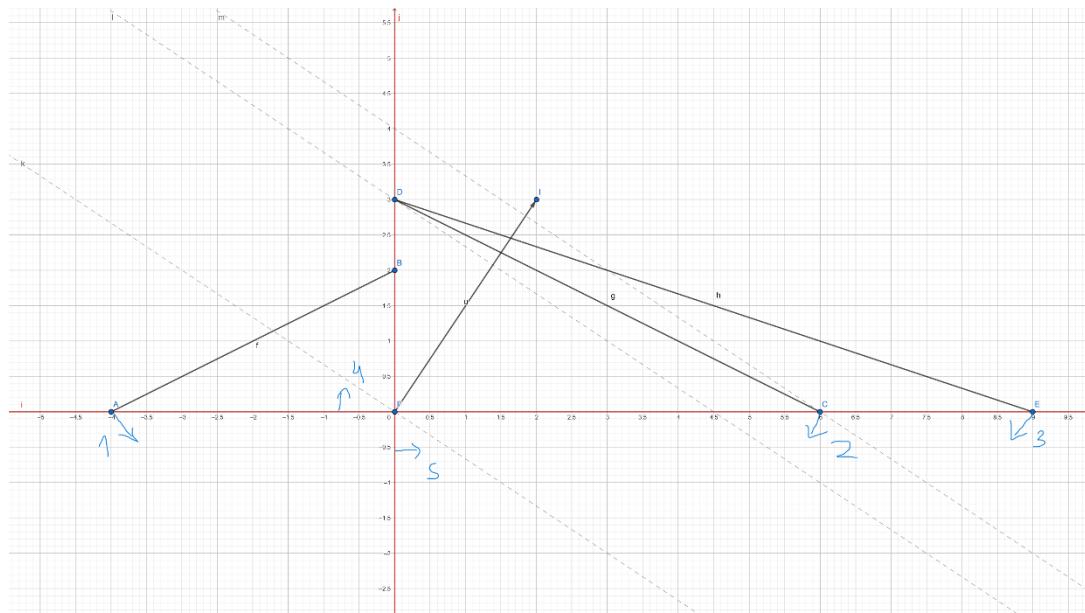


1.1-



$$\text{Maximizar: LUCRO} = 2x_1 + 3x_2$$

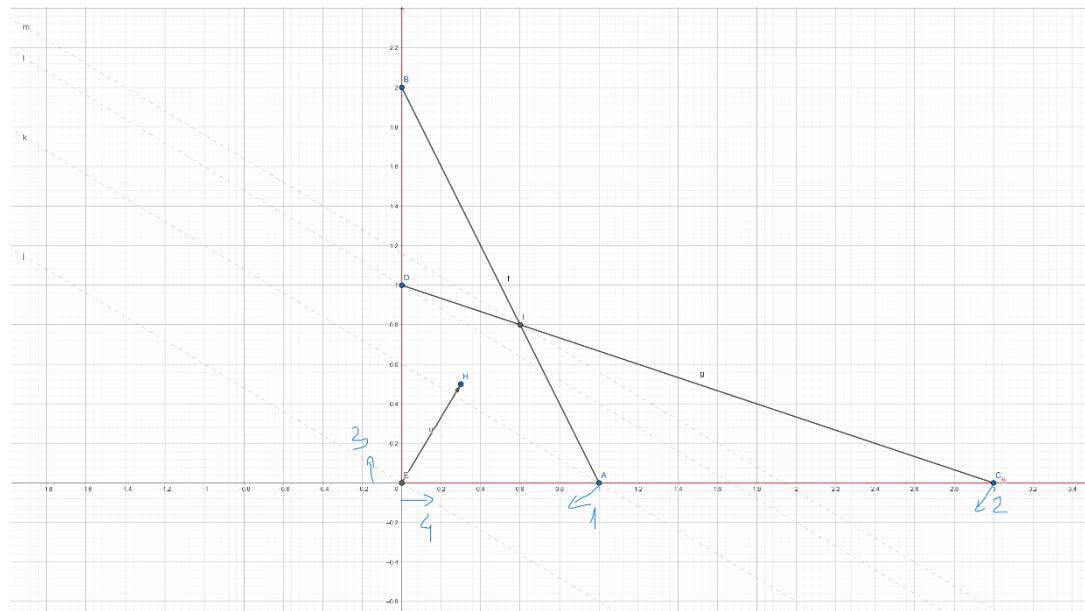
$$x_1 = 6; x_2 = 0$$

$$x_1 = 0; x_2 = 3$$

$$\text{LUCRO} = 12 *$$

$$\text{LUCRO} = 9$$

1.2 –



$$\text{Maximizar: RECEITA} = 0.3x_1 + 0.5x_2$$

$$x_1 = 1; x_2 = 0$$

$$x_1 = 0; x_2 = 1$$

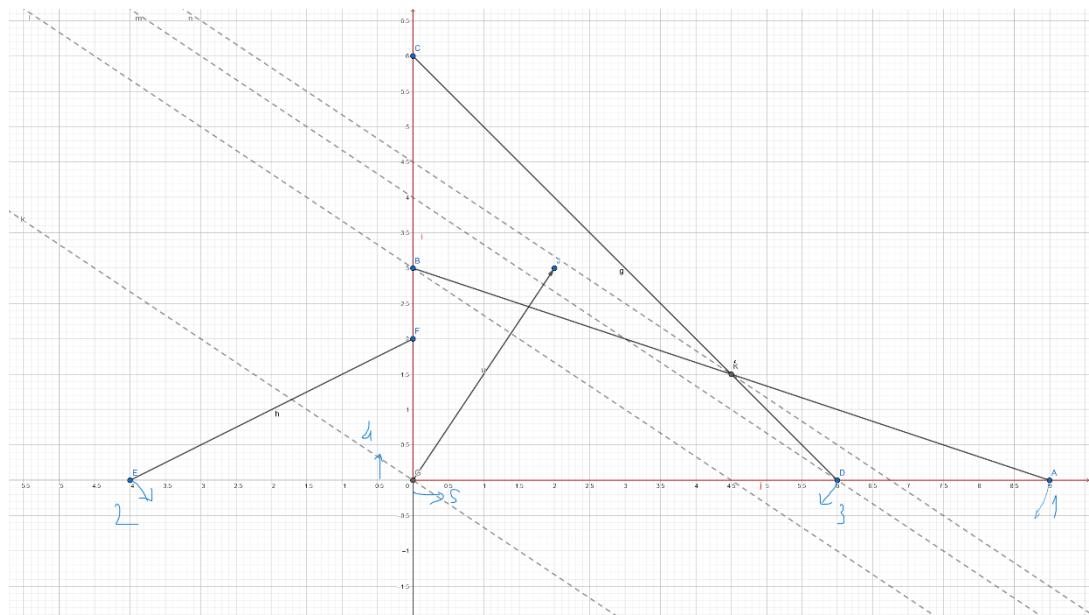
$$x_1 = 0.6; x_2 = 0.8$$

$$\text{RECEITA} = 0.3$$

$$\text{RECEITA} = 0.5$$

$$\text{RECEITA} = 0.58 *$$

1.3 –



$$\text{Maximizar: LUCRO} = 2x_1 + 3x_2$$

$$x_1 = 6; x_2 = 0$$

$$\text{LUCRO} = 12$$

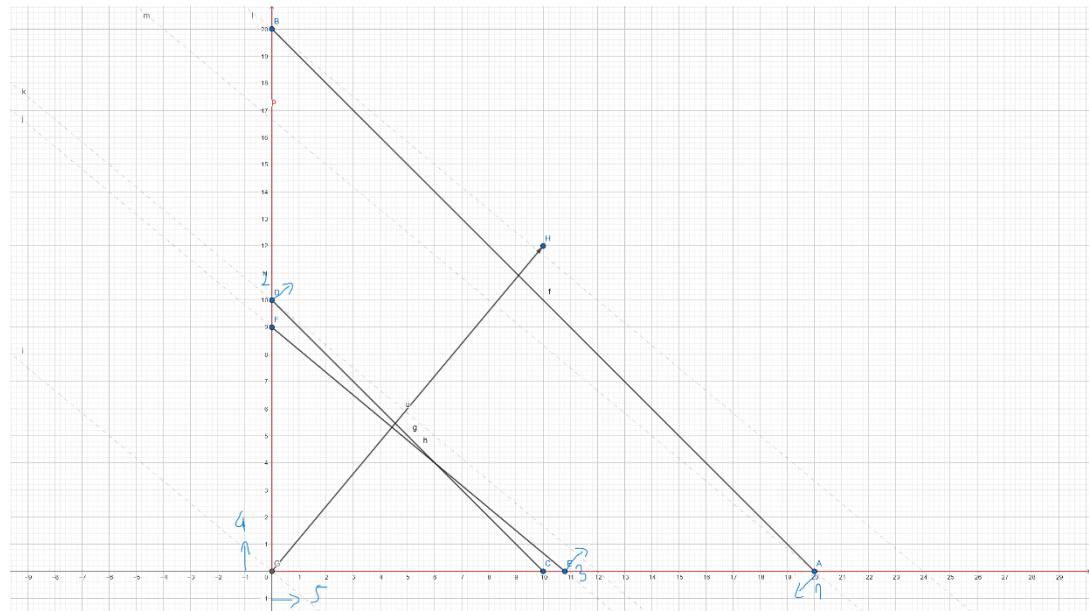
$$x_1 = 0; x_2 = 3$$

$$\text{LUCRO} = 9$$

