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Modeling Aspects of Theory of Mind with Markov Random Fields

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Abstract We propose Markov random fields (MRFs) as a probabilistic mathematical model for incorporating the internal states of other agents, both human and robotic, into robot decision making. By using estimates of Theory of Mind (ToM), the "mental" states of other agents can be incorporated into decision making through statistical inference, allowing robots to balance their own goals and internal objectives with those of other collaborating agents. The MRF model is well-suited to domains in which the joint probability over latent (action) and observed (perceived) variables can be factored into pairwise interactions between these variables. Specifically, these interactions occur through functions that evaluate **"local evidence"** between an observed and latent variable and **"compatibility"** between a pair of latent variables. We will model some experimental findings from ToM research and show how the MRF model can be applied to a social robotics task. We will also describe how to use belief propagation on a multi-robot MRF as a novel approach to multi-robot coordination, with parallels to human collaboration strategies.

Keywords Theory of Mind · Markov Random Fields · Multi-Robot · Belief Propagation

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1 Introduction

Humans often appear to use their expectations about the beliefs and intentions of other agents to inform their decision making. This understanding of beliefs and intentions, commonly referred to in cognitive science as Theory of Mind (ToM), can often be explicitly communicated or inferred from the observed actions of others. Significant previous efforts have hypothesized how one perceives the world and the intentions and beliefs of others [28, 2, 19, 43]. Such work has informed the creation of socially interactive robots [36, 4]. To complement such work, our aim is to develop mathematical models capable of incorporating domain knowledge from ToM (or social processes more generally) with probabilistic inference. Mathematical models of probablistic inference have proven to be viable explanatory frameworks for children's cognitive development [18], and have been applied to problems in theory of mind [17]. Our approach is to treat the intended action of an agent (either robot or human) as a latent random variable that is conditioned on both the agent's own perceptual observations (estimated from their own unique sensing) of the physical world and the agent's inferences about the intentions of other agents.

In the form of a generative statistical model, we will describe ToM probabilistically as a Markov Random Field (MRF), where interactions between variables drive the process. Casting ToM in a MRF, latent variables represent the state of each agent and observed

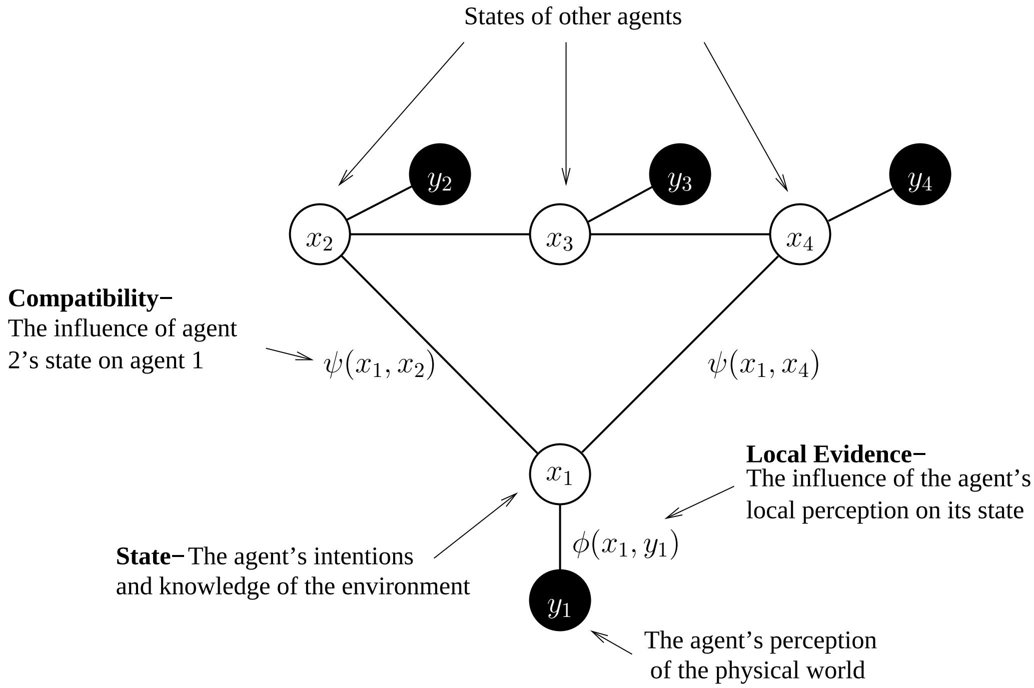

Fig. 1 An illustration of the ToM Markov Random Field model and the related functions from the viewpoint of agent 1

variables represent the observations of each agent. In the form of a graph, these variables are nodes where dependencies between variables are represented by connecting edges. This graph expresses the joint probability of the state of all the agents conditioned on all their observations and permits the use various inference algorithms for coordination [6]. However, the essence of the model is in its defined edge potentials. These edges correspond to local evidence and pairwise compatibility, which dictate interactions between variables. Local evidence edges connecting pairs of latent and observed variables define how the physical observations influence agents' states. Pairwise compatibility edges between pairs of latent variables indicate how agents influence each others' states. Similar to a Bayes filter [41], we hypothesize MRFs can be crafted to describe various ToM processes and adapted to human-robot situations through appropriate selection of evidence and compatibility functions.

We will review the mathematical and computational fundamentals of the MRF model and describe its application to modeling coordination among distributed collections of agents, either human or robot. We consider findings on human interaction as observed in previous Theory of Mind experiments from cognitive science researchers, relating to reliability and uncertainty, which can be modeled within the MRF framework. We additionally present a method for multi-robot coordination using belief propagation on a MRF for the task of visibility chain formation [38]. Our ultimate goal of expanding the use of this model to coordinating human-robot teams could then be achieved by the combination of these two results, replacing some of the robots with humans and using robotic ToM tools to estimate the humans' mental states.

1.1 Related Work

Theory of Mind refers to the ability of humans to understand the intentions and mental processes of themselves and others. Understanding others' mental states, including their desires, intentions, and beliefs, is an important tool for understanding and predicting others' behavior [43]. There is considerable debate among cognitive scientists as to how ToM is represented and the nature of ToM development (for a review, see e.g., Carruthers and Smith [8]). Our intent here is not to speculate on how ToM is represented or acquired in human beings. Rather, our goal is to consider the effect that ToM has on decision making and construct a sound mathematical model for including ToM in the decision making process of robots. However, the form of ToM representation will have an effect on how the state of a robot is defined.

Scasselatti [36] discussed two important models of ToM (those of Leslie [28] and Baron-Cohen [2]) and the implications of these models to robotics. This work also implemented some of the essential skills needed for a robot to have a ToM, such as gaze-tracking and distinguishing between inanimate and animate objects. However, he points out that the interactions between the various modules in the psychological descriptions proposed by Leslie and Baron-Cohen are not suitably well-defined for generalizable robotics implementation. Our belief is that MRFs can be useful in providing a mathematical structure that allows a robot to factor in its estimate of the intentions of other agents as well as its own local computation into its decision-making.

An appeal of the MRF framework is that it allows us to consider recent work in children's ToM, which examines children's credulity and "trust in testimony" [21,11,27,20]. These investigations suggest that children develop ontological commitments about the world, particularly about socially constructed information or about events they cannot directly observe, from information garnered from other people. In doing so, children take into account the nature of how that information is transmitted, particularly in terms of the confidence of the teacher and the reliability of that individual as a source of information.

Significant previous work in social robotics has pursued estimation of the "mental" states of other agents, both in terms of using social cues and simulating differing perspectives. Work in social cue interpretation uses a robot's local perception of human gaze, pointing, and other gestures to estimate their intentions and attentional focus [4,37]. The mental states estimated from the local perception of social cues fit within our MRF framework as distributions over other agents' latent variables. "Perspective taking" [3] occurs where the robot must include estimates of other agent's perspectives (or at least physical state) in their own state, which we phrase as a function of compatibility. The Polyscheme [9] architecture has been used to estimate the distribution over possible states of another agent through simulating situations from the perspective of the other agent [42]. With their Leonardo robot, Breazeal et al. [4,3] have explored various forms of perspective taking. In our MRF framework, we would consider this line of work as ambiguity in the form of the agent's own state variable, similar to how one would express state (or robot pose) for a Bayes filter. Specifically, does the agent maintain physical and/or intentional information about others within their state? Additionally, Hoffman and Breazeal [22] have explored anticipation as a mechanism for robots to predict and more appropriately respond to human partners. Thinking of the Bayes filter again, anticipation is essentially a prediction mechanism about collective state (or situations) that could occur in the future. In the MRF framework, prediction can be conceptualized as a form of compatibility where an agent's estimation about future state is influenced by its own intentions and observations as well as their compatibility with the recognized intentions of other agents.

In autonomous robotics, multi-robot coordination describes the space of methods for controlling groups of autonomous robots. This coordination can often be expressed as multi-robot task allocation (MRTA), which has been well explored in previous work [31,1,25]. Dias and Stentz [14] have proposed auction-based methods for MRTA. They phrase MRTA as a market economy where leaders develop plans for a group of robots and bid for tasks against other leader robots at an auction. Gerkey and Matarić [15] took a large step towards creating a common framework for MRTA problems by phrasing a wide variety of MRTA problems as already well-studied problems in optimization. We previously were able to express several existing approaches to MRTA as forms of inference within MRF models [6] and propose probabilistic belief propagation as an alternative form of distributed inference [38].

Graphical models have been previously used in the coordination of static sensor networks. Techniques have developed which use belief propagation [23,13,33] or junction trees [32] to do distributed inference on sensor networks. However, their extension into non-static robotics domains, especially those involving human interaction, is not quite straightforward. While this sensornet-based work provides a good foundation, the issues of intention recognition, defining compatibility and local evidence functions, non-stationary distributions over time, as well as basic issues of representing state remain open issues.

2 ToM as Markov Random Fields

An MRF [45] is a graphical model that factors a system into a finite set of observed and hidden, or latent, variables with pairwise interactions between them. Variables in our case are vector-valued random variables, meaning that they are multidimensional probability distributions. Each agent i maintains one observed and one hidden variable. The observed variable y_i represents an agent's perception of the physical world (akin to what Leslie [28] calls a Theory of Body, and/or what Wellman and Bartsch [44] consider the input to belief-desire reasoning, see also [43]). These perceptions are derived from a robot's own sensing information and pertain only to information about physical objects in the world. Local evidence only includes information about other agents in a mechanical sense; it does not contain any information about their beliefs or intentions. The **state**, which is represented by the hidden variable x_i, is a vector consisting of variables important to the agent's behavior. The state vector can contain variables representing intended actions for the agent, plans for other agents, knowledge about the environment (such as localization information or object classification), or other high level information.

In our ToM MRF model, the state of an agent is conditional on its observed variable and the states of other agents whose states it can perceive. From the perspective of a single agent, the other nodes are actually the perceived states of other agents. For the purposes of this paper we will make no distinction between perceived state and actual state of others, due to robots being unlikely to access to a human's actual mental state. Because we are concerned with collaboration between agents acting cooperatively, there is no reason for agents to deceive each other and any information about state can be explicitly communicated. Considering state to be perceived state will simplify the model and allow us to model a collaborative team as a single graph with agents mutually affecting each other (Figure 1).

Given the observed and hidden variables, a pairwise MRF factors a collaborative team action x into two functions: pairwise compatibility $\psi_{j,i}(x_j, x_i)$ between each agent pair (ij) and local evidence $\phi_i(x_i, y_i)$. The joint probability distribution can then be stated as follows:

$$Pr(x) = \frac{1}{Z} \prod_{(ij)} \psi_{j,i}(x_j, x_i) \prod_{i} \phi_i(x_i, y_i) \qquad (1)$$

The normalization constant Z ensures that the distribution sums to 1. The formulation in (1) has two key benefits: we factor the global coordination and local computation into distinct terms; and we can express a spectrum of multi-robot action selection methods by modifying these terms.

The **local evidence** $\phi(x_i, y_i)$ is a function that is used to form a distribution for agent i over possible states, x_i, given only its physical observations y_i. Computationally, the local evidence produces a scalar output proportional to how well a particular assignment of state x_i reconciles with percepts of reality y_i. Integrating over all possible states given y_i and then normalizing will produce a distribution over x_i. Formation of this distribution over x_i is analogous to making decisions without any information about the mental states of others (i.e., without a ToM), even though that information is provided by other agents. Representing the state as a distribution over possible values allows the model to contain the uncertainty in decision making and perception. The local evidence function is analogous to likelihood models as they are used in Bayes filters [41] for localization.

The **pairwise compatibility** $\psi_{j,i}(x_j, x_i)$ is a function that encodes the influence of agent j's state on agent i's state. Similarly to local evidence, a compatibility function returns a scalar output proportional to the compatibility of agents i and j taking on specific state assignments. This function can be used to encourage agreement (or disagreement) between perceptual information from multiple agents, to produce compatibility in plans, or inform predictions about changes in the environment (such as those that will be brought about by another agent).

3 Modeling Experimental Findings

We can model findings from research into Theory of Mind with respect to child behavior within our MRF framework. Representing the action of the child as its state and choosing appropriate local evidence and compatibility functions for the task in each experiment, we illustrate that our model generates predictions that agree with the experimental results. As with several previous endeavors to instantiate children's cognition as a probabilistic model [39, 40] we do not believe that this approach desribes the processes by which children engage in their behaviors. Rather, we offer this description as a computational-level analysis [29] of children's cognitive abilities.

3.1 Effects of Uncertainty

A study by Sabbagh and Baldwin [34] showed that the uncertainty in another agent's beliefs affected the beliefs of the child participants receiving the information. Preschoolers were told that a friend of the experimenter's wanted his toy, which was a "blicket." There were two conditions in the experiment. Half of the children were instructed by a "confident" experimenter, who picked up a particular object that was novel to the children, and labeled it a blicket. The other children were instructed by an "uncertain" experimenter, who picked up the same object and gave it the same label but with hesitance and doubt. Children were later tested on their retention of this label, and those that were exposed to the confident experimenter were more likely to be able to correctly identify the toy in later production and comprehension tests.

To model this experiment with an MRF (Figure 3), we consider the state of each agent to be the name given to the toy in question. To simplify the model we will consider only two possibilities, the agent believes the object is a blicket ($x_i = b$) or the agent believes it is not a blicket ($x_i \neq b$). The child is agent

