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Development of a multibody model for go-karts considering frame flexibility

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ABSTRACT

This study focuses on the optimization dynamics of racing go-karts, which is heavily influenced by the frame's stiffness. Lacking suspensions and differentials, go-karts rely on the frame stiffness for wheel balancing and skid prevention by lifting the inner rear wheel during turns. Utilizing a rigid-flexible model in MSC Software ADAMS View, validated by frame deformation measurements, this research integrates finite element analysis with multibody techniques. The model, leverages computer aided design files for frame geometry and employs finite element analysis for frame validation. It facilitates evaluating go-kart dynamics through simulations, aiding in maneuver testing and design optimization. This approach provides a comprehensive framework for advancing go-kart designs.

KEYWORDS

go-kart dynamics, frame stiffness, MSC software ADAMS, multibody simulation

1. INTRODUCTION

Racing go-karts are a special type of vehicle that presents unique challenges to driving dynamics research. The lack of a differential, the wheels rigidly attached to the rear axle, and the absence of shock absorption all give the go-kart unique dynamics. Automotive engineers try to use various methods and approaches to gain insight into the dynamic behavior of go-karts.

Go-karts are often called "three-wheeled vehicles" even though they have four wheels. The reason for this is the turn, when the inner rear wheel of the go-kart must be lifted to prevent the tires from slipping, as the wheels travel on different arc length paths. Due to the elasticity of the frame, the inner rear wheel can be raised when cornering to accommodate different wheel paths and prevent the tires from slipping. This effect is called the wheel lift technique and is a pivotal point in the dynamics of go-karts.

Due to the classified nature of the knowledge of the racing teams, the literature on the set-up and dynamic performance simulation of go-karts is limited. Typically, go-kart mechanics use settings based on experience. Research into the torsional stiffness of the frame is crucial from a vehicle dynamics point of view due to the wheel lift phenomenon. Due to these factors, a suitable simulation methodology is needed [1, 2].

Several literature reviews focus on structural aspects of go-kart frames, e.g., studies focusing on the effect of torsional stiffness [3, 4], while others have investigated vehicle behavior by performing experimental tests or using commercial MultiBody Dynamics (MBD) software [5, 6]. Many papers have also investigated the effect of frame stiffness during cornering on load transfer, which directly affects the down-force on the tires, and thus friction and tire grip [7, 8].

Simulation models for predicting lateral forces of go-karts need to consider the specific parameters and settings of the go-kart, including the effects of frame stiffness [9–16]. Based on the literature, it can be said that the application of flexible MBD simulation is essential for complete vehicle dynamics analysis. The present study aims to confirm this fact.

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MBD simulations and their experimental validation are complex tasks. Some researchers, e.g., Mirone [4], modeled a go-kart with a flexible frame and carried out experimental validations, highlighting the importance of frame stiffness in dynamic performance. Gonçalves [17] researched the comfort and handling optimization of body models by modifying the parameters of the suspension system. Sampayo [18] investigates a coupled simulation methodology in the design of the kart frame, emphasizing the role of frame stiffness due to the lack of a suspension system. The present study also aims to have a validated simulation model.

2. RESEARCH QUESTIONS AND METHODOLOGY

2.1. Research questions

The research examines the correlations between the dynamic performance of the go-kart and the frame's flexibility, with a specific focus on an Italian racing go-kart frame. Additionally, a goal was set to compare the dynamic differences between rigid and flexible MBD models.

The research questions and hypotheses articulated in this article focus on understanding the impact of the stiffness of a go-kart's frame on vehicle dynamics [19]. The investigation addresses the following queries:

- How does the stiffness of a go-kart's frame influence its dynamic behavior?
- How can frame stiffness be efficiently modeled in MBD simulations?
- How can Finite Element (FE) simulations be efficiently integrated with MBD to create a coupled flexible MBD simulation?
- The fundamental hypothesis of this research is that considering the flexibility of the frame in MBD simulation is indispensable for achieving accurate results.

To achieve these objectives, a complete vehicle model is presented using the MSC Software ADAMS View [20], commercial MBD simulation software.

2.2. Methodological approach and workflow

In the model development process, the flexibility of the frame was considered. The mode shapes [11] and necessary mechanical data were extracted from a FE model. The model was used to simulate special maneuvers, allowing the observation of the effects of flexibility on dynamic performance. The workflow consists of the following steps:

1. Creation of the Computer Aided Design (CAD) model of the go-kart frame;
2. Determination of the stiffness and modal properties of the go-kart frame using FE software;
3. Performing dynamic and static simulations, including numerical modal analysis and numerical torsional stiffness analysis;
4. Experimental validation: the experimental and simulated mode shapes and frequencies of the trimmed frame were matched;
5. Exporting a modal neutral file from the validated FE model to the rigid MBD model;
6. Development of the flexible MBD model and comparison of flexible and rigid simulations;
7. Drawing conclusions.

These steps allow an understanding of the dynamic behavior resulting from the torsional stiffness of the frame.

3. MODELING AND VALIDATION

This section covers static and dynamic analyses of the go-kart frame, merging numerical and experimental methods. Dynamic simulations, including modal analysis, are experimentally validated and compared with the literature. Post-validation, a modal file from the FE model, detailing frame elasticity, is used for MBD simulation. This leads to a flexible MBD model in ADAMS View [21]. An advanced FE model, developed using ANSYS 2022/R2 Workbench and ANSA/NASTRAN, is validated through Széchenyi István University's laboratory tests.

3.1. Modeling: numerical modal analysis

Numerical modal analysis of the frame structure, including identifying natural frequencies and mode shapes, is essential for understanding its dynamic behavior and vibration characteristics [22–25]. These results allow the conversion of a rigid body model into a flexible MBD model. The compatibility between MSC Software NASTRAN [26] and MSC ADAMS enhances simulation input for flexible bodies and facilitates data transfer across software programs.

The CAD model of the go-kart frame for Homologation 33/CH/20, Model RY 29 S7, was created in CREO software based on the homologation document [27] provided by the Italian manufacturer BirelART and the actual frame dimensions. This model was then imported into the FE software. During the FE analysis, a total of 54,839 elements were created on the frame structure, which was simple shell elements with 109,555 nodes, using an automatic meshing process with 0.01 m surface triangle mesh elements. The material was AISI 4130, a chromium-molybdenum alloy steel. A free-free boundary condition was assigned to the model, allowing every point of the structure to move freely without any constraints or external forces. This condition enabled the internal forces of the structure to react freely, thus allowing the simulation to model the deformation and vibrations of the structure. Only gravity was considered, with no other loads applied, and damping factors were also neglected.

The settings for the modal analysis were defined, including the frequency range 20–200 Hz. In addition to the six rigid body modes, the analysis discovered seventeen natural modes within the 20–200 Hz frequency range. Figure 1 shows the first two mode shapes of the frame, the

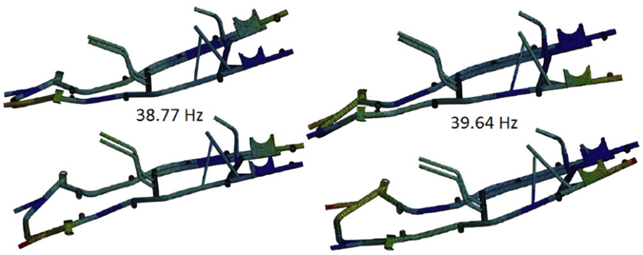

Fig. 1. First two modes of the frame: the two oscillation apexes for each mode

first twisting mode at 38.77 Hz and the first bending mode at 39.64 Hz.

The results in the 0–200 Hz range are typical for go-kart frames. The increase of natural frequencies by improving the frame stiffness is practically not possible. Stiffness adjustments are limited by other requirements related to driving dynamics [13].

3.2. Modeling: torsional stiffness analysis

To determine the torsional stiffness of the go-kart frame, the structure was loaded with a static twisting torque [19] both in experiment and simulation. The deformation of the frame was measured from which the torsional stiffness was calculated. In the static linear FE analysis, the determination of torsional stiffness considers the structure's geometry, material, loading conditions, and boundary conditions. For the torsional stiffness simulation, fixed supports and torques are set, with applied forces of $F_{left} = F_{right} = F = 250$ N, as it is show in Fig. 2. The maximum stresses were far below the yield stress value.

Torsional stiffness is a property that determines how much a material resists torsional loading. The calculation of torsional stiffness, based on displacements and loads, involves determining the torsional moment T. This distance b refers to the span between the supported point and the location of the applied forces:

$$T = 2 \cdot F \cdot b = 500 \text{ N} \cdot 0.685 \text{ m} = 342.5 \text{ Nm}. \qquad (1)$$

The torsional angle θ of twist was calculated from the displacement D of the exertion point of the force F_{left} or F_{right} under torsional loading. The angle θ will be the rotation relative to the original position of the material when torsional load is applied, i.e., the magnitude of the deformation. The torsional angle θ can be calculated as

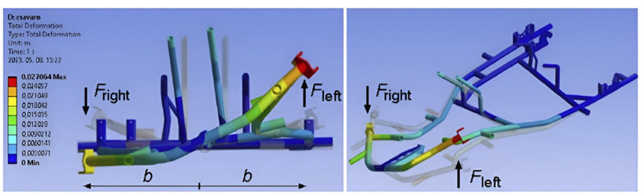

Fig. 2. FE simulation of torsional stiffness test

$$\theta = \tan^{-1}\left(\frac{\mathrm{D}}{\mathrm{b}}\right) = \tan^{-1}\left(\frac{0.0307915 \text{ m}}{0.685 \text{ m}}\right) = 2.573 \text{ deg}. \quad (2)$$

To calculate torsional stiffness K, the following formula can be used

$$K = \frac{T}{\theta} = \frac{342.5 \text{ Nm}}{2.573 \text{ deg}} = 133.11 \text{ Nm/deg}. \qquad (3)$$

3.3. Validation: experimental modal analysis

Experimental Modal Analysis (EMA) tests were carried out in a controlled laboratory environment. This study utilizes a trimmed go-kart frame, previously modeled in simulations, for the experiment. Vibration measurements are performed using an accelerometer and analyzed with Data AcQuisition (DAQ) software, employing a DAQ process through LMS Software SCADAS Mobile Frame (LMS Testlab) software.

Various input excitations and response outputs are crucial for calculating the Frequency Response Function (FRF) [24, 25] during measurements. The FRF plays a key role in modal analysis. FRF provides insight into the dynamic behavior of the system in terms of frequency, and helps to identify vibration modes and their frequencies. During the experiment, the inputs can be quantified, for example, in torque (Nm), and the outputs in torsional displacement (degrees).

The frame of the go-kart was excited with an impulse hammer. The force inputs were measured, i.e., input forces were sensed by the impedance head on the hammer and transmitted to the DAQ system. The response to the impulse on the examined frame structure was measured with accelerometers. The experimental setup assumes free-free boundary conditions, enabling a "floating" state for the test object, thereby excluding external influences. This condition was readily achievable in computer modeling, and was replicated in physical modal tests using elastic rubber bands, facilitating the identification of the first six rigid-body modes due to their low frequency. The setup includes an impulse hammer, a PCB 356a45 triaxial accelerometer, and a signal amplifier-data acquisition and analysis software. During the EMA test, both input and output signals are measured, using a PCB accelerometer for data collection. The LMS SCADAS Mobile Frame data processor and LMS Testlab software are employed for data post-processing. The experimental setup is shown in Fig. 3.

The measurement procedure involves the excitation of the go-kart frame at predetermined points with the impulse hammer to ensure adequate energy transfer. Sensor positions are selected considering anticipated vibration modes and desired response functions, with measurements averaged to account for random noise [26–31]. The accelerometer, a movable three-directional instrument, is strategically placed based on FE simulations and relocated after each excitation. The recorded FRFs are averaged after multiple excitations it can be seen in Fig. 4.

During the test, the quality of FRFs is assessed, evaluating the stability of the relationship between excitation and

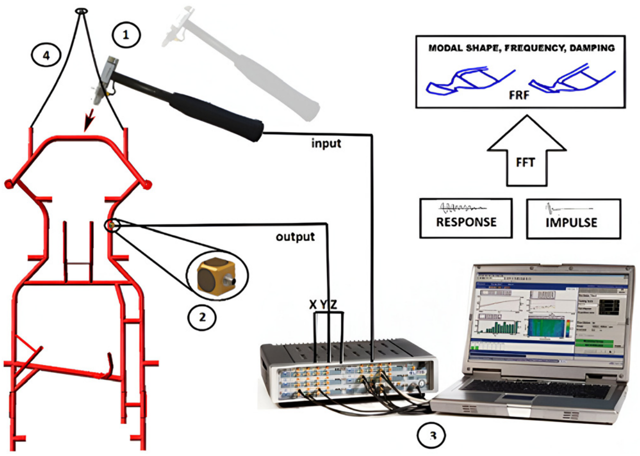

Fig. 3. Modal analysis experimental setup

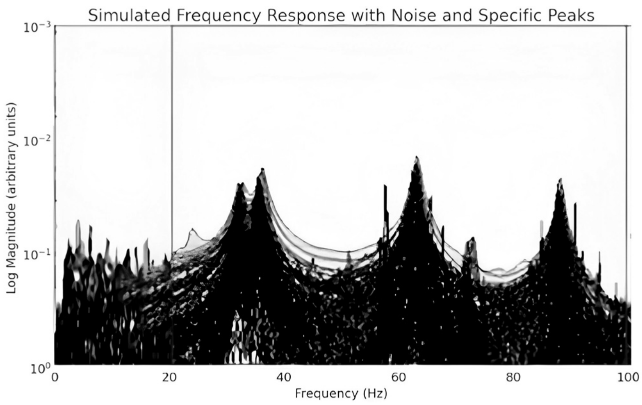

Fig. 4. FRF of the „trimmed" frame

response functions, with coherence values indicating the degree of correlation. In the experimental modal analysis, ten Eigenmodes were extracted in the 0–200 Hz range, compared with simulation results for validating the numerical model [29], see Table 1. Notably, the identified 38.5 Hz torsional and 40.4 Hz bending modes showed very close matches with simulation results. Some modes obtained in FE software were not identified in the experimental analysis. The modal parameters obtained with the Finite Element Analysis (FEA) method are compared with the results from the EMA FRF diagram. The EMA and numerical results are presented together in Table 1. Distinct peaks are observable in the frequency range of up to 200 Hz. The comparison of frequencies and visual modes indicates good agreement between the two techniques. The maximum absolute deviation is 2.34%, while the average deviation is −0.94%. Up to the 100 Hz frequency range, several peaks are observable, with three initial peaks representing three critical modes as indicated by the resonance frequency.

3.4. Validation: equivalent torsional stiffness

Equivalent Torsional Stiffness (ETS) is a dynamic stiffness test [30]. The ETS method is suitable for determining the torsional stiffness of vehicle structures. The method involves subjecting the structure under test to excitation forces of different frequencies and then determining the stiffness based on the response. During the measurement process, the frame structure is excited by forces at specific points and the output acceleration is measured. The relationship between input force and output acceleration can also be described by transfer functions. The angle of deformation φ of the frame structure was measured. The deformation angles at the front and rear excitation points are summed to determine the total deformation angle.

The dynamic torsional stiffness k is calculated as the ratio of the moment T to the total angle of deformation φ. The dynamic torsional stiffness curve obtained in MATLAB is then extrapolated to 0 Hz to determine the torsional stiffness. This value reflects the stiffness of the structure under torsional loading. The mathematical background for the ETS measurement is provided by the relationship between input forces and output accelerations, deformation angles, dynamic torsional stiffness, and torsional stiffness.

The ETS measurement is required as a first step to clean the frame and check for structural damage and cracks. The frame should also be secured with a rubber band. To measure the displacements and applied force, accelerometers and force sensors should be attached to the frame, as it is shown in Fig. 5.

The sensors, i.e., the measuring points, were placed in critical places, for example near the wheel connection points. Excitation was done with electrodynamic shakers near the measurement points. Elastic deformation is induced by periodic signals within a predetermined frequency range of

Table 1. EMA vs. FEA natural frequencies and their % deviation

Nr.	EMA (Hz)	FEA (Hz)	Abs. Dev. (%)	Nr.	EMA (Hz)	FEA (Hz)	Abs. Dev. (%)
1.	38.5	38.77	0.70%	10.	–	136.40	–
2.	40.4	39.64	1.88%	11.	148.8	147.92	0.59%
3.	–	62.18	–	12.	–	155.49	–
4.	72.2	71.13	1.48%	13.	157.8	158.09	0.18%
5.	80.4	79.25	1.43%	14.	–	174.03	–
6.	93.5	93.11	0.42%	15.	–	184.12	–
7.	119.7	119.62	0.07%	16.	–	185.81	–
8.	–	122.67	–	17.	191.9	196.40	2.34%
9.	133.5	133.06	0.33%				

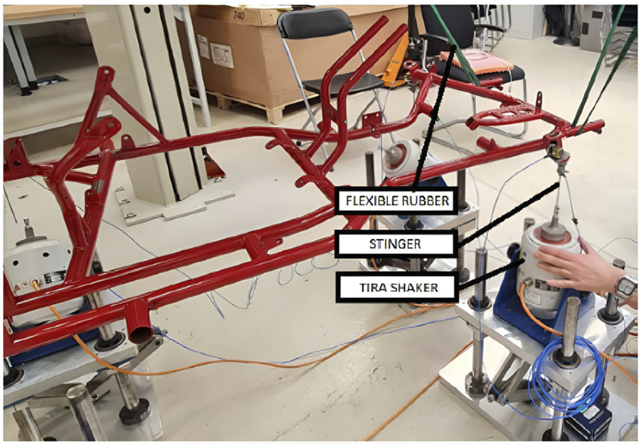

Fig. 5. Performing an equivalent torsional stiffness test

5–100 Hz, allowing the response to be measured at different frequencies. The measurement test setup is shown in Fig. 6.

During the measurement, the data collected by the sensors was analyzed using a MATLAB script. The extrapolated torsional stiffness of approximately 120 (Nm/deg) matches the magnitude of the FE simulation result of 133 (Nm/deg). The slight deviation may come from factors like manufacturing inaccuracies of the frame, welding seams, measurement inaccuracies, or the simulation parameters.

3.5. Flexible MBD model of go-kart

The detailed topology of the go-kart flexible MBD model developed during the research work is shown in Fig. 7. The topology of the vehicle model includes different subsystems and their connections.

The model consists of various sub-assemblies including the front and rear wheels, the Ackermann [31] steering system, and the frame structure connected to the rear axle. The detailed topology model works like a map, clearly detailing the relationships between vehicle components, including joints and constraints.

Special emphasis must be placed on constraints and the definition of connections. For example, the rear axle, attached to the frame with ball bearings, is emulated as a revolute joint in the model. Additionally, the rear wheels are rigidly fixed to the rear axle, ensuring they rotate together (Fig. 8).

Regarding the real behavior of the model, another key feature is the incorporation of elastic properties obtained from FE simulations. Figure 9 shows as a result, the frame appears as a flexible part in the simulation [9].

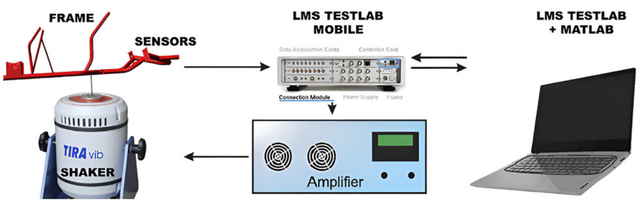

Fig. 6. Equivalent torsional stiffness test setup

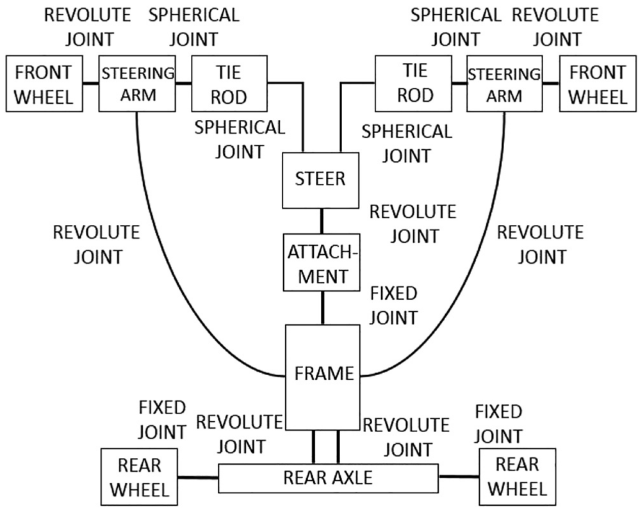

Fig. 7. Topology of the go-kart

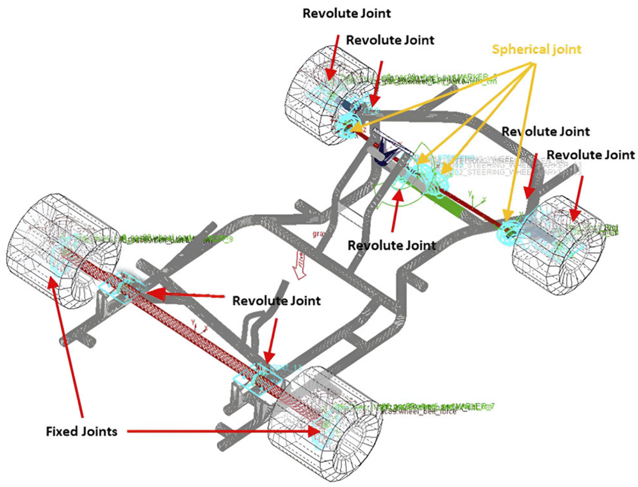

Fig. 8. Joints in ADAMS View environment

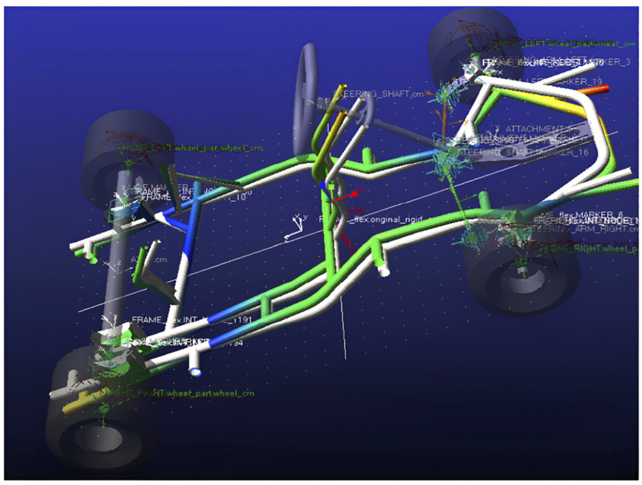

Fig. 9. Flexible go-kart frame in Adams View

All components were rigid except for the frame and tires. The rear axle was modeled as a rigid body. The rear axle is equipped with revolute joints to facilitate movement. The driver's weight was included, approximately 70 kg, bringing the total mass of the MBD model to around 140 kg.

In terms of modeling the front portion, especially the steering mechanism, a simplified approach was adopted, using geometrical dimensions from an existing CAD model. Tire modeling employed the PAC89 model from ADAMS tires module, a Pacejka Magic Formula [32] model that requires relatively few parameters.

The frame was connected and restricted in its movement by 14 joints, which imposed 84 constraints. The detailed connection topology is illustrated in Fig. 7. The model is adept at simulating vehicle handling in dynamic maneuvers.

4. SIMULATION

The primary objective is to thoroughly examine and evaluate the dynamic characteristics of the developed model, with a particular emphasis on comparing its performance to that of a model with a rigid frame. The simulation process begins with the crucial step of setting and configuring the model parameters [5]. This is followed by defining the specific maneuver [33] and the conditions under which the simulation will commence. Throughout the simulation, the focus is on meticulously observing the vehicle's movement and assessing the impact of frame flexibility. The simulation encompasses an extensive examination of various key factors like the vehicle movement, wheel traction, suspension dynamics, and, importantly, the impact of frame flexibility.

Among the specific tests conducted is the drop test simulation, where the flexible frame go-kart is "dropped" from above the road with its steering fixed in a stationary position. This particular test serves to highlight the damping effects resulting from the flexibility of the frame. The damping of the flexible frame is clearly visible in Fig. 10.

Another critical test is the Step Steer Maneuver, executed at a constant speed in a circular path, which allows for the observation of trajectory changes attributable to variations in frame stiffness. During the maneuver, the vehicle is driven at a fixed speed in a straight line for a few seconds to reach a steady state. Once this state is achieved, the vehicle is steered from zero to a predetermined value, typically 15°, and then the steering wheel is held at this angle for the remainder of the simulation. During this event, tire forces are measured.

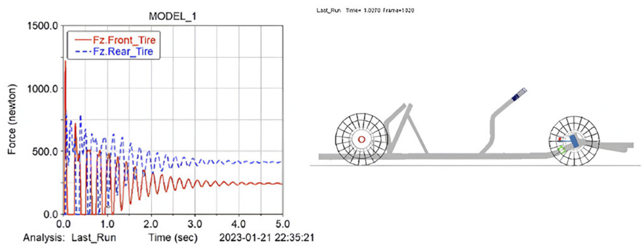

Fig. 10. Fz (N) forces acting on the wheel

In one of the scenarios, the vehicle, traveling initially at 30 km h^{-1}, is quickly steered to a 15-degree angle between 1 and 2 s and then maintains this steering angle until the simulation concludes; with the vehicle's speed gradually decreasing to a halt (see Fig. 11).

4.1. The simulation results

The stiffer the frame, the greater the yaw angle and lateral acceleration. The graphs demonstrate the impact of frame flexibility. During the maneuver, the vertical force on the rear inner tire decreases, while the force on the rear outer tire increases. With a rigid frame, the rear inner tire experiences a greater load compared to a scenario with a flexible frame (see Fig. 12).

The lateral accelerations in both rigid and flexible frame scenarios indicate that a stiffer frame leads to higher lateral acceleration.

The simulation of constant-speed cornering reveals that frame stiffness significantly influences vehicle behavior and handling. By examining critical vehicle dynamics parameters like tire vertical forces, yaw rate, and lateral acceleration.

5. DISCUSSION

A validated go-kart model with a flexible frame was created. The simulation and experimental results showed a good correlation between static stiffness test cases and modal parameters. However, some Eigen-frequencies were not captured by the measurement, which could be improved by increasing the number of sensors and excitation points. The results were obtained similar to several existing studies. Any discrepancies between simulation and experiment are easily explainable by manufacturing inaccuracies of the frame, for example welding, cutting, and assembly errors. The FE simulation and its validation can be considered successful.

The simulation results confirmed the results of the literature research, i.e., the stiffness of the frame significantly affect the dynamic behavior of the vehicle. The stiffer frame provides a more direct response to steering movements and enables greater lateral acceleration.

However, as the frame stiffness decreases, the vehicle reacts more and more in an undesirable way. This clearly shows the importance of the distribution of stiffness along the frame's longitudinal axis.

Furthermore, the study showed that the overall stiffness between the front and rear tires is more important than the

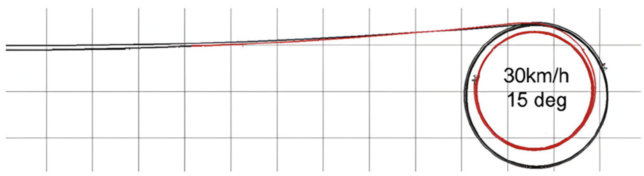

Fig. 11. The effect of the rigidity of the frame on the trajectory: rigid frame with red, flexible frame with black

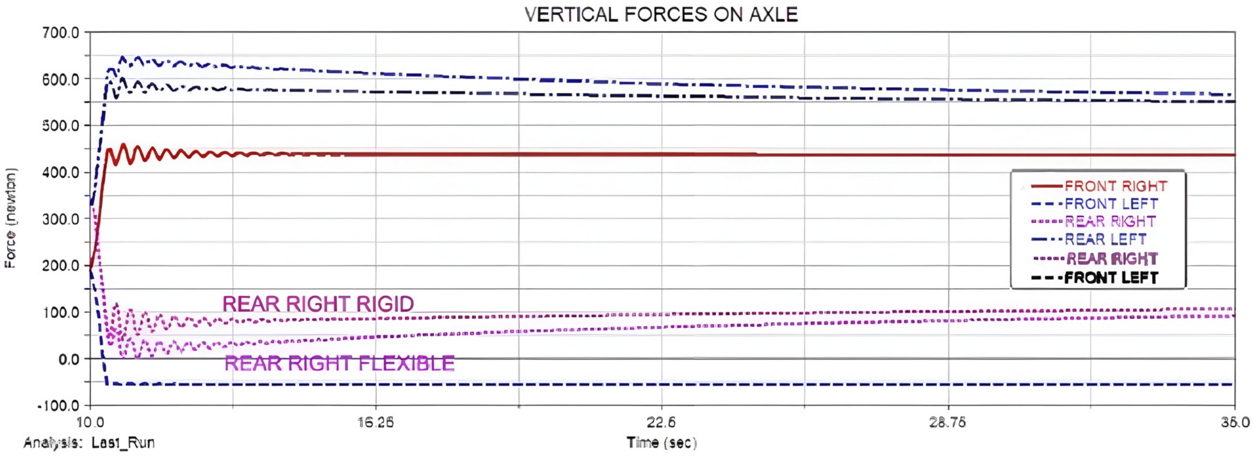

Fig. 12. Vertical forces on the tire

distribution of stiffness along the longitudinal axis of the frame. These results correlate with Mirone [4].

From the simulations of constant-speed cornering, it was observed that the stiffness of the frame significantly affects the vehicle's behavior. The decrease in frame stiffness leads to undesirable responses to driver interventions, making the vehicle more difficult to handle.

It was discovered that the dynamic behavior of a racing go-kart cannot be representatively simulated without including a flexible frame [9].

6. CONCLUSION

In this work, a flexible multibody racing go-kart model was built up. Since the elastic properties of the frame are essential, the FE model of the frame was validated by experimental modal analysis. Specific maneuvers were simulated to gain information on the difference between the rigid and the flexible model of the frame. Based on these evaluations, it is concluded that increasing the frame stiffness may be advantageous for the vehicle's dynamic behavior. This is because it provides a more direct response and allows for greater lateral acceleration.

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