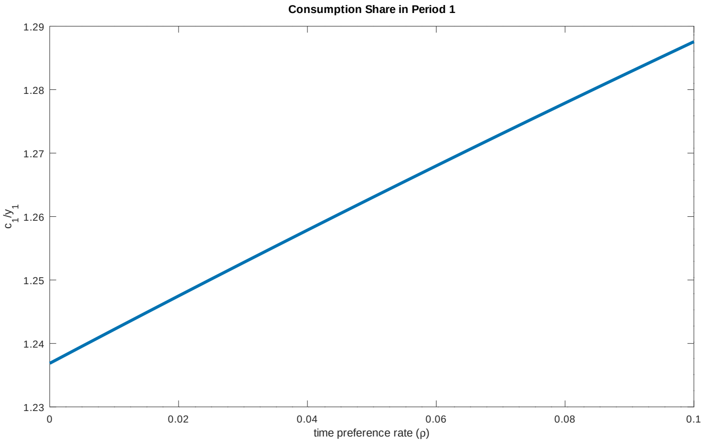
General Equilibrium in the Two-Period Model

- Initial capital stock (k_1) and total factor productivity (A_1, A_2) are exogenous
- 9 endogenous variables: y_1, y_2, c_1, c_2, r_1, r_2, w_1, w_2, k_2
- 9 equations
  - 2 production functions (for period 1 and for period 2, respectively)
  - 2 budget constraints (for period 1 and for period 2, respectively)
  - Euler equation for consumption
  - 4 profit maximizing first-order conditions
- Solve numerically for general equilibrium: `twoperiodmodel.m`

4. Saving and Investment in a Two-Period Model

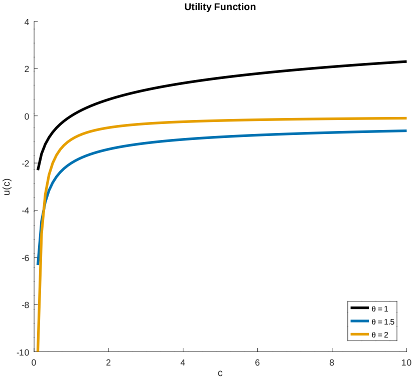
The Higher the Time Preference Rate, the Higher Period 1 Consumption

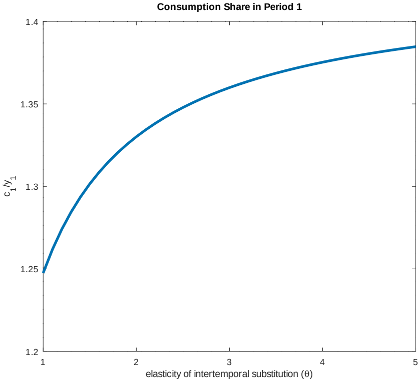
(`twoperiodsensitivity.m`)



The Higher the Elasticity of Intertemporal Substitution, the Higher Period 1 Consumption

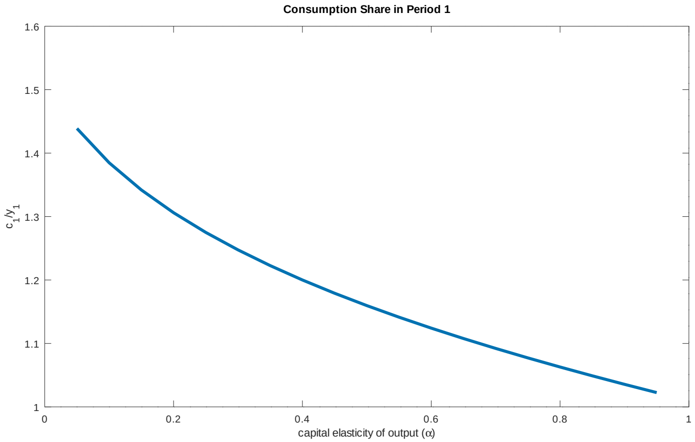
(`twoperiodsensitivity.m`)





