AI534 Implementation 1

Deadline: Sunday, Oct. 13, by 11:59pm

Submission: Submit 1) your completed notebook in ipynb format, and 2) a PDF export of the completed notebook with outputs (the codeblock at the end of the notebook should automatically produce the pdf file).

In this assignment, we will implement and experiment linear regression to predict the price of a house based on features describing the house, using the housing data that you have explored in the warm up assignment.

We will implement two versions, one using the closed-form solution, and one using gradient descent.

You may modify the starter code as you see fit, including changing the signatures of functions and adding/removing helper functions. However, please make sure that your TA can understand what you are doing and why.

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First lets import the necessary packages.

In [108…
```
!pip install nbconvert > /dev/null 2>&1
!pip install pdfkit > /dev/null 2>&1
!apt-get install -y wkhtmltopdf > /dev/null 2>&1
import os
import pdfkit
import contextlib
import sys
from google.colab import files
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
# add more imports if necessary
```

Part 0: (5 pts) data and preprocessing

On canvas, we have provided two different data files for this assignment: ia1_train.csv (for training) and ia1_val.csv(for validation). Download them and upload them to your google drive. Then mount the google drive from your google colab notebook:

In [109…
```
from google.colab import drive
drive.mount('/content/gdrive')

train_path = '/content/gdrive/My Drive/AI534/ia1_train.csv' # DO NOT MODIFY THIS. Please make sure your data has
val_path = '/content/gdrive/My Drive/AI534/ia1_val.csv' # DO NOT MODIFY THIS. Please make sure your data has thi
```

```
Drive already mounted at /content/gdrive; to attempt to forcibly remount, call drive.mount("/content/gdrive", f
orce_remount=True).
```

Now load the training and validation data.

In [110…
```
# your code goes here
```

In [111…
```
train_data = pd.read_csv(train_path)
val_data = pd.read_csv(val_path)
```

In [112…
```
train_data
```

Out[112]:

	id	date	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	...	sqft_abo
0	7972604355	5/21/2014	3	1.00	1020	7874	1.0	0	0	3	...	10
1	8731951130	6/9/2014	3	2.25	2210	8000	2.0	0	0	4	...	22
2	7885800740	2/18/2015	4	2.50	2350	5835	2.0	0	0	3	...	23
3	4232900940	5/22/2014	3	1.50	1660	4800	2.0	0	0	3	...	16
4	3275850190	9/5/2014	3	2.50	2410	9916	2.0	0	0	4	...	24
...	...	...	...	...	...	...	...	...	...	...	...	.
7995	4222500410	2/26/2015	4	1.75	2000	7350	1.0	0	0	3	...	11
7996	1150700170	9/26/2014	4	2.25	1870	6693	2.0	0	0	3	...	18
7997	1959702045	11/19/2014	2	1.00	1240	5500	1.0	0	0	3	...	12
7998	7234601221	10/14/2014	3	1.50	1280	2114	1.5	0	0	3	...	12
7999	3275740030	5/7/2014	3	2.25	1770	8165	2.0	0	0	3	...	17

8000 rows × 21 columns

In [113…

```
val_data
```

Out[113]:

	id	date	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	...	sqft_abo
0	3211200460	8/6/2014	4	1.00	1520	9800	1.5	0	0	4	...	15
1	4124000320	3/16/2015	3	2.25	1800	15903	1.0	0	0	3	...	13
2	7129302800	12/12/2014	3	1.50	1780	5000	1.0	0	4	4	...	10
3	1392800035	6/18/2014	2	1.00	1240	6400	1.0	0	1	4	...	10
4	2154900040	10/30/2014	3	2.25	2190	8834	1.0	0	0	3	...	13
...	...	...	...	...	...	...	...	...	...	...	...	.
1995	5132000140	1/20/2015	6	1.00	1370	5080	1.5	0	0	3	...	11
1996	6624010170	5/8/2014	3	1.75	1390	7399	1.0	0	0	4	...	13
1997	1853080840	2/11/2015	5	3.50	3700	7055	2.0	0	0	3	...	37
1998	2767601311	10/24/2014	3	2.50	1260	1102	3.0	0	0	3	...	12
1999	226059078	2/27/2015	2	1.00	1840	81892	1.0	0	0	3	...	18

2000 rows × 21 columns

Perform the following preprocessing steps.

1. remove the *ID* column from both training and validation data
2. change *date* into 3 numerical features *day*, *month* and *year*, like in the warm up exercise
3. The feature *yr_renovated* is set to 0 if the house has not been renovated. This creates an inconsistent meaning to the numerical values. Replace it with a new feature called *age_since_renovated*:

> if *yr_renovate* != 0
>
> > *age_since_renovated* = *year* - *yr_renovated*
>
> otherwise
>
> > *age_since_renovated* = *year* - *yr_built*

1. Normalize all the feautres using z-score normalization based on the training data. Do not normalize *price* as it is the target. To normalize a feature *x* using z-score normalization, the fomula is

> $z=\frac{x-\mu}{\sigma}$

where μ and σ are the mean and standard deviation of x respectively. The normalized feature will have zero mean and unit standard deviation. Note that you should estimate μ and σ for each feature only using the training data and use the same μ and σ to normalize the features for both training and validation data.

In [114…

```
# Your code goes here
```

In [115…

```
#Removing ID column
train_data_without_id = train_data.drop(columns=['id'])
```

```
val_data_without_id = val_data.drop(columns=['id'])
```

In [116…

```
# change date into 3 numerical features day, month and year.
def convert_date_to_features(df):
  df['date'] = pd.to_datetime(df['date'], format='%m/%d/%Y')
  df['day'] = df['date'].dt.day
  df['month'] = df['date'].dt.month
  df['year'] = df['date'].dt.year
  df.drop(columns=['date'], inplace=True)


convert_date_to_features(train_data_without_id)
convert_date_to_features(val_data_without_id)
```

In [117…

```
#Replacing feature yr_renovated to new feature age_since_renovated
def age_since_renovated(df):
  for index, row in df.iterrows():
    if row['yr_renovated'] != 0:
      row['year'] - row['yr_renovated']
      df.at[index, 'age_since_renovated'] = row['year'] - row['yr_renovated']
    else :
      row['year'] - row['yr_built']
      df.at[index, 'age_since_renovated'] = row['year'] - row['yr_built']
  df = df.drop(columns=['yr_renovated'], inplace = True)

age_since_renovated(train_data_without_id)
age_since_renovated(val_data_without_id)
```

In [118…

```
# Calculate the mean and standard deviation for each column
mean_values = train_data_without_id.drop('price', axis=1).mean()
std_values = train_data_without_id.drop('price', axis=1).std()
```

In [119…

```
#Normalize all the feautres using z-score normalization based on the training data
norm_train=(train_data_without_id.drop('price', axis=1)-mean_values)/std_values
norm_train['price']=train_data_without_id['price']
norm_train
```

Out[119]:

	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	grade	sqft_above	...
0	-0.388520	-1.465613	-1.159110	-0.171139	-0.922332	-0.082432	-0.304487	-0.634184	-0.563015	-0.924206	...
1	-0.388520	0.169158	0.145003	-0.168133	0.917732	-0.082432	-0.304487	0.888924	0.290361	0.507090	...
2	0.649956	0.496112	0.298428	-0.219787	0.917732	-0.082432	-0.304487	-0.634184	0.290361	0.675478	...
3	-0.388520	-0.811705	-0.457738	-0.244481	0.917732	-0.082432	-0.304487	-0.634184	0.290361	-0.154433	...
4	-0.388520	0.496112	0.364182	-0.122419	0.917732	-0.082432	-0.304487	0.888924	1.997115	0.747644	...
...	...	...	...	...	...	...	...	...	...	...	...
7995	0.649956	-0.484751	-0.085134	-0.183641	-0.922332	-0.082432	-0.304487	-0.634184	-0.563015	-0.827984	...
7996	0.649956	0.169158	-0.227600	-0.199316	0.917732	-0.082432	-0.304487	-0.634184	-0.563015	0.098149	...
7997	-1.426996	-1.465613	-0.918013	-0.227780	-0.922332	-0.082432	-0.304487	-0.634184	-0.563015	-0.659596	...
7998	-0.388520	-0.811705	-0.874177	-0.308567	-0.002300	-0.082432	-0.304487	-0.634184	0.290361	-0.611485	...
7999	-0.388520	0.169158	-0.337190	-0.164196	0.917732	-0.082432	-0.304487	-0.634184	-0.563015	-0.022128	...

8000 rows × 22 columns

In [120…

```
#Normalize all the feautres using z-score normalization based on the training data
norm_val=(val_data_without_id.drop('price', axis=1)-mean_values)/std_values
norm_val['price']=val_data_without_id['price']
norm_val
```

Out[120]:

	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	grade	sqft_above	...
0	0.649956	-1.465613	-0.611163	-0.125186	-0.002300	-0.082432	-0.304487	0.888924	-0.563015	-0.322821	...
1	-0.388520	0.169158	-0.304313	0.020425	-0.922332	-0.082432	-0.304487	-0.634184	0.290361	-0.539319	...
2	-0.388520	-0.811705	-0.326231	-0.239710	-0.922332	-0.082432	5.014070	0.888924	-0.563015	-0.912178	...
3	-1.426996	-1.465613	-0.918013	-0.206307	-0.922332	-0.082432	1.025152	0.888924	-0.563015	-0.876095	...
4	-0.388520	0.169158	0.123085	-0.148234	-0.922332	-0.082432	-0.304487	-0.634184	-0.563015	-0.479181	...
...	...	...	...	...	...	...	...	...	...	...	...
1995	2.726909	-1.465613	-0.775547	-0.237801	-0.002300	-0.082432	-0.304487	-0.634184	-1.416392	-0.803929	...
1996	-0.388520	-0.484751	-0.753629	-0.182472	-0.922332	-0.082432	-0.304487	0.888924	-0.563015	-0.479181	...
1997	1.688433	1.803929	1.777884	-0.190679	0.917732	-0.082432	-0.304487	-0.634184	1.143738	2.299217	...
1998	-0.388520	0.496112	-0.896095	-0.332712	2.757797	-0.082432	-0.304487	-0.634184	0.290361	-0.635541	...
1999	-1.426996	-1.465613	-0.260477	1.594857	-0.922332	-0.082432	-0.304487	-0.634184	-1.416392	0.062066	...

2000 rows × 22 columns

Let's do a quick testing of your normalization, please

1. Estimate and print the new mean and standard deviation of the normalized features for the training data --- this should be 0 and 1 respectively.
2. Estimate and print the new mean and standard deviation of the normalized features for the validation data --- these values will not be 0 and 1, but somewhat close

In [121…

```
# Your code goes here
```

In [122…

```
#1. Estimate and print the new mean and standard deviation of the normalized features for the training data ---
print(norm_train.drop('price', axis=1).mean())
print(norm_train.drop('price', axis=1).std())
```

```
bedrooms               1.652012e-16
bathrooms             -1.634248e-16
sqft_living            9.059420e-17
sqft_lot              -1.776357e-18
floors                 4.396483e-17
waterfront             2.464695e-17
view                  -6.394885e-17
condition              6.306067e-17
grade                  1.207923e-16
sqft_above            -9.947598e-17
sqft_basement          5.329071e-18
yr_built               3.400835e-15
zipcode                3.704059e-14
lat                   -4.131451e-14
long                   9.379608e-14
sqft_living15          6.039613e-17
sqft_lot15            -1.776357e-18
day                   -2.142730e-17
month                 -2.264855e-17
year                   1.991887e-13
age_since_renovated   -9.681145e-17
dtype: float64
bedrooms               1.0
bathrooms              1.0
sqft_living            1.0
sqft_lot               1.0
floors                 1.0
waterfront             1.0
view                   1.0
condition              1.0
grade                  1.0
sqft_above             1.0
sqft_basement          1.0
yr_built               1.0
zipcode                1.0
lat                    1.0
long                   1.0
sqft_living15          1.0
sqft_lot15             1.0
day                    1.0
month                  1.0
year                   1.0
age_since_renovated    1.0
dtype: float64
```

In [123…

```
#2. Estimate and print the new mean and standard deviation of the normalized features for the validation data -
print(norm_val.drop('price', axis=1).mean())
print(norm_val.drop('price', axis=1).std())
```

```
bedrooms                 0.005582
bathrooms               -0.011648
sqft_living              0.013909
sqft_lot                 0.005044
floors                   0.022541
waterfront               0.015265
view                     0.002659
condition               -0.055403
grade                    0.057390
sqft_above               0.028274
sqft_basement           -0.024773
yr_built                 0.018428
zipcode                 -0.030558
lat                     -0.018752
long                     0.016222
sqft_living15            0.051889
sqft_lot15              -0.005096
day                     -0.017296
month                   -0.017980
year                     0.009397
age_since_renovated     -0.023726
dtype: float64
bedrooms                 0.893240
bathrooms                1.003430
sqft_living              0.993781
sqft_lot                 0.912514
floors                   0.992549
waterfront               1.088181
view                     1.025579
condition                0.976223
grade                    1.033680
sqft_above               0.996623
sqft_basement            0.981484
yr_built                 1.000355
zipcode                  1.006450
lat                      1.000963
long                     0.986680
sqft_living15            1.051790
sqft_lot15               0.785337
day                      1.008404
month                    0.983898
year                     1.003819
age_since_renovated      0.999477
dtype: float64
```

Question

Why is it import to use the same μ and σ to perform normalization on the training and validation data? What would happen if we use μ and σ estimated using the validation to perform normalization on the validation data?

Your answer goes here:

To avoid poor generalization and overfitting, we need to use same mean and standard deviation to perform normalization on the training and validation data. The goal of using a validation set is to simulate how the model will perform on new data set. If we use mean and standard deviation estimated using the validation to perform normalization on the validation data, the model might not guarantee the performance since it performs poorly and less accurate when a set of new data is inserted. Moreover, training with validation data can cause to overfit on the validation data, meaning that there would be no difference (gap) between training and validation set.