$$x_1 = 4.5; x_2 = 1.5$$

$$\text{LUCRO} = 13.5 *$$

1.4 –



$$\text{Minimizar: CUSTO} = 10x_1 + 12x_2$$

$$x_1 = 0; x_2 = 10$$

$$\text{CUSTO} = 120$$

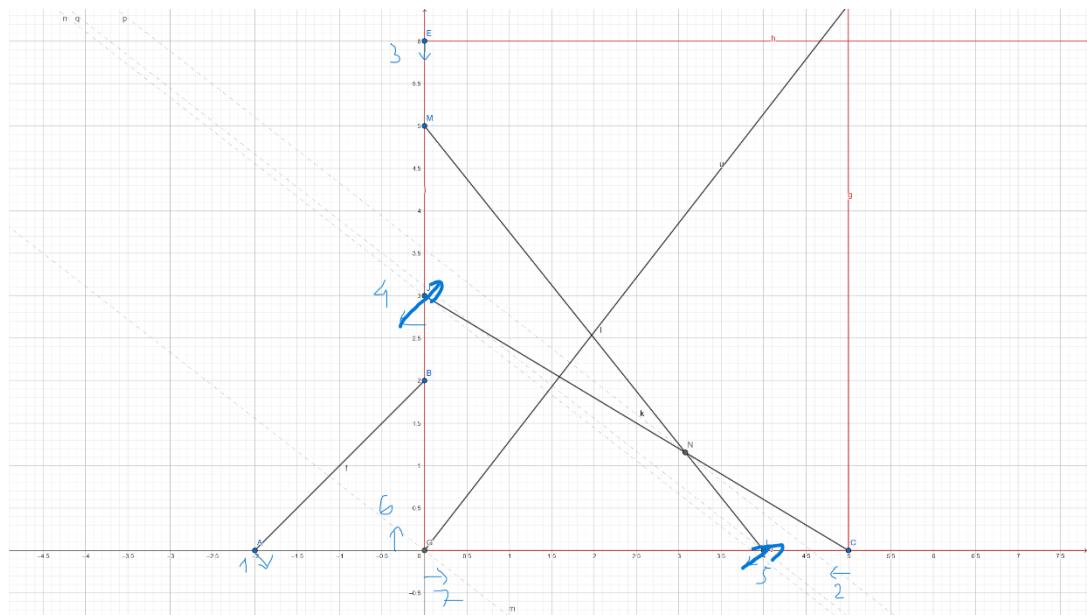
$$x_1 = 6; x_2 = 4$$

$$\text{CUSTO} = 108^*$$

$$x_1 = 10.8; x_2 = 0$$

$$\text{CUSTO} = 108 *$$

1.5 –



$$\text{Minimizar: } Z = 7x_1 + 9x_2$$

$$x_1 = 0; x_2 = 5$$

$$Z = 45$$

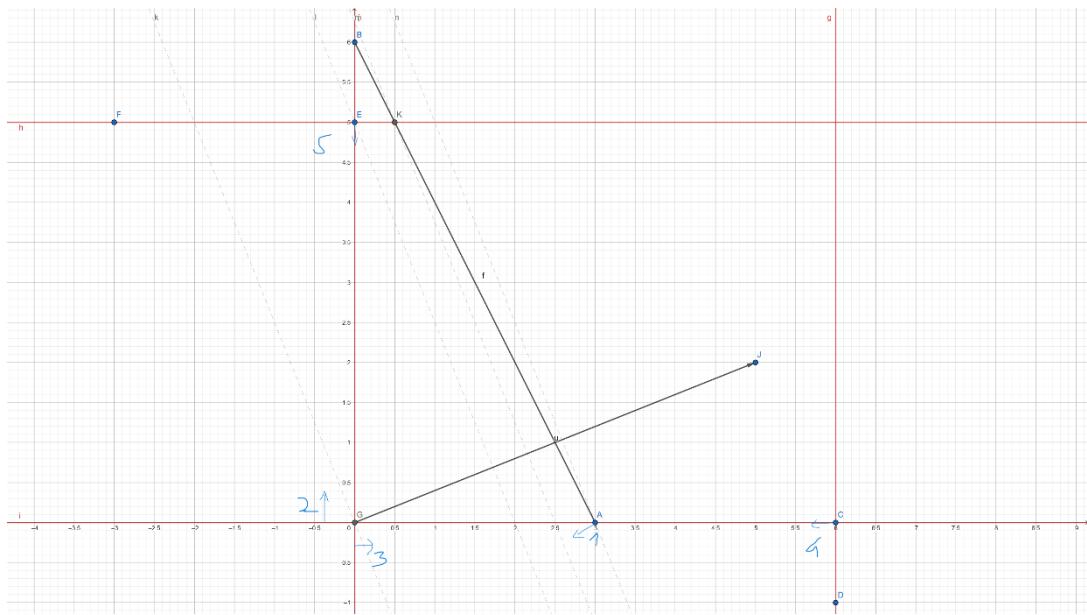
$$x_1 = 5; x_2 = 0$$

$$Z = 35$$

$$x_1 = 3.1; x_2 = 1.2$$

$$Z = 32^*$$

2 –



$$\text{Maximizar: LUCRO} = 5x_1 + 2x_2$$

$$x_1 = 0; x_2 = 5$$

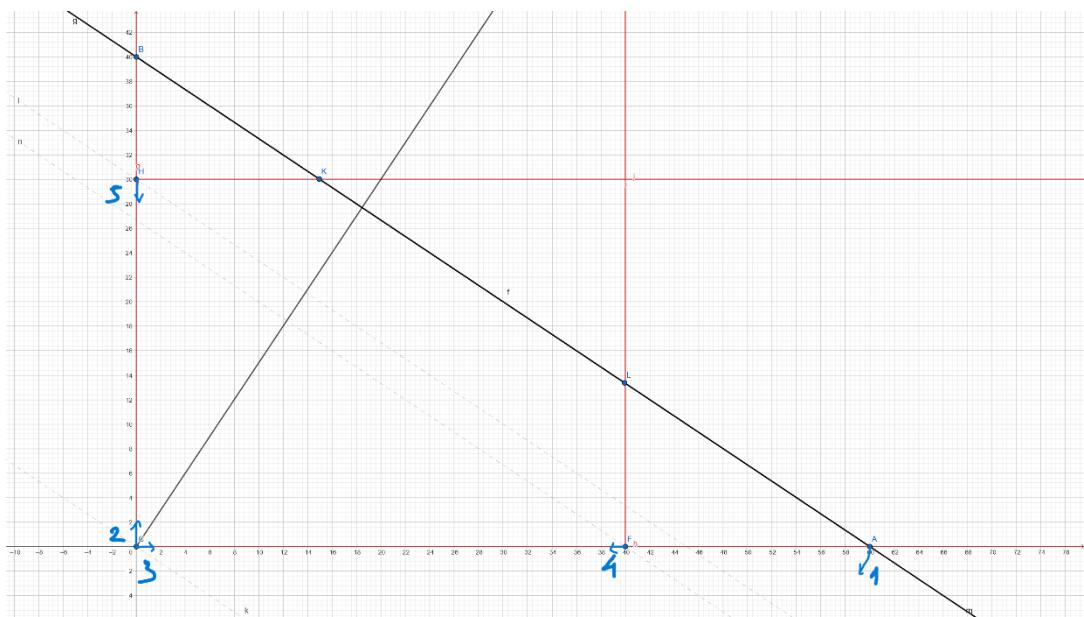
$$\text{LUCRO} = 10$$

$$x_1 = 3; x_2 = 0$$