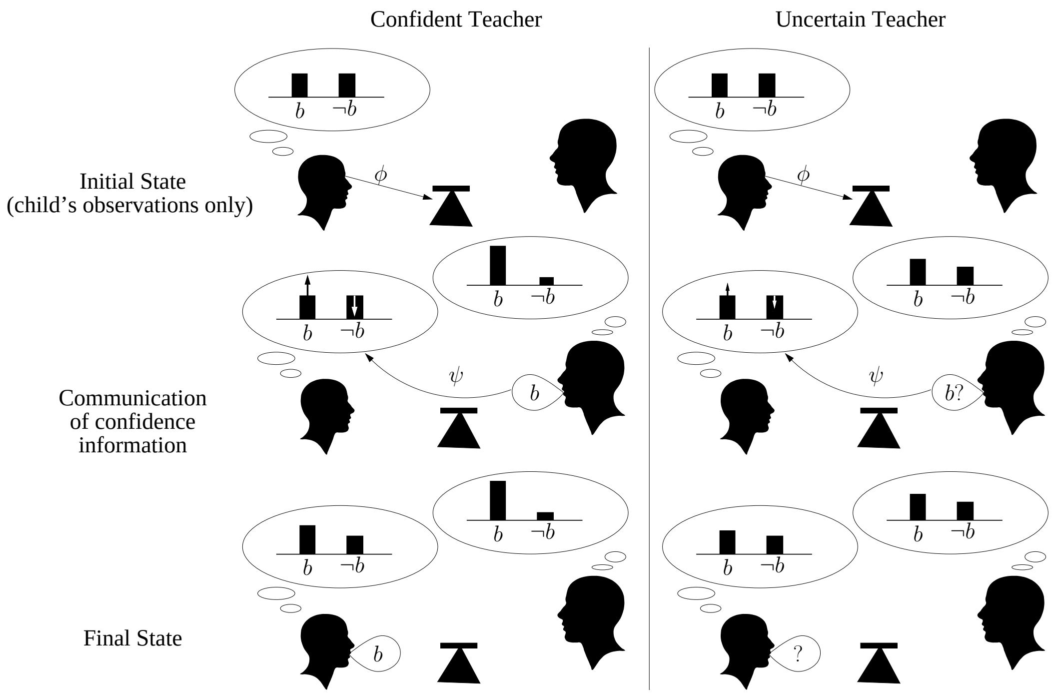

Fig. 2 The thought bubbles in this figure contain the probability distribution over whether or not the toy in the center is a blicket. The child initially has an even distribution, having no idea whether the object is a blicket. However, when the teacher communicates their belief, the compatibility function pushes the child's distribution toward agreeing with the teacher's answer. However, the more confident the teacher is (i.e. the more peaked the teacher's distribution), the greater the influence on the child's distribution.

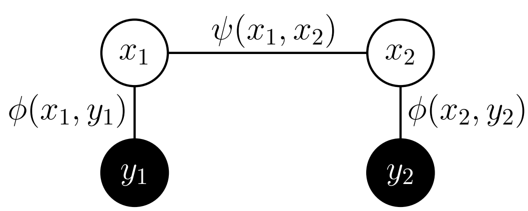

Fig. 3 A graphical representation of the MRF for the toy identification task. Agent 1 is the child and agent 2 is the experimenter. Their states are made up of a single value representing the name they give to the toy. y_1 is the child's observation of the toy, while y_2 represents the script given to the experimenter.

1 and the teacher is agent 2. Because the teacher is scripted, her local evidence function is fixed for both teachers; $\phi(x_2 = b) >> \phi(x_2 \neq b)$ for the certain teacher and $\phi(x_2 = b) > \phi(x_2 \neq b)$ for the uncertain teacher. The object is novel to the child so $\phi(x_1)$ is uniform. Children accept adults as authorities on the names of new objects when there's no information to suggest otherwise [12], so the child's compatibility will be higher when their beliefs agree with those of the teacher ($\psi(x_1 = x_2, x_2) > \psi(x_1 \neq x_2, x_2)$). We can express the certainty with which a child believes that the object is a blicket with Equation 2.

$$Pr(x_1 = b) = \frac{1}{Z}\phi(x_1 = b, y_1) * \\ (\psi(x_1 = b, x_2 = b)\phi(x_2 = b) + \\ \psi(x_1 = b, x_2 \neq b)\phi(x_2 \neq b)) \tag{2}$$

Because children were assigned to the two groups randomly, we assume all the functions that are inherent to the child (i.e., their initial belief about whether the novel object in question is a "blicket") remain unchanged across the two conditions. However, $\phi(x_2)$, the teacher's belief about the toy, is affected by the certainty of the teacher. Plugging in the different values for $\phi(x_2)$ into the equation changes the child's final distribution. Because $\psi(x_1 = b, x_2 = b) > \psi(x_1 = b, x_2 \neq b)$, the children's certainty with the new word blicket increases with the certainty of the teacher, exactly as shown by the experiment (see Figure 2).

