Localization from Pairwise Distance Relationships using Kernel PCA

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1 Introduction

In this paper, we present a method for estimating the relative localization for a set of points from information indicative of pairwise distance. While other methods for localization have been proposed, we propose an alternative that is simple to implement and easily extendable for different datasets of varied dimensionality. This method utilizes the Kernel PCA framework [6] (or equivantly the Multidimensional Scaling framework [8]) for producing localization coordinates. To localize, Kernel PCA is performed on a matrix of pairwise similarity values, assuming similarity values are reflective of the data generation distance metric. We test this localization method on 3D points comprising a human upper body and signal strength data from Mote processor modules.

2 What is Kernel PCA?

As developed by Scholkopf et al., the Kernel PCA (KPCA) is a technique for nonlinear dimension reduction of data with an underlying nonlinear spatial structure. A key insight behind KPCA is to transform the input data into a higher-dimensional *feature space*. The feature space is constructed such that a nonlinear operation can be applied in the input space by applying a linear operation in the feature space. Consequently, standard PCA (a linear operation) can be applied in feature space to perform nonlinear PCA in the input space.

The caveat to operating in feature space, however, is that the mapping from input space to feature space is not uniquely identifiable. The process of explicitly stating the coordinates of the input data in a feature space could be difficult to produce. To avoid this problem, KPCA utilizes the *kernel trick* to perform feature space operations using only dot-product between data points in the feature space. In simpler terms, KPCA requires only the similarity between all pairs to perform nonlinear dimension reduction. Similarity between data pairs can be measured in input space using some chosen similarity measure. KPCA uses kernel functions (e.g., radial-basis functions). Isomap [7], a similar technique, uses geodesic distances. By using pairwise similarity, the relative topology among the data is preserved between the input and feature spaces. However, the specific metric information about the data position, scale, and orientation are lost.

In a larger sense, KPCA provides a means for mapping data between Cartesian spaces and pairwise distance matrices while preserving the relative topological structure of the data. The KPCA framework general procedures for computing pairwise similarity from Cartesian input data (via some all-pairs distance computation) and for computing Cartesian locations of data from pairwise similarity (via a MDS-like procedure). In performing nonlinear dimension reduction, KPCA transforms input data to pairwise distances to Cartesian locations of an embedding such that the embedding locations preserve the topology of the input data while removing the nonlinearity of its underlying structure. We have found that Isomap typically performs better in this capacity [3, 1], but KPCA provides a more grounded explanation of these procedures for subtleties such as feature space centering.

3 Kernel PCA localization

If we focus on the transformation of pair distances to coordinates, this transformation is similar to localizing a set of entities with pairwise distance estimates. For example, consider the case of a group of robots capable of estimating their distance to other robots in the group. The distance measurements could be actual physical distances, possibly from laser range readings [2], or indicators with a proportional relationship to physical distance, such as radio signal strength [4]. The distances between the robot entities are directly or indirectly grounded in the physical world. The topology of this physical grounding is preserved in the pairwise distances between robot entities. However, metric information specific to the physical world is lost. The embedding produced by KPCA specifies the relative locations of the robot entities to each other. KPCA produces feature space eigenvectors. The i^{th} component of the j^{th} feature space eigenvector specifies the location of the i^{th} entity with respect to the j^{th} embedding dimension. We assume the number of dimensions to select for embedding is the same as the dimensionality of the input data (i.e. planar world, 3D coordinates) and is known *a priori*. If landmarks with absolute coordinates in the physical world are included in the KPCA process, an affine transformation can be found to restore world metric information to the embedded coordinates.

For this paper, we assume that all entities can estimate distance to all other entities. This assumption is not true in general and is a potential source of localization error because the localization problem becomes underconstrained.

Our implementation of KPCA is derived from the Kernel PCA toy example for MATLAB provided by the authors of [6], which can be downloaded from http://www.kernel-machines.org/.

4 Results

We tested the Kernel PCA localization technique on two data sources with available ground truth. The first dataset is a set of voxels collected from a volume capture system (Figure 1 1). The second dataset is pairwise signal strength data from multiple communicating radio modules (Figure 2 2).

