\documentclass[11pt, a4paper]{article}

\usepackage[utf8]{inputenc}

\usepackage{amsmath,setspace,geometry}

\usepackage{amsthm}

\usepackage{amsfonts}

\usepackage[shortlabels]{enumitem}

\usepackage{rotating}

\usepackage{pdflscape}

\usepackage{graphicx}

\usepackage{bbm}

\usepackage[dvipsnames]{xcolor}

\usepackage{hyperref}

\hypersetup{colorlinks=true, linkcolor= BrickRed, citecolor = BrickRed, filecolor = BrickRed, urlcolor = BrickRed, hypertexnames = true}

\usepackage[]{natbib}

\bibpunct[:]{(}{)}{,}{a}{}{,}

\geometry{left = 1.0in,right = 1.0in,top = 1.0in,bottom = 1.0in}

\usepackage[english]{babel}

\usepackage{float}

\usepackage{caption}

\usepackage{subcaption}

\usepackage{booktabs}

\usepackage{pdfpages}

\usepackage{threeparttable}

\usepackage{lscape}

\usepackage{bm}

\setstretch{1.4}

%\usepackage[tablesfirst,nolists]{endfloat}

\newtheorem{theorem}{Theorem}

\newtheorem{assumption}{Assumption}

\newtheorem{lemma}{Lemma}

\newtheorem{definition}{Definition}

\newtheorem{proposition}{Proposition}

\newtheorem{claim}{Claim}

\newtheorem{corollary}{Corollary}

\newtheorem{example}{Example}

\DeclareMathOperator{\rank}{rank}

\title{Resolving the Conflict on Conduct Parameter Estimation in Homogeneous Goods Markets between Bresnahan (1982) and Perloff and Shen (2012)}

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%Declarations of interest: none %this is for Economics Letters

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\begin{document}

\maketitle

\begin{abstract}

We revisit conduct parameter estimation in homogeneous goods markets to resolve the conflict between Bresnahan (1982) and Perloff and Shen (2012) regarding the identification and the accuracy of conduct parameter estimation. We point out that Perloff and Shen's (2012) proof is incorrect and its simulation setting is invalid. Our simulation shows that estimation becomes accurate when demand shifters are properly added in the supply estimation and the sample size is increased,

supporting Bresnahan (1982).

\vspace{0.1in}

\noindent\textbf{Keywords:} Conduct parameters, Homogenous goods market, Multicollinearity problem, Monte Carlo simulation

\vspace{0in}

\newline

\noindent\textbf{JEL Codes:} C5, C13, L1

\bigskip

\end{abstract}

\section{Introduction}

Measuring competitiveness is one of the important tasks in the empirical industrial organization literature.

A conduct parameter is considered to be a useful measure of competitiveness.

However, it cannot be directly measured from the data because data generally lack information about marginal costs.

Therefore, researchers endeavor to identify and estimate conduct parameters.

In this regard, there are two conflicting results regarding conduct parameter estimation in homogeneous goods markets in linear demand and marginal cost systems.

On the one hand, \citet{bresnahan1982oligopoly} proposes an approach to identify a conduct parameter using the demand rotation instrument.

With identification guaranteed, the conduct parameter can be estimated using standard linear regression.

This result is extended to nonlinear cases by \citet{lau1982identifying} and differentiated product markets by \citet{nevoIdentificationOligopolySolution1998}.

On the other hand, \citet{perloff2012collinearity} (hereafter, PS) asserted that the linear model considered by \citet{bresnahan1982oligopoly} suffers from the multicollinearity problem when the error terms in the demand and supply equations are zero, implying that the identification of the conduct parameter is impossible for this condition.

Moreover, PS used simulations to demonstrate that the conduct parameter cannot be estimated accurately even when the error terms are nonzero.

This disagreement is a major obstacle in the literature.

Several papers and handbook chapters have referenced PS’ results, such as \citet{claessensWhatDrivesBank2004, coccoreseMultimarketContactCompetition2013, coccoreseWhatAffectsBank2021, garciaMarketStructuresProduction2020, kumbhakarNewMethodEstimating2012, perekhozhukRegionalLevelAnalysisOligopsony2015}, and \citet{shafferMarketPowerCompetition2017}.

We revisit conduct parameter identification and estimation in homogeneous product markets to determine the validity of these results.

First, we show that the proof of the multicollinearity problem in PS is incorrect and that the problem does not occur under standard assumptions reflecting the insights by \citet{bresnahan1982oligopoly}.

Second, the simulations in PS lack an excluded demand shifter in the supply equation estimation; we confirm that the accuracy of estimation holds by including a demand shifter in the supply estimation.

