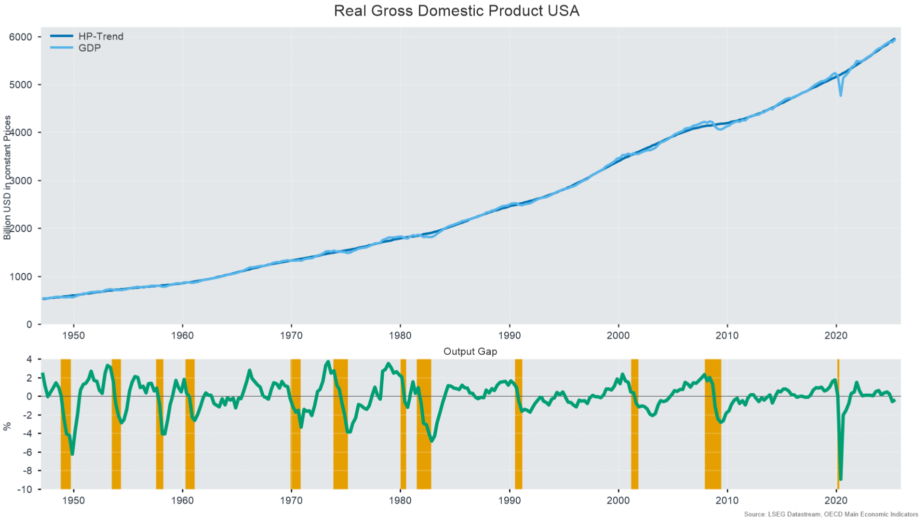
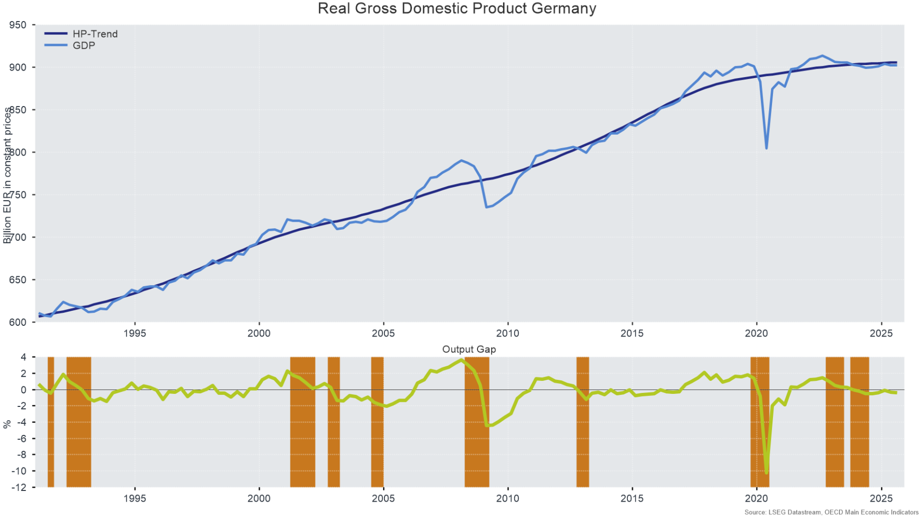
Advanced Macroeconomics

8. Aggregate Fluctuations and Real Business Cycles

Economic Fluctuations



Economic Fluctuations



8. Aggregate Fluctuations and Real Business Cycles

An Extended Ramsey Model to Account for Economic Fluctuations

- In this chapter we will extend the Ramsey growth model to account for economic fluctuations
- We add endogenous labor supply
- We discuss expectations
- We simulate the dynamic adjustment to stochastic productivity shocks
- We compare the model simulations to actual data

Exogenous Variables

- Population growth

$$\ln N_t = \ln N_0 + \gamma_n t$$

- Efficiency of labor

$$\ln A_t = \ln A_0 + \gamma_a t$$

- Total factor productivity

$$\ln Z_t = \eta_z \ln Z_{t-1} + \varepsilon_t^z$$

Model Structure: Production

- Aggregate production function

$$Y_t = Z_t K_t^{\alpha} \left(A_t \underbrace{N_t h_t}_{H_t} \right)^{1-\alpha}, \qquad 0 < \alpha < 1, \qquad 0 \leq h_t \leq 1$$

