Freshly

Freshly is a local business that sells bottled fresh coconut water and fresh orange juice to local groceries on a daily basis. They hired Mr. Mip to help them plan their operations to optimize profit.

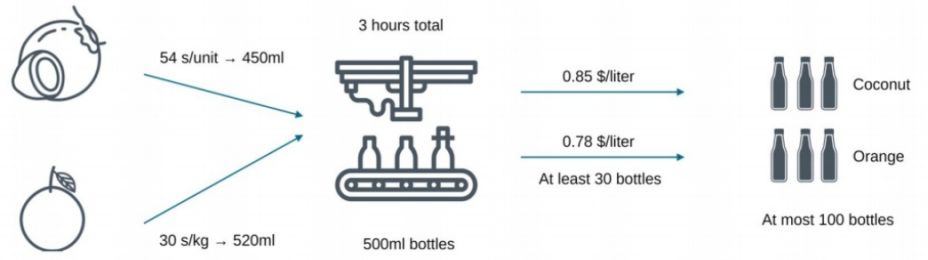
Although it takes only about 54 seconds to process one coconut and about 30 seconds to process one kilogram of oranges, Freshly operates for only 3 hours every morning because they have to ship the bottled products to the market by 8:00 AM. Another important fact is that Freshly has only one piece of equipment to process both coconuts and oranges, so they must process them sequentially.

On average, one coconut yields $450ml$ of coconut water, and one kilogram of oranges yields $520ml$ of orange juice.

In addition, Freshly must ship at least 30 bottles of orange juice and at most 100 bottles in total every day. These are all $500ml$ bottles.

Freshly's profit is 0.85 dollars per litter of coconut wather (i.e., $0.85 \cdot 0.450 = 0.3825$ dollars per coconut) and 0.78 dollars per litter of orange juice (i.e., $0.78 \cdot 0.520 = 0.4056$ dollars per kilogram of orange).

Mr. Mip's job is to prescribe how many coconuts and how many kilograms of oranges Freshly should process daily to maximize their total profit.



Vars: $X_1 \rightarrow$ n° of coconuts

$X_2 \Rightarrow$ n° of Kgs of oranges

F.O.: maximize

$$X_1 \cdot 0,3825 + X_2 \cdot 0,4056$$

Const.: (I) $3h \rightarrow 180\ min. \rightarrow 10800\ sec.$

$$X_1 \cdot 54 + X_2 \cdot 30 \leq 10800 \rightarrow X_1 \cdot 9 + X_2 \cdot 5 \leq 1800$$

(II) 30 bottles of OJ $\rightarrow$ $30 \cdot 500\,(ml) = 15000\ ml$

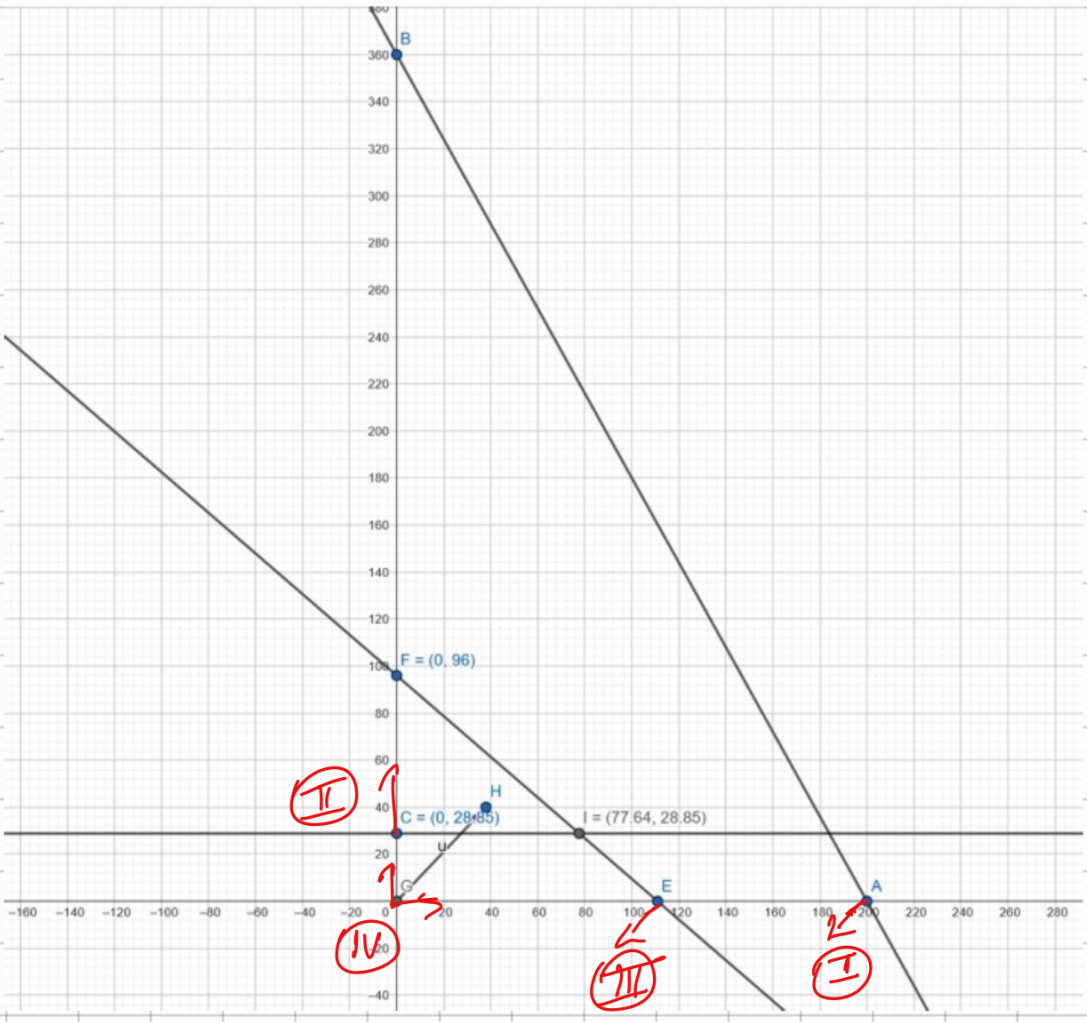
$1Kg = 520\ ml \Rightarrow 15000/520 = 28,85\ Kg$

$$X_2 \geq 28,85$$

(III) $\frac{450}{500} = 0,9$ $\qquad$ $\frac{520}{500} = 1,04$

$$X_1 \cdot 0,9 + X_2 \cdot 1,04 \leq 100$$

(IV) $X_1 ; X_2 \geq 0$



Sol.: $X_1 = 0$ Profit: 38,94 \$
$X_2 = 96$

$X_1 = 0$ Profit: 11,70 \$
$X_2 = 28,85$

$X_1 = 77.64$ Profit: 41,47 \$ *
$X_2 = 28,85$

Freshly - Espaço Vetorial

máx. $0{,}3825\,X_1 + 0{,}4056\,X_2$

$$54X_1 + 30X_2 \leq 10800 \qquad 54X_1 + 30X_2 + X_{f1} = 10800$$
$$0{,}45X_1 + 0{,}52X_2 \leq 50 \rightarrow 0{,}45X_1 + 0{,}52X_2 + X_{f2} = 50$$
$$0{,}52X_2 \geq 15 \qquad 0{,}52X_2 - X_{f3} = 15$$
$$X_1, X_2 \geq 0$$

$$\underset{(3\times 5)}{\begin{bmatrix} 54 & 30 & 1 & 0 & 0 \\ 0{,}45 & 0{,}52 & 0 & 1 & 0 \\ 0 & 0{,}52 & 0 & 0 & -1 \end{bmatrix}} \cdot \underset{(5\times 1)}{\begin{bmatrix} X_1 \\ X_2 \\ X_{f1} \\ X_{f2} \\ X_{f3} \end{bmatrix}} = \underset{(3\times 1)}{\begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix}}$$