$$\text{LUCRO} = 15^*$$

Ociosidade de 30 min/hora

3 –



$$\text{Maximizar: LUCRO} = 100x_1 + 150x_2$$

$$X_1 = 0; x_2 = 30$$

$$\text{LUCRO} = 4500$$

$$X_1 = 40; x_2 = 0$$

$$\text{LUCRO} = 4000$$

$$X_1 = 15; x_2 = 30$$

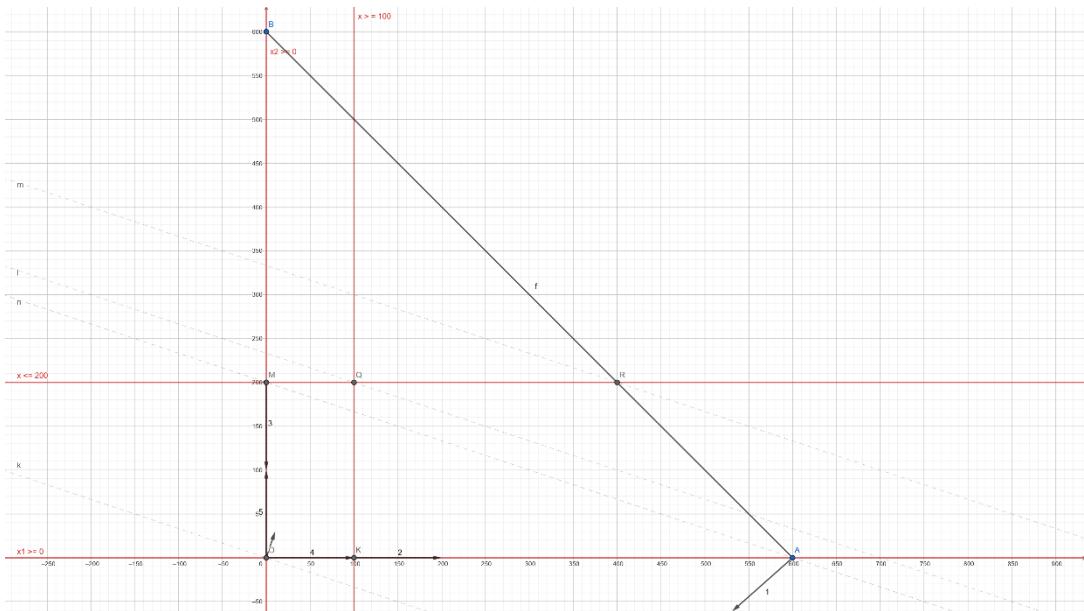
$$\text{LUCRO} = 6000^*$$

$$X_1 = 40; x_2 = 13$$

$$\text{LUCRO} = 5950$$

Ociosidade demanda P1 = 25 unidades

4 –



$$\text{Maximizar: LUCRO} = 10x_1 + 30x_2 + 4000$$

$$X_1 = 100; x_2 = 200$$

$$\text{LUCRO} = 11000$$

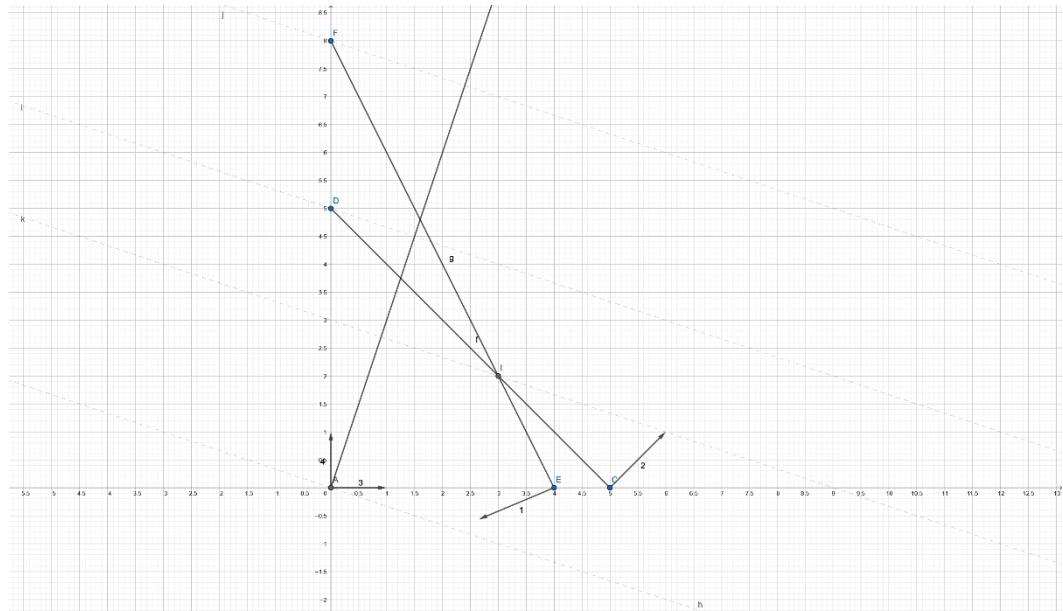
$$X_1 = 400; x_2 = 200$$

$$\text{LUCRO} = 14000^*$$

$$X_1 = 600; x_2 = 0$$

$$\text{LUCRO} = 10000$$

5 –