Part 1 (15 pts) Generate closed-form solution for reference.

Our data now contains 21 numeric features, before we learn a linear regression model using gradient descent, we will first build the closed-form solution as a reference point. So for this part, you need to

1. Implement the close-form solution for linear regression and apply it to the training data to learn the weight vector for your linear regression model. For matrix inversion you can use existing numpy functions. Specifically, we recommend the numpy.linalg.pinv() function for inverting near-singular matrices.
2. Apply your learned linear regression model to the training data to make predictions for all training examples and report the Mean Squared Error.
3. Apply your learned linear regression model to the validation data to make predictions for all the validation examples and report the mean squared error for the validation data.

Your code should print the weight vector, which has 22 dimensions, one for each feature plus one additional w_0. Your code should also report the MSE for the training and validation data respectively.

In [124…
```
# Your code goes here
```

In [125…
```
#1. Closed form to compute weight: w={(XX^T)^-1}X^TY
x_train = norm_train.drop('price', axis=1)
y_train = norm_train['price']

ones_column = np.ones((x_train.shape[0], 1))
x_train_with_bias = np.hstack([ones_column, x_train])


w_train = np.dot(np.linalg.pinv(np.dot(x_train_with_bias.T, x_train_with_bias)), np.dot(x_train_with_bias.T, y_t
print(x_train_with_bias.shape)
print(w_train)
```

```
(8000, 22)
[ 5.36167284 -0.28135266  0.3390716   0.76341998  0.05815041  0.01813676
  0.3281388   0.44675376  0.1998432   1.11544343  0.75623295  0.15546155
 -0.88336171 -0.26341874  0.83661248 -0.30369641  0.14358099 -0.09927428
 -0.05063652  0.05485035  0.17375019 -0.10255779]
```

In [126…
```
x_val = norm_val.drop('price', axis=1)
y_val = norm_val['price']

ones_column = np.ones((x_val.shape[0], 1))
x_val_with_bias = np.hstack([ones_column, x_val])
```

In [127…
```
#2. Apply your learned linear regression model to the training data to make predictions for all training example
n_train = x_train_with_bias.shape[0]
mse_train = (1 / n_train) * np.sum(((np.dot(x_train_with_bias, w_train) - y_train) ** 2))
print("MSE for training data:", mse_train)
```

```
MSE for training data: 3.7578870899545866
```

In [128…
```
#3.Apply your learned linear regression model to the validation data to make predictions for all the validation
n_val = x_val_with_bias.shape[0]
mse_val = (1/n_val)*np.sum(((np.dot(x_val_with_bias, w_train) - y_val) ** 2))
print("MSE for val data:", mse_val)
```

```
MSE for val data: 4.503508105356858
```

Question

The learned feature weights are often used to understand the importance of the features. The sign of the weights indicates if a feature positively or negatively impact the price, and the magnitude suggests the strength of the impact. Does the sign of all the features match your expection based on your common-sense understanding of what makes a house expensive? Please hightlight any surprises from the results.

Your answer goes here

As my expectation, it seems that the number of bedrooms have a positive impact on the price of the house, making the price expensive. Surprisingly, its weight is about 5.36, which is the highest values. On the other hand, not as much as the number of bedrooms, zipcode negatively affect the price of house, which is about -0.89.

Part 2 (40 pts) Implement and experiment with batch gradient descent

Your implementation should take following inputs:

1. the training data (with d features and 1 target variable y),
2. the learning rate γ,
3. the number of iterations T
4. Optional convergence threshold (optional) ϵ_l for the loss or ϵ_g for the norm of the gradient

It should output:

1. the learned $d+1$ - dimensional weight vector

2. the sequence of T MSE losses, one for each training epoch. You will be asked to plot the losses as a function of training epoch later.

In [129…

```
# Your code goes here
```

In [130…

```
# Implementation of batch gradient descent

def batch_gradient_descent(X, y, lr_gamma, T, epsilon_l=None, epsilon_g=None):

  n, d = X.shape
  w = np.zeros(d)
  mse_history = []

  for t in range(1,T):
    errors = np.dot(X, w) - y

    mse_loss = (1 / n) * np.sum(errors**2)
    mse_history.append(mse_loss)

    gradient = (2 / n) * np.dot(errors, X)
    w = w - (lr_gamma * gradient)

    if epsilon_g is not None and np.linalg.norm(gradient) <= epsilon_g:
      print(f'Converged at iteration {t+1} with gradient norm {np.linalg.norm(gradient)}')
      break

    if epsilon_l is not None and t > 0 and abs(mse_history[-1] - mse_history[-2]) <= epsilon_l:
      print(f'Converged at iteration {t+1} with loss change {abs(mse_history[-1] - mse_history[-2])}')
      break

  return w, mse_history
```

You will now experiment with the batch gradient descent algorithm with different learning rate on the provided data.

Please train your model for up to 3000 iterations using different learning rate: $\gamma=10^{-i}$, $i=0,1,...,4$. For each learning rate, you can opt to stop early if it has converged (using the convergence threshold) or diverged (the loss start to grow larger).

For each converging (not necessarily converged yet) learning rate, please compute and report the MSE of the final learned weights on the validation data.

Also please provide a plot that shows the training loss curves (MSE as a function of the # of epochs) for all the converging learning rates. Please use different colors mark different learning rates and provide proper legends for your figure.