4.1 Human Volumes

The human volume dataset was collected from a volume capture system implemented from the description provided by Penny et al.[5]. This dataset consists of a set of points in 3D representative of the occupancy of a human upper body within a voxel grid. The volume of each voxel is approximately $2.5in^3$. The volume points were converted into a matrix of pairwise Euclidean distances to serve as input into the localization and saved for ground truth. The pairwise distance matrix was run through KPCA localization to reproduce volume point locations closely approximating the input volume, but not exact. The variance between the input and localized volumes can be seen by their respective pairwise distance matrices. These matrices illustrate that the embedding process does not completely preserve the pairwise distances, but provides a good approximation. This inexact approximation stems from the fact that KPCA itself is a least-squares approximation in feature space. We hypothesize that the localization can be automatically refined to minimize difference between the pairwise distances of the input and localized volumes, but that is for future work.

4.2 Radio Modules

The radio signal strength and collected from a set of 7 Mote processor modules in a 2D hexagon pattern continuously passing messages between each other. This data and ground truth was graciously provided by Gabe Sibley. Signal strength between each pair of radios was used as an indicator of distance between the pair and formed into a matrix of pairwise distances. However, KPCA cannot be performed directly on this matrix because of its gross nonsymmetries. Several aspects of the KPCA technique assume the matrix is symmetric (or at least close to symmetric). One critical aspect require symmetry is the feature space centering mechanism. We applied two modifications to KPCA localization for the problems associated with nonsymmetric pairwise distances. The first modification scale normalized each column followed by scale normalization of each row. This normalization was used to counter variances between radios with different transmitting strengths and reception capabilities. Making an analogy between radios radial basis kernels, this normalization would be an attempt to make kernels of different widths scale to a common width. The second modification assumes normalization will not bring symmetry to the distance matrix. Thus, feature space centering will not work. This modification removes feature space centering from the localization process. The feature space mean will be represented in the first eigenvector of the embedding. We ignore the first eigenvector and use the second and third to perform normalization. The radio placement produced by KPCA localization was surprisingly representative of their actual hexagon positioning, but far from perfect. Considering the factors obfuscating the structure of the radio positions and much room is left for improving KPCA localization, this result is quite promising.