3.2 Effects of Reliability

A study by Koenig and Harris [26] showed that children depend more heavily on information from sources that have demonstrated reliability in the past. These results point to the fact that in addition to considering another's knowledge states, the agent needs to allow information gained from others' mental states to be factored into decisions differently depending on the nature of the source of the knowledge. In the experiment, preschoolers were shown a set of familiar objects (i.e.,

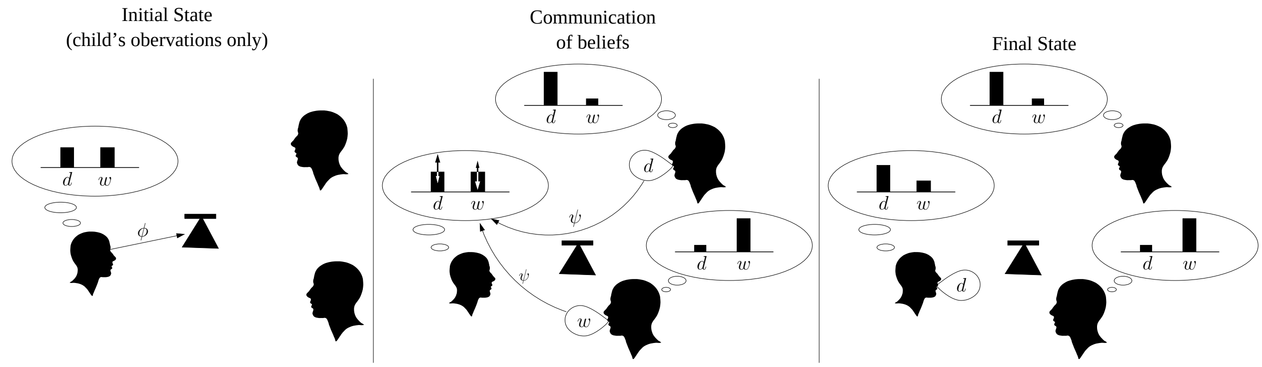

Fig. 4 The thought bubbles in this figure contain the probability distribution over whether the toy in the center is a dax or a wug. The child initially does not have a preference between the two names. However, when the adult confederates communicate their beliefs the difference in compatibility functions means that the reliable teacher has more influence. The resulting distribution leads the child to choose the answer given by the reliable teacher.

objects that children of this age would easily label accurately) and two adult confederates. Each confederate was asked to generate the object's label. One confederate was always accurate (e.g., calling a shoe a shoe), while the other confederate was always inaccurate (e.g., calling the same shoe a ball). After this training, children were shown a novel object, for which they did not know the name. Children were asked which of the two confederates they would ask to provide them with the label for this novel object, and the majority of children picked the reliable confederate. Each confederate also labeled the novel object, with the reliable and unreliable confederates generating different novel labels (e.g., one called it a "dax", the other a "wug"). When asked to endorse one of the two labels, children generally chose the label generated by the reliable confederate.

We can model this experiment with Equation 3.

$$Pr(x_1) = \frac{1}{Z}\phi(x_1, y_1) \prod_{i=\{R,U\}} \sum_{x_i} \psi(x_1, x_i)\phi(x_i) \tag{3}$$

x_1 represents the child, while x_R and x_U represent the reliable and unreliable confederates respectively (Figure 5). Both confederates appear equally certain, so $\phi(x_R) = \phi(x_U)$ when x_R and x_U are the answers given by the respective confederate. The answer to the question (i.e., what the label of the object is) is unknown to the child, so $\phi(x_1)$ will be the same for any answer presented to the child. Because of the past differences in reliability, the child would have different compatibility functions for each confederate. For the unreliable confederate the compatibility function would be nearly uniform and for the reliable confederate it would be peaked where $x_1 = x_R$. The effect of the difference in the compatibility functions is that the child's final distribution is peaked at the answer given by the reliable confederate (see Figure 4). The model predicts the same behavior found in the experiment; children rely more heavily on the reliable individual.

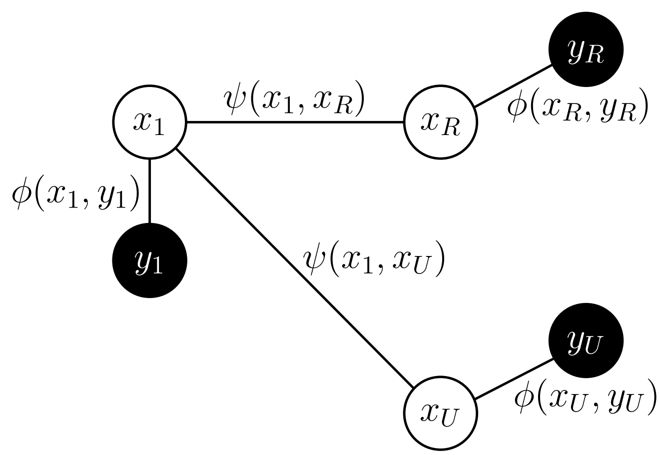

Fig. 5 A graphical representation of the MRF for the reliable and unreliable confederate experiment. The difference in the compatibility functions will make it more probable that $x_1 = x_R$.