We also show that increasing the sample size improves the accuracy of estimation.

Hence, our results support those of \cite{bresnahan1982oligopoly} theoretically and numerically.

\section{Model}

Consider data with $T$ markets with homogeneous products.

Assume that there are $N$ firms in each market.

Let $t = 1,\ldots, T$ be the index of the markets.

Then, we obtain the supply equation as follows:

\begin{align}

P\_t = -\theta\frac{\partial P\_t(Q\_{t})}{\partial Q\_{t}}Q\_{t} + MC\_t(Q\_{t}),\label{eq:supply\_equation}

\end{align}

where $Q\_{t}$ is the aggregate quantity, $P\_t(Q\_{t})$ is the demand function, $MC\_{t}(Q\_{t})$ is the marginal cost function, and $\theta\in[0,1]$, which is called the conduct parameter.

The equation nests perfect competition, $\theta=0$, Cournot competition, $\theta=1/N, N$ firm symmetric perfect collusion, $\theta=1$, etc.\footnote{See \cite{bresnahan1982oligopoly}.}

Consider an econometric model that integrates the above model.

Assume that the demand and marginal cost functions are written as follows:

\begin{align}

P\_t = f(Q\_{t}, Y\_t, \varepsilon^{d}\_{t}, \alpha), \label{eq:demand}\\

MC\_t = g(Q\_{t}, W\_{t}, \varepsilon^{c}\_{t}, \gamma),\label{eq:marginal\_cost}

\end{align}

where $Y\_t$ and $W\_{t}$ are the vector of exogenous variables, $\varepsilon^{d}\_{t}$ and $\varepsilon^{c}\_{t}$ are error terms, and $\alpha$ and $\gamma$ are the vector of the parameters.

Additionally, we have the demand- and supply-side instrument variables, $Z^{d}\_{t}$ and $Z^{c}\_{t}$, and assume that the error terms satisfy the mean independence condition, $E[\varepsilon^{d}\_{t}\mid Y\_t, Z^{d}\_{t}] = E[\varepsilon^{c}\_{t} \mid W\_{t}, Z^{c}\_{t}] =0$.

\subsection{Linear demand and cost}

Assume that linear demand and marginal cost functions are specified as follows:

\begin{align}

P\_t &= \alpha\_0 - (\alpha\_1 + \alpha\_2Z^{R}\_{t})Q\_{t} + \alpha\_3 Y\_t + \varepsilon^{d}\_{t},\label{eq:linear\_demand}\\

MC\_t &= \gamma\_0 + \gamma\_1 Q\_{t} + \gamma\_2 W\_{t} + \gamma\_3 R\_{t} + \varepsilon^{c}\_{t},\label{eq:linear\_marginal\_cost}

\end{align}

where $W\_{t}$ and $R\_{t}$ are excluded cost shifters and $Z^{R}\_{t}$ is Bresnahan's demand rotation instrument.

The supply equation is written as follows:

\begin{align}

P\_t

%&= \gamma\_0 + [\theta(\alpha\_1 + \alpha\_2Z^{R}\_{t})+ \gamma\_1] Q\_{t} + \gamma\_2 W\_{t} + \gamma\_3 R\_{t} + \varepsilon^{c}\_{t}\nonumber\\

&= \gamma\_0 + \theta \alpha\_2 Z^{R}\_tQ\_{t} + (\theta\alpha\_1 + \gamma\_1) Q\_{t} + \gamma\_2 W\_t + \gamma\_3 R\_{t} +\varepsilon^c\_t.\label{eq:linear\_supply\_equation}

\end{align}

By substituting Equation \eqref{eq:linear\_demand} with Equation \eqref{eq:linear\_supply\_equation} and solving it for $P\_t$, we obtain the aggregate quantity $Q\_{t}$ based on the parameters and exogenous variables as follows:

\begin{align}

Q\_{t} = \frac{\alpha\_0 + \alpha\_3 Y\_t - \gamma\_0 - \gamma\_2 W\_{t} - \gamma\_3 R\_{t} + \varepsilon^{d}\_{t} - \varepsilon^{c}\_{t}}{(1 + \theta) (\alpha\_1 + \alpha\_2 Z^{R}\_{t}) + \gamma\_1}.\label{eq:quantity\_linear}

\end{align}

\subsection{Is the multicollinearity problem in PS incorrect?}

To demonstrate the multicollinearity problem, PS attempt to demonstrate linear dependence between the variables in the supply equations.