The Higher Capital Elasticity of Output, the Lower Period 1 Consumption

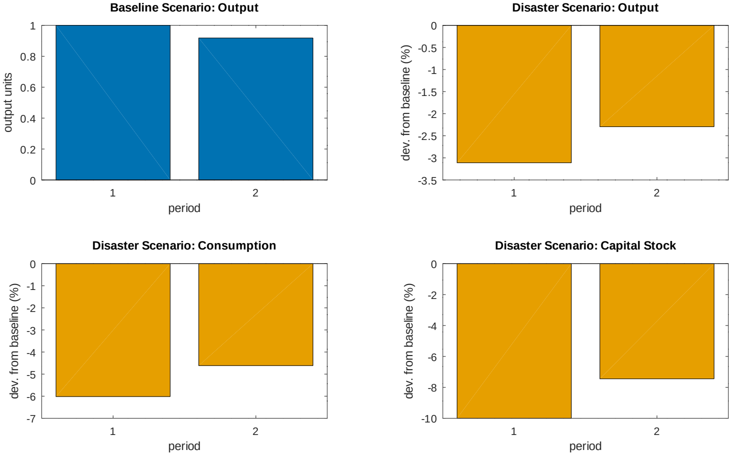
(`twoperiodsensitivity.m`)



4. Saving and Investment in a Two-Period Model

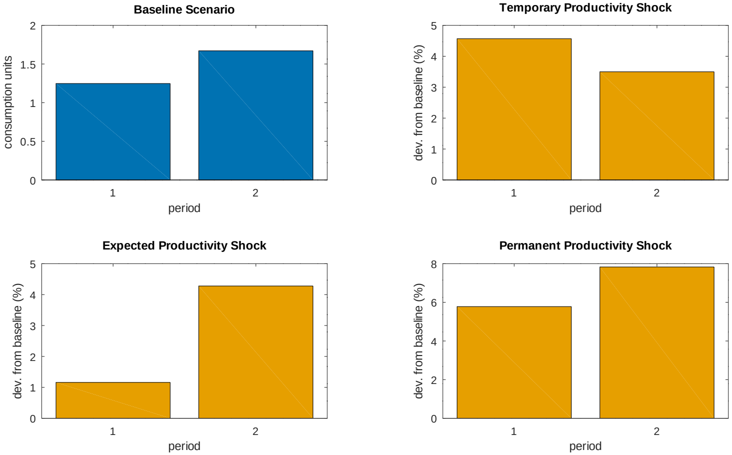
Effects of a Natural Disaster: Rebuilding the Capital Stock

(`twoperioddisaster.m`, baseline: $k_1 = 1$, disaster: $k_1 = 0.9$)



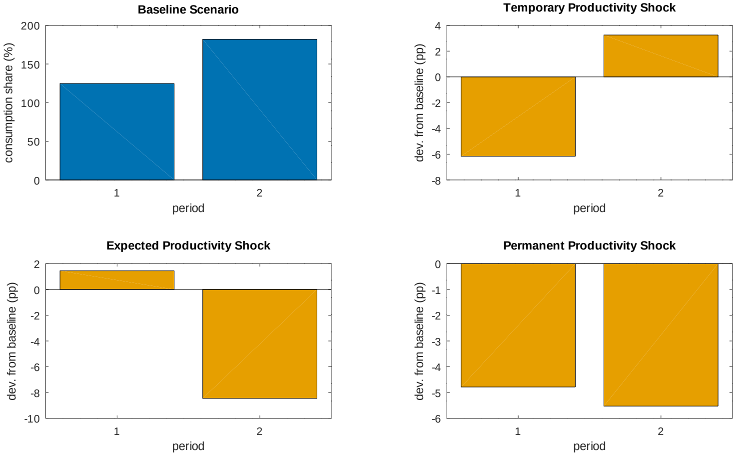
Productivity Shocks and Consumption Smoothing I

(`twoperiodproductivity.m`, Shock Size 10%)



Productivity Shocks and Consumption Smoothing II

(`twoperiodproductivity.m`, Shock Size 10%)



4. Saving and Investment in a Two-Period Model

Utility Function and Budget Constraint

- Intertemporal utility function

$$U = \frac{c_1^{1-\theta}}{1-\theta} - \frac{n_1^{1+\gamma}}{1+\gamma} + \frac{1}{1+\rho}\left(\frac{c_2^{1-\theta}}{1-\theta} - \frac{n_2^{1+\gamma}}{1+\gamma}\right)$$