3.3 Gaze Following

Most researchers in social cognitive development agree that joint attention, particularly in the form of gaze following—attending to an unseen position in space based on the attentional focus of another agent—is an important aspect of developing an adult-like theory of mind [5,7,30,35]. Gaze following has long been considered a precursor to more adult-like mental-state understanding in theory of mind research [2,10,24]. Critically, it is also an aspect of theory of mind that has been implemented in robots with some success. Both Breazeal [4] and Scassellati [36] include gaze following systems on their social robots. In both systems, the robot has

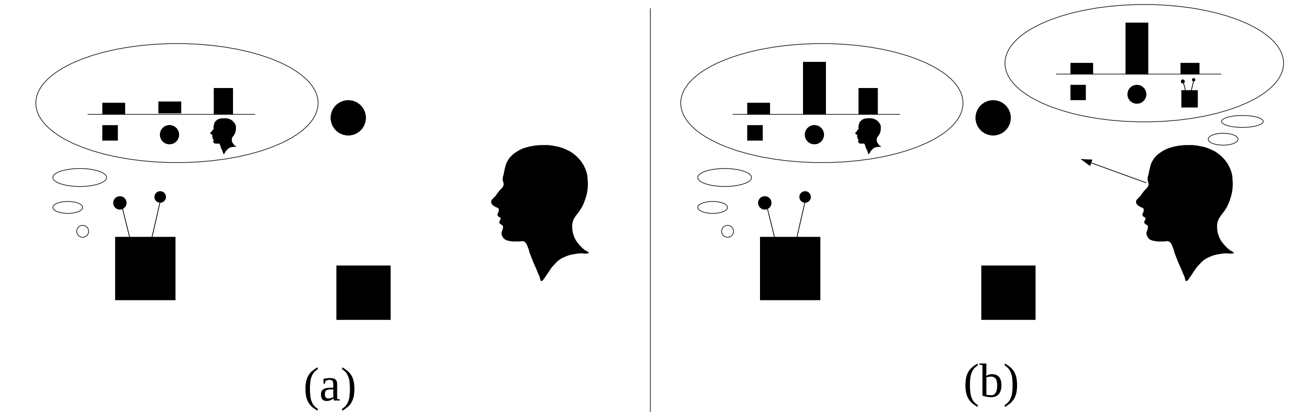

Fig. 6 (a) The robot's distribution over possible objects of focus without any understanding of ToM. The robot has a preference for the human face and two close objects. (b) The robot is using gaze tracking to estimate the human's mental state. The compatibility function gives a high probability to looking at the same object as the human, in this case the ball.

a preference for attending to some objects based on its perception, preferring to look at faces, bright colors, or moving objects. The robot also has a preference for attending to objects that are the focus of other agents. The goal is for the robot to find a balance between these preferences, alternately following the gaze of the human and then looking back to the human face or other objects that attracted its perception.

This behavior can be conceptualized as a natural consequence of casting these preferences into our MRF model. The same two node graph from the uncertainty model (see Figure 3) would be used, but the state variable and the local evidence and compatibility functions would change. The state would be a variable representing the attentional focus of each agent. The preferences based on the perceptual properties of the objects would be encoded into its local evidence function. The compatibility function would express a preference for following the gaze of the human. This is similar to the compatibility function used for the reliable teacher. The compatibility function would be peaked where $x_1 = x_2$ ($\psi(x_1 = x_2, x_2) > \psi(x_1 \neq x_2, x_2)$). The resulting distribution over possible directions of focus, conditioned on the human's state, would have peaks at those locations where the robot perceived objects of interest and also at the location of the human's gaze (see Figure 6). Periodically sampling from this distribution would cause the robot to gaze at these locations of heightened interest.

4 MRF-based Multi-robot Coordination

For multi-robot coordination, an MRF can be used to model a robot team and any other agents with which the team will be interacting. A single framework would be used to incorporate the states of other robots and human collaborators. The compatibility functions and even the methods for obtaining the state would be very different for robotic and human collaborators. The simplest application of the MRF model to multi-robot coordination is where the state is the action to be taken and all the agents are robots. Then to perform action selection with a multi-robot MRF, each robot must compute its "piece" of the joint probability distribution (1). That is, for each robot i, we want to compute the marginal probability $p_i(x_i)$, which expresses the likelihood of robot i taking an action, conditioned on both its own observations and knowledge of the other robots' actions. Naively, this inference procedure would require communicating all the robots' observations to a centralized decision-maker and the required computation would expand exponentially with the number of robots. Instead, we propose exploiting the factored structure of the MRF to apply the belief propagation (BP) algorithm [45], which performs inference in a distributed manner.

4.1 Belief Propagation

BP operates by passing "advice messages" between robots, and using these messages, in combination with local observations, to maintain a belief $b_i(x_i)$ for each robot i. When BP converges, the belief $b_i(x_i)$ is approximately equal to the marginal probability $p_i(x_i)$ that we need for coordinated action selection. Robot i exchanges messages only with its neighbors $N(i)$, ensuring that the algorithm can scale to large teams.