$$\text{Maximizar: } \text{TELE} = 30000x_1 + 10000x_2$$

$$x_1 = 0; x_2 = 5$$

$$x_1 = 0; x_2 = 8$$

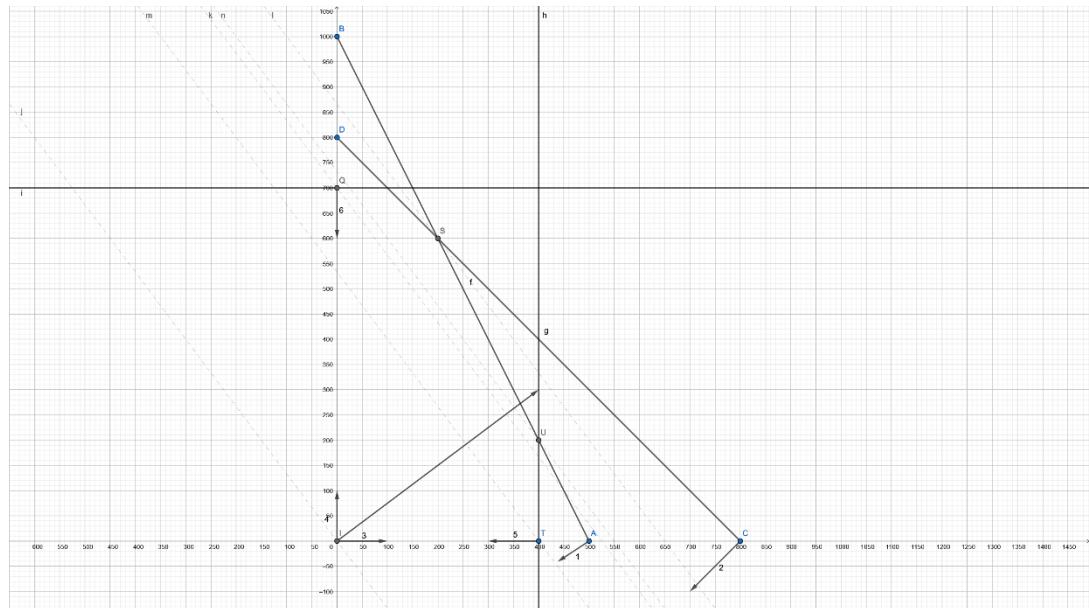
$$x_1 = 3; x_2 = 2$$

$$\text{TELE} = 50000$$

$$\text{TELE} = 80000$$

$$\text{TELE} = 110000^*$$

6 –



$$\text{Maximizar: } \text{LUCRO} = 4x_1 + 3x_2$$

$$x_1 = 0; x_2 = 700$$

$$x_1 = 200; x_2 = 600$$

$$x_1 = 400; x_2 = 200$$

$$x_1 = 400; x_2 = 0$$

$$\text{LUCRO} = 2100$$

$$\text{LUCRO} = 2600^*$$

$$\text{LUCRO} = 2200$$

$$\text{LUCRO} = 1600$$

7-

7-

$x_1 \rightarrow$  días fábrica 1

$x_2 \rightarrow$  días fábrica 2

F.O. minimizar

$$x_1 \cdot 1000 + x_2 \cdot 2000$$

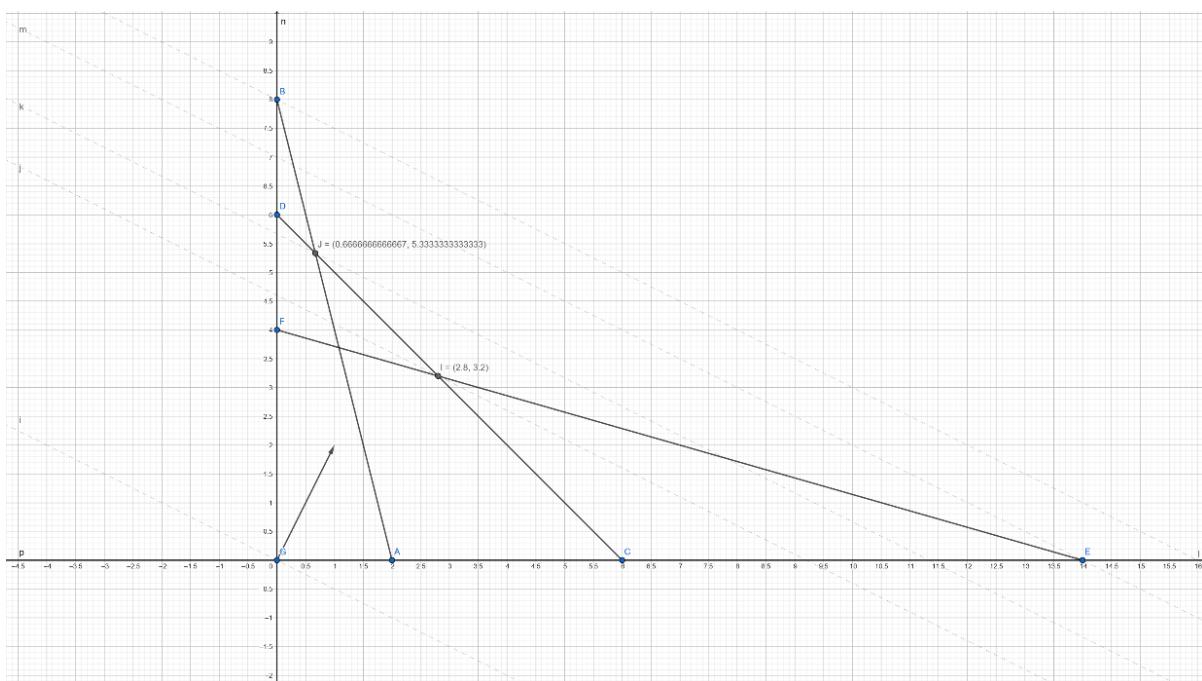
rest.

