

# Exemplo

Maximizar  $Z = 2x_1 + 3x_2 + x_3$

Sujeito a:

$$\begin{cases} x_1 + x_2 + x_3 \leq 40 \\ 2x_1 + x_2 - x_3 \leq 20 \\ 3x_1 + 2x_2 - x_3 \leq 30 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

$$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$$

$$f.o. Z - 2x_1 - 3x_2 - x_3 = 0$$

rest.:

$$x_1 + x_2 + x_3 + x_{f_1} = 40$$

$$2x_1 + x_2 + x_3 + x_{f_2} = 20$$

$$3x_1 + 2x_2 - x_3 + x_{f_3} = 30$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{f_1} \\ x_{f_2} \\ x_{f_3} \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 30 \end{bmatrix}$$

$$x_{f_1} = 40 \quad x_{f_2} = 20 \quad x_{f_3} = 30$$

↓ Entra

	$Z$	$x_1$	$x_2$	$x_3$	$x_{f_1}$	$x_{f_2}$	$x_{f_3}$	b
I	1	-2	-3	-1	0	0	0	0
II	0	1	1	1	0	0	40	
III	0	2	1	1	0	1	0	20
IV	0	3	<u>L2</u>	-1	0	0	1	30

Coef.  $x_2$

$$\begin{aligned} \text{II} & \quad 1 \rightarrow 40/1 = 40 \\ \text{III} & \quad 1 \rightarrow 20/1 = 20 \\ \text{IV} & \quad 2 \rightarrow 30/2 = 15 \rightarrow x_{f_3} = 0 \end{aligned}$$

↳ pivô

nova linha pivô

LP. (-1) + LII

$$0 \frac{3}{2} 1 \frac{1}{2} 0 0 \frac{1}{2} 15$$

$$\begin{aligned} & 0 -\frac{3}{2} -1 \frac{1}{2} 0 0 \frac{1}{2} -15 \\ & + \underline{0 1 1 1 1 0 0 40} \\ & \hline 0 -\frac{1}{2} 0 \frac{3}{2} 1 0 -\frac{1}{2} 25 \end{aligned}$$

LP. 3 + LF

$$\begin{aligned} & 0 \frac{9}{2} 3 \frac{3}{2} 0 0 \frac{3}{2} 45 \\ & + \underline{1 -2 -3 -1 0 0 0 0} \\ & \hline 1 \frac{5}{2} 0 -\frac{5}{2} 0 0 \frac{3}{2} 45 \end{aligned}$$

$$\begin{aligned} & \text{LP. (-1)} + \underline{\text{LIII}} \\ & 0 -\frac{3}{2} -1 \frac{1}{2} 0 0 -\frac{1}{2} -15 \\ & + \underline{0 2 1 -1 0 1 0 20} \\ & \hline 0 \frac{1}{2} 0 -\frac{1}{2} 0 1 -\frac{1}{2} 5 \end{aligned}$$

	$Z$	$X_1$	$X_2$	$X_3$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	$b$
I	1	$\frac{5}{2}$	0	$-\frac{5}{2}$	0	0	$\frac{3}{2}$	45
II	0	$-\frac{1}{2}$	0	$\frac{3}{2}$	1	0	$-\frac{1}{2}$	25
III	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$	5
IV	0	$\frac{3}{2}$	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	15

$$\left[ \begin{array}{ccc|c} 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 5 \\ 1 & 0 & 0 & 15 \end{array} \right] = \left[ \begin{array}{c} X_{f_1} \\ X_{f_2} \\ X_2 \end{array} \right]$$

Próxima variável

$X_3$

$$\frac{45}{-\frac{5}{2}} = -\frac{90}{5} = -18$$

$$\frac{25}{\frac{3}{2}} = \frac{50}{3} = 16,6 * \rightarrow X_{f_1} = 0$$

$$\frac{5}{-\frac{1}{2}} = -10 ; \frac{15}{-\frac{1}{2}} = -30$$

$$LP\left(\frac{1}{2}\right) + L III$$

$$0 -\frac{1}{6} 0 \frac{1}{2} \frac{1}{3} 0 -\frac{1}{6} \frac{25}{3}$$

$$+ 0 \frac{1}{2} 0 -\frac{1}{2} 0 1 -\frac{1}{2} 5$$

$$0 \frac{1}{6} 0 0 \frac{1}{3} 1 -\frac{1}{6} \frac{40}{3}$$

$$LP\left(\frac{5}{2}\right) + L I :$$

$$+ 0 -\frac{5}{6} 0 \frac{5}{2} \frac{10}{6} 0 -\frac{5}{6} \frac{250}{6}$$

$$1 \frac{5}{2} 0 -\frac{5}{2} 0 0 \frac{3}{2} 45$$

$$- 1 \frac{10}{6} 0 0 \frac{10}{6} 0 \frac{4}{6} \frac{520}{6}$$

$$LP\left(\frac{1}{2}\right) + L IV$$

$$0 -\frac{1}{6} 0 \frac{1}{2} \frac{1}{3} 0 -\frac{1}{6} \frac{25}{3}$$

$$0 \frac{3}{2} 1 -\frac{1}{2} 0 0 \frac{1}{2} 15$$

$$0 \frac{8}{6} 1 0 \frac{1}{3} 0 \frac{2}{6} \frac{70}{3}$$

$$X_1 = 0 \quad X_{f_1} = 0 \quad X_{f_3} = 0$$

	$Z$	$X_1$	$X_2$	$X_3$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	$b$
I	1	$\frac{10}{6}$	0	0	$\frac{10}{6}$	0	$\frac{4}{6}$	$\frac{520}{6}$
II	0	$-\frac{1}{3}$	0	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{50}{3}$
III	0	$\frac{2}{6}$	0	0	$\frac{1}{3}$	1	$-\frac{4}{6}$	$\frac{40}{2}$
IV	0	$\frac{8}{6}$	1	0	$\frac{1}{3}$	0	$\frac{2}{6}$	$\frac{70}{3}$

$$\begin{array}{c|ccccccc|c}
 & Z & X_1 & X_2 & X_3 & X_{f_1} & X_{f_2} & X_{f_3} & b \\
 \text{I} & 1 & \frac{10}{6} & 0 & 0 & \frac{10}{6} & 0 & \frac{4}{6} & \frac{520}{6} \\
 \text{II} & & 0 & -\frac{1}{3} & 0 & 1 & \frac{2}{3} & 0 & -\frac{1}{3} & \frac{50}{3} \\
 \text{III} & & 0 & \frac{2}{6} & 0 & 0 & \frac{1}{3} & 1 & -\frac{4}{6} & \frac{40}{3} \\
 \text{IV} & & 0 & \frac{8}{6} & 1 & 0 & \frac{1}{3} & 0 & \frac{2}{6} & \frac{70}{3}
 \end{array}$$

$$\left[ \begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{array} \right] \left[ \begin{array}{c} X_3 \\ X_{f_2} \\ X_2 \end{array} \right] = \left[ \begin{array}{c} \frac{50}{3} \\ \frac{40}{3} \\ \frac{70}{3} \end{array} \right]$$