In [131…

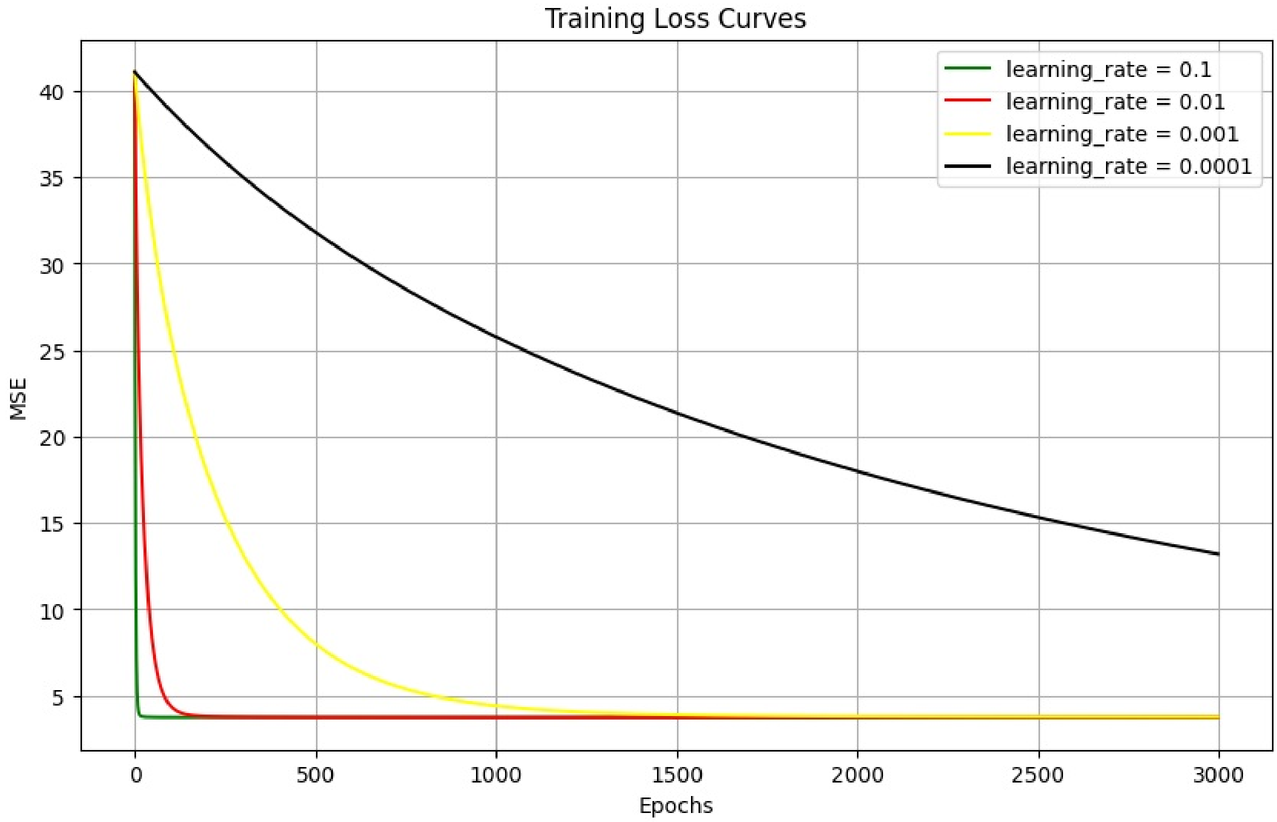
```
# Your code goes here
```

In [132…

```
learning_rate = [1, 0.1, 0.01, 0.001, 0.0001]
Iterations = 3000
epsilon_l = None
epsilon_g = None

weight_set = []
history_set = []
for lr_gamma in learning_rate:
  w, mse_history = batch_gradient_descent(x_train_with_bias, y_train, lr_gamma, Iterations, epsilon_l, epsilon_g
  weight_set.append(w)
  history_set.append(mse_history)


valid_mse_set = []
for w in weight_set:
  valid_mse_value = (1/n_val)*np.sum(((np.dot(x_val_with_bias, w) - y_val) ** 2))
  print("MSE of the final learned weights on the validation data: ", valid_mse_value)
  valid_mse_set.append(valid_mse_value)

epochs_set = []
for i in range(len(history_set)):
  epochs_set.append([x for x in range(len(history_set[i]))])


# A plot shows the training loss curves for all the converging learning rates
plt.figure(figsize=(10, 6))

# This case (learning rate = 1) is diverged.
#plt.plot(epochs_set[0], history_set[0], label="learning_rate = 1", color='blue')

plt.plot(epochs_set[1], history_set[1], label="learning_rate = 0.1", color='green')
plt.plot(epochs_set[2], history_set[2], label="learning_rate = 0.01", color='red')
plt.plot(epochs_set[3], history_set[3], label="learning_rate = 0.001", color='yellow')
plt.plot(epochs_set[4], history_set[4], label="learning_rate = 0.0001", color='black')
```

```
plt.title("Training Loss Curves")
plt.xlabel("Epochs")
plt.ylabel("MSE")

plt.legend()

plt.grid(True)
plt.show()
```

```
/usr/local/lib/python3.10/dist-packages/numpy/core/_methods.py:49: RuntimeWarning: overflow encountered in redu
ce
  return umr_sum(a, axis, dtype, out, keepdims, initial, where)
MSE of the final learned weights on the validation data:  0.0
MSE of the final learned weights on the validation data:  4.503508105356861
MSE of the final learned weights on the validation data:  4.503498432645095
MSE of the final learned weights on the validation data:  4.525769160670072
MSE of the final learned weights on the validation data:  14.469470619134585
```



Question

Which learning rate leads to the best trainng and validation MSE respectively? Do you observe better training MSE tend to correpsond to better validation MSE? How is this different from the trend shown on page 51 of the lecture slides (titled danger of using training loss to select M) regarding overfitting? Is there any issue with using training loss to pick learning rate in this case?

The best training result shows when the learning rate is 0.1, and the best validation MSE is 4.503498xx when learning rate is 0.01, which is slightly better than when learning rate is 0.1. This means better training MSE does not tend to correspond to better validation MSE. Unlike lecture slides page 51, it seems training loss is similar to validation loss. If I use validation data for training, the results will be likely to present overfitting. One issue I have is when learning rate is 1, the training loss and weight display meaningless values and diverged.

Part 3. More exploration.

3(a). (20 pts) Normalization of features: what is the impact?

In part 1, you were asked to perform z-score normalization of all the features. In this part, we will ask you to first conceptually think about what is the impact this operation on the solution and then use some experiments to varify your conceptual understanding.

Questions.

The normalization process applies a linear transformation to each feature, where the transformed feature x' is simply a linear function of original feature x: $x'=\frac{x-\mu}{\sigma}$.

Let's disect the influence of this transformation on our learned linear regression model.

1. How do you think this transformation will influnce the training and validation MSE we get for the closed-form solution? Why?
2. How do you think this will change the magnitude of the weights of the learned model? Why?
3. How do you think this will change the convergence behavior of the batch gradient descent algorithm? Why?

Your answer goes here.

1. In my opinion, the transformation have a little or no influence on the training and validation MSE since normalization is for scaling and no difference of original shape of the function. The reason is that non-normalized features can have relatively much greater values than other smaller features, whereas normalized features are properly in the even range like [0, 1], maintaining or slightly improving the performance of training and validation

2. Normalization changes magnitude of the weightsof the learned model. The benefit of normalization is that all features have even range [0, 1], leading to learn uniform size of weights. If we use non-normalization, some features that has greater range of values are assigned with larger wegiths, others are assigned smaller weights relatively.

3. In general, if the scale of each feature is significantly different, it will negatively affect the convergence behavior of the batch gradient descent algorithm. Non-normalization can cause non-even updates of the weights, degrading the rate of convergence and making it hard to reach optimal solution. Thus, we need to use normalization for the stable rate of learning.

Now please perform the following experiments to verify your answer to the above questions.

1. Apply closed-form solution to data that did not go through the feature normalization step, and report the learned weights and the resulting training and testing MSEs.
2. Apply gradient descent algorithm to data that did not go through the feature normalization step using different learning rate. Note that the learning rate used in previous section will no longer work here. You will need to search for an appropriate learning rate to get some converging behavior. Plot your MSE loss curve as a function of the epochs once you identify a convergent learning rate. Hint: the learning rate needs to be much, much, much, much, much smaller (think about each much as an order of manitude) than what was used in part 2). Also unless you let it run for a long time, it is unlikely to converge to the same level of loss values. So use a upper bound on the # of iterations so that it won't take forever.

In [133…

```
# Your code goes here
```

In [134…

```
# 1.1. Apply closed-form solution to data without normalization step and
#        Report the learned weights
#  = ()-y

def closed_form_linear_regression(X, y):
  w = np.linalg.pinv(np.transpose(X) @ X) @ np.transpose(X) @ y
  return w

non_normalized_X_train = train_data_without_id.drop(columns=['price'])
print(non_normalized_X_train)
non_normalized_X_train_biased = np.column_stack((np.ones(non_normalized_X_train.shape[0]),non_normalized_X_trai
y_train = train_data_without_id['price']
non_normalized_weight = closed_form_linear_regression(non_normalized_X_train_biased, y_train)
print("Learned weights: ", non_normalized_weight)
```

```
      bedrooms  bathrooms  sqft_living  sqft_lot  floors  waterfront  view  \
0            3       1.00         1020      7874     1.0           0     0
1            3       2.25         2210      8000     2.0           0     0
2            4       2.50         2350      5835     2.0           0     0
3            3       1.50         1660      4800     2.0           0     0
4            3       2.50         2410      9916     2.0           0     0
...        ...        ...          ...       ...     ...         ...   ...
7995         4       1.75         2000      7350     1.0           0     0
7996         4       2.25         1870      6693     2.0           0     0
7997         2       1.00         1240      5500     1.0           0     0
7998         3       1.50         1280      2114     1.5           0     0
7999         3       2.25         1770      8165     2.0           0     0

      condition  grade  sqft_above  ...  yr_built  zipcode      lat     long  \
0             3      7        1020  ...      1956    98106  47.5175 -122.346
1             4      8        2210  ...      1969    98023  47.3085 -122.381
2             3      8        2350  ...      2003    98042  47.3494 -122.153
3             3      8        1660  ...      1907    98119  47.6352 -122.358
4             4     10        2410  ...      1989    98052  47.6911 -122.103
...         ...    ...         ...  ...       ...      ...      ...      ...
7995          3      7        1100  ...      1963    98003  47.3428 -122.303
7996          3      7        1870  ...      1996    98003  47.2774 -122.299
7997          3      7        1240  ...      1954    98102  47.6461 -122.317
7998          3      8        1280  ...      1904    98122  47.6174 -122.308
7999          3      7        1770  ...      1977    98034  47.7166 -122.236

      sqft_living15  sqft_lot15  day  month  year  age_since_renovated
0              1290        7320   21      5  2014                 58.0
1              1990        8000    9      6  2014                 45.0
2              3010        5772   18      2  2015                 12.0
3              1690        4000   22      5  2014                107.0
4              2310        8212    5      9  2014                 25.0
...             ...         ...  ...    ...   ...                  ...
7995           1720        7350   26      2  2015                 52.0
7996           1650        6518   26      9  2014                 18.0
7997           2080        4400   19     11  2014                 60.0
7998           1540        1456   14     10  2014                110.0
7999           1650        8165    7      5  2014                 37.0

[8000 rows x 21 columns]
Learned weights:  [ 1.80833199e-04 -2.94943590e-01  4.40041759e-01  9.84276265e-04
  1.34948677e-06  4.48302601e-02  4.01762150e+00  5.98760319e-01
  2.90711785e-01  9.51607367e-01  7.63596476e-04  2.20678812e-04
 -3.01274280e-02 -5.53837326e-03  6.00871553e+00 -2.13593882e+00
  1.94481314e-04 -3.43347452e-06 -6.96949116e-03 -2.32014597e-02
  2.44379981e-02 -3.16835550e-03]
```

In [135…

```python
# 1.2. Apply linear regression model to training data to make predictions for all examples
#       and report Mean Squared Error for training data

def mse(X, y, w):
  n_examples = X.shape[0]

  yhat = np.dot(X, w)
  error = yhat - y
  mse = np.sum(error**2) / n_examples

  return mse

non_normalized_train_mse = mse(non_normalized_X_train_biased, y_train, non_normalized_weight)
print("MSE for non_normalized_train: ", non_normalized_train_mse)
```

```
MSE for non_normalized_train:  3.7690052852155977
```

In [136…

```python
# 1.3. Apply linear regression model to the validation data to make predictions for all examples
#       and  report Mean Squared Error for validation data

non_normalized_X_valid = val_data_without_id.drop(columns=['price'])
non_normalized_X_valid_biased = np.column_stack((np.ones(non_normalized_X_valid.shape[0]),non_normalized_X_valid
y_valid = val_data_without_id['price']

non_normalized_valid_mse = mse(non_normalized_X_valid_biased, y_valid, non_normalized_weight)
print("MSE for non_normalized_valid: ", non_normalized_valid_mse)
```

```
MSE for non_normalized_valid:  4.51614482629364
```

In [137…

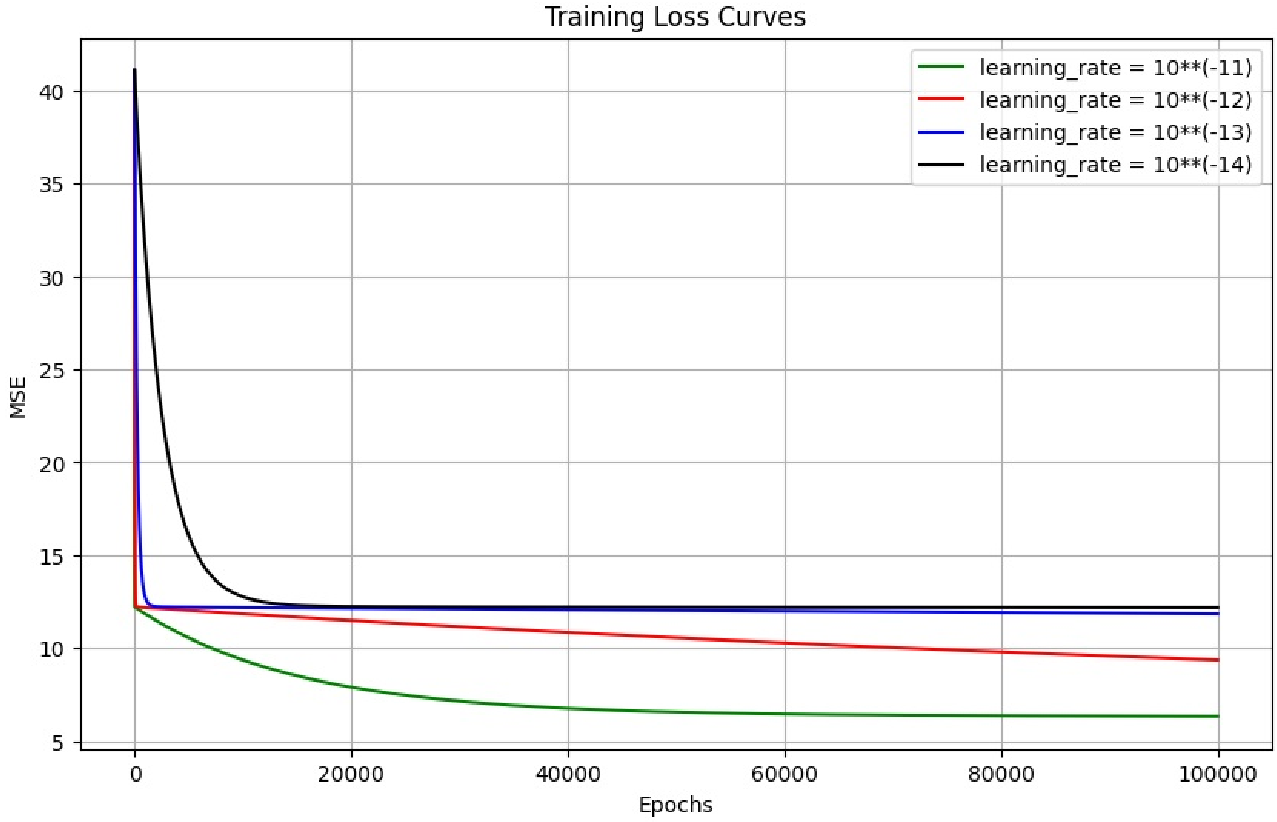
```python
# 2.1. Apply gradient descent algorithm to data that did not go through the feature normalization step using di
#       Note that the learning rate used in previous section will no longer work here.
#       You will need to search for an appropriate learning rate to get some converging behavior.

learning_rate = [10**(-i) for i in range(10, 15)]
Iterations = 100000
epsilon_l = None
epsilon_g = None

non_normalized_weight_set = []
non_normalized_history_set = []

for lr_gamma in learning_rate:
```

```
  non_normalized_w, non_normalized_mse_history = batch_gradient_descent(non_normalized_X_train_biased, y_train,
  non_normalized_weight_set.append(non_normalized_w)
  non_normalized_history_set.append(non_normalized_mse_history)

# print(non_normalized_weight_set)
# print(non_normalized_history_set)

non_normalized_valid_mse_set = []
for w in non_normalized_weight_set:
  non_normalized_valid_mse_value = mse(non_normalized_X_valid_biased, y_valid, w)
  print("MSE of the final learned weights on the non_normalized_validation data: ", non_normalized_valid_mse_va
  non_normalized_valid_mse_set.append(non_normalized_valid_mse_value)

non_normalized_epochs_set = []
for i in range(len(non_normalized_history_set)):
  non_normalized_epochs_set.append([x for x in range(len(non_normalized_history_set[i]))])
```

```
/usr/local/lib/python3.10/dist-packages/numpy/core/_methods.py:49: RuntimeWarning: overflow encountered in redu
ce
  return umr_sum(a, axis, dtype, out, keepdims, initial, where)
<ipython-input-130-fbc1593a31bf>:16: RuntimeWarning: invalid value encountered in subtract
  w = w - (lr_gamma * gradient)
MSE of the final learned weights on the non_normalized_validation data:  0.0
MSE of the final learned weights on the non_normalized_validation data:  7.66227293149035
MSE of the final learned weights on the non_normalized_validation data:  11.26703719953302
MSE of the final learned weights on the non_normalized_validation data:  13.938957608258036
MSE of the final learned weights on the non_normalized_validation data:  14.301011354506715
```

In [138_

```
# 2.2. Plot your MSE loss curve as a function of the epochs once you identify a convergent learning rate.
#       Hint: the learning rate needs to be much, much, much, much, much smaller (think about each much as an o
#             Also unless you let it run for a long time, it is unlikely to converge to the same level of loss 
#             So use a upper bound on the # of iterations so that it won't take forever.

plt.figure(figsize=(10, 6))

#plt.plot(non_normalized_epochs_set[0], non_normalized_history_set[0], label="learning_rate = 10**(-10)", color=
plt.plot(non_normalized_epochs_set[1], non_normalized_history_set[1], label="learning_rate = 10**(-11)", color=
plt.plot(non_normalized_epochs_set[2], non_normalized_history_set[2], label="learning_rate = 10**(-12)", color=
plt.plot(non_normalized_epochs_set[3], non_normalized_history_set[3], label="learning_rate = 10**(-13)", color=
plt.plot(non_normalized_epochs_set[4], non_normalized_history_set[4], label="learning_rate = 10**(-14)", color=


plt.title("Training Loss Curves")
plt.xlabel("Epochs")
plt.ylabel("MSE")

plt.legend()

plt.grid(True)
plt.show()
```



Questions

Please revisit the questions above. Does your experiment confirm your expectation? Can you provide explanations to the observed differences (or lack of differences) between the normalized data and unnormalized data? Based on these observations and your understanding of them, please comment on the benefits of normalizing the input features in learning for linear regressions.

Your answer goes here

Normalization slightly influenced on the training and validation MSE since the values of each case of MSE are very close to each other like training is about 3.5 and validation about 4. Normalized case showed faster convergence behaviors than non-normalized cases displayed within 3,000 iterations (non-normalized cases took much more time to converge). Even though the learning rates are small enough and the number of iterations of non-normalization cases are incredibly larger than normalized cases, the speed of learning shows that normalized cases are very fast. From these observations, we can guess that the benefit of the normalization of features is to provide faster learning for linear regression and proper performance than non-normalization.

3(b). (20 pts) Explore the impact of correlated features

In the warm up exercise, you all have seen some features are highly correlated with one another. For example, there are multiple squared footage related features that are strongly correlated (e.g., *sqft_above* and *sqrt_living* has a correlation coefficient of 0.878). This is referred to as multicollinearity phenomeon, where two or more features are correlated.

There are numerous consequences from multicollinearity. It makes it more challenging to estimate the weights of the features accurately. The weights may become unstable, and their interpretation becomes less clear.

In this part you will work with the pre-processed training set, and perform the following experiments **using the closed-form solution**. Specifically, your code should:

1. Create five slighly different training sets, each of which is obtained by randomly subsample 75% of the orginial training set.
2. Use the closed-form solution of linear regression to fit the model on each of the five training sets.
3. For each model, report the learned weight vector in a table. The table should have five rows (one for each model) and a column for each feature’s weight. Include a header row to clearly label the feature names for each column.
4. Compute the variance of the learned weight coefficients across the five models for each feature. This variance will serve as a measure of the **stability** of the weight assigned to each feature.

In [139…

```
# Your code goes here
```

In [140…

```
weight_vectors = []

for i in range(5):
    df = pd.DataFrame(train_data_without_id)
    # Subsample 75% of the data
    df_subsample = df.sample(frac=0.75, random_state=i)

    # Calculate the mean and standard deviation for each column
    mean_values = df_subsample.drop('price', axis=1).mean()
    std_values = df_subsample.drop('price', axis=1).std()
    # Normalize sub samples
    norm_train_sub=(df_subsample.drop('price', axis=1)-mean_values)/std_values
    norm_train_sub['price']=df_subsample['price']

    # Extract features (X) and target (y)
    x_train = norm_train_sub.drop('price', axis=1).values
    y_train = norm_train_sub['price'].values


    ones_column = np.ones((x_train.shape[0], 1))
    x_train_with_bias = np.hstack([ones_column, x_train])

    weight = np.dot(np.linalg.pinv(np.dot(np.transpose(x_train_with_bias), x_train_with_bias)), np.dot(np.trans

    weight_vectors.append(weight)


#print(weight_vectors)
weights_df = pd.DataFrame(weight_vectors, columns=['bias'] + list(df.drop('price', axis=1).columns))

print("\nWeights for each feature:\n")
weights_df
```

Weights for each feature:

Out[140]:

	bias	bedrooms	bathrooms	sqft_living	sqft_lot	floors	waterfront	view	condition	grade	...	yr_bui
0	5.356910	-0.283985	0.337227	0.820976	0.034697	-0.018337	0.363491	0.428266	0.170816	1.125794	...	-0.9478
1	5.361843	-0.235392	0.265663	0.676916	0.066376	0.062349	0.372320	0.369623	0.198685	1.242794	...	-0.9355
2	5.374270	-0.278849	0.356035	0.748614	0.062915	0.025501	0.221981	0.483853	0.176067	1.128752	...	-0.8177
3	5.352730	-0.289905	0.239916	0.725911	0.059279	0.082372	0.323469	0.449814	0.225081	1.129508	...	-0.9067
4	5.333906	-0.262785	0.346543	0.796533	0.065370	-0.028110	0.336579	0.508875	0.192850	1.047385	...	-0.8490

5 rows × 22 columns

In [141…

```
weights_variance = weights_df.var()
weights_variance
```

Out[141]:

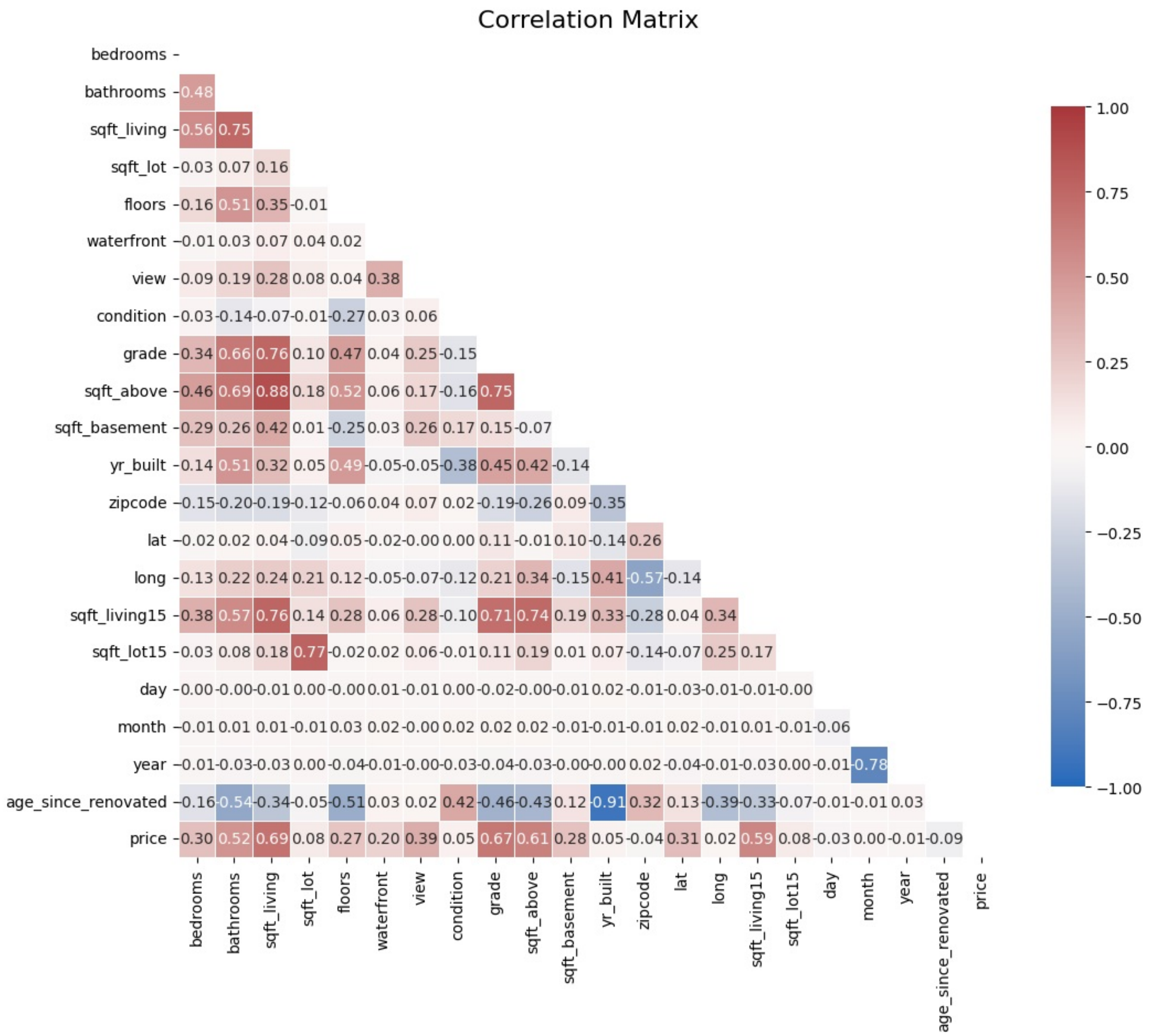
	0
bias	0.000217
bedrooms	0.000480
bathrooms	0.002767
sqft_living	0.003264
sqft_lot	0.000173
floors	0.002346
waterfront	0.003615
view	0.002882
condition	0.000460
grade	0.004862
sqft_above	0.005732
sqft_basement	0.001271
yr_built	0.003147
zipcode	0.000160
lat	0.000182
long	0.000823
sqft_living15	0.004674
sqft_lot15	0.000245
day	0.000130
month	0.000248
year	0.000490
age_since_renovated	0.003743