$$x_1 \cdot 8 + x_2 \cdot 2 \geq 16$$

$$x_1 \cdot 1 + x_2 \cdot 1 \geq 6$$

$$x_1 \cdot 2 + x_2 \cdot 7 \geq 28$$

$$x_1, x_2 \geq 0$$



Minimizar: MINIMIZAR =  $1000 x_1 + 2000 x_2$

$$x_1 = 0.67; x_2 = 5.33$$

$$x_1 = 2.8; x_2 = 3.2$$

$$\text{LUCRO} = 11330$$

$$\text{LUCRO} = 9200^*$$

8 -

8 -

$x_1 \rightarrow$  nº de caminhões do tipo A

$x_2 \rightarrow$  nº de caminhões do tipo B

f.o. minimizar

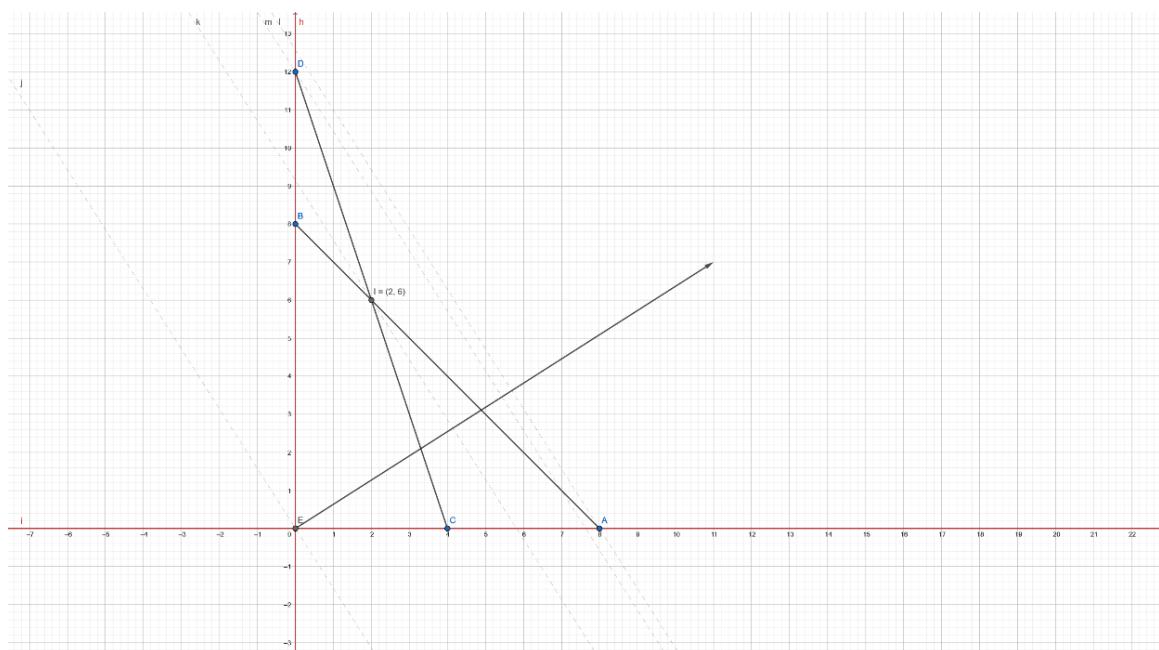
$$x_1 \cdot 1100 + x_2 \cdot 700$$

rest.

$$x_1 \cdot 2 + x_2 \cdot 2 \geq 16$$

$$x_1 \cdot 3 + x_2 \cdot 1 \geq 12$$

$$x_1, x_2 \geq 0$$



MINIMIZAR: CUSTO =  $1100 x_1 + 750 x_2$

$x_1=2$   $x_2=6$

CUSTO = 6700\*

9.1 -

9 -  $x_1 \rightarrow$  quantidade de P<sub>1</sub>

$x_2 \rightarrow$  quantidade de P<sub>2</sub>

A) ~~Maximizar~~

P.O ~~Maximizar~~

$$x_1 \cdot 1900 + x_2 \cdot 2100$$

rest.

$$x_1 \cdot 4 + x_2 \cdot 2 \leq 20$$

$$x_1 \cdot 2 + x_2 \cdot 3 \leq 10$$

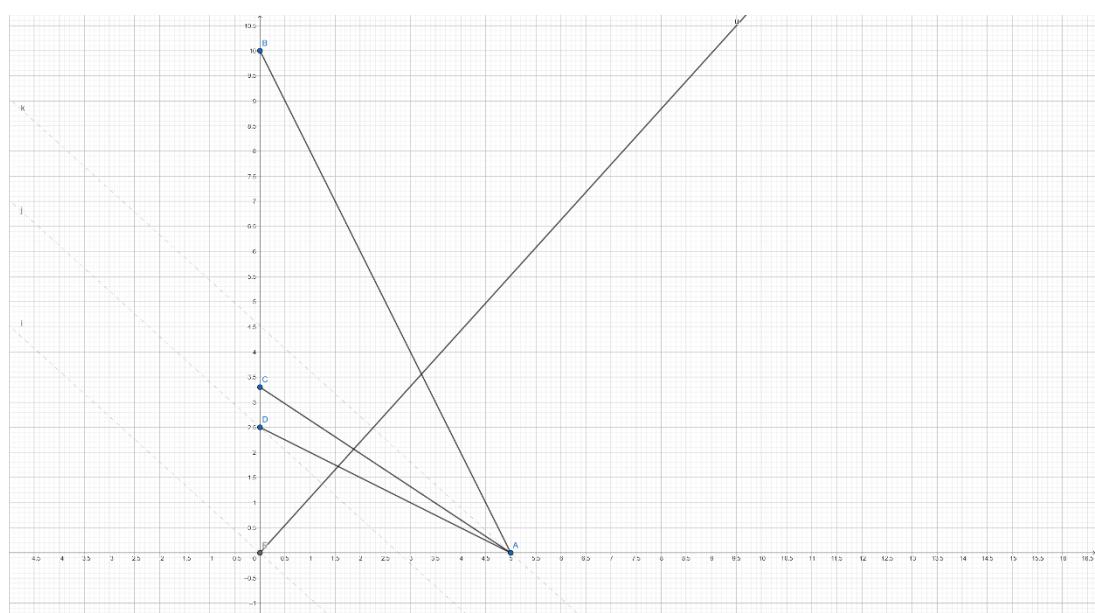
$$x_1 \cdot 100 + x_2 \cdot 200 \leq 500$$

$$x_1, x_2 \geq 0$$

B)

$$x_1 \cdot (1900 - 600 - 200 - 100) + x_2 \cdot (2100 - 300 - 300 - 200)$$

$$x_1 \cdot 1000 + x_2 \cdot 1300$$



Maximizar: LUCRO =  $1900x_1 + 2100x_2$

$$x_1=0; x_2=3.3$$

$$x_1 = 0; x_2 = 2.5$$

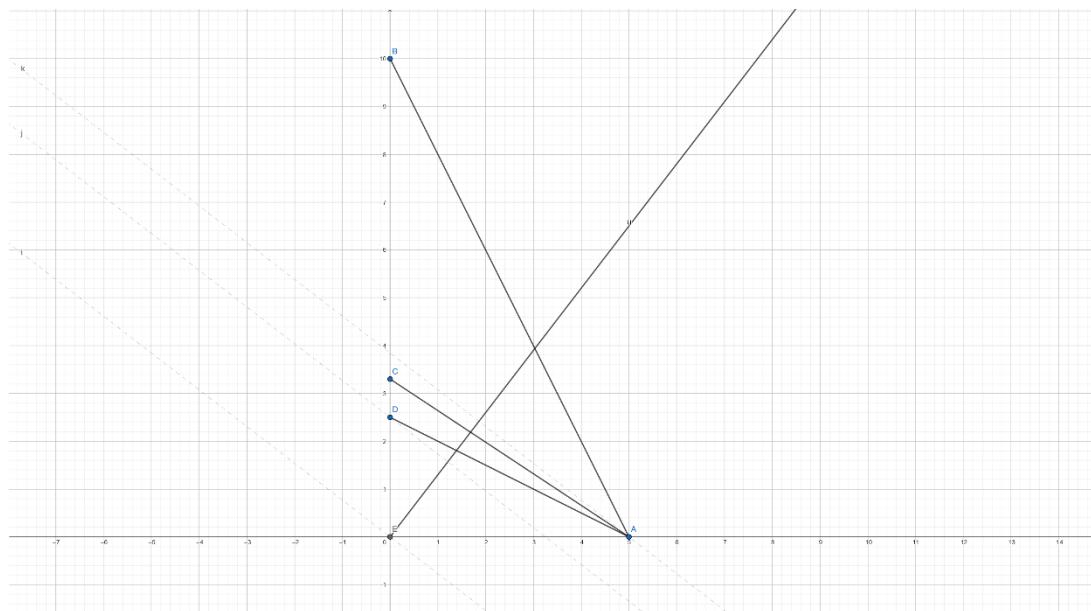
$$x_1 = 5; x_2 = 0$$

$$\text{LUCRO} = 6930$$

$$\text{LUCRO} = 5250$$

$$\text{LUCRO} = 9500^*$$

9.2 -



$$\text{Maximizar: LUCRO} = 1000x_1 + 1300x_2$$

$$x_1=0; x_2=3.3$$

$$x_1 = 0; x_2 = 2.5$$

$$x_1 = 5; x_2 = 0$$

$$\text{LUCRO} = 42900$$

$$\text{LUCRO} = 3250$$

$$\text{LUCRO} = 5000^*$$

10 -

10 -

$x_1 \rightarrow$  quantidade de gasolina

$x_2 \rightarrow$  quantidade de gasóleo

f.o.

maximizar

$$x_1 \cdot 7 + x_2 \cdot 5$$

rest.

$$x_1 \leq 400\ 000$$

$$x_2 \leq 450\ 000$$

$$a \cdot x_1 + b \cdot x_2 \leq c$$

$$x_2 = 0 \rightarrow x_1 \leq \frac{c}{a} = \frac{c}{7} = 500\ 000$$

$$x_1 = 0 \rightarrow x_2 \leq \frac{c}{b} = \frac{c}{5} = 600\ 000$$

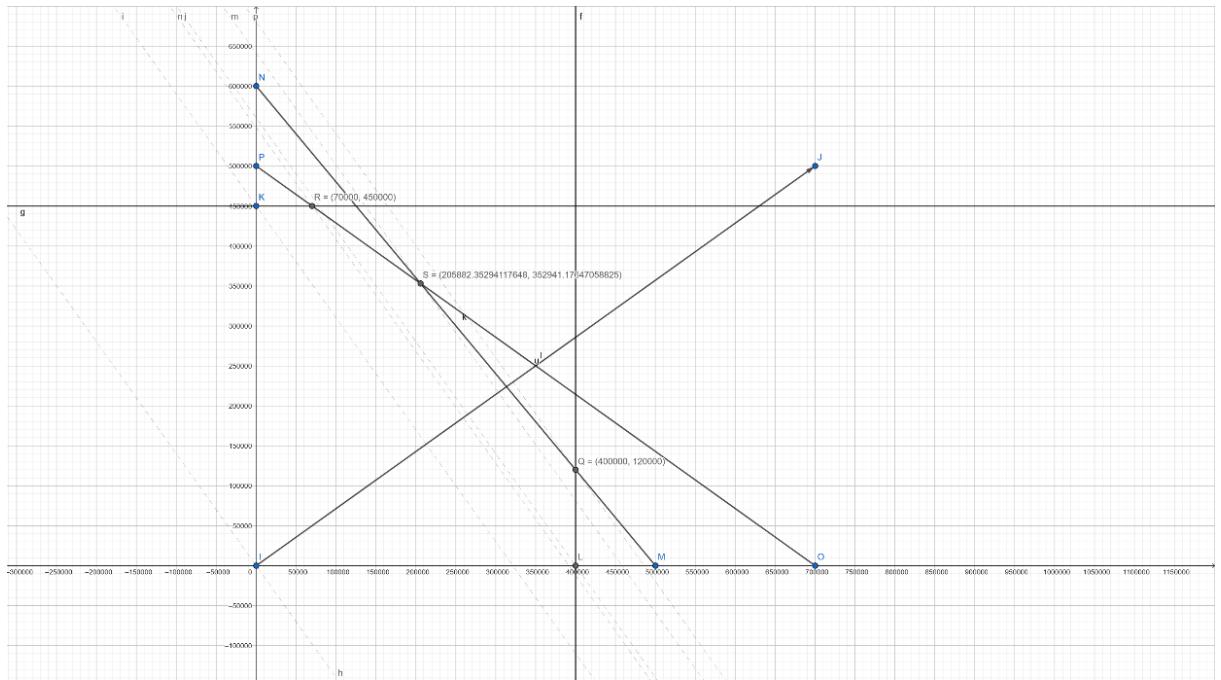
$$\Rightarrow \frac{a}{c} x_1 + \frac{b}{c} x_2 \leq 1$$

$$\frac{x_1}{500\ 000} + \frac{x_2}{600\ 000} \leq 1$$

$$x_2 = 0 \rightarrow x_1 \leq \frac{c}{a} = \frac{c}{7} = 700\ 000$$

$$x_1 = 0 \rightarrow x_2 \leq \frac{c}{b} = \frac{c}{5} = 500\ 000$$

$$\Rightarrow \frac{x_1}{700\ 000} + \frac{x_2}{500\ 000} \leq 1$$



Maximizar: LUCRO =  $7x_1 + 5x_2$

$$X_1=70K; x_2=450K$$

$$\text{LUCRO}=2740000$$

$$X_1=206K; x_2=353K$$

$$\text{LUCRO}=3207000$$

$$X_1=400K; x_2=120K$$

$$\text{LUCRO}=3400000$$

# Freshly

Freshly is a local business that sells bottled fresh coconut water and fresh orange juice to local groceries on a daily basis. They hired Mr. Mip to help them plan their operations to optimize profit.

Although it takes only about 54 seconds to process one coconut and about 30 seconds to process one kilogram of oranges, Freshly operates for only 3 hours every morning because they have to ship the bottled products to the market by 8:00 AM. Another important fact is that Freshly has only one piece of equipment to process both coconuts and oranges, so they must process them sequentially.

On average, one coconut yields 450ml of coconut water, and one kilogram of oranges yields 520ml of orange juice.

