Applied Mechanics September 2012 Vol.57 No.26: 3430–3435
doi: 10.1007/s11434-012-5386-6

Dynamic prediction of building subsidence deformation with data-based mechanistic self-memory model

WANG Wei[1,2], SU JingYu[1*], HOU BenWei[2], TIAN Jie[2] & MA DongHui[1]

[1] *Institute of Earthquake Resistance and Disaster Reduction, Beijing University of Technology, Beijing 100124, China;*
[2] *College of Civil Engineering, Beijing University of Technology, Beijing 100124, China*

Received May 11, 2012; accepted June 12, 2012

This paper describes a building subsidence deformation prediction model with the self-memorization principle. According to the non-linear specificity and monotonic growth characteristics of the time series of building subsidence deformation, a data-based mechanistic self-memory model considering randomness and dynamic features of building subsidence deformation is established based on the dynamic data retrieved method and the self-memorization equation. This model first deduces the differential equation of the building subsidence deformation system using the dynamic retrieved method, which treats the monitored time series data as particular solutions of the nonlinear dynamic system. Then, the differential equation is evolved into a difference-integral equation by the self-memory function to establish the self-memory model of dynamic system for predicting nonlinear building subsidence deformation. As the memory coefficients of the proposed model are calculated with historical data, which contain useful information for the prediction and overcome the shortcomings of the average prediction, the model can predict extreme values of a system and provide higher fitting precision and prediction accuracy than deterministic or random statistical prediction methods. The model was applied to subsidence deformation prediction of a building in Xi'an. It was shown that the model is valid and feasible in predicting building subsidence deformation with good accuracy.

data-based mechanistic, self-memorization equation, building subsidence deformation, retrieved modeling, dynamic prediction

Citation: Wang W, Su J Y, Hou B W, et al. Dynamic prediction of building subsidence deformation with data-based mechanistic self-memory model. Chin Sci Bull, 2012, 57: 3430–3435, doi: 10.1007/s11434-012-5386-6

The fast development of economics promotes the urbanization of China. More and more high-rises appear in our cities [1–16]. In order to ensure the service life and security of constructional works, it is necessary to monitor the constructional work systematically during its building and operating. Due to the diversity, complexity and non-deterministic characters of inducing factors of the building subsidence deformation, the mechanical behavior and deformation trend of building subsidence also reflect non-linear characteristics with the coexistence of the uncertainty and randomness [17–34]. Existing methods for building subsidence deformation prediction usually establish the time series analysis model by utilizing monitored data [35–40]. However, most of these models are of the "parameter model" class which may not have clear physical meaning of parameters and discard effective parts of monitoring data. On the basis of the retrieved modeling, self-memorization principle has been recently developed based on the mechanism of time series data. As a mathematic realization on integration of deterministic and random theory, the self-memory model is a statistical-dynamical method to solve problems in nonlinear dynamic systems [41–43]. The core method of the model is to transform differential equations into difference-integral equations by introduction of the self-memory function. For dynamic systems described by differential equations, prediction model can be established directly by self-memorization equation. As for dynamic systems which cannot be described by differential equations, as long as the

*Corresponding author (email: jysu@bjut.edu.cn)

system has a certain length of time series data which can be treated as particular solutions of the nonlinear dynamic system, the approximate nonlinear differential equation describing the dynamic system can be deduced by retrieved modeling. With the help of differential equation, system prediction can be easily conducted then. Turbulence and explosion [44–46] are rapid and intense mechanical process different from slowly sinking activities in this paper, but they can also be described by the dynamical evolution. Some micro-activities, such as unsaturated soil volume change will have an impact. Therefore, to improve the accuracy of prediction, the self-memory model not only combines dynamics calculations and estimate parameters of historical data, but also utilizes the strength of extracting information from historical data in statistics [47–49].

In view of advantages of the self-memorization principle in prediction, this study established a building subsidence deformation prediction model with the self-memory function. The model takes the evolution of building subsidence deformation as that of dynamical systems and utilizes the self-memorization equation and its prediction model of dynamic system to conduct the non-linear displacement time series analysis and prediction. As an illustration, the subsidence deformation of a building in Xi'an was predicted in case study. It was shown that the prediction results are consistent with the monitored data; the proposed model is valid and feasible in predicting building subsidence deformation.