Coef. de I  $> 0$        $X_1 = 0$      $X_2 = \frac{70}{3}$      $X_3 = \frac{50}{3}$

$$Z = \frac{520}{6} = 86,67$$

$$1.1. \text{Max. Receita} = 10x_1 + 12x_2$$

Sujeito a:

$$\begin{cases} x_1 + x_2 \leq 100 \\ 2x_1 + 3x_2 \leq 270 \\ x_1 \geq 0, x_2 \geq 0 \end{cases}$$

$$r - 10x_1 - 12x_2 = 0$$

$$x_1 + x_2 + X_{f1} = 100$$

$$2x_1 + 3x_2 + X_{f2} = 270$$

	$x_1$	$x_2$	$X_{f1}$	$X_{f2}$	b	
I	1	-10	-12	0	0	0
II	0	1	1	1	0	$\frac{100}{1} = 100$
III	0	2	<u>3</u>	0	1	$\frac{270}{3} = 90^*$

$$LP \rightarrow LIII \cdot \frac{1}{3} \rightarrow 0 \ 2/3 \ 1 \ 0 \ 1/3 \ 90$$

$$LP \cdot (12) + LI$$

$$+ \begin{array}{r} 0 \ 8 \ 12 \ 0 \ 4 \ 1080 \\ 1 \ -10 \ -12 \ 0 \ 0 \ 0 \\ \hline 1 \ -2 \ 0 \ 0 \ 4 \ 1080 \end{array}$$

$$LP \cdot (-1) + LIV$$

$$+ \begin{array}{r} 0 \ -2/3 \ -1 \ 0 \ -1/3 \ -90 \\ 0 \ 1 \ 1 \ 1 \ 0 \ 100 \\ \hline 0 \ 1/3 \ 0 \ 1 \ -1/3 \ 10 \end{array}$$

	$x_1$	$x_2$	$X_{f1}$	$X_{f2}$	b	
I	1	-2	0	0	4	1080
II	0	<u><math>1/3</math></u>	0	1	$-\frac{1}{3}$	10
III	0	$\frac{2}{3}$	1	0	$\frac{1}{3}$	90

$$x_1; X_{f2} = 0$$

$$X_2 = 90; X_{f1} = 10$$

$$LP \rightarrow LII \cdot 3 \rightarrow 0 \ 1 \ 0 \ 3 \ -1 \ 30$$

$$LP \cdot (2) + LI$$

$$+ \begin{array}{r} 0 \ 2 \ 0 \ 6 \ -2 \ 60 \\ 1 \ -2 \ 0 \ 0 \ 1 \ 1080 \\ \hline 1 \ 0 \ 0 \ 6 \ 2 \ 1140 \end{array}$$

$$LP \cdot (-\frac{2}{3}) + LIII$$

$$+ \begin{array}{r} 0 \ -\frac{2}{3} \ 0 \ -2 \ \frac{1}{3} \ -20 \\ 0 \ \frac{2}{3} \ 1 \ 0 \ \frac{1}{3} \ 90 \\ \hline 0 \ 0 \ 1 \ -2 \ 1 \ 70 \end{array}$$

	$x_1$	$x_2$	$X_{f1}$	$X_{f2}$	b	
I	1	0	0	6	2	1140
II	0	1	0	3	-1	30
III	0	0	1	-2	1	70

$$X_{f1} = 0; X_{f2} = 0 \quad r = 1140$$

$$X_1 = 30; X_2 = 70$$

$$1.2. \text{Max. Lucro} = 2x_1 + 3x_2 + 4x_3$$

Sujeito a:

$$\begin{cases} x_1 + x_2 + x_3 \leq 100 \\ 2x_1 + x_2 \leq 210 \\ x_1 \leq 80 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

$$L - 2x_1 - 3x_2 - 4x_3 = 0$$

$$x_1 + x_2 + x_3 + x_{f_1} = 100$$

$$2x_1 + x_2 + x_{f_2} = 210$$

$$x_1 + x_{f_3} = 80$$

L	$x_1$	$x_2$	$x_3$	$x_{f_1}$	$x_{f_2}$	$x_{f_3}$	R
1	-2	-3	-4	0	0	0	0
0	1	1	<u>1</u>	1	0	0	100
0	2	1	0	0	1	0	210
0	1	0	0	0	0	1	80

$$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$$

$$x_{f_1} = 100 \quad x_{f_2} = 210 \quad x_{f_3} = 80$$

$$L = 0$$

LP

$$0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 100$$

LP.4 + LI

$$+ \begin{array}{r} 0 \ 4 \ 4 \ 4 \ 4 \ 0 \ 0 \ 400 \\ 1 \ -2 \ -3 \ -4 \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ 2 \ 1 \ 0 \ 4 \ 0 \ 0 \ 400 \end{array}$$

$$L \ x_1 \ x_2 \ x_3 \ x_{f_1} \ x_{f_2} \ x_{f_3} \ R$$

$$1 \ 2 \ 1 \ 0 \ 4 \ 0 \ 0 \ 400$$

$$0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 100$$

$$x_1 = 0 \quad x_2 = 0 \quad x_{f_1} = 0$$

$$0 \ 2 \ 1 \ 0 \ 0 \ 1 \ 0 \ 210$$

$$x_3 = 100 \quad x_{f_2} = 210 \quad x_{f_3} = 80$$

$$0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 80$$

$$L = 400$$

Coeff. LI > 0 ; resposta ótima

$$1.3. \text{Max. } z = 0,2x_1 + 2x_2 + 4x_3$$

$$z - 0,2x_1 - 2x_2 - 4x_3 = 0$$

$$x_1 + 2x_2 + x_{f1} = 20$$

$$3x_1 + x_3 + x_{f2} = 50$$

$$x_1 + x_2 - x_3 + x_{f3} = 15$$

Sujeito a:

$$\begin{cases} x_1 + 2x_2 \leq 20 \\ 3x_1 + x_3 \leq 50 \\ x_1 + x_2 - x_3 \leq 15 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

$$z \ x_1 \ x_2 \ x_3 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$$

$$I \ 1 \ -0,2 \ -2 \ -4 \ 0 \ 0 \ 0 \ 0$$

$$x_{f1} = 20 \quad x_{f2} = 50 \quad x_{f3} = 15$$

$$II \ 0 \ 1 \ 2 \ 0 \ 1 \ 0 \ 0 \ 20$$

$$z = 0$$

$$III \ 0 \ 3 \ 0 \ 1 \ 0 \ 1 \ 0 \ 50$$

$$IV \ 0 \ 1 \ 1 \ -1 \ 0 \ 0 \ 1 \ 15$$

$$\frac{50}{1} = 50 \quad \frac{15}{-1} = -15$$

Linha pivô

$$0 \ 3 \ 0 \ 1 \ 0 \ 1 \ 0 \ 50$$

LP(4) + LI

LP(1) + L IV

$$\begin{array}{r} 0 \ 12 \ 0 \ 4 \ 0 \ 4 \ 0 \ 200 \\ + 1 \ -0,2 \ -2 \ -4 \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ 11,8 \ -2 \ 0 \ 0 \ 4 \ 0 \ 200 \end{array}$$

$$\begin{array}{r} 0 \ 3 \ 0 \ 1 \ 0 \ 1 \ 0 \ 50 \\ + 0 \ 1 \ 1 \ -1 \ 0 \ 0 \ 1 \ 15 \\ \hline 0 \ 4 \ 1 \ 0 \ 0 \ 1 \ 1 \ 65 \end{array}$$

$$z \ x_1 \ x_2 \ x_3 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1 = 0 \quad x_2 = 0 \quad x_{f2} = 0$$

$$I \ 1 \ 11,8 \ -2 \ 0 \ 0 \ 4 \ 0 \ 200$$

$$x_3 = 50 \quad x_{f1} = 20 \quad x_{f3} = 65$$

$$II \ 0 \ 1 \ 2 \ 0 \ 1 \ 0 \ 0 \ 20$$

$$III \ 0 \ 3 \ 0 \ 1 \ 0 \ 1 \ 0 \ 50$$

$$IV \ 0 \ 4 \ 1 \ 0 \ 0 \ 1 \ 1 \ 65$$

$Z$	$X_1$	$X_2$	$X_3$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	$R$	$\frac{20}{2} = 10$	$\frac{65}{1} = 65$
I	1	11,8	-2	0	0	4	0	200	
II	0	1	<u>2</u>	0	1	0	0	20	
III	0	3	0	1	0	1	0	50	
IV	0	4	1	0	0	1	1	65	