Robot i's belief $b_i(x_i)$ is given by the following product, with normalization constant Z:

$$b_i(x_i) = Z\phi_i(x_i, y_i) \prod_{j \in N(i)} m_{j,i}(x_i) \qquad (4)$$

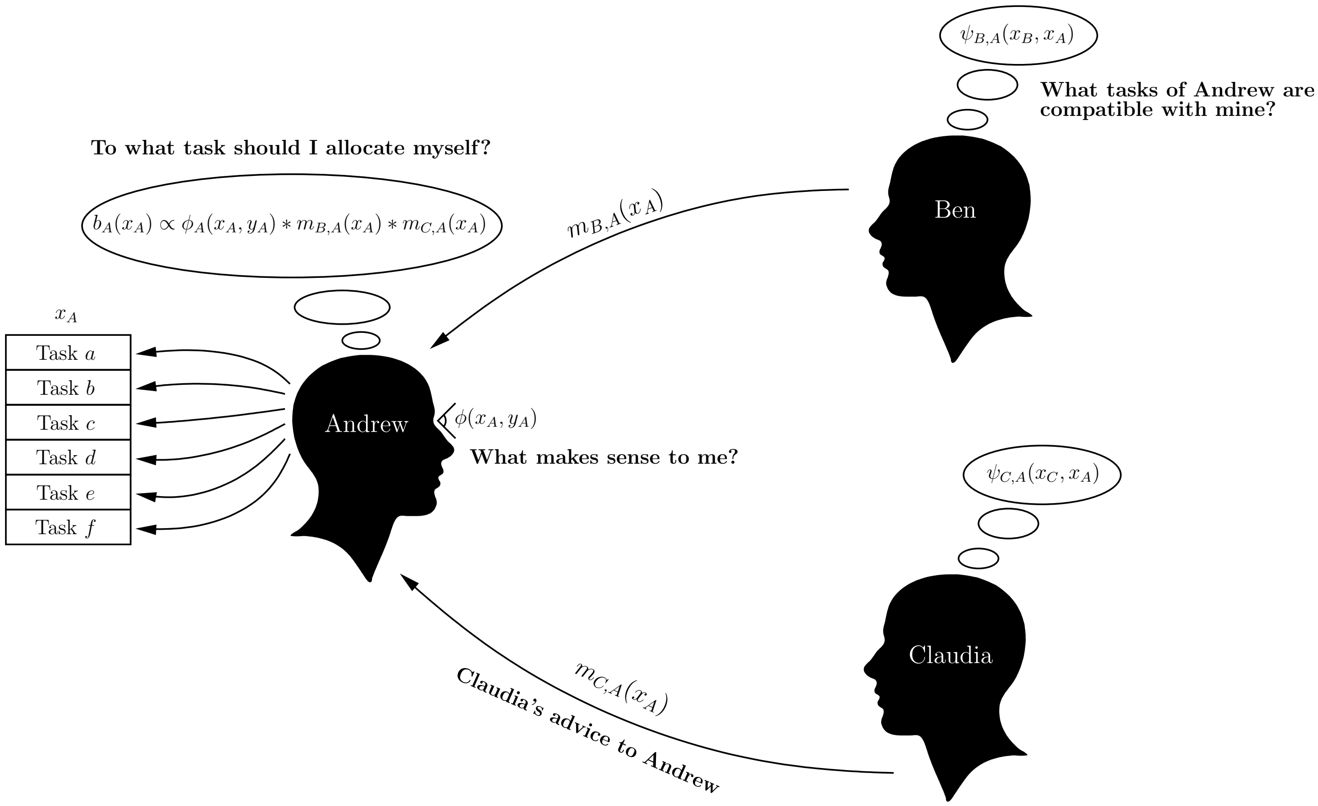

Fig. 7 A graphical representation of the how ToM may be incorporated into decision making through belief propagation. Andrew is listening to incoming advice messages updating his beliefs about the probability he should be doing each task. He will then transmit advice messages back to Claudia and Ben reflecting his updated beliefs.

The term $m_{j,i}(x_i)$ is an advice message from robot j to robot i suggesting how robot i should act, given what robot j knows about the world. Robot j computes its message to a neighboring robot i as a product of robot j's local evidence, the pairwise compatibility between their actions, and incoming messages from robot j's neighborhood (except for those coming from robot i), summed over all possible actions x_j:

$$m_{j,i}(x_i) = \sum_{x_j} \phi_j(x_j, y_j)\psi_{j,i}(x_j, x_i) \prod_{k \in N(j) \setminus i} m_{k,j}(x_j) \quad (5)$$

Multi-robot belief propagation is performed through selective updating of inter-robot messages. The essence of inference with belief propagation lies in the pairwise messages $m_{j,i}(x_i)$. Messages between robots are continually but selectively updated according to (5) and domain-specific compatibility and likelihood functions. Beliefs are computed only when needed to make a decision. While belief propagation is a commonly used algorithm for inference on MRFs, in this context it also takes on a social interpretation (see Fig. 7). Robots continually send each other advice messages about their intentions while modifying their strategy to take into account their neighbors' plans. This continues until convergence, at which point the team has reached a stable strategy.

The result of multi-robot belief propagation will be an *action posterior*, a probabilistic belief distribution over actions for each robot. Whenever an individual robot needs to make a new action decision, it computes its action posterior as a belief from local evidence and incoming messages, according to (4). This belief distribution can then be sampled (e.g., using expectation or MAP estimators) to select a specific action for the robot to execute.

4.2 Multi-Robot Experiments

In our results, we found we were able to use BP with the MRF model to control teams of robots to work together to form chains of sight (COS). In COS, we assume that multiple robots spread out in an environment where there is a known start location and a goal object at an unknown location. The group of robots searches for the goal object and once found, forms a visibility chain between the start and the goal. Individual robots can communicate within some range and perceive other robots and the goal location through their field of view.