In addition, Freshly must ship at least 30 bottles of orange juice and at most 100 bottles in total every day. These are all 500ml bottles.

Freshly's profit is 0.85 dollars per liter of coconut water (i.e.,  $0.85 \cdot 0.450 = 0.3825$  dollars per coconut) and 0.78 dollars per liter of orange juice (i.e.,  $0.78 \cdot 0.520 = 0.4056$  dollars per kilogram of orange).

Mr. Mip's job is to prescribe how many coconuts and how many kilograms of oranges Freshly should process daily to maximize their total profit.



Var :  $x_1 \rightarrow \text{no of coconuts}$   
 $x_2 \rightarrow \text{no of Kgs of oranges}$

F.O. : maximize  
 $x_1 \cdot 0,3825 + x_2 \cdot 0,4056$

Const.: (I)  $3h \rightarrow 180 \text{ min.} \rightarrow 10800 \text{ sec.}$   
 $x_1 \cdot 54 + x_2 \cdot 30 \leq 10800 \rightarrow x_1 \cdot 9 + x_2 \cdot 5 \leq 1800$

(II) 30 bottles of OJ  $\rightarrow 30 \cdot 500 \text{ (ml)} = 15000 \text{ ml}$   
 $1 \text{ Kg} = 520 \text{ ml} \Rightarrow 15000 / 520 = 28,85 \text{ Kg}$   
 $x_2 \geq 28,85$

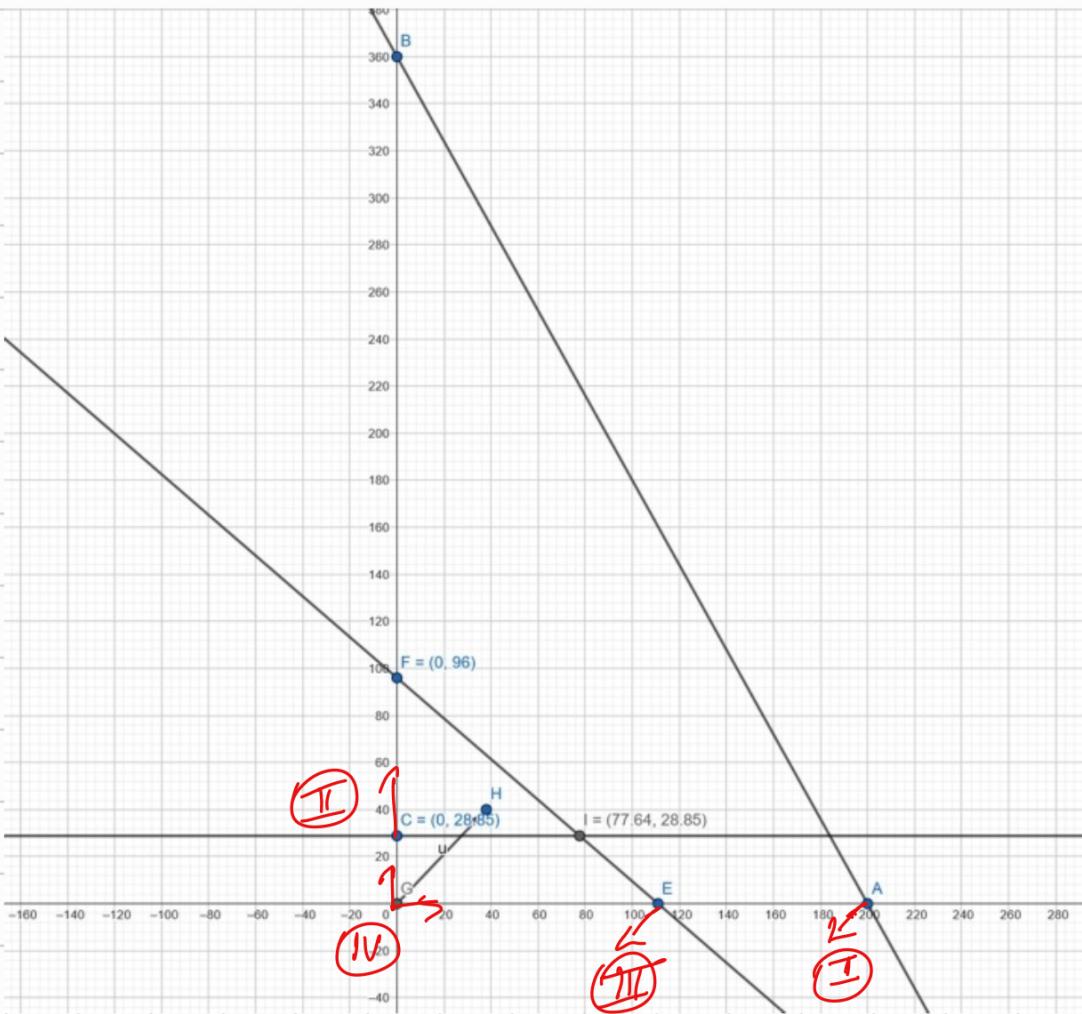
(III)

$\frac{450}{500} = 0,9$	$\frac{520}{500} = 1,04$
-------------------------	--------------------------

$$x_1 \cdot 0,9 + x_2 \cdot 1,04 \leq 100$$

(IV)

$$x_1, x_2 \geq 0$$



Sol.:  $X_1 = 0$    Profit: 38,94 \$  
 $X_2 = 96$

$X_1 = 0$    Profit: 11,70 \$  
 $X_2 = 28,85$

$X_1 = 77,64$    Profit: 41,47 \$ \*  
 $X_2 = 28,85$

# Freshly - Espeço Vetalcial

$$\text{max. } 0,3825 X_1 + 0,4056 X_2$$

$$54 X_1 + 30 X_2 \leq 10800$$

$$0,45 X_1 + 0,52 X_2 \leq 50 \rightarrow 0,45 X_1 + 0,52 X_2 + X_{f_1} = 50$$

$$0,52 X_2 \geq 15$$

$$0,52 X_2 - X_{f_3} = 15$$

$$X_1, X_2 \geq 0$$

$$\begin{array}{c} \left[ \begin{array}{ccccc} 54 & 30 & 1 & 0 & 0 \\ 0,45 & 0,52 & 0 & 1 & 0 \\ 0 & 0,52 & 0 & 0 & -1 \end{array} \right] \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_{f_1} \\ X_{f_2} \\ X_{f_3} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix} \quad (3 \times 1) \\ (3 \times 5) \end{array}$$

$X_{f_1}, X_{f_2}, X_{f_3} \rightarrow \text{base}$

$X_1, X_2 \rightarrow \text{não base}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X_{e_1} \\ X_{e_2} \\ X_{e_3} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix} \rightarrow \begin{array}{l} X_{f_1} = 10800 \\ X_{f_2} = 50 \\ X_{f_3} = -15 \end{array}$$