1 Establish the differential equation for the dynamic prediction of building subsidence deformation by bi-directional difference method

If the time series of building subsidence deformation are $x(t)$, and x_t=$x(t)$, then the time-varying differential equation can be described as [47,48]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a_1 x_t + a_2 x_{t-1} + \cdots + a_p x_{t-p+1} + b_1 x_t^2 + b_2 x_{t-1}^2 + \cdots + b_p x_{t-p+1}^2, \tag{1}$$

where a_1, a_2,$\cdots a_p$, b_1, b_2,$\cdots b_p$ are undetermined coefficients; x denotes the building subsidence deformation, x_t, x_{t-1},$\cdots x_{t-p+1}$ are the values of x at time t, t–1,$\cdots t$–p+1, respectively; p is the retrospective order of the equation, which denotes that the change of x is related to the values at time t, t–1,$\cdots t$–p+1.

If $x(t)$ is a sample of same time interval, and if Δt= (t+1)–t=1, then the errors of bi-directional in eq. (1) can be expressed respectively as

$$\varepsilon_{bk} = \left(x_k - x_{k-1}\right) - \left(a_1 x_{k-1} + \ldots + a_p x_{k-p} + b_1 x_{k-1}^2 + \ldots + b_p x_{k-p}^2\right), \tag{2}$$

$$\varepsilon_{fk} = \left(x_{k+1} - x_k\right) - \left(a_1 x_k + \ldots + a_p x_{k-p+1} + b_1 x_k^2 + \ldots + b_p x_{k-p+1}^2\right). \tag{3}$$

Then the objective function to get the minimum of eqs. (2) and (3) can be expressed as

$$\min\ \varepsilon^2 = \sum_{k=1}^{n}\left(\varepsilon_{bk}^2 + \varepsilon_{fk}^2\right), \tag{4}$$

where n=m–p–1, m is the time-series length that will be used to solve relevant parameters.

The coefficients a_1, a_2,$\cdots a_p$ and b_1, b_2,$\cdots b_p$ in eq. (4) are evaluated by the least squares method. The retrospective order p should be a smaller value to ensure the robustness of the model. Meanwhile, the number of terms in the right-hand side of eq. (1) can be reduced by the size of relative variance. Let c_k=[a_k b_k]; the criterion can be expressed as

$$\sigma_k = c_k^2 \Big/ \sum_k c_k^2 . \tag{5}$$

When σ_k is less than the specified value σ_c (σ_c is a pre-given smaller value and assumed to be 0.01 in this paper), this term plays a minor role in eq. (1) and can be deleted. Then, the term numbers and coefficients in eq. (1) can be determined.

2 Dynamic prediction with data-based mechanistic self-memory model

Introducing a memory function into a differential equation will produce a difference-integral equation, that is, the self-memorization equation [47–50]. The differential equation dx/dt, which has been determined by solving eqs. (1)–(5), is considered to be the system dynamic equation of building subsidence deformation:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = F\left(x, \lambda, t\right), \tag{6}$$

where λ is a parameter, t is time interval series, and $F(x, \lambda, t)$ is the dynamic kernel. Introduce a memory function $\beta(t)$, and define an inner product in the Hilbert space:

$$\left(f, g\right) = \int_a^b f\left(\xi\right) g\left(\xi\right) \mathrm{d}\xi \quad \left(f, g \in L^2\right). \tag{7}$$

Applying the above inner product operation to eq. (6) and supposing variables x, β are continuous, differentiable and integrable, we can obtain the analytic formula of eq. (6) as

$$\int_{t_{-p}}^{t_{-p+1}} \beta\left(t\right)\frac{\partial x}{\partial t}\mathrm{d}t + \int_{t_{-p+1}}^{t_{-p+2}} \beta\left(t\right)\frac{\partial x}{\partial t}\mathrm{d}t + \cdots + \int_{t_0}^{t_1} \beta\left(t\right)\frac{\partial x}{\partial t}\mathrm{d}t = \int_{t_{-p}}^{t} \beta\left(t\right) F\left(x, \lambda, t\right)\mathrm{d}t. \tag{8}$$