- Households can borrow and lend on a competitive financial market at interest rate r
- Budget constaint period 1

$$w_1 n_1 - c_1 = a_2$$

- Budget constraint period 2

$$c_2 = (1+r)a_2 + w_2 n_2$$

- Combine the two constraints: intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = w_1 n_1 + \frac{w_2 n_2}{1+r}$$

Utility Maximization

- Lagrange function

$$\mathcal{L} = \frac{c_1^{1-\theta}}{1-\theta} - \frac{n_1^{1+\gamma}}{1+\gamma} + \frac{1}{1+\rho}\left(\frac{c_2^{1-\theta}}{1-\theta} - \frac{n_2^{1+\gamma}}{1+\gamma}\right) - \lambda\left(c_1 + \frac{c_2}{1+r} - w_1 n_1 - \frac{w_2 n_2}{1+r}\right)$$

- First-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= c_1^{-\theta} - \lambda = 0 &\Rightarrow\quad & c_1^{-\theta} = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_2} &= \frac{c_2^{-\theta}}{1+\rho} - \frac{\lambda}{1+r} = 0 &\Rightarrow\quad & \frac{c_1^{-\theta}}{1+\rho} = \frac{\lambda}{1+r} \\
\frac{\partial \mathcal{L}}{\partial n_1} &= n_1^{\gamma} - \lambda w_1 = 0 &\Rightarrow\quad & n_1^{\gamma} = \lambda w_1 \\
\frac{\partial \mathcal{L}}{\partial n_1} &= \frac{n_2^{\gamma}}{1+\rho} - \frac{\lambda}{1+\rho} w_2 = 0 &\Rightarrow\quad & \frac{n_2^{\gamma}}{1+\rho} = \frac{\lambda}{1+\rho} w_2
\end{aligned}$$

Household Equilibrium Conditions

- Consumption EULER equation

$$\frac{1}{1+\rho}\left(\frac{c_2}{c_1}\right)^{-\theta} = \frac{1}{1+r}$$

- Labor supply EULER equation

$$\frac{1}{1+\rho}\left(\frac{n_2}{n_1}\right)^{\gamma} = \frac{1}{1+r}\frac{w_2}{w_1}$$

- Labor supply in period 1

$$\frac{n_1^{\gamma}}{c_1^{-\theta}} = w_1$$

- Labor supply in period 2

$$\frac{n_2^{\gamma}}{c_2^{-\theta}} = w_2$$

Profit Maximization

- Firms maximize the present value of profits

$$\mathcal{P} = F(n_1) - w_1 n_1 + \frac{1}{1+\rho}\left(F(n_2) - w_2 n_2\right)$$

- First-order conditions

$$\begin{aligned} w_1 &= F_n(n_1) \\ w_2 &= F_n(n_2) \end{aligned}$$

- For linear production $y_t = A_t n_t$:

$$\begin{aligned} w_1 &= A_1 \\ w_2 &= A_2 \end{aligned}$$

General Equilibrium in the Two-Period Model

- Productivity (A_1, A_2) is exogenous
- 9 endogenous variables: $y_1, y_2, c_1, c_2, r, w_1, w_2, n_1, n_2$
- 9 equations
  - 2 resource constraints: $y_1 = c_1$ and $y_2 = c_2$
  - Euler equation for consumption
  - 2 labor supply equations (for period 1 and period 2, respectively)
  - 2 profit maximizing first-order conditions (labor demand)
  - 2 production functions (for period 1 and for period 2, respectively)
- Solve numerically for general equilibrium: `twoperiodlabourmodel.m`

4. Saving and Investment in a Two-Period Model

Summary

- Consumption smoothing: less consumption today facilitates more consumption tomorrow
- Effects of productivity shocks depend on shock persistence
  - Temporary shocks: consumption increases but less than productivity, households save to smooth consumption
  - Expected shocks: consumption increases immediately, households save less
- The two-period model of savings and investment is the basis of most one-sector models of economic growth (Chapters 5 to 7) and of competitive models of aggregate fluctuations (Chapter 8)
- The two-period model with endogenous labor supply is the basis of DSGE models of aggregate fluctuations (Chapters 9 and 10)

Literature

- Alogoskoufis, George (2019): Dynamic Macroeconomics, MIT Press, Chapter 2
- Fisher, Irving (1930): The Theory of Interest, Macmillan
- Friedman, Milton (1957): A Theory of the Consumption Function, Princeton University Press