Linha pivô

$$0 \frac{1}{2} 1 0 \frac{1}{2} 0 0 10$$

LP.(2) + LI

$$\begin{array}{r}
 0 1 2 0 1 0 0 20 \\
 + 1 11,8 -2 0 0 4 0 200 \\
 \hline
 1 12,8 0 0 1 4 0 220
 \end{array}$$

LP(-1) + LIV

$$\begin{array}{r}
 0 -\frac{1}{2} -1 0 -\frac{1}{2} 0 0 -10 \\
 + 0 4 1 0 0 1 1 65 \\
 \hline
 0 \frac{3}{2} 0 0 -\frac{1}{2} 1 1 55
 \end{array}$$

$$Z \ X_1 \ X_2 \ X_3 \ X_{f_1} \ X_{f_2} \ X_{f_3} \ R$$

$$X_1 = 0 \quad X_{f_1} = 0 \quad X_{f_2} = 0$$

$$I \ 1 \ 12,8 \ 0 \ 0 \ 1 \ 4 \ 0 \ 220$$

$$X_2 = 10 \quad X_3 = 50 \quad X_{f_3} = 55$$

$$II \ 0 \ \frac{1}{2} \ 1 \ 0 \ \frac{1}{2} \ 0 \ 0 \ 10$$

$$Z = 220$$

$$III \ 0 \ 3 \ 0 \ 1 \ 0 \ 1 \ 0 \ 50$$

$$IV \ 0 \ \frac{3}{2} \ 0 \ 0 \ -\frac{1}{2} \ 1 \ 1 \ 55$$

Cof. da Linha I  $\neq 0$ ; solução ótima

1.4. Max.  $z = 5x_1 - 3x_2 + 4x_3 - x_4$

Sujeito a:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 \leq 600 \\ 2x_1 + x_3 \leq 280 \\ x_2 + 3x_4 \leq 150 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \end{cases}$$

$$\begin{aligned} z - 5x_1 + 3x_2 - 4x_3 + x_4 &= 0 \\ x_1 + x_2 + x_3 + x_4 + x_{f_1} &= 600 \\ 2x_1 + x_3 + x_{f_2} &= 280 \\ x_2 + 3x_4 + x_{f_3} &= 150 \end{aligned}$$

$$\begin{array}{ccccccccc|c} z & x_1 & x_2 & x_3 & x_4 & x_{f_1} & x_{f_2} & x_{f_3} & R \\ \hline I & 1 & -5 & 3 & -4 & 1 & 0 & 0 & 0 & 0 \\ II & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 600 & \\ III & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 280 \\ IV & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 1 & 150 \end{array}$$

$$x_1 \cup x_4 = 0$$

$$x_{f_1} = 600 \quad x_{f_2} = 280 \quad x_{f_3} = 150$$

$$z = 0$$

$$600/1 = 600 \quad 280/2 = 140$$

$$\text{Linha } P_{1,0} \rightarrow LIII/2 = 0 \ 1 \ 0 \ \frac{1}{2} \ 0 \ 0 \ \frac{1}{2} \ 0 \ 140$$

$$LP(5) + LI$$

$$LP(-1) + LII$$

$$\begin{array}{ccccccccc|c} & 0 & 5 & 0 & \frac{5}{2} & 0 & 0 & \frac{5}{2} & 0 & 700 \\ + & 1 & -5 & 3 & -4 & 1 & 0 & 0 & 0 & 0 \\ \hline & 1 & 0 & 3 & -\frac{3}{2} & 1 & 0 & \frac{5}{2} & 0 & 700 \end{array} \quad \begin{array}{ccccccccc|c} & 0 & -1 & 0 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 - 140 \\ + & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 600 \\ \hline & 0 & 0 & 1 & \frac{1}{2} & 1 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 460 \end{array}$$

$$z \ x_1 \ x_2 \ x_3 \ x_4 \ x_{f_1} \ x_{f_2} \ x_{f_3} \ R \quad x_2 \cup x_4 = 0 \quad x_{f_2} = 0$$

$$I \ 1 \ 0 \ 3 \ -\frac{3}{2} \ 1 \ 0 \ \frac{5}{2} \ 0 \ 700$$

$$x_1 = 140 \quad x_{f_1} = 460 \quad x_{f_3} = 150$$

$$II \ 0 \ 0 \ 1 \ \frac{1}{2} \ 1 \ 1 \ -\frac{1}{2} \ 0 \ 460$$

$$z = 700$$

$$III \ 0 \ 1 \ 0 \ \frac{1}{2} \ 0 \ 0 \ \frac{1}{2} \ 0 \ 140$$

$$\frac{460}{1/2} = 920 \quad \frac{140}{1/2} = 280 *$$

$$IV \ 0 \ 0 \ 1 \ 0 \ 3 \ 0 \ 0 \ 1 \ 150$$

$Z$	$X_1$	$X_2$	$X_3$	$X_4$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R	
I	1	0	3	$-\frac{3}{2}$	1	0	$\frac{5}{2}$	0	700
II	0	0	1	$\frac{1}{2}$	1	1	$-\frac{1}{2}$	0	460
III	0	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	140
IV	0	0	1	0	3	0	0	1	150

Linha pivô  $\rightarrow L \text{ III } 1(\frac{1}{2}) \rightarrow 0 \ 2 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 280$

$LP(3/2) + LI$

$$\begin{array}{ccccccccc} 0 & 3 & 0 & \frac{3}{2} & 0 & 0 & \frac{3}{2} & 0 & 480 \\ + & 1 & 0 & 3 & -\frac{3}{2} & 1 & 0 & \frac{5}{2} & 0 & 700 \\ \hline 1 & 3 & 3 & 0 & 1 & 0 & 4 & 0 & 1120 \end{array}$$

$LP(-1/2) + L\text{II}$

$$\begin{array}{ccccccccc} 0 & -1 & 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 0 & -40 \\ + & 0 & 0 & 1 & \frac{1}{2} & 1 & 1 & -\frac{1}{2} & 0 & 460 \\ \hline 0 & -1 & 1 & 0 & 1 & 1 & -1 & 0 & 320 \end{array}$$

$Z$	$X_1$	$X_2$	$X_3$	$X_4$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R	
I	1	3	3	0	1	0	4	0	1120
II	0	-1	1	0	1	1	-1	0	320
III	0	2	0	1	0	0	1	0	280
IV	0	0	1	0	3	0	0	1	150

$$X_1; X_2; X_4; X_{f_2} = 0$$

$$X_3 = 280 \quad X_{f_1} = 320 \quad X_{f_3} = 150$$

$$Z = 1120$$

Coef. LII  $> 0$ ; solução ótima

$$1.5. \text{Max. } z = 2x_1 + 4x_3$$

Sujeito a:

$$\begin{cases} x_1 + 2x_2 + x_3 \leq 8.000 \\ 2x_1 \leq 6.000 \\ x_2 + x_3 \leq 620 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

$$z - 2x_1 - 4x_3 = 0$$

$$x_1 + 2x_2 + x_3 + X_{f1} = 8000$$

$$2x_1 + X_{f2} = 6000$$

$$x_2 + x_3 + X_{f3} = 620$$

$$\begin{array}{ccccccccc|c} z & x_1 & x_2 & x_3 & x_{f1} & x_{f2} & x_{f3} & R \\ \hline I & 1 & -2 & 0 & -4 & 0 & 0 & 0 & 0 \\ II & 0 & 1 & 2 & 1 & 1 & 0 & 0 & 8000 \\ III & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 6000 \\ IV & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 620 \end{array}$$