For every integral term in the left-hand side of eq. (8), after following calculus, making an integration by parts, applying the median theorem, and performing algebra operation, we can obtain a difference-integral equation which is called a self-memorization equation with the retrospective order p:

$$x_t=\frac{1}{\beta_t}\left[\beta_{-p}x_{-p}+\sum_{i=-p}^{0}x_i^m\left(\beta_{i-1}-\beta_i\right)+\int_{t_{-p}}^{t}\beta\left(\tau\right)F\left(x,\lambda,\tau\right)\mathrm{d}\tau\right],\qquad(9)$$

where $\beta_t\equiv\beta(t), x_t=x(t),\ \ \beta_i\equiv\beta(t_i), x_i=x(t_i);\ i=-p,\ -p+1,\cdots0;$ $x_i^m\equiv x(t_m),\ \ t_i<t_m<t_{i+1}$.

As the first and second terms in eq. (9) denote the relative contributions to the value of variable x, they are defined as the self-memory terms. The third term is the total contribution of the function $F(x, \lambda, t)$ in the retrospective time interval $[t_{-p}, t_0]$, and it is defined as the exogenous effect term. eq. (8) emphasizes serial correlation of the system by itself, i.e. the self-memorization of the system. Therefore eq. (9) is called the self-memory equation of the system.

If integral operation is substituted by summation and differential is transformed into difference in eq. (9), then the mean value x_i^m is replaced simply by two values of different times, namely

$$x_i^m=\left(x_{i+1}+x_i\right)/2\equiv y_i.\qquad(10)$$

Take equidistance time interval $\Delta t_i=t_{i-1}-t_i=1$, and merge β_t and β_i together. Letting $x_{-p}\equiv x_{-p-1}^m$ and $\beta_{-p-1}=0$, we have a discrete-type of eq. (9) as

$$x_t=\sum_{i=-p-1}^{-1}\alpha_i y_i+\sum_{i=-p}^{0}\theta_i F\left(x,\lambda,i\right),\qquad(11)$$

where $\alpha_i=(\beta_{i+1}-\beta_i)/\beta_t$, $\theta_i=\beta_i/\beta_t$, $F(x, \lambda, t)$ is determined by eq. (1). The memory coefficients α_i and θ_i are estimated by least squares method [47,50]. Namely, eq. (11) may be used to dynamically predict the building subsidence deformation.

3 Case study

To verify that the proposed model can be used for nonlinear subsidence deformation prediction of buildings, the subsidence data of a high-rise residential building in Chang'an University was selected for modeling and analysis. Table 1 lists the cumulative subsidence data [40] of monitoring point 3#.

In the establishment of a self-memory model, intervals No. 1–26 were taken as the training samples, intervals No. 27–30 were chosen as the testing samples for prediction test. The retrospective order p was determined by trial calculation method. That is, p=2, 3, 4, 5 were selected respectively in turn to construct the prediction model. The smaller relative errors of fitting and multi-step extrapolation prediction were chosen as the criterion to filter the retrospective order p. The calculation results were shown in Table 2.

According to the relationship between the retrospective order p and errors of prediction in Table 2, it could be seen that the average relative error of prediction was reduced with the increase of retrospective order p, which is basically consistent with the modeling experience of trial calculation method. When the retrospective order p=2, the average relative error of prediction was the least, so the value of retrospective order was selected as p=2. Then, the time-varying differential equation of building subsidence deformation could be inversely derived as

$$\frac{\mathrm{d}x}{\mathrm{d}t}=0.5735x_t-0.4862x_{t-1}+0.0344x_t^2-0.0282x_{t-1}^2.\qquad(12)$$

If the right-side terms of the inversely derived differential equation are regard as the dynamic kernel F, then $\mathrm{d}x/\mathrm{d}t=F$. The self-memorization equation of building subsidence

