(a)

$$f(n) = 12n - 5 \Rightarrow f(n) = Cn$$

$$g(n) = 1235813n + 2017 \Rightarrow g(n) = Cn$$

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{C}{C} = 1 \Rightarrow \underline{\Theta(g)}\ \#$$

(b)

$$f(n) = n\log n \quad , f'(n) = \log n + n\frac{1}{n} = \log n + 1$$

$$g(n) = 0.0\cdots1n \Rightarrow g(n) = Cn \quad , g'(n) = C$$

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{\log n}{C} = \infty \Rightarrow \underline{\Omega(g)}\ \#$$

(c) $f(n) = n^{\frac{2}{3}} \Rightarrow f(n) = n^k \Rightarrow f'(n) = kn^{k-1}$

$$g(n) = 7n^{\frac{3}{4}} + n^{\frac{1}{10}} \Rightarrow g(n) = Cn^k + n^k$$

$$= (C+1)n^k \Rightarrow g'(n) = (C+1)kn^{k-1}$$

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{kn^{k-1}}{(C+1)kn^{k-1}} = \frac{1}{C+1} \Rightarrow \underline{\Theta(g)}\ \#$$

(d)

$$f(n) = n^{1.00001} \Rightarrow f(n) = n^k \Rightarrow f'(n) = kn^{k-1}$$

$$g(n) = n\log n \Rightarrow g'(n) = \log n + 1$$

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{kn^{k-1}}{\log n} = \infty \Rightarrow \Omega(g)$$

$\#$

(e)

$$f(n) = n6^n \Rightarrow f(n) = na^n \Rightarrow f'(n) = a^n + na^n \log a$$

$$g(n) = 3^{n^2} = 3^{2n} \Rightarrow g(n) = a^{bn} \Rightarrow g'(n) = b a^{bn} \log a$$

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{na^n \log a}{ba^{bn}\log a} = \lim_{n\to\infty} \frac{na^n}{ba^{bn}}$$

$$= \lim_{n\to\infty} \frac{na^n}{b(a^n)^b} = \lim_{n\to\infty} \frac{n}{b(a^n)^{b-1}} = 0$$

$$\Rightarrow O(g)$$

$\#$

結論:

$$\lim_{n\to\infty} \frac{f'(n)}{g'(n)} \begin{cases} 0 & \Rightarrow O(g) \\ \text{constant} & \Rightarrow \Theta(g) \\ \infty & \Rightarrow \Omega(g) \end{cases}$$