Casting the problem into the MRF framework, the state of the robot was taken to be its intended location (limited to discrete values within a finite-size environment). The local evidence function was the product of three functions:

$$\phi_i(x_i, y_i) = D(x_i, y_i)G(x_i, y_i)O(x_i, y_i) \tag{6}$$

The factor $D(x_i, y_i)$ represents traveling distance and expresses a robot's preference for closer locations, $G(x_i, y_i)$ stands for goal attraction and assures that a robots does not move if it sees the goal and $O(x_i, y_i)$ expresses occupancy of locations by other robots or obstacles. The compatibility function expresses a high compatibility between locations that are visible to each other and lie on the shortest path toward the start location. Belief propagation is used to find the conditional probabilities and each robot chooses its most probable action, so robots coordinated without using any centralized planning; instead the robots only use information about each other's "intentions".

In this work, we conducted 50 COS trials in simulation (Player/Gazebo [16]) with five Pioneer robots. In each trial, inference using MRF produced successful action allocations, causing the team to produce a chain of sight from a start to a goal location (Figures 8 and 9). The 50 trials consisted of test runs on five different configurations of start, goal and robot locations, resulting in 10 trials per configuration. The number of total messages transmitted before reaching a correct arrangement ranged from 10 messages, in situations where only two robots were required for the COS, to 80 messages for complicated arrangements. The number of messages needed also varied over multiple runs on the same scenario, an indication of the randomness involved in the problem.

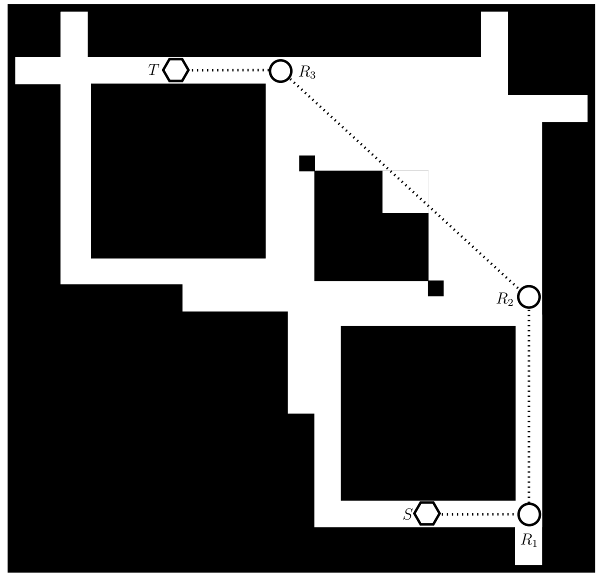

Fig. 8 A map showing three robots forming a successful chain of sight formation

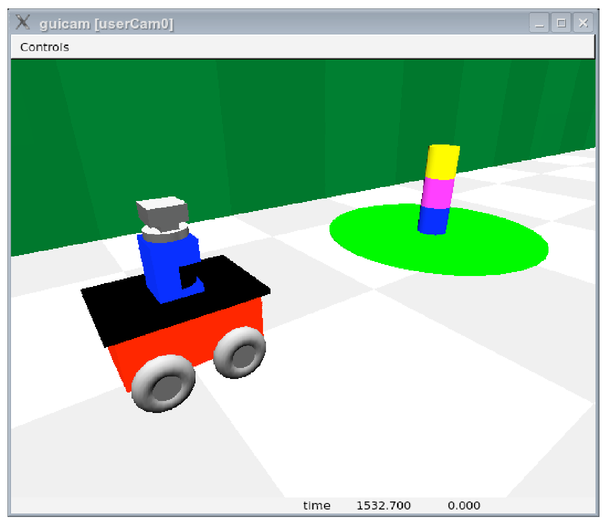

Fig. 9 A screen shot from the Player/Gazebo [16] simulator showing one of the Pioneer robots and a goal beacon

5 Conclusion

We have proposed MRFs as a probabilistic model for incorporating domain knowledge from ToM into robotic decision making. We have shown how MRFs can model some simple situations which require the use of ToM for decision making. Choosing intuitive local likelihood and compatibility functions, we find the predictions from our model match experimental results from ToM research with young children. Although we recognize that other computational models might also potentially account for these findings, we believe the MRF framework has significant potential for phrasing hypotheses and predicting results from experimentation. For instance, describing how the child's belief states change as a function of their compatibility with speakers (based on their reliability) suggests that there might be domains of knowledge that are unaffected by reliability. We are currently exploring this possibility empirically with preschoolers.

Local evidence and compatibility functions factor the decision influences into internal objectives and the influences of external agents. Although our application of MRFs to issues in robotic theory of mind is still developing—for instance, we have not described exactly how to define what the state vector of each agent should contain—we did show how the model could be applied to a limited task like gaze following. We also demonstrated how this model was used for multi-robot coordination in simulation. Our goal in the future is to apply the model to other multi-robot coordination tasks, as well as expand into the area of human-robot collaboration using estimates of state derived from robotic ToM mechanisms as they develop.

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