PS begin the proof on page 137 in their appendix by stating the following (we modify the notations):

\begin{quote}

``We demonstrate that the $W\_{t}, R\_{t}, Z^{R}\_{t}Q\_{t}$, and $Q\_{t}$ terms in Eq.4 are perfectly collinear for $\varepsilon\_{t}^{d} = \varepsilon\_{t}^{c} = 0$. We show this result by demonstrating that there exist nonzero coefficients $\chi\_1,\chi\_2,\chi\_3,\chi\_4$, and $\chi\_5$ such that

\begin{align\*}

Z^{R}\_{t} Q\_{t} + \chi\_1 Q\_{t} + \chi\_2 W\_{t} + \chi\_3 R\_{t} + \chi\_4 Y\_{t} + \chi\_5 = 0 \quad (\text{A1})."

\end{align\*}

\end{quote}

Eq.4 in the quotation corresponds to the supply equation \eqref{eq:linear\_supply\_equation}.

Therefore, PS show that there exists a nonzero vector of $\chi\_1, \ldots, \chi\_5$ that satisfies (A1).

An incorrect detail in the proof is that while attempting to demonstrate linear dependence between $Z^{R}\_{t}Q\_{t}, Q\_{t}, W\_{t}$, and $R\_{t}$, they show linear dependence between $W\_{t}, R\_{t}, Z^{R}\_{t}Q\_{t}, Q\_{t}$, and $Y\_t$.

However, linear dependence between $W\_{t}, R\_{t}, Z^{R}\_{t}Q\_{t}, Q\_{t}$, and $Y\_t$ does not always imply the linear dependence between $Z^{R}\_{t}Q\_{t}, Q\_{t}, W\_{t}$, and $R\_{t}$.

Therefore, we contend that the multicollinearity problem does not occur under the additional standard assumptions in Proposition 1.

\begin{proposition}

Assume that (i) $\alpha\_2$ and $\alpha\_3$ are nonzero and (ii) $Z^R\_t, W\_t, R\_t$, and $Y\_t$ are linearly independent.

Then, $Z^{R}\_{t}Q\_{t}, Q\_{t}, W\_{t}$, and $R\_{t}$ are linearly independent.

\end{proposition}

See Appendix \ref{sec:appendix} for the proof.

Equation \eqref{eq:linear\_supply\_equation} implies that the main challenge is separately identifying the conduct parameter and the slope of marginal cost.

As quantity is endogenous, this requires two excluded instruments.

Assumption (i) makes the demand rotation instrument and the demand shifter relevant and Assumption (ii) ensures that these instruments and the other cost shifters do not covary.

Under these assumptions, identification of the conduct parameter is possible.

In the context of differentiated products markets, \cite{magnolfi2022falsifying} discuss similar issues concerning instrument requirements for falsifying models with upward sloping marginal cost.

They build on the results of \cite{berry2014identification}, who show that with instruments, falsification of models of conduct with flexible cost functions is possible.

\section{Simulation results}\label{sec:results}

Table \ref{tb:linear\_linear\_sigma\_1} presents the results of the linear model with the demand shifter.\footnote{See Appendix \ref{sec:appendix} for simulation details and additional results.}

Panel (a) shows that when the standard deviation of the error terms in the demand and supply equations is $\sigma = 0.001$, estimation of all parameters is extremely accurate.

When sample size is large, the root-mean-squared error (RMSE) of all parameters are less than or equal to 0.001.

Panel (c) shows the case with $\sigma = 2.0$.

As sample size increases, the RMSE sharply decreases.

Thus, the imprecise results reported by PS are due to the lack of demand shifters and the small sample size.

\begin{table}[!htbp]

\begin{center}

\caption{Results of the linear model with demand shifter}

\label{tb:linear\_linear\_sigma\_1}

\subfloat[$\sigma=0.001$]{\input{figuretable/linear\_linear\_sigma\_0.001\_bias\_rmse.tex}}\\

\subfloat[$\sigma=0.5$]{\input{figuretable/linear\_linear\_sigma\_0.5\_bias\_rmse}}\\

\subfloat[$\sigma=2.0$]{\input{figuretable/linear\_linear\_sigma\_2\_bias\_rmse}}

\end{center}

\footnotesize

Note: The error terms in the demand and supply equation are drawn from a normal distribution, $N(0,\sigma)$.

\end{table}

\section{Conclusion}

We revisit the conduct parameter estimation in homogeneous goods markets.

There is a conflict between \citet{bresnahan1982oligopoly} and \citet{perloff2012collinearity} in terms of identification and estimation.

We highlight the problems in the proof and simulation in \citet{perloff2012collinearity}.

Our simulation shows that estimation of the conduct parameter becomes accurate when demand shifters are appropriately introduced in the supply estimation and the sample size is increased.

Based on our theoretical and numerical investigation, we support the argument made by \citet{bresnahan1982oligopoly}.

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\appendix

\end{document}