$X_2, X_{e_1}, X_{e_2} \rightarrow \text{base}$

$X_1, X_{e_3} \rightarrow \text{não base}$

$$\begin{bmatrix} 30 & 1 & 0 \\ 0,52 & 0 & 1 \\ 0,52 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_{e_1} \\ X_{e_2} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix} \quad \begin{array}{l} 30 X_2 + X_{f_1} = 10800 \\ 0,52 X_2 + X_{f_2} = 50 \\ 0,52 X_2 = 15 \end{array}$$

$$X_2 = \frac{15}{0,52} = 28,84$$

$$30 (X_2 = 28,84) + X_{f_1} = 10800$$

$$X_{f_1} = 10800 - 865,20$$

$$X_{f_2} = 50 - (X_2 = 28,84) \cdot 0,52$$

$$X_{f_2} = 9934,60$$

$$X_{f_3} = 35$$

$X_1, X_2, X_{f_1} \rightarrow$  base  
 $X_{f_2}, X_{f_3} \rightarrow$  nad base

$$\left[ \begin{array}{ccc} 54 & 30 & 1 \\ 0,45 & 0,52 & 0 \\ 0 & 0,52 & 0 \end{array} \right] \left[ \begin{array}{c} X_1 \\ X_2 \\ X_{f_1} \end{array} \right] = \left[ \begin{array}{c} 10800 \\ 50 \\ 15 \end{array} \right]$$

I  $54X_1 + 30X_2 + X_{f_1} = 10800$   
 II  $0,45X_1 + 0,52X_2 = 50$   
 III  $0,52X_2 = 15$

$$\text{III} \rightarrow X_c = \frac{15}{0,52} = \frac{1500}{52}$$

$$\text{II} \rightarrow 0,45X_1 + \frac{52}{100} \cdot \frac{1500}{52} = 50 \rightarrow \frac{45}{100}X_1 = 50 - 15$$

$$\frac{45}{100}X_1 = 35 \rightarrow X_1 = \frac{3500}{45:5} = \frac{700}{9}$$

$$\text{I} \Rightarrow \cancel{54} \cdot \frac{700}{9} + 30 \cdot \frac{1500}{52} + X_{f_1} = 10800$$

$$4200 + \frac{45000}{52} + X_{f_1} = 10800$$

$$X_{f_1} = 10800 - 4200 - \frac{45000}{52} = \frac{6600 \cdot 52}{52} - \frac{45000}{52} = \frac{298200}{52}$$

$$X_1 = \frac{700}{9} = 77,78 \quad X_2 = \frac{1500}{52} = 28,84 \quad X_{f_1} = 5739,61$$

# Pink Pig

Pink Pig is a local pig farm. Among the challenges they face in raising healthy animals is to compose a nutritious diet as cheaply as possible.

A good pig diet must contain sufficient amount of energy and protein. Pink Pig relies on a mix of rice and corn to supply these nutrients.

The tables below contain the cost of each ingredient, the minimum and maximum recommended daily intake of each nutrient, and the quantity of each nutrient in each ingredient.

- Foods

Food Name	Cost (\$/Kg)
rice	1.50
corn	2.23

- Nutrients

Nutrient Name	Min Intake (Kg)	Max Intake (Kg)
carbohydrates	6.0	7.5
protein	0.9	1.6

- Nutrients per Foods

Food ID	Nutrient ID	Quantity (Kg)
rice	carbohydrates	0.77
rice	protein	0.08
corn	carbohydrates	0.66
corn	protein	0.14

Var.:  $x_1 \rightarrow$  rice amount

$x_2 \rightarrow$  corn amount

TF. → minimize

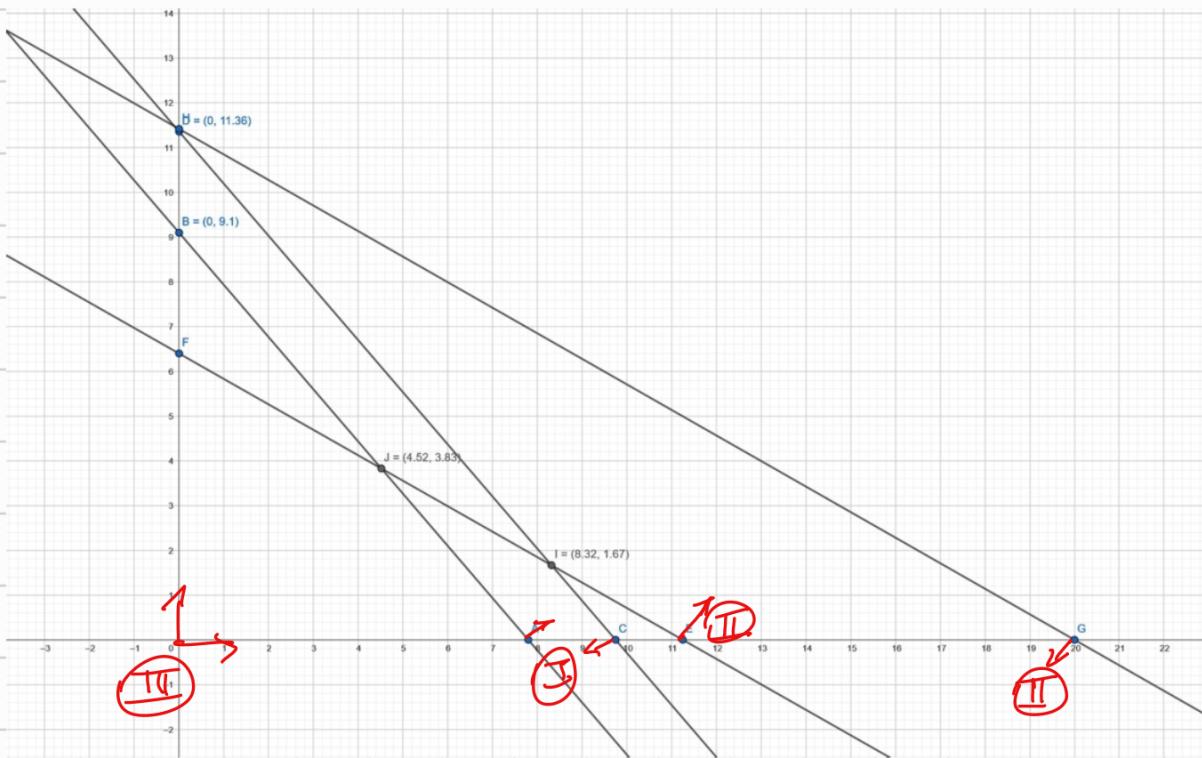
$$\text{Cost: } x_1 \cdot 1,5 + x_2 \cdot 2,23$$

Const.:

(I) Carbo.:  $6 \leq x_1 \cdot 0,77 + x_2 \cdot 0,66 \leq 7,5$

(II) Prot.:  $0,9 \leq x_1 \cdot 0,08 + x_2 \cdot 0,14 \leq 1,6$

(III)  $x_1, x_2 \geq 0$



SoJ.:  $x_1 = 0$  Cost: 25, 33

$$x_{II} = 11,36$$

$x_1 = 0$  Cost: 20, 29

$$x_{II} = 9,1$$

$x_1 = 4,52$  Cost: 15, 32 \*

$$x_{II} = 3,83$$

$x_1 = 8,32$  Cost: 21, 06

$$x_{II} = 1,67$$