$$x_1, x_2, x_3 = 0$$

$$X_{f1} = 8000 \quad X_{f2} = 6000 \quad X_{f3} = 620$$

$$R = 0$$

$$\frac{8000}{1} = 8000 \quad \frac{620}{1} = 620$$

Linha Pivô  $\rightarrow LIV$

$$LP(4) + LI$$

$$\begin{array}{r} + \\ \hline \end{array} \begin{array}{ccccccccc|c} 0 & 0 & 4 & 4 & 0 & 0 & 4 & 2480 \\ 1 & -2 & 0 & -4 & 0 & 0 & 0 & 0 \\ \hline 1 & -2 & 4 & 0 & 0 & 0 & 4 & 2480 \end{array}$$

$$LP(-1) + LII$$

$$\begin{array}{r} + \\ \hline \end{array} \begin{array}{ccccccccc|c} 0 & 0 & -1 & -1 & 0 & 0 & -1 & -620 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 & 8000 \\ \hline 0 & 1 & 1 & 0 & 1 & 0 & -1 & 7380 \end{array}$$

$$\begin{array}{ccccccccc|c} z & x_1 & x_2 & x_3 & x_{f1} & x_{f2} & x_{f3} & R \\ \hline I & 1 & -2 & 4 & 0 & 0 & 0 & 4 & 2480 \\ II & 0 & 1 & 1 & 0 & 1 & 0 & -1 & 7380 \\ III & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 6000 \\ IV & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 620 \end{array}$$

$$x_1, x_2, x_{f3} = 0$$

$$X_3 = 620 \quad X_{f1} = 7380 \quad X_{f2} = 6000$$

$$\frac{7380}{1} = 7380 \quad \frac{6000}{2} = 3000$$

$$z = 2480$$

Linha Pivô  $\rightarrow LIII/2$

$$0 \quad 1 \quad 0 \quad 0 \quad 0 \quad \frac{1}{2} \quad 0 \quad 3000$$

$$LP(2) + LI$$

$$\begin{array}{r} + \\ \hline \end{array} \begin{array}{ccccccccc|c} 0 & 2 & 0 & 0 & 0 & 1 & 0 & 6000 \\ 1 & -2 & 4 & 0 & 0 & 0 & 4 & 2480 \\ \hline 1 & 0 & 4 & 0 & 0 & 1 & 4 & 8480 \end{array}$$

$$LP(-1) + LIF$$

$$\begin{array}{r} + \\ \hline \end{array} \begin{array}{ccccccccc|c} 0 & -1 & 0 & 0 & 0 & -\frac{1}{2} & 0 & -3000 \\ 0 & 1 & 1 & 0 & 1 & 0 & -1 & 7380 \\ \hline 0 & 0 & 1 & 0 & 1 & -\frac{1}{2} & -1 & 4380 \end{array}$$

$Z$	$X_1$	$X_2$	$X_3$	$X_{f1}$	$X_{f2}$	$X_{f3}$	R	
I	1	0	4	0	0	1	4	8480
II	0	0	1	0	1	$-\frac{1}{2}$	-1	4380
III	0	1	0	0	0	$\frac{1}{2}$	0	3000
IV	0	0	1	1	0	0	1	620

$$X_2, X_{f2}, X_{f3} = 0$$

$$X_1 = 3000 \quad X_3 = 620 \quad X_{f1} = 4380$$

$$Z = 8480$$

Coef. de L.I > 0 ; solução ótima

$$1.6. \text{Max. } z = 2x_1 + 4x_2 + 6x_3$$

Sujeito a:

$$\begin{cases} x_1 + x_2 + x_3 \leq 100 \\ 2x_1 - x_2 + 5x_3 \leq 50 \\ 3x_1 + x_3 \leq 200 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{cases}$$

$$z - z x_1 - 4x_2 - 6x_3 = 0$$

$$x_1 + x_2 + x_3 + x_{f1} = 100$$

$$2x_1 - x_2 + 5x_3 + x_{f2} = 50$$

$$3x_1 + x_3 + x_{f3} = 200$$

$$z \ x_1 \ x_2 \ x_3 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1, x_2, x_3 = 0$$

$$I \ 1 \ -2 \ -4 \ -6 \ 0 \ 0 \ 0 \ 0$$

$$x_{f1} = 100 \quad x_{f2} = 50 \quad x_{f3} = 200$$

$$II \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 100 \frac{100}{1} = 100$$

$$z = 0$$

$$III \ 0 \ 2 \ -1 \ 5 \ 0 \ 1 \ 0 \ 50 \frac{50}{5} = 10^*$$

$$IV \ 0 \ 3 \ 0 \ 1 \ 0 \ 0 \ 1 \ 200 \frac{200}{1} = 200$$

$$\text{Linha pivô} \rightarrow LIII/5 \quad LIII \ 0 \ \frac{2}{5} \ -\frac{1}{5} \ 1 \ 0 \ \frac{1}{5} \ 0 \ 10$$

$$LP(6) + LI$$

$$+ \begin{array}{r} 0 \ \frac{12}{5} \ -\frac{6}{5} \ 6 \ 0 \ \frac{6}{5} \ 0 \ 60 \\ 1 \ -2 \ -4 \ -6 \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ \frac{2}{5} \ -\frac{26}{5} \ 0 \ 0 \ \frac{6}{5} \ 0 \ 60 \end{array}$$

$$LP(-1) + LII$$

$$+ \begin{array}{r} 0 \ -\frac{2}{5} \ \frac{1}{5} \ -1 \ 0 \ -\frac{1}{5} \ 0 \ -10 \\ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 100 \\ \hline 0 \ \frac{3}{5} \ \frac{6}{5} \ 0 \ 1 \ -\frac{1}{5} \ 0 \ 90 \end{array}$$

$$LP(-1) + LIV$$

$$+ \begin{array}{r} 0 \ -\frac{2}{5} \ \frac{1}{5} \ -1 \ 0 \ -\frac{1}{5} \ 0 \ -10 \\ 0 \ 3 \ 0 \ 1 \ 0 \ 0 \ 1 \ 200 \\ \hline 0 \ \frac{13}{5} \ \frac{1}{5} \ 0 \ 0 \ -\frac{1}{5} \ 1 \ 190 \end{array}$$

$$z \ x_1 \ x_2 \ x_3 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1, x_2, x_{f2} = 0$$

$$I \ 1 \ \frac{2}{5} \ -\frac{26}{5} \ 0 \ 0 \ \frac{6}{5} \ 0 \ 60$$

$$x_3 = 10 \quad x_{f1} = 90 \quad x_{f3} = 190$$

$$II \ 0 \ \frac{3}{5} \ \frac{6}{5} \ 0 \ 1 \ -\frac{1}{5} \ 0 \ 90$$

$$z = 60$$

$$III \ 0 \ \frac{2}{5} \ -\frac{1}{5} \ 1 \ 0 \ \frac{1}{5} \ 0 \ 10$$

$$IV \ 0 \ \frac{13}{5} \ \frac{1}{5} \ 0 \ 0 \ -\frac{1}{5} \ 1 \ 190$$

	$Z$	$X_1$	$X_2$	$X_3$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R	
I	1	$\frac{2}{5}$	$-\frac{2}{5}$	0	0	$\frac{6}{5}$	0	60	$\frac{90}{6} = \frac{150}{6} *$
II	0	$\frac{3}{5}$	$\frac{6}{5}$	0	1	$-\frac{1}{5}$	0	90 *	$\frac{10}{-1} = -50$
III	0	$\frac{2}{5}$	$-\frac{1}{5}$	1	0	$\frac{1}{5}$	0	10	$\frac{1}{-1} = -10$
IV	0	$\frac{13}{5}$	$\frac{1}{5}$	0	0	$-\frac{1}{5}$	1	190	$\frac{190}{1} = 190$

$$\text{Linha pivô} \rightarrow LII \cdot (\frac{5}{6}) \rightarrow 0 \frac{1}{2} 1 0 \frac{5}{6} -\frac{1}{6} 0 \frac{450}{6}$$

LP( $\frac{26}{5}$ ) + LI

$$+ \begin{array}{l} 0 \frac{26}{10} \frac{26}{5} 0 \frac{26}{6} \frac{26}{30} 0 390 \\ 1 \frac{2}{5} \frac{-2}{5} 0 0 \frac{6}{5} 0 60 \end{array} + \begin{array}{l} 0 \frac{1}{10} \frac{1}{5} 0 \frac{1}{6} -\frac{1}{30} 0 15 \\ 0 \frac{3}{5} \frac{-1}{5} 1 0 \frac{1}{5} 0 10 \end{array}$$

LP( $-\frac{1}{5}$ ) + LIV

$$+ \begin{array}{l} 0 -\frac{1}{10} -\frac{1}{5} 0 -\frac{1}{6} \frac{1}{30} 0 -15 \\ 0 \frac{13}{5} \frac{1}{5} 0 0 -\frac{1}{5} 1 190 \end{array}$$

$$\begin{array}{l} Z X_1 X_2 X_3 X_{f_1} X_{f_2} X_{f_3} R \\ I 1 3 0 0 \frac{26}{6} \frac{1}{3} 0 450 \\ II 0 \frac{1}{2} 1 0 \frac{5}{6} -\frac{1}{6} 0 \frac{450}{6} \\ III 0 \frac{1}{2} 0 1 \frac{1}{6} \frac{1}{6} 0 25 \\ IV 0 \frac{25}{10} 0 0 -\frac{1}{6} -\frac{1}{6} 1 175 \end{array}$$

$$X_1, X_{f_1}, X_{f_2} = 0$$

$$X_2 = \frac{450}{6} \quad X_3 = 25 \quad X_{f_3} = 175$$

$$6$$

$$= 75$$

Coef. de LI > 0 ; solução ótima

## E.1 Problema 1 - Lista 1

$$L = 5x_1 + 2x_2$$

$$2x_1 + x_2 \leq 6$$

$$x_1 \leq 6$$

$$x_2 \leq 5$$

$$L - 5x_1 - 2x_2 = 0$$

$$2x_1 + x_2 + x_{f_1} = 6$$

$$x_1 + x_{f_2} = 6$$

$$x_2 + x_{f_3} = 5$$

	$x_1$	$x_2$	$x_{f_1}$	$x_{f_2}$	$x_{f_3}$	R	$x_1, x_2 = 0$
I	1	-5	-2	0	0	0	$x_{f_1} = 6$
II	0	1	2	1	0	6	$x_{f_2} = 6$
III	0	0	1	0	1	5	$x_{f_3} = 5$
				$\frac{b}{2} = 3^*$			$L = 0$
				$b/1 = 6$			
						X	

$$\text{Linha } \text{pivo} \rightarrow \text{LII}/2 \rightarrow 0 \ 1 \ \frac{1}{2} \ \frac{1}{2} \ 0 \ 0 \ 3$$

$$LP(5) + LI$$

$$+ \begin{array}{cccccc} 0 & 5 & \frac{5}{2} & \frac{5}{2} & 0 & 0 & 15 \\ 1 & -5 & -2 & 0 & 0 & 0 & 0 \end{array} \quad \underline{+ \quad \quad \quad}$$

$$LP(-1) + LIII$$

$$+ \begin{array}{cccccc} 0 & -1 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & 0 & 6 \end{array} \quad \underline{+ \quad \quad \quad}$$

	$x_1$	$x_2$	$x_{f_1}$	$x_{f_2}$	$x_{f_3}$	R	$x_2, x_{f_1} = 0$
I	1	0	$\frac{1}{2}$	$\frac{5}{2}$	0	0	15
II	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	0	3
III	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	3
	0	0	1	0	0	1	5

Coeff. LII  $> 0$ ; solução ótima

22 - Problema 2 Lista 1

$$L = 100x_1 + 150x_2$$

$$L - 100x_1 - 150x_2 = 0$$

$$2x_1 + 3x_2 \leq 120$$

$$2x_1 + 3x_2 + x_{f1} = 120$$

$$x_1 \leq 40$$

$$x_1 + x_{f2} = 40$$

$$x_2 \leq 30$$

$$x_2 + x_{f3} = 30$$

$$L \ x_1 \ x_2 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1, x_2 = 0$$

$$I \ 1 \ -100 \ -150 \ 0 \ 0 \ 0 \quad x_{f1} = 120 \quad x_{f2} = 40 \quad x_{f3} = 30$$

$$II \ 0 \ 2 \ 3 \ 1 \ 0 \ 0 \ 120 \quad \frac{120}{3} = 40 \quad L = 0$$

$$III \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 40 \quad X$$

$$IV \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 30 \quad \frac{30}{1} = 30^*$$

Linha pivô  $\rightarrow$  LIV

LP(150) + LI

$$\begin{array}{r} + \\ \hline 0 \ 0 \ 150 \ 0 \ 0 \ 150 \ 4500 \\ 1 \ -100 \ -150 \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ -100 \ 0 \ 0 \ 0 \ 150 \ 4500 \end{array}$$

$$0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 30$$

LP(-3) + LII

$$\begin{array}{r} + \\ \hline 0 \ 0 \ -3 \ 0 \ 0 \ -3 \ -90 \\ 0 \ 2 \ 3 \ 1 \ 0 \ 0 \ 120 \\ \hline 0 \ 2 \ 0 \ 1 \ 0 \ -3 \ 30 \end{array}$$

$$L \ x_1 \ x_2 \ x_{f1} \ x_{f2} \ x_{f3} \ R$$

$$x_1, x_{f3} = 0$$

$$I \ 1 \ -100 \ 0 \ 0 \ 0 \ 150 \ 4500$$

$$II \ 0 \ 1 \ 0 \ 1 \ 0 \ -3 \ 30 \quad \frac{30}{1} = 30^* \quad x_2 = 30 \quad x_{f1} = 30 \quad x_{f2} = 40$$

$$III \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 40 \quad \frac{40}{1} = 40 \quad L = 4500$$

$$IV \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 30 \quad X$$

Linha pivô  $\rightarrow$  LIV/2

LP(100) + LI

$$\begin{array}{r} + \\ \hline 0 \ 100 \ 0 \ 50 \ 0 \ -150 \ 1500 \\ 1 \ -100 \ 0 \ 0 \ 0 \ 150 \ 4500 \\ \hline 1 \ 0 \ 0 \ 50 \ 0 \ 0 \ 6000 \end{array}$$

$$0 \ 1 \ 0 \ \frac{1}{2} \ 0 \ -\frac{3}{2} \ 15$$

LP(-1) + LIII

$$\begin{array}{r} + \\ \hline 0 \ -1 \ 0 \ -\frac{1}{2} \ 0 \ \frac{3}{2} \ -15 \\ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 40 \\ \hline 0 \ 0 \ 0 \ -\frac{1}{2} \ 1 \ \frac{3}{2} \ 30 \end{array}$$

$L$	$X_1$	$X_2$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	$R$	$X_{f_1}, X_{f_3} = 0$
I	1	0	0	50	0	0	6000
II	0	1	0	$\frac{1}{2}$	0	$-\frac{3}{2}$	15
III	0	0	0	$-\frac{1}{2}$	1	$\frac{3}{2}$	30
IV	0	0	1	0	0	1	30
							$X_1 = 15 \quad X_2 = 30 \quad X_{f_2} = 30$ $b = 6000$

Coef.  $bI > 0$  ; solución óptima

2.3 - Problema 3 Lista 1

$$L = 10x_1 + 30x_2 + 4000$$

$$L - 10x_1 - 30x_2 + M_2 a_2 = 4000$$

$$x_1 + x_2 \leq 600$$

$$x_1 + x_2 + x_{f1} = 600$$

$$x_1 \geq 100 \rightarrow a_2$$

$$x_1 - x_{f2} + a_2 = 100$$

$$x_2 \leq 200$$

$$x_2 + x_{f3} = 200$$

$$\begin{array}{ccccccc|c} L & x_1 & x_2 & x_{f1} & x_{f2} & x_{f3} & a_2 & R \\ I & 1 & -10 & -30 & 0 & 0 & 0 & M_2 & 4000 \end{array}$$

$$x_1, x_2, x_{f2} = 0$$

$$\begin{array}{ccccccc|c} II & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 600 \frac{600}{1} = 600 \end{array}$$

$$x_{f1} = 600 \quad a_2 = 100 \quad x_{f3} = 200$$

$$\begin{array}{ccccccc|c} III & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 100 X \end{array}$$

$$L = 4000$$

$$\begin{array}{ccccccc|c} IV & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 600 \frac{200}{1} = 200 * \end{array}$$

Linha pivot  $\rightarrow$  LIV

LP(30) + LI

$$\begin{array}{ccccccc|c} + & 0 & 0 & 30 & 0 & 0 & 30 & 0 & 6000 \\ \underline{1 & -10 & -30 & 0 & 0 & 0 & 0 & M_2 & 4000} \\ 1 & -10 & 0 & 0 & 0 & 30 & M_2 & 10000 \end{array}$$

LP(-1) + LII

$$\begin{array}{ccccccc|c} + & 0 & 0 & -1 & 0 & 0 & -1 & 0 & -200 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 600 \\ \hline 0 & 1 & 0 & 1 & 0 & -1 & 0 & 400 \end{array}$$

$$L \ x_1 \ x_2 \ x_{f1} \ x_{f2} \ x_{f3} \ a_2 \ R$$

$$x_1, x_{f2}, x_{f3} = 0$$

$$I \ 1 \ -10 \ 0 \ 0 \ 0 \ 30 \ M_2 \ 10000$$

$$II \ 0 \ 1 \ 0 \ 1 \ 0 \ -1 \ 0 \ 400 \frac{400}{1} = 400 \quad x_2 = 200 \quad x_{f1} = 400 \quad a_2 = 100$$

$$III \ 0 \ \underline{1} \ 0 \ 0 \ -1 \ 0 \ 1 \ 100 \frac{100}{1} = 100 *$$

$$L = 10000$$

$$IV \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 200$$

Linha pivot  $\rightarrow$  LIII

LP.(10) + LI

$$\begin{array}{ccccccc|c} + & 0 & 10 & 0 & 0 & -10 & 0 & 10 & 1000 \\ 1 & -10 & 0 & 0 & 0 & 30 & M_2 & 10000 \\ \hline 1 & 0 & 0 & 0 & -10 & 30 & M_2 & 11000 \end{array}$$

LP(-1) + LII

$$\begin{array}{ccccccc|c} + & 0 & -1 & 0 & 0 & 1 & 0 & -1 & -100 \\ 0 & 1 & 0 & 1 & 0 & -1 & 0 & 400 \\ \hline 0 & 0 & 0 & 1 & 1 & -1 & -1 & 300 \end{array}$$

L	$X_1$	$X_2$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R
I	1	0	0	0	-10	30
II	0	0	0	1	1	-1
III	0	1	0	0	-1	0
IV	0	0	1	0	0	1
						200

$$X_{f_2}, X_{f_3}, \alpha_2 = 0$$

$$X_1 = 100 \quad X_2 = 200 \quad X_{f_1} = 300$$

$$L = 11000$$

L	$X_1$	$X_2$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R
I	1	0	0	0	-10	30
II	0	0	0	1	1	-1
III	0	1	0	0	-1	0
IV	0	0	1	0	0	1
						200
						X

Linha pivô  $\rightarrow$  LII

$$LP(10) + LI$$

$$\begin{array}{r}
 + 0 \ 0 \ 0 \ 10 \ 10 \ -10 \ 300 \\
 \hline
 1 \ 0 \ 0 \ 0 \ -10 \ 30 \ 11K
 \end{array}$$

$$0 \ 0 \ 0 \ 1 \ 1 \ -1 \ 300$$

$$LP(1) + LIII$$

$$\begin{array}{r}
 + 0 \ 0 \ 0 \ 1 \ 1 \ -1 \ 300 \\
 \hline
 0 \ 1 \ 0 \ 0 \ -1 \ 0 \ 100
 \end{array}$$

L	$X_1$	$X_2$	$X_{f_1}$	$X_{f_2}$	$X_{f_3}$	R
I	1	0	0	10	0	20
II	0	0	0	1	1	-1
III	0	1	0	1	0	-1
IV	0	0	1	0	0	1
						200

$$X_{f_1}, X_{f_3} = 0$$

$$X_1 = 100 \quad X_2 = 200 \quad X_{f_2} = 300$$

$$L = 14000$$

Coef. de LI > 0 ; solução ótima

3. Um fabricante de fantasias tem em estoque 32 m de brim, 22 m de seda e 30 m de cetim e pretende fabricar dois modelos de fantasias. O primeiro modelo (M1) consome 4 m de brim, 2 m de seda e 2 m de cetim. O segundo modelo (M2) consome 2 m de brim, 4 m de seda e 6 m de cetim. Se M1 é vendido a 6.000 u.m. e M2 a 10.000 u.m., quantas peças de cada tipo o fabricante deve fazer para obter a receita máxima?

Var.;  $X_1 \rightarrow$  quantidade fantasia M<sub>1</sub>

$X_2 \rightarrow$  quantidade fantasia M<sub>2</sub>

F.O: maximizar  $R = 6000X_1 + 10000X_2$

rest:

$$X_1, X_2 \geq 0$$

$$\rightarrow R - 6000X_1 - 10000X_2 = 0$$

$$4X_1 + 2X_2 \leq 32$$

$$4X_1 + 2X_2 + X_{f1} = 32$$

$$2X_1 + 4X_2 \leq 22$$

$$2X_1 + 4X_2 + X_{f2} = 22$$

$$2X_1 + 6X_2 \leq 30$$

$$2X_1 + 6X_2 + X_{f3} = 30$$

$$R \quad X_1 \quad X_2 \quad X_{f1} \quad X_{f2} \quad X_{f3} \quad R \quad X_1 = 0 \quad X_2 = 0$$

$$I \quad 1 \quad -6K \quad -10K \quad 0 \quad 0 \quad 0 \quad 0 \quad X_{f1} = 32 \quad X_{f2} = 22 \quad X_{f3} = 30$$

$$II \quad 0 \quad 4 \quad 2 \quad 1 \quad 0 \quad 0 \quad 32 \quad \frac{32}{2} = 16 \quad R = 0$$

$$III \quad 0 \quad 2 \quad 4 \quad 0 \quad 1 \quad 0 \quad 22 \quad \frac{22}{4} = 1\frac{1}{2}$$

$$IV \quad 0 \quad 2 \quad 16 \quad 0 \quad 0 \quad 1 \quad 30 \quad \frac{30}{16} = 5^*$$

$$\text{Linha } IV \rightarrow LIV/16 \rightarrow 0 \quad \frac{1}{3} \quad 1 \quad 0 \quad 0 \quad \frac{1}{6} \quad 5$$

$$LP(10K) + LI$$

$$LP(-2) + LII$$

$$+ \begin{matrix} 0 & \frac{10K}{3} & 10K & 0 & 0 & \frac{10K}{6} & 50K \\ 1 & -6K & -10K & 0 & 0 & 0 & 0 \end{matrix} \quad \underline{-} \quad \begin{matrix} 1 & -\frac{8K}{3} & 0 & 0 & 0 & \frac{10K}{6} & 50K \end{matrix}$$

$$+ \begin{matrix} 0 & -\frac{2}{3} & -2 & 0 & 0 & -\frac{1}{3} & -10 \\ 0 & 4 & 2 & 1 & 0 & 0 & 32 \end{matrix} \quad \underline{\quad} \quad \begin{matrix} 0 & \frac{10}{3} & 0 & 1 & 0 & -\frac{1}{3} & 22 \end{matrix}$$

$$LP(-4) + LIII$$

$$+ \begin{matrix} 0 & -\frac{4}{3} & -4 & 0 & 0 & -\frac{2}{3} & -20 \\ 0 & 2 & 4 & 0 & 1 & 0 & 22 \end{matrix} \quad \underline{\quad} \quad \begin{matrix} 0 & \frac{2}{3} & 0 & 0 & 1 & -\frac{2}{3} & 2 \end{matrix}$$

	$\gamma$	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	R	$X_1, X_{f3} = 0$
I	1	$-\frac{8K}{3}$	0	0	0	$\frac{10K}{6}$	$50K$	$X_2 = 5$ $X_{f1} = 22$ $X_{f2} = 2$
II	0	$\frac{10}{3}$	0	1	0	$-\frac{1}{3}$	22	$\frac{22}{10/3} = \frac{66}{10}$ $T = 50000$
III	0	$\frac{8}{3}$	0	0	1	$-\frac{2}{3}$	2	$\frac{2}{8/3} = 3^*$
IV	0	$\frac{1}{3}$	1	0	0	$\frac{1}{6}$	5	$\frac{5}{1/3} = 15$

$$\text{Linha pivô} \rightarrow L_{\text{III}} \cdot (3/2) \rightarrow 0 \ 1 \ 0 \ 0 \ \frac{3}{2} \ -1 \ 3$$

$$LP\left(\frac{8K}{3}\right) + LI$$

$$+ \begin{array}{r} 0 \ \frac{8K}{3} \ 0 \ 0 \ 4K \ -\frac{8K}{3} \ 8K \\ 1 \ -\frac{8K}{3} \ 0 \ 0 \ 0 \ \frac{10K}{6} \ 50K \\ \hline 1 \ 0 \ 0 \ 0 \ 4K \ -1K \ 58K \end{array}$$

$$LP\left(-\frac{10}{3}\right) + LII$$

$$+ \begin{array}{r} 0 \ -\frac{10}{3} \ 0 \ 0 \ -5 \ \frac{10}{3} \ -10 \\ 0 \ \frac{10}{3} \ 0 \ 1 \ 0 \ -\frac{1}{3} \ 22 \\ \hline 0 \ 0 \ 0 \ 1 \ -5 \ 3 \ 12 \end{array}$$

$$LP\left(-\frac{1}{3}\right) + LIV$$

$$+ \begin{array}{r} 0 \ -\frac{1}{3} \ 0 \ 0 \ -\frac{1}{2} \ \frac{1}{3} \ -1 \\ 0 \ \frac{1}{3} \ 1 \ 0 \ 0 \ \frac{1}{6} \ 5 \\ \hline 0 \ 0 \ 1 \ 0 \ -\frac{1}{2} \ \frac{1}{2} \ 4 \end{array}$$

	$\gamma$	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	R	$X_{f2}, X_{f3} = 0$
I	1	0	0	0	$4K$	$-1K$	$58K$	
II	0	0	0	1	$-5$	$3$	12	$\frac{12}{3} = 4^*$ $X_1 = 3$ $X_2 = 4$ $X_{f1} = 12$
III	0	1	0	0	$3/2$	$-1$	3	$\frac{3}{2} = -3$ $T = 10000$
IV	0	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	4	$\frac{4}{1/2} = 8$

$$\text{Linha pivô} \rightarrow L_{\text{II}}/3 \rightarrow 0 \ 0 \ 0 \ \frac{1}{3} \ -\frac{5}{3} \ 1 \ 4$$

$$LP(1) + LI$$

$$+ \begin{array}{r} 0 \ 0 \ 0 \ \frac{1K}{3} \ -\frac{5K}{3} \ 11K \ 4K \\ 1 \ 0 \ 0 \ 0 \ 4K \ -1K \ 58K \\ \hline 1 \ 0 \ 0 \ \frac{1K}{3} \ -\frac{5K}{3} \ 0 \ 62K \end{array}$$

$$LP(1) + LIII$$

$$+ \begin{array}{r} 0 \ 0 \ 0 \ \frac{1}{3} \ -\frac{5}{3} \ 1 \ 4 \\ 0 \ 1 \ 0 \ 0 \ \frac{3}{2} \ -1 \ 3 \\ \hline 0 \ 1 \ 0 \ \frac{1}{3} \ -\frac{1}{6} \ 0 \ 7 \end{array}$$

$$LP\left(-\frac{1}{2}\right) + LIV$$

$$+ \begin{array}{r} 0 \ 0 \ 0 \ -\frac{1}{6} \ -\frac{5}{6} \ -\frac{1}{2} \ -2 \\ 0 \ 0 \ 1 \ 0 \ -\frac{1}{2} \ \frac{1}{2} \ 1 \\ \hline 0 \ 0 \ 1 \ -\frac{1}{6} \ -\frac{8}{6} \ 0 \ 2 \end{array}$$

	$x_1$	$x_2$	$x_{f_1}$	$x_{f_2}$	$x_{f_3}$	R	
I	1	0	0	$\frac{1}{3}$	$\frac{7}{3}$	0	$62K$
II	0	1	0	$\frac{1}{3}$	$-\frac{1}{6}$	0	$\neq$
III	0	0	0	$\frac{1}{3}$	$-\frac{5}{3}$	1	4
IV	0	0	1	$-\frac{1}{6}$	$-\frac{8}{6}$	0	2

$$x_{f_1} = 0 \quad x_{f_2} = 0$$

$$x_1 = 7 \quad x_2 = 2 \quad x_{f_3} = 1$$

Coef. LI > 0 ; solución óptima

4. O problema consiste em programar a produção de dois itens P1 e P2 a partir dos recursos produtivos R1, R2 e R3. Os dados colhidos nos vários setores da empresa são os seguintes:

Uso dos recursos produtivos

Produtos	R1 por unidade	R2 por unidade	R3 por unidade
P1	2	4	1
P2	3	2	5
Disponibilidades mensais	3.000	4.000	4.500
Produtos	Custo unitário	Custo de venda	Preço de venda
P1	20	20%	50
P2	30	20%	70

Demandas conjuntas dos produtos: 1.000 unidades/mês. O objetivo é maximizar o lucro.

Var.:  $x_1 \rightarrow$  quantidade de P1  
 $x_2 \rightarrow$  quantidade de P2

F.O.:  $L = x_1 \cdot (50 - (0,2 \cdot 50) - 20) + x_2 (70 - (0,2 \cdot 70) - 30)$   
 $= x_1 (50 - 10 - 20) + x_2 (70 - 14 - 30)$   
 $= x_1 \cdot 20 + x_2 \cdot 26$

rest.:  $x_1, x_2 \geq 0$   $L - 20x_1 - 26x_2 = 0$   
 $x_1 + x_2 \leq 1000$   $x_1 + x_2 + x_{f1} = 1000$   
 $x_1 \cdot 2 + x_2 \cdot 3 \leq 3000$   $2x_1 + 3x_2 + x_{f2} = 3000$   
 $x_1 \cdot 4 + x_2 \cdot 2 \leq 4000$   $4x_1 + 2x_2 + x_{f3} = 4000$   
 $x_1 \cdot 1 + x_2 \cdot 5 \leq 4500$   $x_1 + 5x_2 + x_{f4} = 4500$

	$L$	$x_1$	$x_2$	$x_{f1}$	$x_{f2}$	$x_{f3}$	$x_{f4}$	$R$	
I	1	-20	-26	0	0	0	0	0	$x_1, x_2 = 0$
II	0	1	1	1	0	0	0	1K	$x_{f1} = 1K$ $x_{f2} = 3K$ $x_{f3} = 4K$ $x_{f4} = 1,5K$
III	0	2	3	0	1	0	0	3K	$1K$
IV	0	4	2	0	0	1	0	4K	$2K$
V	0	1	15	0	0	0	1	4,5K	$900^*$

L	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	$X_{f4}$	R
I	1	-20	-26	0	0	0	0
II	0	1	1	1	0	0	1K
III	0	2	3	0	1	0	0
IV	0	4	2	0	0	1	0
V	0	1	15	0	0	0	1 4,5K 900*

Linha pivô  $\rightarrow$  LIV/5  $\rightarrow$   $0 \frac{1}{5} 1 0 0 0 \frac{1}{5} 900$

LP(26) + LI

$$+ \begin{matrix} 0 & \frac{26}{5} & 26 & 0 & 0 & 0 & \frac{26}{5} & 234K \\ 1 & -20 & -26 & 0 & 0 & 0 & 0 & 0 \end{matrix} \underline{\underline{1 - \frac{74}{5} 0 0 0 0 \frac{26}{5} 234K}}$$

LP(-1) + LII

$$+ \begin{matrix} 0 & -\frac{1}{5} & -1 & 0 & 0 & 0 & -\frac{1}{5} & -900 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1000 \end{matrix} \underline{\underline{0 \frac{4}{5} 0 1 0 0 -\frac{1}{5} 100}}$$

LP(-3) + LIII

$$+ \begin{matrix} 0 & -\frac{3}{5} & -3 & 0 & 0 & 0 & -\frac{3}{5} & -2700 \\ 0 & 2 & 3 & 0 & 1 & 0 & 0 & 3000 \end{matrix} \underline{\underline{0 \frac{7}{5} 0 0 1 0 -\frac{3}{5} 300}}$$

LP(-2) + LIV

$$+ \begin{matrix} 0 & \frac{2}{5} & -2 & 0 & 0 & 0 & -\frac{2}{5} & -1800 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 4000 \end{matrix} \underline{\underline{0 \frac{18}{5} 0 0 0 1 -\frac{2}{5} 2200}}$$

L	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	$X_{f4}$	R
I	1	$-\frac{74}{5}$	0	0	0	$\frac{26}{5}$	$23400 \times$
II	0	$\frac{4}{5}$	0	1	0	0	$-\frac{1}{5} 100$
III	0	$\frac{7}{5}$	0	0	1	0	$-\frac{3}{5} 300$
IV	0	$\frac{18}{5}$	0	0	0	1	$-\frac{2}{5} 2200$
V	0	$\frac{1}{5}$	1	0	0	0	$\frac{1}{5} 900$

$$X_1, X_{f4} = 0$$

$$X_2 = 900 \quad X_{f1} = 100$$

$$X_{f2} = 300 \quad X_{f3} = 2200$$

L2

L	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	$X_{eq}$	R
I	1	$\frac{7}{5}$	0	0	0	$\frac{26}{5}$	23400
II	0	$\frac{1}{5}$	0	1	0	$-\frac{1}{5}$	100
III	0	$\frac{7}{5}$	0	0	1	$-\frac{3}{5}$	300
IV	0	$\frac{18}{5}$	0	0	0	$1 - \frac{2}{5}$	2200
V	0	$\frac{1}{5}$	1	0	0	$\frac{1}{5}$	900

$$X_1, X_{eq} = 0$$

$$\frac{100}{\frac{1}{5}} = 125 *$$

$$X_2 = 900 \quad X_{f1} = 100$$

$$\frac{300}{\frac{7}{5}} = 1500$$

$$X_{f2} = 300 \quad X_{f3} = 2200$$

$$L_c$$

$$\text{Linha pivô} \rightarrow L_{II} \cdot \left(\frac{5}{6}\right)$$

$$LP. \left(\frac{7}{5}\right) + LI$$

+	0	$\frac{7}{5}$	0	$\frac{37}{2}$	0	0	$-\frac{37}{10}$	1850
	1	$\frac{7}{5}$	0	0	0	$\frac{26}{5}$	23400	
	1	0	0	$\frac{37}{2}$	0	0	$\frac{1}{2}$	25250

$$\rightarrow 0 \ 1 \ 0 \ \frac{5}{4} \ 0 \ 0 \ -\frac{1}{4} \ 125$$

$$LP \left(-\frac{7}{5}\right) + L_{III}$$

$$+ 0 \ -\frac{7}{5} \ 0 \ -\frac{7}{4} \ 0 \ 0 \ \frac{7}{20} \ -175$$

$$+ 0 \ \frac{7}{5} \ 0 \ 0 \ 1 \ 0 \ -\frac{3}{5} \ 300$$

$$\underline{0 \ 0 \ 0 \ -\frac{7}{4} \ 1 \ 0 \ -\frac{1}{4} \ 225}$$

$$LP \left(-\frac{18}{5}\right) + L_{IV}$$

+	0	$-\frac{18}{5}$	0	$-\frac{18}{4}$	0	0	$-\frac{18}{20}$	-450
	0	$\frac{18}{5}$	0	0	0	1	$-\frac{2}{5}$	2200
	0	0	0	$-\frac{18}{4}$	0	1	$-\frac{15}{10}$	1750

$$LP \left(-\frac{1}{5}\right) + L_{IV}$$

$$+ 0 \ -\frac{1}{5} \ 0 \ -\frac{1}{4} \ 0 \ 0 \ \frac{1}{20} \ -25$$

$$+ 0 \ \frac{1}{5} \ 1 \ 0 \ 0 \ 0 \ \frac{1}{5} \ 900$$

$$\underline{0 \ 0 \ 1 \ -\frac{1}{4} \ 0 \ 0 \ \frac{1}{4} \ 875}$$

L	$X_1$	$X_2$	$X_{f1}$	$X_{f2}$	$X_{f3}$	$X_{eq}$	R	
I	1	0	0	$\frac{37}{2}$	0	0	$\frac{1}{2}$	25250
II	0	1	0	$\frac{5}{4}$	0	0	$-\frac{1}{4}$	125
III	0	0	0	$-\frac{7}{4}$	1	0	$-\frac{1}{4}$	225
IV	0	0	0	$-\frac{18}{4}$	0	1	$-\frac{15}{10}$	1750
V	0	0	1	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	875

$$X_{f1}, X_{eq} = 0$$

$$X_1 = 125 \quad X_2 = 875$$

$$X_{f2} = 225 \quad X_{f3} = 1750$$

$$L = 25250$$