Chebyshev Polynomials

In Exercise 10 of Chapter 9, the problem provides a hint that we can use the k-th order Chebyshev polynomials, denoted by $T_k(\cdot)$. The polynomial mentioned in the hint is the k-th order Chebyshev polynomial of the first kind: $T_k(x)$ is **the k-th polynomial** satisfying

$$T_k(x) = \begin{cases} \cos(k\cos^{-1}(x)), & \text{if } |x| \le 1 \\ \cosh(k\cosh^{-1}(x)), & \text{if } x > 1 \\ (-1)^k \cosh(k\cosh^{-1}(-x)), & \text{if } x < -1 \end{cases}$$

where $\cosh(x) \triangleq \dfrac{e^x + e^{-x}}{2}$. In the question, we can set P_{k-1} in the error bound such that the k-th order polynomial $1+\lambda P_{k-1}(\lambda)$ is equivalent to $T_k\left(\frac{A+a-2\lambda}{A-a}\right)/T_k\left(\frac{A+a}{A-a}\right)$; note that division by $T_k\left(\frac{A+a}{A-a}\right)$ is intended to make the constant term in the polynomial equal to 1. After this, you can use the above expression for $T_k(x)$ to find a simple upper bound of the error bound. One hint for the question is that $|\cos(k\cos^{-1}(x))| \le 1$ for $x \in [-1, 1]$.