

Arthur's El Farol bar problem [2] is a perfect example of a congestion game. In this problem each agent  $i$  decides whether to attend a bar by predicting, based on its previous experience, whether the bar will be too crowded to be "rewarding" at that time, as quantified by a system reward  $G$ . The congestion game structure means that if most agents think the attendance will be low (and therefore choose to attend), the attendance will actually be high, and vice-versa.

For this assignment, we use a modified version of the bar problem where the  $N$  agents pick one out of  $K$  nights to attend the bar every week [1]. The system reward in any particular week is:

$$G(z) \equiv \sum_{k=1}^K x_k(z) e^{\frac{-x_k(z)}{b}}, \quad (1)$$

where  $x_k(z)$  is the total attendance on night  $k$ , and  $b$  is a real-valued parameter representing the optimal number of people in the bar. The system dynamics are as follows:

Initialize: week  $\leftarrow 0$

For week  $<$  Time Limit

- Each agent chooses an action.
- Those actions lead to a system state.
- The system state leads to a system reward.
- Each agent receives a reward (i.e., agent reward) .
- week  $\leftarrow$  week + 1

**Learning Agents:** Each agent  $i$  keeps a  $K$ -dimensional vector of its estimates of the reward it would receive for taking each of those  $K$  actions (action-value learning). Each week, each agent  $i$  picks the night to attend based on sampling this vector.

**Problem 1:** Consider a simple local reward where each agent is rewarded based on the night  $k$  they chose to attend the bar:  $L(z) = x_k(z) e^{\frac{-x_k(z)}{b}}$ . Discuss the alignment and sensitivity of this reward. How do you think this reward would work?

**Problem 2:** Derive a difference reward for each agent. What is a good counterfactual  $c_i$  for this case? For at least two values of  $c_i$ , discuss the locality of the information and the alignment and sensitivity of the resulting difference rewards.

**Problem 3:** Perform a simulation for this problem with the following parameters:

- a) There are 25 agents in the system,  $b=5$ , and  $k=7$ .
- b) There are 40 agents in the system,  $b=4$ , and  $k=6$ .

Plot the performance of three agent rewards ( $G$ , difference, and local) and a histogram of sample attendance profiles. How do the rewards in Problems 1 and 2 perform in the two cases? Discuss the simulation results. The report should be in research paper format, and clearly describe your algorithms, results and analysis. Please submit your report as a PDF, and submit your code in a separate zip file.

## References

- [1] A. K. Agogino and K. Tumer, Analyzing and visualizing multiagent rewards in dynamic and stochastic environments, *Journal of Autonomous Agents and Multi Agent Systems*, 17:320–338, 2008.
- [2] W. B. Arthur. Complexity in economic theory: Inductive reasoning and bounded rationality. *The American Economic Review*, 84(2):406–411, May 1994.