5 Acknowledgements

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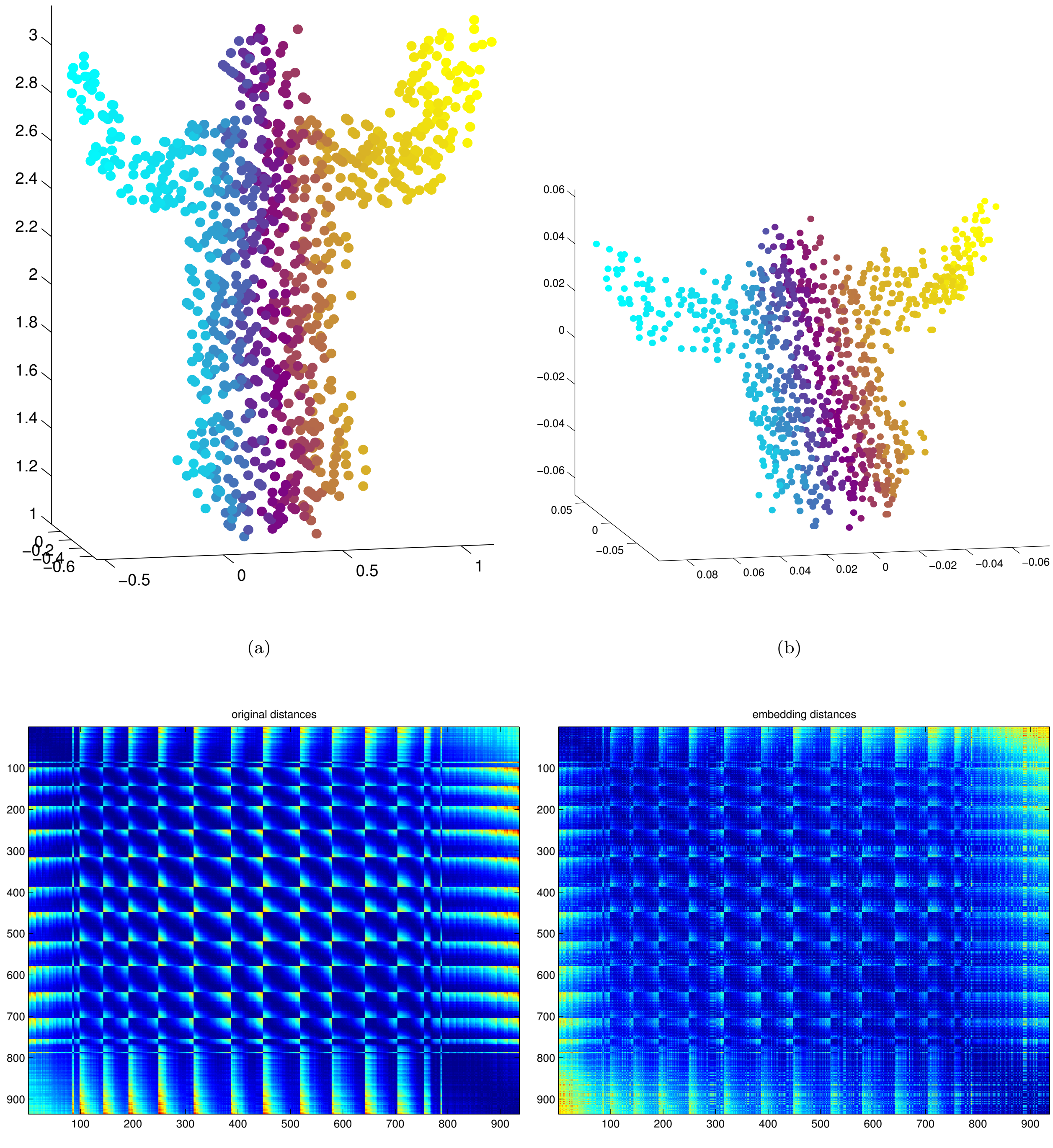
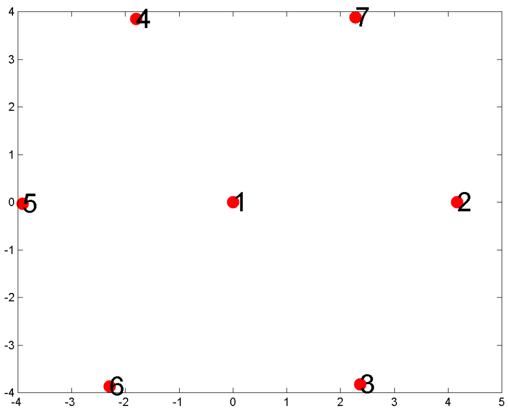
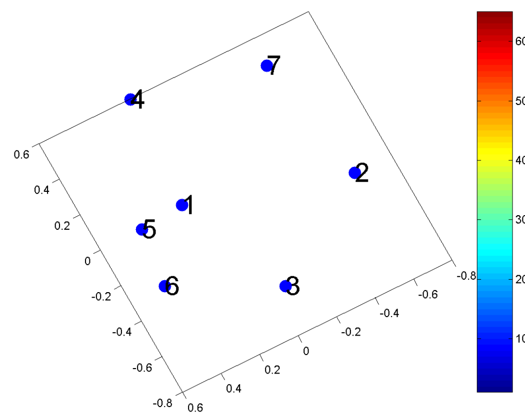


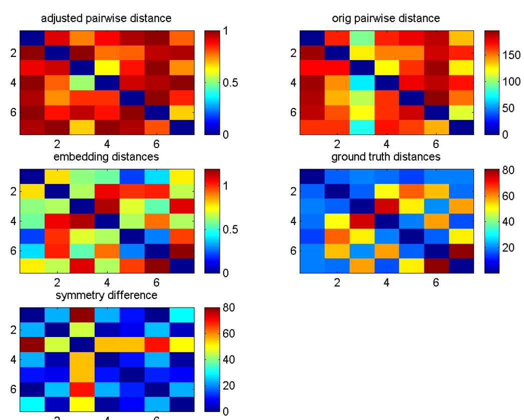
Figure 1:



(a)



(b)


adjusted pairwise distance
orig pairwise distance
embedding distances
ground truth distances
symmetry difference

(c)

Figure 2: