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avg00 Team Note

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Based on KACTL
https://github.com/kth-competitive-programming/kactl

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Contest (1)

template.cpp 27 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

#ifdef OHSOLUTION
#define ce(t) cerr<<t
#define AT cerr << "\n=================ANS=================\n"
#define AE cerr << "\n=====================================\n"
#define DB(a) cerr << __LINE__ << ": " << #a << " = " << (a) <<
    endl;
#define __builtin_popcount __popcnt
#define __builtin_popcountll __popcnt64
const LL LNF = 0x3f3f3f3f3f3f3f3f;
const int INF = 0x3f3f3f3f;
template<typename T, typename U> void ckmax(T& a, U b) { a = a
    < b ? b : a; }
template<typename T, typename U> void ckmin(T& a, U b) { a = a
    > b ? b : a; }
template<typename T, typename U> void MOD(T& a, U b) { a += b;
    if (a >= mod) a -= mod; };
#else
#define AT
#define AE
#define ce(t)
#endif
```

Mathematics (2)

2.1 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.2 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.3 Fibonacci Series

$\sum_{i=1}^{n} F_i = F_{n+2} - 1$ $\quad$ $\sum_{i=1}^{n} F_{2i} = F_{2n+1} - 1$

$\sum_{i=1}^{n} F_{2i-1} = F_{2n}$ $\quad$ $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$

$gcd(F_n, F_m) = F_{gcd(n,m)}$ $\quad$ $F_{2n-1} = F_n^2 + F_{n-1}^2$

$F_{2n} = (F_{n-1} + F_{n+1})F_n = (2F_{n-1} + F_n)F_n$

2.4 Pick's theorem

Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points that are interior to the polygon, and let b be the number of integer points on its boundary (including vertices as well as points along the sides of the polygon). Then the area A of this polygon is

$$A = i + \frac{b}{2} - 1.$$

2.5 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.5.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.5.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $U(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).
Usage: `hash_map<int, int> table({}, {}, {}, {}, {1 <<16});`

1d6e64, 21 lines

```
#include <bits/extc++.h>

struct splitmix64_hash {
  // http://xorshift.di.unimi.it/splitmix64.c
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    return splitmix64(x + 0x2425260000242526);
  }
};

template <typename K, typename V>
using hash_map = __gnu_pbds::gp_hash_table<K, V,
     splitmix64_hash>;

template <typename K>
using hash_set = hash_map<K, __gnu_pbds::null_type>;
```

LazySegmentTree.h

Description: Lazy Segment Tree

d3e94e, 75 lines

```
//1-base
struct SegTree{
    typedef int T; // Change this if you need
    T id = 0; // identity element (sum:0, mul:1, ...etc)
    T initval = 0, unused = 0;

    int S;
    vector<T> tree;
    vector<T> lazy;
    T combine(T a, T b){
        // Real value, parent <- combine(child1, child2)
        return a + b;
    }
    T lazy_prop(T a, T b){
        // Lazy value, child <- lazy_prop(child, parent) & node
            <- lazy_prop(node, val)
        return a + b;
    }
    void unlazy(int idx, int nl, int nr){
        // update Real value when its lazy is applied
        // ex) tree[idx] = lazy[idx] * (nr-nl+1)
        tree[idx] = lazy[idx] * (nr - nl + 1);
    }

    SegTree(int N){
        for(S=1;S<N;S<<=1);
        tree.resize(S*2, initval);
        lazy.resize(S*2, unused);
    }
    void propagate(int idx, int nl, int nr){
        if(lazy[idx] == unused) return;
        if(idx < S){
            lazy[idx*2]   = lazy_prop(lazy[idx*2], lazy[idx]);
            lazy[idx*2+1] = lazy_prop(lazy[idx*2+1], lazy[idx])
                ;
        }
        unlazy(idx, nl, nr);
        lazy[idx] = unused;
    }
    void update(int l, int r, T val){ update(l, r, val, 1, 1, S
        ); }
    void update(int l, int r, T val, int idx, int nl, int nr){
        propagate(idx, nl, nr);
        if(r<nl || nr<l) return;
        if(l<=nl && nr<=r){
            lazy[idx] = lazy_prop(lazy[idx], val);
            propagate(idx, nl, nr);
            return;
        }
        int mid = (nl + nr) / 2;
        update(l, r, val, idx*2, nl, mid);
        update(l, r, val, idx*2+1, mid+1, nr);
        tree[idx] = combine(tree[idx*2], tree[idx*2+1]);
    }
    void set_value(int i, T val){ set_value(i, val, 1, 1, S); }
    void set_value(int i, T val, int idx, int nl, int nr){
        propagate(idx, nl, nr);
        if(i<nl || nr<i) return;
        if(nl==i && nr==i){
            tree[idx] = val;
            lazy[idx] = unused;
            propagate(idx, nl, nr);
            return;
        }
        int mid = (nl + nr) / 2;
        set_value(i, val, idx*2, nl, mid);
        set_value(i, val, idx*2+1, mid+1, nr);
        tree[idx] = combine(tree[idx*2], tree[idx*2+1]);
    }
    T query(int l, int r){ return query(l, r, 1, 1, S); }
    T query(int l, int r, int idx, int nl, int nr){
        propagate(idx, nl, nr);
        if(r<nl || nr<l) return id;
        if(l<=nl && nr<=r) return tree[idx];
        int mid = (nl + nr) / 2;
        return combine( query(l, r, idx*2, nl, mid) , query(l,
            r, idx*2+1, mid+1, nr) );
    }
};
```

SegmentTree-GoldMine.h

Description: Segment Tree - Gold Mine
Usage: `'e' : identity element`
`'op': unite two nodes`
Time: $\mathcal{O}(\log N)$.

46a7fd, 62 lines

```
struct node_t {
  ll lmax, cmax, rmax, sum;
};

template <typename node_t>
class segtree {
  const node_t e {}; // change
  const size_t n, height, size;
  vector<node_t> tree;

public:
  segtree(size_t n) : n(n), height(n ? __lg(n - 1) + 1 : 0),
      size(1 << height), tree(size << 1, e) {}

  node_t& operator[](size_t i) { return tree[size + i]; }
  void build() {
    for (size_t i = size; i--;) {
      pull(i);
    }
  }
  void set(size_t idx, node_t val) {
    assert(0 <= idx and idx < n);
    tree[idx += size] = val;
    while (idx >>= 1) pull(idx);
  }
  node_t prod(size_t l, size_t r) const {
    assert(0 <= l and l <= r and r <= n);
    node_t lval = e, rval = e;
    for (l += size, r += size; l != r; l >>= 1, r >>= 1) {
      if (l & 1) lval = op(lval, tree[l++]);
      if (r & 1) rval = op(tree[--r], rval);
    }
    return op(lval, rval);
  }
  ll all_prod() const {
    return tree[1].cmax;
  }
```

```
  void clear() {
    fill(tree.begin(), tree.end(), e);
  }

private:
  inline int get_index(node_t& node) const { return &node -
      tree.data(); }
  inline int get_depth(node_t& node) const { return __lg(
      get_index(node)); }
  inline int get_height(node_t& node) const { return height -
      get_depth(node); }
  inline int get_length(node_t& node) const { return 1 <<
      get_height(node); }
  inline int get_first(node_t& node) const {
    int idx = get_index(node);
    int dep = __lg(idx);
    int len = 1 << height - dep;
    return len * (idx ^ 1 << dep);
  }
  void pull(size_t i) {
    tree[i] = op(tree[i << 1], tree[i << 1 | 1]);
  }
  node_t op(node_t l, node_t r) const {
    // return node_t {
    //      .lmax = max(l.lmax, l.sum + r.lmax),
    //      .cmax = max({l.cmax, r.cmax, l.rmax + r.lmax}),
    //      .rmax = max(r.rmax, r.sum + l.rmax),
    //      .sum = l.sum + r.sum};
  }
};
```

DisjointSet.h

Description: Disjoint-set data structure with undo?
Usage: `TOTO`
Time: TODO

f90a56, 93 lines

```
struct disjoint_set {
  vector<int> par, enemy;

  disjoint_set(int n) : par(n, -1), enemy(n, -1) {}
  int find(int u) {
    return par[u] < 0 ? u : par[u] = find(par[u]);
  }
  int merge(int u, int v) {
    if (u == -1) return v;
    if (v == -1) return u;
    u = find(u), v = find(v);
    if (u == v) return u;
    if (par[u] > par[v]) swap(u, v);
    par[u] += par[v];
    par[v] = u;
    return u;
  }
  bool ack(int u, int v) {
    u = find(u), v = find(v);
    if (enemy[u] == v) return false;
    int a = merge(u, v), b = merge(enemy[u], enemy[v]);
    enemy[a] = b;
    if (~b) enemy[b] = a;
    return true;
  }
  bool dis(int u, int v) {
    u = find(u), v = find(v);
    if (u == v) return false;
    int a = merge(u, enemy[v]), b = merge(v, enemy[u]);
    enemy[a] = b, enemy[b] = a;
    return true;
  }
};
```

```
// offline dynamic connectivity
struct disjoint_set {
  vector<int> par;
  vector<pair<int, int>> stk;

  disjoint_set(int n) : par(n, -1) {}
  int find(int u) {
    while (par[u] >= 0) u = par[u];
    return u;
  }
  bool merge(int u, int v) {
    u = find(u), v = find(v);
    if (u == v) return false;
    if (par[u] > par[v]) swap(u, v);
    stk.emplace_back(v, par[v]);
    par[u] += par[v];
    par[v] = u;
    return true;
  }
  void roll_back(size_t check_point) {
    for (; stk.size() != check_point; stk.pop_back()) {
      const auto& [u, sz] = stk.back();
      par[par[u]] -= sz, par[u] = sz;
    }
  }
};

// minimize maximum weight in path
template <typename T, typename F = less<T>>
class disjoint_set {
  const T e = 0x3f3f3f3f; // change this
  const F cmp {};
  const int n;
  vector<int> par;
  vector<T> weight;

public:
  disjoint_set(int n) : n(n), par(n, -1), weight(n, e) {}
  int find(int u) {
    while (par[u] >= 0) u = par[u];
    return u;
  }
  void unite(int u, int v, T w) {
    u = find(u), v = find(v);
    if (u == v) return;
    if (par[u] > par[v]) swap(u, v);
    par[u] += par[v];
    par[v] = u;
    weight[v] = w;
  }
  T query(int u, int v) {
    T ret = e;
    for (; u != v; u = par[u]) {
      if (cmp(weight[v], weight[u])) swap(u, v);
      ret = weight[u];
    }
    return ret;
  }
};
```

Matrix.h

Description: Basic operations on square matrices.
Usage:
```
Matrix<int, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;
```

c43c7d, 26 lines

```
template<class T, int N> struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;
      p >>= 1;
    }
    return a;
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

BinaryIndexedTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Usage: `TODO`

Time: Both operations are $\mathcal{O}(\log N)$.

806f55, 47 lines

```
template <typename T>
class binary_indexed_tree {
  const size_t n;
  vector<T> tree;

public:
  binary_indexed_tree(size_t n) : n(n), tree(n + 1) {}
  // a[i] += val
  void update(size_t i, T val) {
    assert(0 <= i and i <= n);
    for (++i; i <= n; i += i & -i)
      tree[i] += val;
  }
  // return the sum of the range [0, i)
  T query(size_t i) const {
    assert(0 <= i and i <= n);
    T ret = 0;
    for (; i; i &= i - 1)
      ret += tree[i];
    return ret;
  }
  // return the sum of the range [l, r)
  T query(size_t l, size_t r) const {
    return query(r) - query(l);
  }
  // return a[i]
  T get(size_t i) const {
    assert(0 <= i and i < n);
    return i & 1 ? query(i, i + 1) : tree[i + 1];
  }
  // return minimum i s.t. sum[0...i] >= k
  size_t lower_bound(T k) const {
    size_t x = 0;
    for (size_t pw = 1 << 25; pw; pw >>= 1)
      if ((x | pw) <= n && tree[x | pw] < k)
        k -= tree[x |= pw];
    return x;
  }
  // return minimum i s.t. sum[0...i] > k
  size_t upper_bound(T k) const {
    size_t x = 0;
    for (size_t pw = 1 << 25; pw; pw >>= 1)
      if ((x | pw) <= n && tree[x | pw] <= k)
        k -= tree[x |= pw];
    return x;
  }
};
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. record in time and out time in dfs. the path of (u, v), $in_u \leq \in_v$ is ... if $u = lca$, $[in_u, in_v]$. if $u \neq lca$, $[out_u, in_v] + in_{lca}$

Usage: `if array: just use add(), del().`
`if tree: NEVER USE add(), del(). only use flip() for both`

Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

349caa, 85 lines

```
struct query_t {
  int l, r, id, lca;
};

void add(int id) {}
void del(int id) {}
int calc() {}

// < if tree >
vector<int> adj[MX_N];
int sz[MX_N], in[MX_N], out[MX_N], par[MX_N], top[MX_N], tour[
    MX_N << 1];
int tick;
bitset<MX_N> visited {};
// </if tree >

void dfs(int u) {
  sz[u] = 1;
  for (auto& v : adj[u]) {
    par[v] = u;
    adj[v].erase(find(adj[v].begin(), adj[v].end(), u)); // if
        bidirectional
    dfs(v);
    sz[u] += sz[v];
    if (sz[v] > sz[adj[u][0]]) {
      swap(v, adj[u][0]);
    }
  }
}

void hld(int u) {
  in[u] = tick, tour[tick] = u;
  ++tick;
  bool heavy = true;
  for (const auto& v : adj[u]) {
    top[v] = heavy ? top[u] : v;
    hld(v);
    heavy = false;
  }
  out[u] = tick, tour[tick] = u;
  ++tick;
}

int get_lca(int u, int v) {
  for (; top[u] != top[v]; u = par[top[u]]) {
    if (sz[top[u]] > sz[top[v]])
      swap(u, v);
  }
  return in[u] < in[v] ? u : v;
}

void flip(int id) {
  // if tree
  visited[id] ? del(id) : add(id);
  visited[id].flip();
}

int main() {
  // example of Mo's on tree
  // how to initialize queries
  vector<query_t> q(m);
  for (int i = 0, u, v; i < m; ++i) {
    cin >> u >> v;
    if (in[u] > in[v]) swap(u, v);
    auto lca = get_lca(u, v);
    u == lca ? (q[i].l = in[u], q[i].lca = -1) : (q[i].l = out[
        u], q[i].lca = lca);
    q[i].r = in[v] + 1, q[i].id = i;
  }
  // how to sort...
  constexpr int sq = 350;
  sort(q.begin(), q.end(), [&](auto& a, auto& b) {
    if (a.l / sq != b.l / sq) return a.l < b.l;
    return a.l / sq & 1 ? a.r > b.r : a.r < b.r;
  });
  // how to calculate answer...
  vector<int> ans(m);
  int pl = q[0].l, pr = q[0].l;
  for (const auto [l, r, id, lca] : q) {
    while (l < pl) flip(tour[--pl]);
    while (pr < r) flip(tour[pr++]);
    while (pl < l) flip(tour[pl++]);
    while (r < pr) flip(tour[--pr]);
    if (~lca) flip(lca);
    ans[id] = calc();
    if (~lca) flip(lca);
  }
}
```

PST.h

Description: Persistent Segment Tree

Usage: `a = minimum number of node at leaf level`
`b = total number of updating operation`
`rn = maximum number of root`
`To get the value of Kth segtree use qry(T.root[k],..)`

568afb, 50 lines

```
struct PST{
    struct Node{
        int sum, L, R;
    };
    vector<Node> tree;
    vector<int> root;
    int h, cnt, base;
    PST(int a, int b, int rn){
        h=cnt=base=1;
        while(base<a) base<<=1, h++;
        int tmp=base*2+h*b+5;
        tree.resize(tmp);
        root.resize(rn+5);
        root[0]=setup(1,base);
    }
    int setup(int ns, int nf){
        int k=cnt++;
        tree[k].sum=0;
        if(ns<nf){
            int mid=(ns+nf)>>1;
            tree[k].L=setup(ns,mid);
            tree[k].R=setup(mid+1,nf);
        }
        return k;
    }
    void update_Kth_tree(int k, int idx=-1, int v=-1){
        if(root[k]==0) root[k]=root[k-1];
        if(idx!=-1) root[k]=make(root[k],idx,v);
    }
    int make(int bef, int idx, int v, int ns=1, int nf=-1){
        if(nf==-1) nf=base;
        if(nf<idx || idx<ns) return bef;
        int k=cnt++;
        if(ns==nf) tree[k].sum=tree[bef].sum+v;
        else{
            int mid=(ns+nf)>>1;
            tree[k].L=make(tree[bef].L,idx,v,ns,mid);
            tree[k].R=make(tree[bef].R,idx,v,mid+1,nf);
            tree[k].sum=tree[tree[k].L].sum+tree[tree[k].R].sum
                ;
        }
        return k;
    }
    int qry(int num, int st, int fn, int ns=1, int nf=-1){
        if(nf==-1) nf=base;
        if(nf<st || fn<ns) return 0;
        if(st<=ns && nf<=fn) return tree[num].sum;
        int mid=(ns+nf)>>1;
        return qry(tree[num].L,st,fn,ns,mid)+qry(tree[num].R,st
            ,fn,mid+1,nf);
    }
};
```

LazyLiChao.h 770cfe, 101 lines

```
const ll inf = 4e18;
struct LiChao  // Minimum line management If you want maximum
    management, you can put -ax-b instead of ax+b.
{
  struct Node {
    int l, r; ll a, b, mn, aa, bb;
    Node() { l = 0; r = 0; a = 0; b = inf; mn = inf; aa = 0; bb
        = 0; }
  };
  vector<Node> seg;
  ll _l, _r;
  LiChao(ll l, ll r) {
    seg.resize(2);
    _l = l; _r = r;
  }

  void propagate(int n, ll l, ll r) {
    if (seg[n].aa || seg[n].bb) {
      if (l != r) {
        if (seg[n].l == 0) seg[n].l = seg.size(), seg.push_back
            (Node());
        if (seg[n].r == 0) seg[n].r = seg.size(), seg.push_back
            (Node());
        seg[seg[n].l].aa += seg[n].aa, seg[seg[n].l].bb += seg[
            n].bb;
        seg[seg[n].r].aa += seg[n].aa, seg[seg[n].r].bb += seg[
            n].bb;
      }
      seg[n].mn += seg[n].bb;
      seg[n].a += seg[n].aa, seg[n].b += seg[n].bb;
      seg[n].aa = seg[n].bb = 0;
    }
  }

  void insert(ll L, ll R, ll a, ll b, int n, ll l, ll r) {
    if (r < L || R < l || L > R) return;
    if (seg[n].l == 0) seg[n].l = seg.size(), seg.push_back(
        Node());
    if (seg[n].r == 0) seg[n].r = seg.size(), seg.push_back(
        Node());
    propagate(n, l, r);
    seg[n].mn = min({ seg[n].mn, a * max(l,L) + b, a * min(r,R)
        + b });
    ll m = l + r >> 1;
    if (l < L || R < r) {
      if (L <= m) insert(L, R, a, b, seg[n].l, l, m);
      if (m + 1 <= R) insert(L, R, a, b, seg[n].r, m + 1, r);
      return;
    }
    ll& sa = seg[n].a, & sb = seg[n].b;
    if (a * l + b < sa * l + sb) swap(a, sa), swap(b, sb);
    if (a * r + b >= sa * r + sb) return;
    if (a * m + b < sa * m + sb) {
      swap(a, sa), swap(b, sb);
      insert(L, R, a, b, seg[n].l, l, m);
    }
    else insert(L, R, a, b, seg[n].r, m + 1, r);
  }

  ll get(ll x, int n, ll l, ll r) {
    if (n == 0) return inf;
    propagate(n, l, r);
    ll ret = seg[n].a * x + seg[n].b, m = l + r >> 1;
    if (x <= m) return min(ret, get(x, seg[n].l, l, m));
    return min(ret, get(x, seg[n].r, m + 1, r));
  }
  ll get(ll L, ll R, int n, ll l, ll r) {
    if (n == 0) return inf;
    if (r < L || R < l || L > R) return inf;
    propagate(n, l, r);
    if (L <= l && r <= R) return seg[n].mn;
    ll m = l + r >> 1;
    return min({ seg[n].a * max(l,L) + seg[n].b, seg[n].a * min
        (r,R) + seg[n].b, get(L, R, seg[n].l, l, m), get(L, R,
         seg[n].r, m + 1, r) });
  }
  // [l,r] insert ax+b
  void insert(ll L, ll R, ll a, ll b) {
    insert(L, R, a, b, 1, _l, _r);
  }

  ll get(ll x) {
    return get(x, 1, _l, _r);
  }

  ll get(ll L, ll R) {
    return get(L, R, 1, _l, _r);
  }
};

int main() {
  LiChao tree(-1e12, 1e12); // range setting

  int q; ci(q);

  while (q--) {
    int tp; ci(tp);

    // insert ax+b
    if (tp & 1) {
      LL a, b; ci(a >> b);
      tree.insert(-1e12, 1e12, -a, -b);
    } else {
      // get maximum y at point x
      LL x; ci(x);
      co(-tree.get(x)<<"\n");
    }
  }

  return 0;
}
```

CDQ.h 1fae3d, 71 lines

```
struct Node {
  LL t, x, y, v, i, sgn;
};

vector<Node> vi;
vector<LL> ans;
LL arr[max_v];
int w;

void cdq(int l, int r) {
  if (l +1 == r) return;

  int mid = l + r >> 1;

  cdq(l, mid); cdq(mid, r);

  vector<Node> tmp;
  vector<setl> his;

  int a = l, b = mid;
  while (a < mid && b < r) {
    if (vi[a].x <= vi[b].x) {
      upd(vi[a].y, vi[a].v); // fenwick
      his.push_back({ vi[a].y,-vi[a].v });
      tmp.push_back(vi[a++]);
    } else {
      ans[vi[b].i] += vi[b].sgn * query(vi[b].y);
      tmp.push_back(vi[b++]);
    }
  }

  while (a < mid) tmp.push_back(vi[a++]);
  while (b < r) {
    ans[vi[b].i] += vi[b].sgn * query(vi[b].y);
    tmp.push_back(vi[b++]);
  }

  fa(i, l, r) vi[i] = tmp[i - l];
  for (auto& x : his) upd(x.first, x.second); // roll-back
}

int main() {
  ans.push_back(0);

  while (1) {
    ci(tp);

    if (tp == 3) break;

    // update point
    if (tp & 1) {
      int x, y, val; ci(x >> y >> val);
      vi.push_back({ ct,x,y,val,0,0 });
    } else {
      // count inner point in ractangle
      int lx, ly, rx, ry; ci(lx >> ly >> rx >> ry);
      vi.push_back({ ct,lx - 1,ly - 1,0,(LL)ans.size(),1 });
      vi.push_back({ ct,rx,ry ,0,(LL)ans.size(),1 });
      vi.push_back({ ct,lx - 1,ry,0,(LL)ans.size(),-1 });
      vi.push_back({ ct,rx,ly - 1,0,(LL)ans.size(),-1 }); //
          idx,(x,y) cod, number of add point, ans save idx,
          sign
      ans.push_back(0);
    }
    ++ct;
  }

  cdq(0, vi.size());
  fa(i, 1, ans.size()) co(ans[i] << "\n");

  return 0;
}
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h c9b7b0, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
```

```
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" b00bfe, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
}
```

BerlekampMassey.h

Description: abcd

903649, 82 lines

```
using lint = long long;

lint ipow(lint x, lint p) {
  lint ret = 1, piv = x;
  while (p)
  {
    if (p & 1) ret = ret * piv % mod;
    piv = piv * piv % mod;
    p >>= 1;
  }
  return ret;
}
vector<int> berlekamp_massey(vector<int> x) {
  vector<int> ls, cur;
  int lf, d;
  for (int i = 0; i < x.size(); i++) {
    lint t = 0;
    for (int j = 0; j < cur.size(); j++) {
      t = (t + 1ll * x[i - j - 1] * cur[j]) % mod;
    }
    if ((t - x[i]) % mod == 0) continue;
    if (cur.empty()) {
      cur.resize(i + 1);
      lf = i;
      d = (t - x[i]) % mod;
      continue;
    }
    lint k = -(x[i] - t) * ipow(d, mod - 2) % mod;
    vector<int> c(i - lf - 1);
    c.push_back(k);
    for (auto& j : ls) c.push_back(-j * k % mod);
    if (c.size() < cur.size()) c.resize(cur.size());
    for (int j = 0; j < cur.size(); j++) {
      c[j] = (c[j] + cur[j]) % mod;
    }
    if (i - lf + (int)ls.size() >= (int)cur.size()) {
      tie(ls, lf, d) = make_tuple(cur, i, (t - x[i]) % mod);
    }
    cur = c;
  }
  for (auto& i : cur) i = (i % mod + mod) % mod;
  return cur;
}
int get_nth(vector<int> rec, vector<int> dp, lint n) {
  int m = rec.size();
  vector<int> s(m), t(m);
  s[0] = 1;
  if (m != 1) t[1] = 1;
  else t[0] = rec[0];
  auto mul = [&rec](vector<int> v, vector<int> w) {
    int m = v.size();
    vector<int> t(2 * m);
    for (int j = 0; j < m; j++) {
      for (int k = 0; k < m; k++) {
        t[j + k] += 1ll * v[j] * w[k] % mod;
        if (t[j + k] >= mod) t[j + k] -= mod;
      }
    }
    for (int j = 2 * m - 1; j >= m; j--) {
      for (int k = 1; k <= m; k++) {
        t[j - k] += 1ll * t[j] * rec[k - 1] % mod;
        if (t[j - k] >= mod) t[j - k] -= mod;
      }
    }
    t.resize(m);
    return t;
  };
  while (n) {
    if (n & 1) s = mul(s, t);
    t = mul(t, t);
    n >>= 1;
  }
  lint ret = 0;
  for (int i = 0; i < m; i++) ret += 1ll * s[i] * dp[i] % mod;
  return ret % mod;
}
int guess_nth_term(vector<int> x, lint n) {
  if (n < x.size()) return x[n];
  vector<int> v = berlekamp_massey(x);
  if (v.empty()) return 0;
  return get_nth(v, x, n);
}
```

sosdp.h

Description: abcd

eef3c8, 7 lines