Table 1 Cumulative subsidence data

No.	1	2	3	4	5	6	7	8	9	10
Monitoring data (mm)	0.20	0.50	–0.10	–0.70	–0.90	–1.10	–1.20	–1.30	–1.80	–2.20
No.	11	12	13	14	15	16	17	18	19	20
Monitoring data (mm)	–2.40	–2.60	–3.10	–3.60	–3.80	–3.90	–4.00	–4.20	–4.70	–5.10
No.	21	22	23	24	25	26	27	28	29	30
Monitoring data (mm)	–5.30	–5.40	–6.20	–6.80	–7.20	–7.20	–7.40	–7.80	–8.10	–8.20

Table 2 Relationship between the retrospective order p and errors of prediction and fitting

	p	2	3	4	5
Error of fitting	root mean square error (mm)	0.1711	0.1658	0.1617	0.1374
	average relative error (%)	4.2700	4.4200	3.4200	2.6200
The average relative error of prediction (%)		3.2200	6.9700	10.46	10.50

deformation can be established. After the differential equation is discretely dealt with, the memory coefficients can be solved by least squares method. Then the prediction equation of building subsidence deformation can be expressed as

$$x_t = \sum_{-3}^{-1} \alpha_i y_i + \sum_{i=-2}^{0} \theta_i F(x, \lambda, i), \tag{13}$$

where α_{-3}=0.1786, α_{-2}=−0.9301, α_{-1}=2.6195, θ_{-2}=−1.7527, θ_{-1}=−0.5815, and θ_0=2.0125.

Eq. (13) was used to calculate and fit the monitoring data No. 5–26 of Table 1. There were not fitting values for intervals No. 1–4 because of the retrospective order. Meanwhile, the subsidence data of intervals No. 27–30 were predicted to test the prediction model. The contrast between the fitting results of self-memory model and observed values are shown in Figure 1. Figure 2 and Table 3 show the comparison of the prediction results and observed values.

It is shown in Figures 2, 3 and Table 3 that the fitted relative errors of 22 samples are from 1.26% to 18.86% with the average of 4.27%. There are 17 samples with the relative errors less than 5.00%, which account for 77.3% of the total; there is no sample with relative error greater than 10.00%. In the test calculation of four prediction steps, the minimum and maximum relative errors of the single-step prediction method are 0.58% and 2.39%, respectively, the average relative error is 1.76%. The minimum and maximum relative errors of the multi-step prediction method are 0.58% and 10.87%, respectively; the average relative error is 3.22%. In [40], the relative errors of prediction were 6.07%, 1.84% and 1.18% for the maximum Lyapunov index prediction method, first-order local prediction method and weighted first-order local prediction method, respectively. From the comparative analysis, it can be known that the prediction accuracy of the proposed self-memory model is higher than that of the maximum Lyapunov index prediction method and is similar to that of the weighted first-order local prediction method. In addition, the average relative error of the single-step prediction method is less than that of the multi-step prediction method, because the single-step prediction method makes full use of the latest monitoring information to improve the prediction accuracy. According to the prediction theory, error generally increases rapidly with the increase of prediction time steps. Compared with the single-step prediction method, the average relative error increase of the multi-step prediction method is only 1.46%, which is in the controllable range. The above analysis shows that a building subsidence deformation prediction model with the self-memory model has certain advantages and can well predict multi-step displacements to meet engineering requirements. The model can be applied to building subsidence deformation prediction, building construction

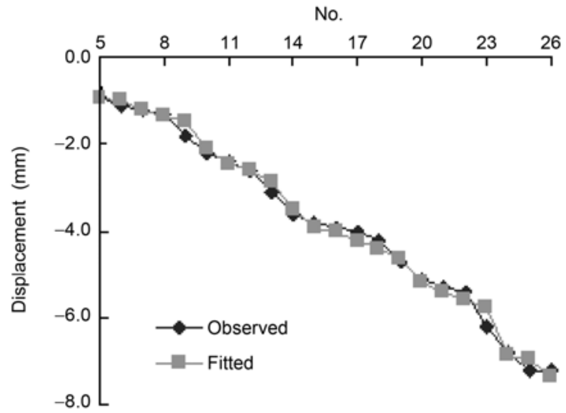

Figure 1 Comparison between fitted and observed values.

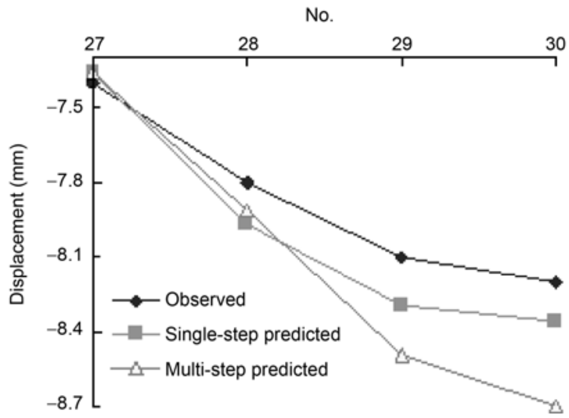

Figure 2 Comparison between predicted and observed values.

Table 3 Comparison between predicted and observed values

No.	Observed data (mm)	Multi-step prediction		Single-step prediction		The maximum Lyaunov index prediction method		The first-order local prediction method		Weighted first-order local prediction method	
		Prediction value (mm)	Relative error (%)	Prediction value (mm)	Relative error (%)	Prediction value (mm)	Relative error (%)	Prediction value (mm)	Relative error (%)	Prediction value (mm)	Relative error (%)
27	−7.40	−7.36	0.58	−7.36	0.58	−8.06	8.93	−7.57	2.29	−7.48	1.13
28	−7.80	−7.91	1.43	−7.97	2.16	−8.05	3.15	−7.76	0.48	−7.66	1.81
29	−8.10	−8.49	10.87	−8.29	2.39	−8.34	2.94	−8.21	1.31	−8.09	0.10
30	−8.20	−8.70	6.04	−8.36	1.91	−8.96	9.28	−8.47	3.28	−8.34	1.68
Average value		–	3.22	–	1.76	–	6.07	–	1.84	–	1.18

management and building safety warning.

4 Conclusion

The nonlinear time series of building subsidence deformation is regarded as the evolution of dynamic system in this paper, and building subsidence deformation dynamic prediction with data-based mechanistic self-memory model is established. Because the time series of building subsidence deformation reflect the nonlinear characteristics of the building subsidence trend, the self-memory equation is a dynamic equation and its memory coefficient is estimated by statistical methods. Therefore, the self-memory model can describe the nonlinear dynamic system of building subsidence deformation and analyze the sequential subsidence deformation effectively. Case study shows that the method has good prediction accuracy and superiority, and has important application value.

This work was supported by the Twelfth Five National Key Technology R&D Program of China (*2009BAJ28B04, 2011BAK07B01, 2011BAJ08B03, 2011BAJ08B05*), *the National Natural Science Foundation of China* (*51108428*), *Beijing Postdoctoral Research Foundation* (*2012ZZ-17*) *and China Postdoctoral Science Foundation* (*2011M500199*).

1 Qi K. Analyzing architecture articles. Sci China Tech Sci, 2010, 53: 1739–1742
2 Zhou Y, Sun Y N. Basic study on architectural planning of psychiatric hospital. Sci China Tech Sci, 2010, 53: 1755–1767
3 Ding X Y. Generative computer simulation for parallel layout of multi-story row house cluster. Sci China Tech Sci, 2010, 53: 1785–1791
4 Yang J Y, Chen W. Research on the evaluation model of the modernization level of urban construction in Jiangsu Province, China. Sci China Tech Sci, 2010, 53: 2510–2514
5 Zhang H, Leimer H P. Low energy certificate —— An exploration on optimization and evaluation of energy-efficient building envelope. Sci China Tech Sci, 2011, 54: 1639–1644
6 Wang J G, Zhang Y, Feng H. A decision-making model of development intensity based on similarity relationship between land attributes intervened by urban design. Sci China Tech Sci, 2010, 53: 1743–1754
7 Weng J, Hu Y K, Ying W. Study on calculation model of road lighting visibility. Sci China Tech Sci, 2010, 53: 1768–1773
8 Zhao Y Z, Wu S X, Qiu J Z, et al. Sound intensity radiated by Gaohu. Sci China Tech Sci, 2010, 53: 1792–1797
9 Tian W, Song W G, Lü W, et al. Experiment and analysis on microscopic characteristics of pedestrian movement in building bottleneck. Sci China Tech Sci, 2011, 54: 1730–1736
10 Liu G H, Guo W, Li H N. An effective and practical method for solving an unnegligible problem inherent in the current calculation model for multi-support seismic analysis of structures. Sci China Tech Sci, 2010, 53: 1774–1784
11 Wang D, Li J. Physical random function model of ground motions for engineering purposes. Sci China Tech Sci, 2011, 54: 175–182
12 Li D S, Hu Q, Ou J P, et al. Fatigue damage characterization of carbon fiber reinforced polymer bridge cables: Wavelet transform analysis for clustering acoustic emission data. Sci China Tech Sci, 2011, 54: 379–387
13 Li X C, Wang Y X, Li G W, et al. Experimental investigation of vortex-induced vibrations of long free spans near seabed. Sci China Tech Sci, 2011, 54: 698–704
14 Ye J H, Zhang Z Q, Chu Y. Strength behavior and collapse of spatial-reticulated structures under multi-support excitation. Sci China Tech Sci, 2011, 54: 1624–1638
15 Rong H, Qian C X, Wang R X. A cementation method of loose particles based on microbe-based cement. Sci China Tech Sci, 2011, 54: 1722–1729
16 Li H J, Wang J R, Hu S-L J, et al. Model updating based on incomplete modal data. Sci China Tech Sci, 2011, 54: 1737–1747
17 Chun Q, Yue Z, Pan J W. Experimental study on seismic characteristics of typical mortise-tenon joints of Chinese southern traditional timber frame buildings. Sci China Tech Sci, 2011, 54: 2404–2411
18 Xu S L, Yu W T, Song S D. Numerical simulation and experimental study on electrothermal properties of carbon/glass fiber hybrid textile reinforced concrete. Sci China Tech Sci, 2011, 54: 2421–2428
19 Gu M, Quan Y. Across-wind loads and effects of super-tall buildings and structures. Sci China Tech Sci, 2011, 54: 2531–2541
20 Teng J, Li Z H, Ou J P, et al. Fiber damage analysis model for RC beam-column based on EEP super-convergent computation. Sci China Tech Sci, 2011, 54: 2542–2548
21 Lu X, Lu X Z, Zhang W K, et al. Collapse simulation of a super high-rise building subjected to extremely strong earthquakes. Sci China Tech Sci, 2011, 54: 2549–2560
22 Hu R, Chen Y F, Zhou C B. Modeling of coupled deformation, water flow and gas transport in soil slopes subjected to rain infiltration. Sci China Tech Sci, 2011, 54: 2561–2575
23 Chen W C, Song L L, Zhi S Q, et al. Analysis on gust factor of tropical cyclone strong wind over different underlying surfaces. Sci China Tech Sci, 2011, 54: 2576–2586
24 Li Z N, Luo D F, Shi W H, et al. Field measurement of wind-induced stress on glass facade of a coastal high-rise building. Sci China Tech Sci, 2011, 54: 2587–2596
25 Yang Q S, Tan F, Wang X F. Loading and wrinkling analysis of membrane structures. Sci China Tech Sci, 2011, 54: 2597–2604
26 Bai J L, Ou J P. Seismic failure mode improvement of RC frame structure based on multiple lateral load patterns of pushover analyses. Sci China Tech Sci, 2011, 54: 2825–2833
27 Ye J H, Liu W Z, Pan R. Research on failure scenarios of domes based on form vulnerability. Sci China Tech Sci, 2011, 54: 2834–2853
28 Liu H W, Zhang S J, Chen J, et al. Simulation analysis theory and experimental verification of air-cushion isolation control of high concrete dams. Sci China Tech Sci, 2011, 54: 2854–2868
29 Tian Y J, Yang Q S, Yang N, et al. Statistical spectrum model of wind velocity at Beijing Meteorological Tower. Sci China Tech Sci, 2011, 54: 2869–2877
30 Ye X G, Wang D C. Selection of real earthquake accelerograms for structural dynamic analysis and energy evaluation. Sci China Tech Sci, 2011, 54: 2878–2885
31 Zheng S S, Wang B, Li L, et al. Study on seismic damage of SRHSC frame columns. Sci China Tech Sci, 2011, 54: 2886–2895
32 Deng Y, Ding Y L, Li A Q, et al. Fatigue reliability assessment for bridge welded details using long-term monitoring data. Sci China Tech Sci, 2011, 54: 3371–3381
33 Ju Y, Liu H B, Liu J H, et al. Investigation on thermophysical properties of reactive powder concrete. Sci China Tech Sci, 2011, 54: 3382–3403
34 Zhang X J, Xiong C H, Ding Y, et al. Variable-step integration method for milling chatter stability prediction with multiple delays. Sci China Tech Sci, 2011, 54: 3137–3154
35 Sun Z X, Pang Y Q, Hang T. Application of improved grey model for forecasting subsidence of the engineering buildings (in Chinese). Eng Survey Map, 2010, 19: 59–62
36 Wang J G, Hu Y H. The integrated forecasting of building subsidence based on least square support vector machine regression (in Chinese). Sci Survey Map, 2010, 35: 96–97
37 Liu D F, Wei X L, Sheng H Z. Prediction of remnant volatile matter in the semicokes from coal partial gasification. Sci China Tech Sci, 2011, 54: 3017–3021

38 Gao Y L, Pan J Y, Ji G L, et al. A time-series modeling method based on the boosting gradient-descent theory. Sci China Tech Sci, 2011, 54: 1325–1337
39 Zhu Y F, Yang C X, Chen X M. Prediction of building settlement based on genetic algorithms (in Chinese). J Liaoning Tech Univ, 2007, 26: 225–227
40 Bi L Z. Chaos theory's application in the construction's deformation forecasting (in Chinese). Dissertation of Master Degree. Xi'an: Chang'an University, 2009
41 Song B, Lee C H. A Favré averaged transition prediction model for hypersonic flows. Sci China Tech Sci, 2010, 53: 2049–2056
42 Phienwej N, Thepparak S, Giao P H. Prediction of differential settlement of buildings induced by land subsidence from deep well pumping. Geotech Eng, 2005, 36: 69–75
43 Chen X D, Xia J, Xu Q. Differential Hydrological Grey Model (DHGM) with self-memory function and its application to flood forecasting. Sci China Tech Sci, 2009, 52: 1039–1049
44 Ran Z. Multiscales and cascading in isotropic turbulence. Chin Sci Bull, 2011, 56: 2889–2892
45 Yang G T. Computational explosion mechanics and related progress. Chin Sci Bull, 2011, 56: 3610–3613
46 Cai G Q, Zhao C G, Liu Y, et al. Volume change behavior of unsaturated soils under non-isothermal conditions. Chin Sci Bull, 2011, 56: 2495–2504
47 Cao H X. Memorial Dynamics of Systems——Forecasting and Computation Application (in Chinese). Beijing: Geological Publishing House, 2002
48 Zhang S H, Huang Q, Sun T R. The study on system self-memory model of hydrologic time series (in Chinese). J Northwest A&F Univ (Nat Sci), 2006, 34: 221–225
49 Liu X P, Xie X B, Luo Y Z. Self-memorization model of dynamic system for predicting nonlinear displacement of slopes (in Chinese). Chin J Geotech Eng, 2010, 32: 1535–1542
50 Feng G L, Cao H X, Gao X Q, et al. Prediction of precipitation during summer monsoon with self-memorial model. Adv Atmos Sci, 2001, 18: 701–709