- Production per-capita

$$\frac{Y_t}{N_t} = Z_t \frac{K_t^{\alpha}}{N_t^{\alpha}} \frac{(A_t N_t h_t)^{1-\alpha}}{N_t^{1-\alpha}} = Z_t \left(\frac{K_t}{N_t} \right)^{\alpha} (A_t h_t)^{1-\alpha}$$

Model Structure: Expenditure

- Aggregate expenditure

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t$$

or

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t$$

- Expenditure per capita

$$\frac{Y_t}{N_t} = \frac{C_t}{N_t} + \frac{K_{t+1}(1+\gamma_n)}{N_t(1+\gamma_n)} - (1-\delta)\frac{K_t}{N_t} = \frac{C_t}{N_t} + \frac{K_{t+1}(1+\gamma_n)}{N_{t+1}} - (1-\delta)\frac{K_t}{N_t}$$

Market Structure: Perfect Competition

- Labor is paid its marginal product

$$w_t = MPL = (1-\alpha) Z_t K_t^{\alpha} (A_t H_t)^{-\alpha} A_t = (1-\alpha) Z_t \left(\frac{K_t}{A_t H_t} \right)^{\alpha} A_t$$

- Capital is paid its marginal product less depreciation

$$r_t = MPK - \delta = \alpha Z_t K_t^{\alpha-1} (A_t H_t)^{1-\alpha} - \delta = \alpha Z_t \left(\frac{A_t H_t}{K_t} \right)^{1-\alpha} - \delta$$

The Representative Household

- Intertemporal utility function

$$U_t = E_t \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho} \right)^{t+s} N_{t+s} u \left(\frac{C_{t+s}}{N_{t+s}}, 1 - h_{t+s} \right)$$

where h is employment per worker in hours in relation to total time

- Instantaneous utility function

$$u = \ln \left(\frac{C_t}{N_t} \right) + b \ln(1 - h_t), \qquad b > 0$$

Labor Supply by the Representative Household

- Intra-period labor decision: Maximize utility

$$\max_{c,h} \ln c + b \ln(1-h)$$

- subject to the budget constraint

$$c = wh$$

- Lagrange function

$$\mathcal{L} = \ln c + b \ln(1-h) + \lambda(wh - c)$$

- First-order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c} &= \frac{1}{c} - \lambda = 0 \qquad \Rightarrow \qquad \lambda = \frac{1}{wh} \\ \frac{\partial \mathcal{L}}{\partial h} &= -\frac{b}{1-h} + \lambda w = 0 \end{aligned}$$

Intra-Period Optimality Condition for Labor Supply

- Substitute for λ in second FOC

$$-\frac{b}{1-h} + \frac{1}{h} = 0 \qquad \Rightarrow \qquad h = \frac{1}{1+b}$$

- Log-utility: labor supply independent from real wage (income effect $=$ substitution effect)
- Use b to calibrate steady-state hours worked by worker

Intertemporal Substitution in Labor Supply

- Two-period budget constraint

$$c_1 + \frac{c_2}{1+r} = w_1 h_1 + \frac{w_2 h_2}{1+r}$$

- Lagrange function

$$\mathcal{L} = \ln c_1 + b \ln(1-h_1) + \frac{1}{1+\rho}\left(\ln c_2 + b\ln(1-h_2)\right) + \lambda\left[w_1 h_1 + \frac{w_2 h_2}{1+r} - c_1 - \frac{c_2}{1+r}\right]$$

- First-order conditions for labor supply in periods 1 and 2

$$\begin{aligned}
\frac{\mathcal{L}}{\partial h_1} &= -\frac{b}{1-h_1} + \lambda w_1 = 0 \quad &\Rightarrow \quad \frac{b}{1-h_1} &= \lambda w_1 \\
\frac{\mathcal{L}}{\partial h_2} &= -\frac{b}{(1-h_2)(1+\rho)} + \frac{1}{1+r}\lambda w_2 = 0 \quad &\Rightarrow \quad \frac{b}{1-h_2} &= \frac{1+\rho}{1+r}\lambda w_2
\end{aligned}$$

Intertemporal Optimality Condition for Labor Supply

- Combine the two first-order conditions

$$\frac{1-h_1}{1-h_2} = \frac{1+\rho}{1+r}\frac{w_2}{w_1}$$

- The higher the relative wage, the higher labor supply
- The higher the interest rate (r), the higher labor supply in period 1
- Labor supply fluctuates if wages or interest rate fluctuate

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Deterministic Solution under Perfect Foresight

- Lagrangian ($c_t = C_t/N_t$ and $k_t = K_t/N_t$, $N_0 = 1$)

$$\begin{aligned}\mathcal{L} \quad = \quad & \sum_{s=0}^{\infty} \left(\frac{1}{1+\rho}\right)^s \left[(1+\gamma_n)^s \left(\ln c_{t+s} + b\ln(1-h_{t+s})\right)\right. \\ & \left. + \lambda_{t+s}\left(w_{t+s}h_{t+s} + (r_{t+s}+\delta)k_{t+s} - c_{t+s} - k_{t+s+1}(1+\gamma_n) + (1-\delta)k_{t+s}\right)\right]\end{aligned}$$

- First-order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c_{t+s}} &= \frac{(1+\gamma_n)^s}{c_{t+s}} - \lambda_{t+s} = 0 \qquad \Rightarrow \qquad \lambda_{t+s} = \frac{(1+\gamma_n)^s}{c_{t+s}} \\ \frac{\partial \mathcal{L}}{\partial h_{t+s}} &= -\frac{(1+\gamma_n)^s b}{1-h_{t+s}} + \lambda_{t+s}w_{t+s} = 0 \\ \frac{\partial \mathcal{L}}{\partial k_{t+s}} &= \beta^s\lambda_{t+s}(r_{t+s}+\delta) - \beta^{s-1}\lambda_{t+s-1}(1+\gamma_n) + \beta^s\lambda_{t+s}(1-\delta) = 0\end{aligned}$$

Household Optimality Condition

- Consumption Euler equation ($s = 1$)

$$\beta(1 + r_{t+1})\frac{c_t}{c_{t+1}} = 1, \qquad \beta = \frac{1}{1+\rho}$$

- Optimal labor supply

$$\frac{c_{t+s}}{1 - h_{t+s}} = \frac{w_{t+s}}{b}$$

Steady State

- From consumption Euler equation ($c_{t+1} = (1+\gamma_a)c_t$)

$$\frac{1+r}{1+\rho}\frac{1}{1+\gamma_a} = 1 \qquad \Rightarrow \qquad r \approx \rho + \gamma_a$$

- Output-capital ratio ($Z = 1$)

$$\frac{Y}{K} = \frac{K^\alpha (AH)^{1-\alpha}}{K} = \left(\frac{AH}{K}\right)^{1-\alpha} = \frac{r+\delta}{\alpha} = \frac{\rho+\gamma_a+\delta}{\alpha}$$

- Efficient-labor-to-capital ratio

$$\frac{AH}{K} = \left(\frac{\rho+\gamma_a+\delta}{\alpha}\right)^{\frac{1}{1-\alpha}}$$

- Consumption share ($K_{t+1} = (1+\gamma_a)(1+\gamma_n)K_t$)

$$\frac{K_{t+1}}{Y_t} = (1-\delta)\frac{K_t}{Y_t} + 1 - \frac{C_t}{Y_t} \qquad \Rightarrow \qquad \frac{C}{Y} = 1 - \alpha\frac{\gamma_n+\gamma_a+\delta}{\rho+\gamma_a+\delta}$$

Extended Ramsey Model: Summary

(`ramseygrowthext_temp.mod`)

- 2 exogenous variables: A_t, N_t
- 6 core endogenous variables: $\tilde{k}_t, h_t, \tilde{y}_t, \tilde{c}_t, \tilde{w}_t, r_t$

$$\tilde{k}_t = \frac{K_t}{\overline{A}_t N_t}, \quad \tilde{y}_t = \frac{Y_t}{\overline{A}_t N_t}, \quad \tilde{c}_t = \frac{C_t}{\overline{A}_t N_t}, \quad \tilde{w}_t = \frac{w_t}{\overline{A}_t}$$

- 6 core equations:
  1. Consumption Euler equation
  2. Labor supply optimality condition
  3. Production function
  4. Capital accumulation
  5. Profit-maximizing condition for real wage
  6. Profit-maximizing condition for real interest rate
- Additional equations: $C_t = c_t N_t$, $K_t = k_t N_t$, $I_t = Y_t - C_t$

Stochastic Ramsey Model I

- Stochastic efficiency of labor

$$A_t = \exp(a_0) \cdot \exp(\gamma_a t) \cdot \exp(a_t) = \overline{A}_t \cdot \exp(a_t)$$

- Consumption Euler equation

$$\beta(1 + r_{t+1}) \frac{c_t}{c_{t+1}} \frac{\overline{A}_{t+1}}{\overline{A}_t(1+\gamma_a)} = 1 \quad \Rightarrow \quad \beta(1 + r_{t+1}) \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \frac{1}{1+\gamma_a} = 1$$

- Labor supply optimality condition

$$\frac{c_{t+s}/\overline{A}_{t+s}}{1 - h_{t+s}} = \frac{w_{t+s}/\overline{A}_{t+s}}{b} \quad \Rightarrow \quad \frac{\tilde{c}_{t+s}}{1 - h_{t+s}} = \frac{\tilde{w}_{t+s}}{b}$$

- Production function

$$Y_t = K_t^{\alpha}(A_t N_t h_t)^{1-\alpha} \quad \Rightarrow \quad \tilde{y}_t = K_t^{\alpha}(\overline{A}_t N_t)^{-\alpha}(\exp(a_t) h_t)^{1-\alpha} = \tilde{k}_t^{\alpha}(\exp(a_t) h_t)^{1-\alpha}$$

Stochastic Ramsey Model II

- Capital accumulation

$$\frac{(1+\gamma_n)(1+\gamma_a)K_{t+1}}{\overline{A}_{t+1}N_{t+1}} = (1-\delta)\frac{K_t}{\overline{A}_t N_t} + \frac{Y_t}{\overline{A}_t N_t} - \frac{C_t}{\overline{A}_t N_t}$$

$$(1+\gamma_n)(1+\gamma_a)\tilde{k}_{t+1} = (1-\delta)\tilde{k}_t + \tilde{y}_t - \tilde{c}_t$$

- Wage

$$\tilde{w}_t = (1-\alpha)\left(\frac{K_t}{\overline{A}_t N_t}\right)^{\alpha}(\exp(a_t)h_t)^{-\alpha}\exp(a_t) = (1-\alpha)k_t^{\alpha}(\exp(a_t)h_t)^{1-\alpha}h_t^{-1} = (1-\alpha)\frac{\tilde{y}_t}{h_t}$$

- Interest rate

$$r_t = \alpha\left(\frac{\overline{A}_t N_t}{K_t}\right)^{1-\alpha}(\exp(a_t)h_t)^{1-\alpha} - \delta = \alpha\tilde{k}_t^{\alpha-1}(\exp(a_t)h_t)^{1-\alpha} - \delta = \alpha\frac{\tilde{y}_t}{\tilde{k}_t} - \delta$$

Solving Dynamic Stochastic General Equilibrium Models

- Specify expectation formation: rational expectations (Muth 1961)

$$E_t x_{t+1} = E(x_t|\Omega_t), \qquad \Omega_t = \{x_{t-i}, z_{t-i}, i = 0, 1, 2, \ldots, \infty\}$$

- Dynare solves linear rational expectation models

$$y_t = aE_t y_{t+1} + bx_t$$

- The solution of the model is a recursive law of motion of the endogenous variables which can be used to simulate data or to derive impulse responses