```
// dp2[t][i] = sigma dp[t][subset i]

fa(t, 1, 18)
{
    fa(i, 0, 1 << 17) dp2[t][i] = dp[t][i];
    for (int j = 0; j < 17; j++) for (int i = 0; i < (1 << 17);
         i++) if (i & (1 << j)) MOD(dp2[t][i] , dp2[t][i ^ (1
        << j)]);
}
```

4.2 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

3313dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2 m\right)$

44c9ab, 38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
```

```
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}(n^2 m)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f68f, 36 lines

```
int matInv(vector<vector<ll>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
        ]);
    swap(col[i], col[c]);
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
      rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    }
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    ll v = A[j][i];
    rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  }

  rep(i,0,n) rep(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
        : 0);
  return n;
}
```

4.3 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
```

```
  if (a.empty() || b.empty()) return {};
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
  vector<C> L(n), R(n), outs(n), outl(n);
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
  fft(L), fft(R);
  rep(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  }
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
}
```

NTT.h

Description: abcd

<cstdio> 0ac2a6, 66 lines

```
/*
Numeric FFT
P == MOD == A*2^B + 1
R  A  B  P
5  3  30 3221225473
3  17 27 2281701377
31 15 27 2013265921
3  7  26 469762049
*/

const int A = 7, B = 26, P = A << B | 1, R = 3;
const int SZ = 20, N = 1 << SZ;

int Pow(int x, int y) {
    int r = 1;
    while (y) {
        if (y & 1) r = (long long)r * x % P;
        x = (long long)x * x % P;
        y >>= 1;
    }
    return r;
}

void FFT(int *a, bool f) {
    int i, j, k, x, y, z;
    j = 0;
    for (i = 1; i < N; i++) {
        for (k = N >> 1; j >= k; k >>= 1) j -= k;
        j += k;
        if (i < j) {
            k = a[i];
            a[i] = a[j];
            a[j] = k;
        }
    }
    for (i = 1; i < N; i <<= 1) {
        x = Pow(f ? Pow(R, P - 2) : R, P / i >> 1);
        for (j = 0; j < N; j += i << 1) {
            y = 1;
            for (k = 0; k < i; k++) {
                z = (long long)a[i | j | k] * y % P;
                a[i | j | k] = a[j | k] - z;
                if (a[i | j | k] < 0) a[i | j | k] += P;
                a[j | k] += z;
                if (a[j | k] >= P) a[j | k] -= P;
                y = (long long)y * x % P;
            }
        }
    }
    if (f) {
        j = Pow(N, P - 2);
        for (i = 0; i < N; i++) a[i] = (long long)a[i] * j % P;
    }
}

int X[N];

int main() {
    int i, n;
    scanf("%d", &n);
    for (i = 0; i <= n; i++) scanf("%d", &X[i]);
    FFT(X, false);
    for (i = 0; i < N; i++) X[i] = (long long)X[i] * X[i] % P;
    FFT(X, true);
    for (i = 0; i <= n + n; i++) printf("%d ", X[i]);
}
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.

Time: $\mathcal{O}(N \log N)$

464cf3, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" 35bfea, 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

6f684f, 3 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

ModPow.h

b83e45, 8 lines

```
const ll mod = 1000000007; // faster if const

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;
  return ans;
}
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod m$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.

Time: $\mathcal{O}(\sqrt{m})$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.

`modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c) \% m$. `divsum` is similar but for floored division.

Time: $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}\left(\log p\right)$ for most p

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.
Time: $\mathcal{O}\left(N\right)$

89939f, 21 lines

```
const int LIM = 1e6;
vector<int> pr; // prime set
int sp[LIM]; // minimum prime
int cnt[LIM]; // 2 ^ (prime_num)?
int mu[LIM]; // Mobius

void get_sieve()
{
  cnt[1] = 1;
  mu[1] = 1;
  for(int i = 2; i < LIM; ++i) {
    if(!sp[i]) pr.push_back(i), cnt[i] = 2, mu[i] = -1;
    for(auto& x : pr) {
      if(x * i >= LIM) break;
      sp[x * i] = x;
      cnt[x * i] = i % x == 0 ? cnt[i] : cnt[i] + 1;
      mu[x * i] = (i % x != 0) * (-mu[i]);
      if(i % x == 0) break;
    }
  }
}
```

PrimalityTest.h

Description: Miller-Rabin and Pollard's rho
Time: isprime(n) : $\mathcal{O}\left(\log n\right)$, factorize(n) : $\mathcal{O}\left(n^{1/4}\right)$

5bdb20, 61 lines

```
class primality_test {
  using num = unsigned long long;
  const vector<num> base_small = {2, 7, 61},
            base_large = {2, 325, 9375, 28178, 450775, 9780504,
                  1795265022};

public:
  bool is_prime(num n) const {
    if (n < 2) return false;
    if (n == 2 || n == 3) return true;
    if (n % 6 != 1 && n % 6 != 5) return false;
    const auto& base = n < 4759123141ULL ? base_small :
        base_large;
    const int s = __builtin_ctzll(n - 1);
    const num d = n >> s;
    for (const auto& b : base) {
      if (b >= n) break;
      if (check_composite(n, b, d, s)) return false;
    }
    return true;
  }
  vector<num> factorize(num n) const {
    if (n == 1) return {};
    if (is_prime(n)) return {n};
    const num x = pollard(n);
    auto l = factorize(x), r = factorize(n / x);
    decltype(l) ret(l.size() + r.size());
    merge(l.begin(), l.end(), r.begin(), r.end(), ret.begin());
    return ret;
  }

private:
  num pow_mod(num a, num p, num m) const {
    num ret = 1;
    for (; p; p >>= 1) {
      if (p & 1) ret = mul_mod(ret, a, m);
      a = mul_mod(a, a, m);
    }
    return ret;
  }
  num mul_mod(num a, num b, num m) const {
    int64_t ret = a * b - m * num(1.L / m * a * b);
    return ret + m * (ret < 0) - m * (ret >= int64_t(m));
  }
  bool check_composite(num n, num x, num d, int s) const {
    x = pow_mod(x, d, n);
    if (x == 1 || x == n - 1) return false;
    while (--s) {
      x = mul_mod(x, x, n);
      if (x == n - 1) return false;
    }
    return true;
  };
  num pollard(num n) const {
    auto f = [&](num x) { return mul_mod(x, x, n) + 1; };
    num x = 0, y = 0, prd = 2, i = 1, q;
    for (int t = 30; t++ % 40 || gcd(prd, n) == 1; x = f(x), y
        = f(f(y))) {
      if (x == y) x = ++i, y = f(x);
      if ((q = mul_mod(prd, x > y ? x - y : y - x, n))) prd = q
          ;
    }
    return gcd(prd, n);
  }
};
```

5.3 Divisibility

euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f, 6 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
// x2 = x + k * b/gcd(a,b) y2 = y - k * a/gcd(a,b)
```

CRT.h

Description: Chinese Remainder Theorem.
crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
Time: $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) :=$ # of positive integers $\le n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$.
$\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

5.4 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.5 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

6.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0} \binom{n_i}{m_i} \pmod p$.

6.2.3 Binomials

BinomialCoefficient.h

Description: Finds binomial coefficient. MOD must be prime.
Usage: `MAXN < MOD -> init(); bi_coeff(n, r)`
`MAXN > MOD -> MAXN = MOD; init(); bi_coeff_lucas(n, r);`
Time: MAXN < MOD -> $\mathcal{O}(N)$ when init, $\mathcal{O}(1)$ to get MAXN > MOD -> $\mathcal{O}(MOD)$ when init, $\mathcal{O}(logN)$ to get

dabe69, 33 lines

```
constexpr ll MAXN = 1000000, MOD = 1000000007;
ll fact[MAXN + 1], invfact[MAXN + 1];
ll pw(ll a, ll b) {
  ll res = 1;
  while(b > 0) {
    if(b & 1) res = res * a % MOD;
    a = a * a % MOD;
    b >>= 1;
  }
  return res;
}
void init(){
  fact[0] = 1;
  for(int i = 1; i <= MAXN; ++i) fact[i] = fact[i - 1] * i %
      MOD;
  invfact[MAXN] = pw(fact[MAXN], MOD - 2);
  for(int i = MAXN - 1; i >= 0; --i) invfact[i] = invfact[i +
      1] * (i + 1) % MOD;
}
ll bi_coeff(int n, int r) {
  ll factn = fact[n];
  ll invrnr = invfact[r] * invfact[n - r] % MOD;
  return factn * invrnr % MOD;
}
ll bi_coeff_lucas(ll n, ll r) {
  ll res = 1;
  while(n > 0 || r > 0) {
    ll a = n % MOD;
    ll b = r % MOD;
    res *= bi_coeff(a, b);
    res %= MOD;
    n /= MOD; r /= MOD;
  }
  return res;
}
```

multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.

a0a312, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k) x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \ge j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

6.3.8 Catalan Convolution

$$C_n^{(k)} = \frac{k+1}{n+k+1}\binom{2n+k}{n} = \binom{2n+k}{n} - \binom{2n+k}{n-1}$$

- count of balanced parentheses sequences consisting of $n+k$ pairs of parentheses where the first k symbols are open brackets.
- ex) n=3, k=2 ((AAABBBBB

Graph (7)

7.1 Fundamentals

SPFA.h
Description: Calculates shortest paths from st in a graph that might have negative edge weights. Return false if the graph has a negative cycle.
2061ea, 34 lines

