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Problem Set 2

Advanced Macroeconomics

Winter 2024/25

1 Analysis with Economic Focus

1. Show that
$$Y = K^{\alpha}(AN)^{1-\alpha}, \qquad 0 < \alpha < 1$$
   (a) has diminishing marginal products of capital and labor.
   (b) has constant returns to scale.
   (c) can be written as $y = k^{\alpha}$, where $y := \frac{Y}{AN}$ and $k := \frac{K}{AN}$ are output and capital in efficiency units.

   Plot the production function in efficiency units in Matlab or Octave.

2. Derive the following CES (constant elasticity of substitution) production function with respect to labor input N:
$$Y = \left[\gamma^{\frac{1}{\sigma}} K^{\frac{\sigma-1}{\sigma}} + (1-\gamma)^{\frac{1}{\sigma}} (AN)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

3. Show that the following utility function has a decreasing positive marginal product of goods c_1 and c_2:
$$U(c_1, c_2) = a\ln(c_1) + b\sqrt{c_2}, \qquad 0 < a, b < 1$$

4. What is the output elasticity of an input factor? Show, that for a standard Cobb-Douglas production function
$$Y = AK^{\alpha}N^{1-\alpha}$$
   it equals α for capital and $1-\alpha$ for labor. Derive also the elasticity of substitution between the two input factors and explain it.

5. Derive the log-linear form of the following production function:
$$Y = AK^{\alpha}\left[(1-\Gamma)N\right]^{1-\alpha}$$
   Define $x \equiv \ln(X)$ for $x = Y, A, K, N$ and $\gamma \equiv \ln\left[(1-\Gamma)^{1-\alpha}\right]$.

2 Solving Non-Linear Equations

Solve the following non-linear equations numerically and graphically in Matlab or Octave: determine the price at which the quantity demanded equals 15, given the following demand functions.

- Case 1:

$$q(p) = 10p^{-0.2}$$

- Case 2:

$$q(p) = 5p^{-0.2} - 5p^{-0.5}$$