- Dynare can linearize non-linear models

- `ramseygrowthtext_stoch.mod`

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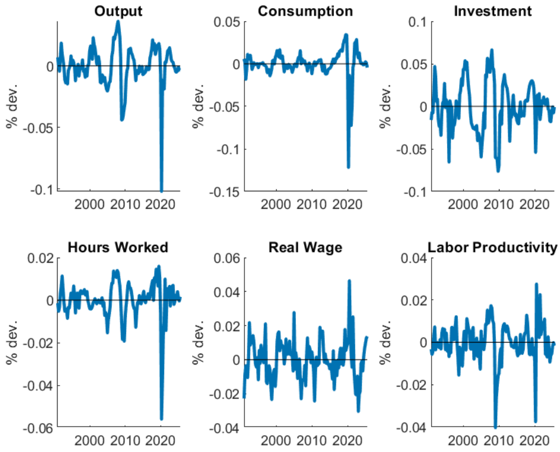
Business Cycle Fluctuations in Germany

(Federal Statistical Office, own calculations, HP-filtered)



Calibration of RBC-Models

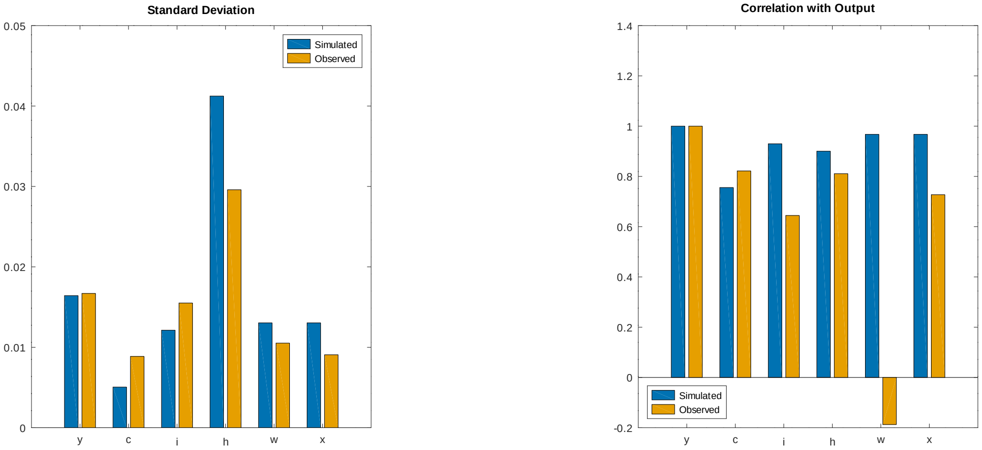
- How well do RBC models explain observed data?
  - Specify the model
  - Specify the parameters
  - Simulate exogenous variables using a random number generator, e.g.

$$\{a_t\}_{t=1}^{T}$$

  - Compute endogenous variables using the model
  - Compare simulated and observed data
- Adjust the parameters until the fit is as good as possible: steady-state (time share spent for work, consumption share, labor share) and simulated trajectories (standard deviations, correlations)

Calibrated RBC Model

(`RBC.m`)



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Dominance of Technological Shocks

- How important are productivity shocks as drivers of business cylces?
  - Productivity is strongly correlated with the cyclical component of output
  - Many things affect productivity: new regulations, strikes, natural disasters, price changes of imported intermediate goods ...
- Productivity, however, is not exogenous
- Other important shocks that are neglected by RBC theory:
  - Demand shocks
  - Cost push shocks

Implications for the Labor Market

- In RBC models, unemployment is voluntary
- Basic RBC models: substitution effect dominates income effect

  Changes in the real wage induce fluctuations in employment: in periods with high productivity, the real wage is higher and people voluntarily chose to work more; in recessions, the real wage is lower and people voluntarily chose to work less.
- However:
  - Empirically observed wage elasticity of employment is too low to explain observed fluctuations in employment
  - In a recession, not all people reduce working hours but some become unemployed while others keep on working the same hours (distributional implications)

Neutrality of Money

- General equilibrium in RBC models independent of monetary variables
- Extension:
  - Money market: $M^S = M^D$
  - FISHER equation
- However, empirical evidence reveals real effects of monetary policy and other nominal shocks
- Add price rigidities to the model in order to include real effects of nominal shocks

Effectiveness of Economic Policy

- In RBC models, business cycle fluctuations are efficient adjustments to productivity shocks
- Stabilizing monetary policy does not increase welfare (aggregate utility)
- Economic policy may increase productivity or its growth rate

8. Aggregate Fluctuations and Real Business Cycles

Summary

- RBC models imply that aggregate fluctuations are caused by real disturbances such as productivity shocks.
- These models are based on the optimizing behavior of households and firms on competitive markets with fully flexible price.
- Policy recommendation of RBC models: focus on the supply side of the economy.
- Weaknesses: In reality, frictions are important.
- Next chapter: Modify market structure and pricing behavior of firms.