```
constexpr ll INF = 9999999999999999;

vector<pair<int, ll>> g[1001];
ll dst[1001];
bool inq[1001];
int n, cycle[1001];

bool spfa(int st) {
  for(int i = 0; i < n; ++i) dst[i] = INF;
  dst[st] = 0;
  queue<int> q;
  q.push(st);
  while(q.empty() == false) {
    int cur = q.front();
    q.pop();
    inq[cur] = false;
    for(auto& nx : g[cur]) {
      int nxt = nx.first;
      ll cost = nx.second;
      if(dst[nxt] > dst[cur] + cost) {
        dst[nxt] = dst[cur] + cost;
        if(inq[nxt] == false) {
          q.push(nxt);
          inq[nxt] = true;
        }
        cycle[nxt]++;
        if(cycle[nxt] > n) {
          return false;
        }
      }
    }
  }
  return true;
}
```

TopoSort.h
Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.
Time: $\mathcal{O}(|V| + |E|)$
66a137, 14 lines

```
vi topoSort(const vector<vi>& gr) {
  vi indeg(sz(gr)), ret;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  queue<int> q; // use priority_queue for lexic. largest ans.
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
  while (!q.empty()) {
    int i = q.front(); // top() for priority queue
    ret.push_back(i);
    q.pop();
    for (int x : gr[i])
      if (--indeg[x] == 0) q.push(x);
  }
  return ret;
}
```

7.2 Network flow

Dinic.h
Description: Dinic algorithm
85a76f, 58 lines

```
using flow_t = int;

struct edge {
  int v, rev;
  flow_t capa;
  edge(int _v, int _rev, flow_t _capa) : v(_v), rev(_rev), capa
       (_capa) {}
};

const flow_t FLOW_MAX = numeric_limits<flow_t>::max();
int n, src = -1, sink = -1;

vector<vector<edge>> adj(n);
vector<int> level(n), ptr(n);

void add_edge(int u, int v, flow_t c) {
  assert(0 <= u and u < n and 0 <= v and v < n);
  adj[u].emplace_back(v, adj[v].size(), c);
  adj[v].emplace_back(u, adj[u].size() - 1, 0);
}

int bfs() {
  fill(level.begin(), level.end(), 0);
  level[src] = 1;
  queue<int> q;
  q.emplace(src);
  while (!q.empty()) {
    const auto u = q.front();
    q.pop();
    for (const auto& [v, rev, capa] : adj[u])
      if (capa && !level[v]) {
        level[v] = level[u] + 1;
        q.emplace(v);
      }
  }
  return level[sink];
}
flow_t dfs(int u, flow_t f) {
  if (u == sink) return f;
  for (int &i = ptr[u], sz = adj[u].size(); i < sz; ++i) {
    auto& [v, rev, capa] = adj[u][i];
    if (capa && level[u] + 1 == level[v])
      if (flow_t d = dfs(v, min(f, capa)); d) {
        capa -= d;
        adj[v][rev].capa += d;
        return d;
      }
  }
  return 0;
}

flow_t max_flow() {
  flow_t ret = 0;
  for (flow_t f; bfs();) {
    fill(ptr.begin(), ptr.end(), 0);
    while ((f = dfs(src, FLOW_MAX))) ret += f;
  }
  return ret;
}
```

MinCostMaxFlow.h
Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}(E^2)$
fe85cc, 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
  VL dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap),
```

```
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    rep(i,0,N) rep(j,0,N) totcost += cost[i][j] * flow[i][j];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

MCMF-OH.h

Description: Dinic-style Min-cost max-flow.

2321e8, 76 lines

```
struct edge {
  int a, b, cap, flow, cost;
};

vector<edge> ve;
vector<int> adj[max_v];
int idx[max_v], dist[max_v],inq[max_v],vist[max_v],S,T;

// addedge (u,v,capicity,cost)
// then run

auto addedge = [&](int a, int b, int cap, int cost) {
  edge e1 = { a,b,cap,0,cost };
  edge e2 = { b,a,0,0,-cost };
  adj[a].push_back(ve.size());
  ve.push_back(e1);
  adj[b].push_back(ve.size());
  ve.push_back(e2);
};

auto spfa = [&]() {
  memset(dist, 0x3f, sizeof(dist));
  memset(inq, 0, sizeof(inq));

  queue<int> bq; bq.push(S);
  dist[S] = 0; inq[S] = 1;
  while (bq.size()) {
    int u = bq.front(); bq.pop(); inq[u] = 0;

    for (auto& v : adj[u]) {
      auto c = ve[v];
      if (c.flow < c.cap && (dist[c.b] > dist[u] + c.cost)) {
        dist[c.b] = dist[u] + c.cost;
        if (!inq[c.b])bq.push(c.b), inq[c.b] = 1;
      }
    }
  }

  return dist[T] != INF;
};

function<int(int, int)> dfs = [&](int u, int f) {
  if (!f) return 0;
  vist[u] = 1;
  if (u == T) return f;

  for (; idx[u] < adj[u].size(); ++idx[u]) {
    int v = adj[u][idx[u]];
    auto c = ve[v];

    if (dist[c.b] != dist[u] + c.cost || vist[c.b]) continue;

    if (int flow = dfs(c.b, min(f, c.cap - c.flow))) {
      ve[v].flow += flow;
      ve[v ^ 1].flow -= flow;
      return flow;
    }
  }

  return 0;
};

auto run = [&]() {
  int total_cost = 0;
  int total_flow = 0;
  while (spfa()) {
    memset(idx, 0, sizeof(idx));
    memset(vist, 0, sizeof(vist));
    while (int f = dfs(S, INF)) {
      total_cost += dist[T] * f;
      total_flow += f;
      memset(vist, 0, sizeof(vist));
    }
  }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

7.3 Matching

7.3.1 L-R max flow

1. Add new source and sink, S' and T' (note that, original source and sink are S and T)
2. Change given edge's capacity to $R - L$ (where lower bound= L, upper bound= R, direction is v_1 to v_2)
3. Add new edge from S' to v_2, and from v_1 to T' and its capacity is L
4. Add new edge from T to S with capacity infinity.
5. To check its validity, execute flow algoritm from S' to T'. If maximum flow from S' to T' is equal to sum of lower bound capacity, there exists solution under the constraint.
6. To find maximum flow, execute flow algorithm from original source to original sink. That is, from S to T.

7.3.2 Hall's Theorem

Let G be a finite bipartite graph with bipartite sets X and Y (i.e. $G := (X + Y, E)$). An X-perfect matching (also called: X-saturating matching) is a matching which covers every vertex in X. For a subset W of X, let $N_G(W)$ denote the neighborhood of W in G, i.e., the set of all vertices in Y adjacent to some element of W. The marriage theorem in this formulation states that there is an X-perfect matching if and only if for every subset W of X:

$$|W| \leq |N_G(W)|$$

In other words: every subset W of X has sufficiently many adjacent vertices in Y.

hopcroftKarp.h

Time: $\mathcal{O}\left(\sqrt{V}E\right)$ 0e7756, 62 lines

```
vector <int> adj[max_v];
int n,m, match[max_v], dist[max_v];

bool bfs() {
  queue<int>bq;
  fa(i, 1, n + 1) {
    if (!match[i]) dist[i] = 0, bq.push(i);
    else dist[i] = INF;
  }

  dist[0] = INF;

  while (bq.size()) {
    int u = bq.front(); bq.pop();

    if (u) for (auto& x : adj[u]){
      if (dist[match[x]] == INF) {
        dist[match[x]] = dist[u] + 1;
        bq.push(match[x]);
      }
    }
  }

  return dist[0] != INF;
}

bool dfs(int u) {
  if (!u) return true;

  for (auto& x : adj[u]) {
    if (dist[match[x]] == dist[u] + 1 && dfs(match[x])) {
      match[x] = u, match[u] =x;
      return true;
    }
  }

  dist[u] = INF;
  return false;
}

int main() {
  ci(n >> m);

  // match i(left) to i+n(right)
  fa(i, 1, n + 1) {
    int x; ci(x);
    while (x--) {
      int y; ci(y); y += n;
      adj[i].push_back(y);
    }
  }

  int ans = 0;

  while (bfs()) {
    fa(i, 1, n + 1) if (!match[i] && dfs(i)) ++ans;
  }

  co(ans);

  return 0;
}
```

DFSMatching-PO.h

Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$ 7810e6, 60 lines

```
int n, m;
vector<vector<int>> adj(n);
vector<int> match(n), rev(m);
vector<bool> visited(n);

bool dfs(int u) {
  visited[u] = true;
  for (const auto& v : adj[u]) {
    if (rev[v] == -1 || (!visited[rev[v]] && dfs(rev[v]))) {
      match[u] = v, rev[v] = u;
      return true;
    }
  }
  return false;
}

int maximum_matching() {
  for (bool update = true; update;) {
    fill(visited.begin(), visited.end(), false);
    update = false;
    for (int i = 0; i < n; ++i)
      if (match[i] == -1 && dfs(i))
        update = true;
  }
  return n - count(match.begin(), match.end(), -1);
}

// if index >= 0 -> left group
//    index < 0 -> right group
vector<int> minimum_vertex_cover() {
  vector<char> check(m);
  auto bfs = [&](int src) {
    queue<int> q;
    q.emplace(src);
    visited[src] = true;
    while (!q.empty()) {
      const auto u = q.front();
      q.pop();
      for (const auto& v : adj[u])
        if (~rev[v] && !visited[rev[v]] && match[u] != v) {
          check[v] = 1;
          visited[rev[v]] = true;
          q.emplace(rev[v]);
        }
    }
  };
  fill(visited.begin(), visited.end(), false);
  for (int i = 0; i < n; ++i)
    if (match[i] == -1 && !visited[i])
      bfs(i);
  vector<int> ret;
  ret.reserve(n - count(match.begin(), match.end(), -1));
  for (int i = 0; i < n; ++i)
    if (!visited[i])
      ret.emplace_back(int(i));
  for (int i = 0; i < m; ++i)
    if (check[i])
      ret.emplace_back(~int(i));
  return ret;
}
```

WeightedMatching.h

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.
Time: $\mathcal{O}\left(N^2M\right)$ 1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

7.4 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(g, n);`
`sccIdx[node] or sccs({0, 1, 3}, {2, 4}, ...)`
Time: $\mathcal{O}(E + V)$ b39228, 27 lines

```
vector<vi> sccs;
vi d, st, sccIdx;
int dNum;
int dfs(vector<vi>& g, int cur) {
  d[cur] = dNum++;
  st.push_back(cur);
  int ret = d[cur];
```

```
  for(int nxt : g[cur]) {
    if(sccIdx[nxt] < 0) ret = min(ret, d[nxt] ? : dfs(g, nxt));
  }
  if(ret == d[cur]) {
    int top;
    sccs.push_back({});
    auto& scc = sccs.back();
    do {
      top = st.back(); st.pop_back();
      scc.push_back(top);
      sccIdx[top] = sccs.size();
    } while(top != cur);
  }
  return ret;
}
void scc(vector<vi>& g, int n)
{
  d.assign(n, 0); sccIdx.assign(n, -1); dNum = 1;
  rep(i,0,n) if (sccIdx[i] < 0) dfs(g, i);
}
```

BCC.h

Time: $\mathcal{O}(E+V)$

f80e60, 37 lines

```
int dfn[max_v], low[max_v], cn, ccn;
vector<int> adj[max_v];
vector<vector<int>> bcc;
vector<pair<int, int>> st;

function<void(int, int)> dfs = [&](int u, int p)
{
  dfn[u] = low[u] = cn++;

  for (auto& v : adj[u]) if (v != p)
  {
    if (dfn[v] < dfn[u]) st.push_back({ u,v });

    if (dfn[v]) ckmin(low[u], dfn[v]);
    else
    {
      dfs(v, u);
      ckmin(low[u], low[v]);
      if (low[v] >= dfn[u])
      {
        if (st.back().first == u && st.back().second == v) bcc[
            ccn].push_back(v);

        while (1)
        {
          pair<int,int> cur = st.back(); st.pop_back();
          bcc[ccn].push_back(cur.first);
          if (cur.first == u && cur.second == v) break;
        }

        ++ccn;
      }

    }
  }
};

for(int i=0;i<n;++i) if (!dfn[i]) dfs(i, -1);
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

5f9706, 61 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};

// a^b == (~a || ~b) & (a||b)
// a eq b == (~a||b) &(a||~b)
// a=> b == (~a||b)
// (a+b+c<=1) == (~a||~b) & (~a||~c) &(~b||~c)
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

7.5 Coloring

7.6 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
}
```

HLD.h

Usage: `dfs(0); hld(0);`

"../data-structures/LazySegmentTree.h" d00f40, 43 lines

```
vector<vector<int>> adj(n);
vector<int> sz(n), in(n), par(n), top(n);
int tick = 0;
```

```cpp
void dfs(int u) {
  sz[u] = 1;
  for (auto& v : adj[u]) {
    par[v] = u;
    adj[v].erase(find(adj[v].begin(), adj[v].end(), u)); // if bidirectional
    dfs(v);
    sz[u] += sz[v];
    if (sz[v] > sz[adj[u][0]]) {
      swap(v, adj[u][0]);
    }
  }
}

void hld(int u) {
  in[u] = tick++;
  bool heavy = true;
  for (const auto& v : adj[u]) {
    top[v] = heavy ? top[u] : v;
    hld(v);
    heavy = false;
  }
}

int query_path(int u, int v) {
  // int ret = 0;
  for (; top[u] != top[v]; u = par[top[u]]) {
    if (sz[top[u]] > sz[top[v]])
      swap(u, v);
    // ret += query(in[top[u]], in[u] + 1);
  }
  if (in[u] > in[v]) swap(u, v);
  // ret += query(in[u], in[v] + 1);     if vertex
  // ret += query(in[u] + 1, in[v] + 1);  if edge
  // return u;                            if lca
}

int query_subtree(int u) {
  // return query(in[u], in[u] + sz[u]);
}
```

Centroid.h

1dc13b, 76 lines

```cpp
vector<pair<int,int>> adj[max_v]; // nxt, dist pair
int vist[max_v],sz[max_v];
int cp[max_v]; // centroid tree parent

// caution: when using hld together, it must not overlap with the sz array used in hld
int dfsz(int u, int par = -1) {
  sz[u] = 1;
  for (auto& v : adj[u]) if (v.first != par && !vist[v.first]) sz[u] += dfsz(v.first, u);
  return sz[u];
}

int fc(int u, int csz, int par = -1){
  for (auto& v : adj[u]) if (v.first != par && !vist[v.first] && sz[v.first] > csz) return fc(v.first, csz, u);
  return u;
}

void go(int u,int trp){
  int csz = dfsz(u);
  int cen = fc(u, csz/2); // find centroid

  vist[cen] = 1;
  cp[cen] = trp; // setting parent centroid of cur cen
  vector<int> cur;
  // After collecting the information of the subtrees into a map in several places with centroid as a disconnect point,
  // the merge can be performed on the logn.

  function<void(int,int)> getsub = [&](int u,int par) {
    cur.push_back(u);
    for (auto& v : adj[u]) if (v.first != par && !vist[v.first]) getsub(v.first, u);
  };

  for (auto& v : adj[cen]) if (!vist[v.first]) {
    getsub(v.first, u);
    for (auto& x : cur) cout << x << " "; // print v.first subtree node
    cur.clear();
  }

  for (auto& v : adj[cen]) if (!vist[v.first]) go(v.first,cen);
       // go nxt centroid
}

// When given a white vertex v, the shortest distance from the other vertex.
int color[max_v];
multiset <int> xset[max_v]; // The set that collects the distances of the white vertices from the vertex.
int p[20][max_v],d[max_v];

int getdist(int u, int v) {
  return d[u] + d[v] - 2 * d[lca(u, v)];
}

// Change the color of node v of the centroid tree update.
void upd(int v) {
  color[v] = !color[v];
  int i = v;
  int ct = 0;
  while (~i) {
    int dist = getdist(i, v);
    if (color[v]) xset[i].insert(dist); // if color is white than ins
    else xset[i].erase(xset[i].find(dist)); // if changed color is black then erase
    i = cp[i]; // move to parent cent
  }
}

// Centroid tree query. Find the black vertex v and the shortest white vertex.
int query(int v) {
  int i = v;
  int ret = INF;
  int ct = 0;
  while (~i) {
    int dist = getdist(i, v); // distance with current cent to v
    if (xset[i].size()) ckmin(ret, dist + *xset[i].begin()); // saved white point distance
    i = cp[i];
  }

  return ret == INF ? -1 : ret;
}
```

ManhattanMST.h

Description: return candidate edges(w, u, v) of Manhattan MST (<= 4n)
Usage: `T(distace type), U(point type)`
run Kruskal's to get the 'true' ManhattanMST

69808c, 29 lines

```cpp
template <typename T, typename U>
vector<tuple<T, int, int>> manhattan_MST(const vector<U>& a) {
  vector<int> id(a.size());
  iota(id.begin(), id.end(), 0);
  vector<tuple<T, int, int>> edges;
  edges.reserve(n << 2);
  for (int t = 0; t < 4; ++t) {
    sort(id.begin(), id.end(), [&](auto& lhs, auto& rhs) {
      return a[lhs].x - a[rhs].x < a[rhs].y - a[lhs].y;
    });
    map<T, int, greater<T>> sweep;
    for (const auto& i : id) {
      for (auto it = sweep.lower_bound(a[i].y); it != sweep.end(); it = sweep.erase(it)) {
        int j = it->y;
        T dx = a[i].x - a[j].x, dy = a[i].y - a[j].y;
        if (dy > dx) break;
        edges.emplace_back(dx + dy, i, j);
      }
      sweep[a[i].y] = i;
    }
    for (auto& [x, y] : a) {
      if (t & 1) {
        x = -x;
      } else {
        swap(x, y);
      }
    }
  }
}
```

7.7 Math

7.7.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

7.7.2 Erdős–Gallai theorem

Source: https://en.wikipedia.org/wiki/ErdTest: stress-tests/graph/erdos-gallai.cpp A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

Geometry (8)

8.1 Geometric primitives

Point.h

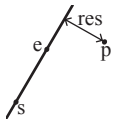
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

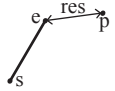


"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



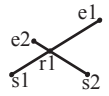
Usage:
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



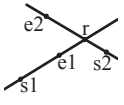
Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage:
```
bool left = sideOf(p1,p2,q)==1;
```

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

OnSegment.h

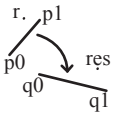
Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively
oriented triangles with vertices at 0 and i
```

0f0602, 35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

Time: $\mathcal{O}(n)$

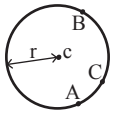
"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

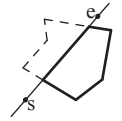
"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d4, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:

Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;
```

```
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf5, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
```

```
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

Time: $\mathcal{O}\left(n^2\right)$

"Point3D.h" 5b45fc, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h

Description: KMP algorithm

Time: $\mathcal{O}\left(n\right)$

eda7d4, 22 lines

```
vector<int> lps(const string& s) {
  vector<int> vt(s.size());
  for (int i = 1, j = 0; i < int(s.size()); ++i) {
    while (j && s[i] != s[j]) j = vt[j - 1];
    if (s[i] == s[j]) vt[i] = ++j;
  }
```

```
  return vt;
}

vector<int> match(const string& s, const string& k) {
  const auto fail = lps(k);
  const int n = s.size(), m = k.size();
  vector<int> ret;
  for (int i = 0, j = 0; i < n; ++i) {
    while (j && s[i] != k[j]) j = fail[j - 1];
    if (s[i] == k[j] && ++j == m) {
      ret.emplace_back(i - m + 1);
      j = fail[m - 1];
    }
  }
  return ret;
}
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

3ae526, 12 lines

```
vi Z(string S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Manacher.h

Description: abcd

e87037, 29 lines

```
int cir[max_v];

int main()
{
    string t; ci(t);

    string s = "#";
    reverse(t.begin(), t.end());
    while (t.size()) s += t.back(), s += '#', t.pop_back();

    int n = s.size();

    auto mana = [&]()
    {
        int r = 0, p = 0;
        fa(i,0,n)
        {
            cir[i] = i <= r ? min(cir[2*p-i],r-(int)i): 0;

            while (i - cir[i] - 1 >= 0 && i + cir[i] + 1 < n &&
                  s[i - cir[i] - 1] == s[i + cir[i] + 1]) ++cir
                 [i];

            if (r < i + cir[i]) r = i + cir[i], p = i;
        }
    };

    mana();
    fa(i, 1, n-1) co(cir[i] << " ");
    return 0;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

SuffixArray.h

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

d3fc67, 57 lines

```
struct SuffixArray {
  vi sa, lcp;

  vi ori, lg2;
  vector<vi> st;
  SuffixArray(string& s, int lim=256) { // or basic_string<int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)), y(n), ws(max(n, lim)), rank(n);
    x.push_back('\0');
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) {
        a = sa[i - 1], b = sa[i];
        x[b] = (y[a] == y[b] && a+j<n && b+j<n && y[a + j] == y
             [b + j]) ? p - 1 : p++;
      }
    }
    rep(i,1,n) rank[sa[i]] = i;
    for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
      for (k && k--, j = sa[rank[i] - 1];
        i+k<n-1 && j+k<n-1 && s[i + k] == s[j + k]; k++);

    // lcp RMQ build
    lg2.resize(n + 1);
    lg2[0] = lg2[1] = 0;
    rep(i,2,n+1) lg2[i] = lg2[i >> 1] + 1;

    ori.resize(n);
    int dep = lg2[n];
    st.resize(n);
    rep(i,0,n) {
      ori[sa[i]] = i;
      st[i].resize(dep + 1);
      st[i][0] = lcp[i];
    }
    rep(j,1,dep+1) {
      for(int i = 0; i + (1 << (j - 1)) < n; ++i) {
        st[i][j] = min(st[i][j - 1], st[i + (1 << (j - 1))][j -
            1]);
      }
    }
  }
  int get_lcp(int l, int r) {
    if(l == r) return sa.size() - l - 1;
    l = ori[l], r = ori[r];
    if(l > r) swap(l, r);
    int j = lg2[r - l];
    return min(st[l + 1][j], st[r - (1 << j) + 1][j]);
  }
};
// sa[0] = str.size(), sa.size() = str.size() + 1
// lcp[i] = lcp(sa[i - 1], sa[i]), lcp[0] = 0
```

Hashing.h

Description: Self-explanatory methods for string hashing.

3f02d8, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
  (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) : B); return r; }
  OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
  H operator-(H o) { return *this + ~o.x; }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

HashStr.h

Description: Get substring of hash.
Usage: `HashStr hs(str); v = hs.substr(0, 10);`
Time: $\mathcal{O}(n)$ *wheninit*, $\mathcal{O}(1)$ *toget*

2c2881, 43 lines

```
template <ll h1 = 3137, ll m1 = 998244353, ll h2 = 53, ll m2 =
    1610612741>
struct StrHash {
  vector<ll> hv, hpow;
```

```
  vector<ll> hv2, hpow2;

  void build(const string& str) {
    int n = str.size();
    hv.resize(n);
    hpow.resize(n);

    hv[0] = str[0];
    hpow[0] = 1;
    for(int i = 1; i < n; ++i) {
      hv[i] = (hv[i - 1] * h1 + str[i]) % m1;
      hpow[i] = (hpow[i - 1] * h1) % m1;
    }

    hv2.resize(n);
    hpow2.resize(n);

    hv2[0] = str[0];
    hpow2[0] = 1;
    for(int i = 1; i < n; ++i) {
      hv2[i] = (hv2[i - 1] * h2 + str[i]) % m2;
      hpow2[i] = (hpow2[i - 1] * h2) % m2;
    }
  }

  // [l, r]
  ll substr(int l, int r) {
    ll res = hv[r-1];
    if(l > 0) {
      res -= hv[l - 1] * hpow[r - l];
      res = ((res % m1) + m1) % m1;
    }
    ll res2 = hv2[r-1];
    if(l > 0) {
      res2 -= hv2[l - 1] * hpow2[r - l];
      res2 = ((res2 % m2) + m2) % m2;
    }
    return (res << 32) | res2;
  }
};
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

f35677, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

edce47, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

9e9d8d, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

MaxQueryDeque.h

Description: Get longest segment that range max value - min value <= k.
Time: $\mathcal{O}(N)$

14142e, 25 lines

```
long long n, k;
vector<int> arr(n);

int l = 0;
int ans = 0;
deque<int> uq, bq;

auto insert = [&](int idx) {
  while (uq.size() && arr[uq.back()] <= arr[idx]) uq.pop_back()
      ;
  while (bq.size() && arr[bq.back()] >= arr[idx]) bq.pop_back()
      ;
  uq.push_back(idx), bq.push_back(idx);
};

auto del = [&](int idx) {
  if (uq.front() == idx) uq.pop_front();
  if (bq.front() == idx) bq.pop_front();
};

for(int i=0;i<n;++i) {
  insert(i);
  while (arr[uq.front()] - arr[bq.front()] > k) del(l++);
  ckmax(ans, i - l + 1);
}

// return ans;
```

HalfPlaneIntersection.h

Description: Half-plane Intersection
Left half-plane for vector from (x1,y1) to (x2,y2) is equal to
a=y2-y1, b=x1-x2, c=x1*y2-x2*y1
To move L[i] parellel to original line by d
long double h=hypot(L[i].a,L[i].b);
long double dx=d*L[i].a/h, dy=d*L[i].b/h;
L[i].c-=dx*L[i].a+dy*L[i].b; // note that -=
HPI function returns HPI polygon

<bits/stdc++.h> 5f222c, 54 lines

```
#define fi first
#define se second

using namespace std;
typedef long long ll;

const double eps = 1e-8;
typedef pair<long double, long double> pi;
long double ccw(pi p1, pi p2, pi p3){
    p2.fi-=p1.fi; p2.se-=p1.se;
    p3.fi-=p1.fi; p3.se-=p1.se;
    return p2.fi*p3.se - p2.se*p3.fi;
}
bool z(long double x){ return fabs(x) < eps; }
struct line{
  long double a, b, c;
  bool operator<(const line &l)const{
    bool flag1 = pi(a, b) > pi(0, 0);
    bool flag2 = pi(l.a, l.b) > pi(0, 0);
    if(flag1 != flag2) return flag1 > flag2;
    long double t = ccw(pi(0, 0), pi(a, b), pi(l.a, l.b));
    return z(t) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : t >
        0;
  }
  pi slope(){ return pi(a, b); }
};
pi cross(line a, line b){
  long double det = a.a * b.b - b.a * a.b;
  return pi((a.c * b.b - a.b * b.c) / det, (a.a * b.c - a.c * b
      .a) / det);
}
bool bad(line a, line b, line c){
  if(ccw(pi(0, 0), a.slope(), b.slope()) <= 0) return false;
  pi crs = cross(a, b);
  return crs.first * c.a + crs.second * c.b >= c.c;
}
bool HPI(vector<line> v, vector<pi> &solution){ // ax + by <= c
    ;
  sort(v.begin(), v.end());
  deque<line> dq;
  for(auto &i : v){
    if(!dq.empty() && z(ccw(pi(0, 0), dq.back().slope(), i.
        slope()))) continue;
    while(dq.size() >= 2 && bad(dq[dq.size()-2], dq.back(), i))
        dq.pop_back();
    while(dq.size() >= 2 && bad(i, dq[0], dq[1])) dq.pop_front
        ();
    dq.push_back(i);
  }
  while(dq.size() > 2 && bad(dq[dq.size()-2], dq.back(), dq[0])
      ) dq.pop_back();
  while(dq.size() > 2 && bad(dq.back(), dq[0], dq[1])) dq.
      pop_front();
  vector<pi> tmp;
  for(int i=0; i<dq.size(); i++){
    line cur = dq[i], nxt = dq[(i+1)%dq.size()];
    if(ccw(pi(0, 0), cur.slope(), nxt.slope()) <= eps) return
        false;
    tmp.push_back(cross(cur, nxt));
  }
  solution = tmp;
  return true;
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}(N^2)$

ca3b93, 29 lines

```
int n, m[5005], sum[5005];
int dp[5005][5005], pos[5005][5005];
const int INF=1e9;

int main()
{
    scanf("%d",&n);
    for(int i=1 ; i<=n ; i++){
        scanf("%d",&m[i]);
        pos[i][i]=i;
        sum[i]=sum[i-1]+m[i];
    }
    //calculate DP diagonally
    for(int len=2 ; len<=n ; len++){
        for(int i=1 ; i<=n-len+1 ; i++){
            dp[i][i+len-1]=INF;
            int s=pos[i][i+len-2], f=pos[i+1][i+len-1];
            for(int j=s ; j<=f ; j++){
                if(j<n && dp[i][i+len-1]>dp[i][j]+dp[j+1][i+len
                    -1]){
                    pos[i][i+len-1]=j;
                    dp[i][i+len-1]=dp[i][j]+dp[j+1][i+len-1];
                }
            }
            dp[i][i+len-1]+=sum[i+len-1]-sum[i-1];
        }
    }
    printf("%d\n",dp[1][n]);
    return 0;
}
```

DnCOptimization.h

Description: Divide and Conquar Optimization DP
D[i][j] = min(D[i-1][k]+C[k][j]) (where k<j)
For a<=b<=c<=d, C[a][c]+C[b][d]<=C[a][d]+C[b][c] (C is Monge array) or optimal k for each j has monotonicity

3a3171, 43 lines

```
function<void(int, int, int, int, int)> dnc = [&](int lev, int
    l, int r, int s, int e) {
  if (l > r || s > e) return;

  int mid = l + r >> 1;
  int opt = -1;
  dp[lev][mid] = LNF;

  fa(i, s, min(mid,e) + 1) {
    LL t = dp[lev - 1][i] + cost(i + 1, mid);

    if (dp[lev][mid] > t) {
      dp[lev][mid] = t;
      opt = i;
    }
  }

  dnc(lev, l, mid - 1, s, opt);
  dnc(lev, mid + 1, r, opt, e);
};

function<void(int, int, int, int)> dnc = [&](int l, int r, int
    s, int e) {
```

```
  if (l > r ||s > e) return;

  int mid = l + r >> 1;
  int opt = -1;
  LL maxi = -LNF;

  fa(i, s, e+1) {
    LL dx = b[i].first - a[mid].first;
    LL dy = b[i].second - a[mid].second;

    LL ret = (dx < 0 && dy < 0) ? 0 : dx * dy;

    if (ret > maxi) {
      maxi = ret;
      opt = i;
    }
  }

  ckmax(ans, maxi);

  dnc(l, mid - 1, s, opt+1); dnc(mid + 1, r, opt-1, e);
};
```

10.4 Debugging tricks

Debug.h

Description: Code for debugging

200d1a, 33 lines

```
#ifdef palilo
template <typename C, typename T = typename enable_if<!is_same<
    C, string>::value, typename C::value_type>::type>
ostream& operator<<(ostream& os, const C& container) {
  os << '[';
  bool first = true;
  for (const auto& x : container) {
    if (!first) os << ", ";
    os << x;
    first = false;
  }
  return os << ']';
}

template <typename T1, typename T2>
ostream& operator<<(ostream& os, const pair<T1, T2>& p) {
  return os << '(' << p.first << ", " << p.second << ')';
}

template <typename T>
void debug_msg(string name, T arg) {
  cerr << name << " = " << arg << endl;
}

template <typename T1, typename... T2>
void debug_msg(string names, T1 arg, T2... args) {
  cerr << names.substr(0, names.find(',')) << " = " << arg << "
       | ";
  debug_msg(names.substr(names.find(',') + 2), args...);
}

#define debug(...) cerr << '(' << __LINE__ << ')' << ' ',
    debug_msg(#__VA_ARGS__, __VA_ARGS__)
#else
#define debug(...)
#endif
```

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod b$ in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: `./a.out < input.txt`

Time: About 5x as fast as cin/scanf.

7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

appendix (A)

A.1 Mobius Example

$$\sum_{i=1}^{n}\sum_{j=1}^{m}[gcd(i,j)=d]$$

$$\sum_{a=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{b=1}^{\lfloor\frac{m}{d}\rfloor}[gcd(a,b)=1] \qquad (i=ad, j=bd)$$

$$\sum_{a=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{b=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{d|gcd(a,b)}\mu(d)$$

$$\sum_{a=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{b=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{k=1}^{p}[k|gcd(a,b)]\mu(k) \qquad (p=min(\lfloor\frac{n}{d}\rfloor,\lfloor\frac{m}{d}\rfloor))$$

$$\sum_{a=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{b=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{k=1}^{p}[k|a][k|b]\mu(k)$$

$$\sum_{k=1}^{p}\mu(k)\sum_{a=1}^{\lfloor\frac{n}{d}\rfloor}[k|a]\sum_{b=1}^{\lfloor\frac{m}{d}\rfloor}[k|b]$$

$$\sum_{k=1}^{p}\mu(k)\lfloor\frac{n}{kd}\rfloor\lfloor\frac{m}{kd}\rfloor$$

A.2 Temp

FindCycle.h

Description: Description: simple cycle detection algorithm. implemented as non-reculsive way. return the vector of vertices on cycle. note that first and last vertex are repeated i.e. cycle.front() == cycle.back(). 4ec3e0, 39 lines

```
vector<int> find_cycle(vector<vector<int>>& adj) {
    vector<char> colour(adj.size());
    vector<int> cycle, eid;
    cycle.reserve(adj.size());
    eid.reserve(adj.size());
    auto dfs = [&](int u) -> void {
        colour[u] = 'g';
        cycle.emplace_back(u);
        eid.emplace_back(0);
        while (!cycle.empty()) {
            for (auto &u = cycle.back(), &i = eid.back();; ++i)
                {
                if (i == int(adj[u].size())) {
                    colour[u] = 'b';
                    cycle.pop_back();
                    eid.pop_back();
                    break;
                } else if (!colour[adj[u][i]]) {
                    colour[adj[u][i]] = 'g';
                    cycle.emplace_back(adj[u][i]);
                    eid.emplace_back(0);
                    break;
                } else if (colour[adj[u][i]] == 'g') {
                    cycle.emplace_back(adj[u][i]);
                    return;
                }
            }
        }
    };
    for (int i = 0; i < int(adj.size()); ++i) {
        if (!colour[i]) {
            dfs(i);
            if (!cycle.empty()) {
                cycle.erase(cycle.begin(), find(cycle.begin(),
                    cycle.end(), cycle.back()));
                return cycle;
            }
        }
    }
    return {};
};
```