dtype: float64

In [142…

```
corr_matrix = norm_train.corr()

import seaborn as sns
import matplotlib.pyplot as plt
# Compute the correlation matrix
corr_matrix = norm_train.corr()

# Create a mask to hide the upper triangle (since it's symmetric)
mask = np.triu(np.ones_like(corr_matrix, dtype=bool))

# Set up the figure size
plt.figure(figsize=(12, 10))

# Draw the heatmap with annotations and mask
sns.heatmap(corr_matrix,
            mask=mask,
            annot=True,       # Display correlation values
            fmt=".2f",        # Format numbers with 2 decimal places
            cmap='vlag',      # Color map for better contrast
            vmin=-1, vmax=1, # Set color scale from -1 to 1
            cbar_kws={"shrink": .8}, # Adjust color bar size
            linewidths=0.5,   # Add spacing between cells
            square=True)      # Keep cells square for visual consistency

plt.title('Correlation Matrix', size=16)
plt.show()
```

```
print(corr_matrix.head())
```




```
             bedrooms  bathrooms  sqft_living  sqft_lot    floors  waterfront  \
bedrooms     1.000000   0.484003     0.561627  0.025321  0.164536   -0.011420
bathrooms    0.484003   1.000000     0.749852  0.071261  0.510925    0.026924
sqft_living  0.561627   0.749852     1.000000  0.164651  0.354314    0.067505
sqft_lot     0.025321   0.071261     0.164651  1.000000 -0.013066    0.035171
floors       0.164536   0.510925     0.354314 -0.013066  1.000000    0.015261

                 view  condition     grade  sqft_above  ...   zipcode  \
bedrooms     0.086933   0.027444  0.338616    0.462793  ... -0.151735
bathrooms    0.186133  -0.139804  0.663571    0.685121  ... -0.198928
sqft_living  0.281220  -0.065189  0.757528    0.878699  ... -0.193850
sqft_lot     0.078483  -0.013092  0.101992    0.176956  ... -0.124270
floors       0.035545  -0.266865  0.465624    0.521707  ... -0.059987

                  lat      long  sqft_living15  sqft_lot15       day  \
bedrooms    -0.019846  0.127421       0.382183    0.025811  0.002175
bathrooms    0.018968  0.219782       0.566942    0.082791 -0.001908
sqft_living  0.037479  0.239505       0.762189    0.178888 -0.009500
sqft_lot    -0.086827  0.208841       0.139211    0.774133  0.004721
floors       0.051138  0.122226       0.279685   -0.019085 -0.003069

                month      year  age_since_renovated     price
bedrooms    -0.009635 -0.005201            -0.156510  0.300435
bathrooms    0.005400 -0.031899            -0.536833  0.519835
sqft_living  0.009835 -0.029894            -0.335183  0.694152
sqft_lot    -0.007479  0.004510            -0.047963  0.084298
floors       0.030949 -0.040355            -0.505122  0.266400

[5 rows x 22 columns]
```

Questions

Ideally, we would like the weight coefficients to be stable across different runs, as this increases confidence in the model's reliability. Do highly correlated features tend to exhibit more instability in their weights across different training sets compared to less correlated features? Discuss any trend you observe based on the variance of the weight coefficients. How does the stability of these features relate to the multicollinearity issue present in this dataset?

Your answer goes here.

1. Do highly correlated features tend to exhibit more instability in their weights across different training sets compared to less correlated features?

- Highly correlated features often exhibit more instability in their weights across different training sets compared to less correlated features.
- This is because when features are highly correlated, the model struggles to distinguish their individual contributions to the target variable. Small changes in the training data can lead to significant shifts in how the model allocates weights among these correlated features.

- In the data, we can see high correlations between:

  - sqft_living and sqft_above (0.88)
  - bathrooms and sqft_living (0.75)
  - bathrooms and sqft_above (0.69)

  - These features are likely contributing to the instability observed in their corresponding weights.

  - For example, sqft_above has a high variance in its weight (0.005732), and this feature is strongly correlated with sqft_living and bathrooms.
- This suggests instability in how the model is distributing importance among these features.

2. Discuss any trend you observe based on the variance of the weight coefficients.

- A trend can be seen when comparing the variance of the weight coefficients with their correlations. Features that are more correlated with each other tend to exhibit higher variances in their weights. For instance:

  - sqft_above (variance: 0.005732) is highly correlated with sqft_living and bathrooms, and both these features also have relatively high weight variances (sqft_living: 0.003264, bathrooms: 0.002767).
  - On the other hand, features like zipcode, which are less correlated with other features (low correlations), have lower weight variances (variance: 0.000160).
- This suggests that multicollinearity between features likely increases the instability of the weight coefficients.

3. How does the stability of these features relate to the multicollinearity issue present in this dataset?

- The instability in the weights, especially for highly correlated features, is a direct manifestation of multicollinearity.
- When multicollinearity exists, the model cannot uniquely assign the effect of a change in the target variable to any single feature, leading to variability in the assigned weights across different training sets.
- This is why highly correlated features like sqft_living, sqft_above, and bathrooms have higher variances in their weights.

- Multicollinearity inflates the variance of the estimated coefficients, making them less stable. Regularization techniques like Ridge or Lasso regression can help mitigate this issue by imposing penalties on the size of the coefficients, reducing variance, and improving stability.

Bonus. In-class competition (5 bonus pts)

We will host a in-class competition using the IA1 data, where you are encouraged to explore different ways to improve the prediction performance by manipunating the data. This could include: feature engineering such as removing, transforming features, constructing new features based on existing ones, using different encoding for the discrete features; data manipulation such as identifying and removing potential outliers; and target manipulation such as log transforming the price target. This is where you can get creative and test your ideas out.

To participate in this competition, use the following link:
https://www.kaggle.com/t/7a885211273e48968e3a5f1b556cb685

You should continue working in the same team for this competition. The training and validation data provided on the kaggle site are the same as the IA1 assignment. To participate, you will need to train your model and apply it to testing data provided on kaggle, and submit prediction files to be scored.

Your scoring will have two parts, the performance on the public leader board as well as the private leader board. The results on the public leader board is visible through out the competition so that you can gauge how well your model is performing in comparison to others. The private leader board shows the final evaluation performance and will be released only once after the competition is closed.

Each team will be allowed to submit 3 final entries to be evaluated. You can use the public leaderboard performance to pick which models to use for your final evaluation entries.

Assginment of the bonus points:

Performance bonus: the top 3 teams on the **private** leader board will recieve 5 bonus points.

Participation bonus: the 5 teams that submitted the most entries (with different performances) will recieve 3 bonus points. Also any team that participated the competition and got non-trivial performance will receive 2 bonus points.

Bonus points are capped at 5.

Please provide the team name on the kaggle competition here _. Leave it blank if you opt not to participate.

In [144_

```
#running this code block will convert this notebook and its outputs into a pdf report.
!jupyter nbconvert --to html /content/gdrive/MyDrive/AI534/lab1/IA1-2024_Final.ipynb  # you might need to change

input_html = '/content/gdrive/MyDrive/AI534/lab1/IA1-2024_Final.html' #you might need to change this path accord
output_pdf = '/content/gdrive/MyDrive/AI534/lab1/IA1output.pdf' #you might need to change this path or name acc

# Convert HTML to PDF
pdfkit.from_file(input_html, output_pdf)

# Download the generated PDF
files.download(output_pdf)
```

```
[NbConvertApp] Converting notebook /content/gdrive/MyDrive/AI534/lab1/IA1-2024_Final.ipynb to html
[NbConvertApp] Writing 1142737 bytes to /content/gdrive/MyDrive/AI534/lab1/IA1-2024_Final.html
```

In [143_