Literature

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Hodrick-Prescott-Filter

(Hodrick and Prescott 1997)

- Flexible Trend

$$x_t = \widehat{x}_t^{trend} + \widehat{x}_t^{cycle}$$

- Weighing good approximation of actual time series (x_t) and smoothness:

$$\min_{\{\widehat{x}_t^{trend}\}_{t=1}^{T}} \sum_{t=1}^{T} (x_t - \widehat{x}_t^{trend})^2 + \lambda \sum_{t=2}^{T-1} \left((\widehat{x}_{t+1}^{trend} - \widehat{x}_t^{trend}) - (\widehat{x}_t^{trend} - \widehat{x}_{t-1}^{trend}) \right)^2 ,$$

- The larger λ, the smoother the trend (usually $\lambda = 1600$ for quarterly data)
- Percentage deviations from trend: filter $\ln x_t$ instead of x_t

Linearization of Non-linear Equations

- Non-linear equation

$$X_{t+1} = g(X_t, Y_t)$$

- Taylor approximation around steady state

$$X_{t+1} \approx g(\overline{X}, \overline{Y}) + \frac{\partial g(\overline{X}, \overline{Y})}{\partial X} \cdot (X_t - \overline{X}) + \frac{\partial g(\overline{X}, \overline{Y})}{\partial Y} \cdot (Y_t - \overline{Y})$$

or

$$X_{t+1} - \overline{X} \approx \frac{\partial g(\overline{X}, \overline{Y})}{\partial X} \cdot (X_t - \overline{X}) + \frac{\partial g(\overline{X}, \overline{Y})}{\partial Y} \cdot (Y_t - \overline{Y})$$

- Often, we are interested in percentage deviations from steady state: take logs

$$x_{t+1} = \ln X_{t+1} = \ln g(X_t, Y_t) = \ln g\left(\exp(x_t), \exp(y_t)\right)$$

such that

$$\underbrace{x_{t+1} - \overline{x}}_{\widehat{x}_{t+1}} \approx \frac{\partial \ln g(\overline{x}, \overline{y})}{\partial x} \cdot \underbrace{(x_t - \overline{x})}_{\widehat{x}_t} + \frac{\partial \ln g(\overline{x}, \overline{y})}{\partial y} \cdot \underbrace{(y_t - \overline{y})}_{\widehat{y}_t}$$

Log-Linearization: Examples

- $Z_t = a \cdot X_t$

$$z_t - \overline{z} = \ln a + x_t - (\ln a + \overline{x}) = x_t - \overline{x}$$

- $Z_t = X_t + Y_t$

$$\begin{aligned} \widehat{z}_t &\approx \frac{\partial \ln\left(\exp(\overline{x}) + \exp(\overline{y})\right)}{\partial x} \cdot \widehat{x}_t + \frac{\partial \ln\left(\exp(\overline{x}) + \exp(\overline{y})\right)}{\partial y} \cdot \widehat{y}_t \\ &= \frac{\overline{X}}{\overline{X} + \overline{Y}} \cdot \widehat{x}_t + \frac{\overline{Y}}{\overline{X} + \overline{Y}} \cdot \widehat{y}_t \end{aligned}$$

Deriving the Correlation Matrix from the Covariance Matrix

(Rinne 2003, p. 88)

- Dynare provides the $(m \times m)$-covariance matrix Σ of m simulated variables in `oo_.var`
- The variances (σ_i^2) of the variables are given on the main diagonal of Σ
- Define the inverse matrix of standard deviations

$$\Sigma_{dia}^{-1/2} = \begin{pmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ 0 & 1/\sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/\sigma_m \end{pmatrix}$$

- The correlation matrix is given by

$$R = \Sigma_{dia}^{-1/2} \Sigma \Sigma_{dia}^{-1/2}$$