X_{f1}, X_{f2}, X_{f3} → base

X_1, X_2 → não base

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X_{f1} \\ X_{f2} \\ X_{f3} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix} \rightarrow \begin{matrix} X_{f1} = 10800 \\ X_{f2} = 50 \\ X_{f3} = -15 \end{matrix}$$

X_2, X_{f1}, X_{f2} → base

X_1, X_{f3} → não base

$$\begin{bmatrix} 30 & 1 & 0 \\ 0{,}52 & 0 & 1 \\ 0{,}52 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_2 \\ X_{f1} \\ X_{f2} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix}$$

$$30X_2 + X_{f1} = 10800$$
$$0{,}52X_2 + X_{f2} = 50$$
$$0{,}52X_2 = 15$$

$$X_2 = \frac{15}{0{,}52} = 28{,}84$$

$$X_{f2} = 50 - (X_2 = 28{,}84) \cdot 0{,}52$$
$$X_{f2} = 35$$

$$30\,(X_2 = 28{,}84) + X_{f1} = 10800$$
$$X_{f1} = 10800 - 865{,}20$$
$$X_{f1} = 9934{,}80$$

X_1, X_2, X_{e1} → base

X_{f2}, X_{f3} → nad base

$$\begin{bmatrix} 54 & 30 & 1 \\ 0{,}45 & 0{,}52 & 0 \\ 0 & 0{,}52 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_{f1} \end{bmatrix} = \begin{bmatrix} 10800 \\ 50 \\ 15 \end{bmatrix}$$

I $54X_1 + 30X_2 + X_{f1} = 10800$

II $0{,}45X_1 + 0{,}52X_2 = 50$

III $0{,}52X_2 = 15$

III → $X_2 = \frac{15}{0{,}52} = \frac{1500}{52}$

II → $0{,}45X_1 + \frac{52}{100} \cdot \frac{1500}{52} = 50 \rightarrow \frac{45}{100} X_1 = 50 - 15$

$\frac{45}{100} X_1 = 35 \rightarrow X_1 = \frac{3500}{45} \overset{:5}{=} \frac{700}{9}$

I ⇒ $\overset{6}{54} \cdot \frac{700}{9} + 30 \cdot \frac{1500}{52} + X_{f1} = 10800$

$4200 + \frac{45000}{52} + X_{f1} = 10800$

$X_{f1} = 10800 - 4200 - \frac{45000}{52} = \frac{6600 \cdot 52}{52} - \frac{45000}{52} = \frac{298200}{52}$

$X_1 = \frac{700}{9} = 77{,}78 \quad X_2 = \frac{1500}{52} = 28{,}84 \quad X_{f1} = 5739{,}61$

PinK Pig

PinkPig is a local pig farm. Among the challenges they face in rising healthy animals is to compose a nutritious diet as cheaply as possible.

A good pig diet must contain sufficient amount of energy and protein. PinkPig relly on a mix of rice and corn to supply these nutrients.

The tables below contain the cost of each ingredient, the minimum and maximum recommended daily intake of each nutrient, and the quantity of each nutrient in each ingredient.

- Foods

Food Name	Cost ($/Kg)
rice	1.50
corn	2.23

- Nutrients

Nutrient Name	Min Intake (Kg)	Max Intake (Kg)
carbohydrates	6.0	7.5
protein	0.9	1.6

- Nutrients per Foods

Food ID	Nutrient ID	Quantity (Kg)
rice	carbohydrates	0.77
rice	protein	0.08
corn	carbohydrates	0.66
corn	protein	0.14

Var.: $X_1 \rightarrow$ rice amount

$X_2 \rightarrow$ corn amount

TF.: minimize

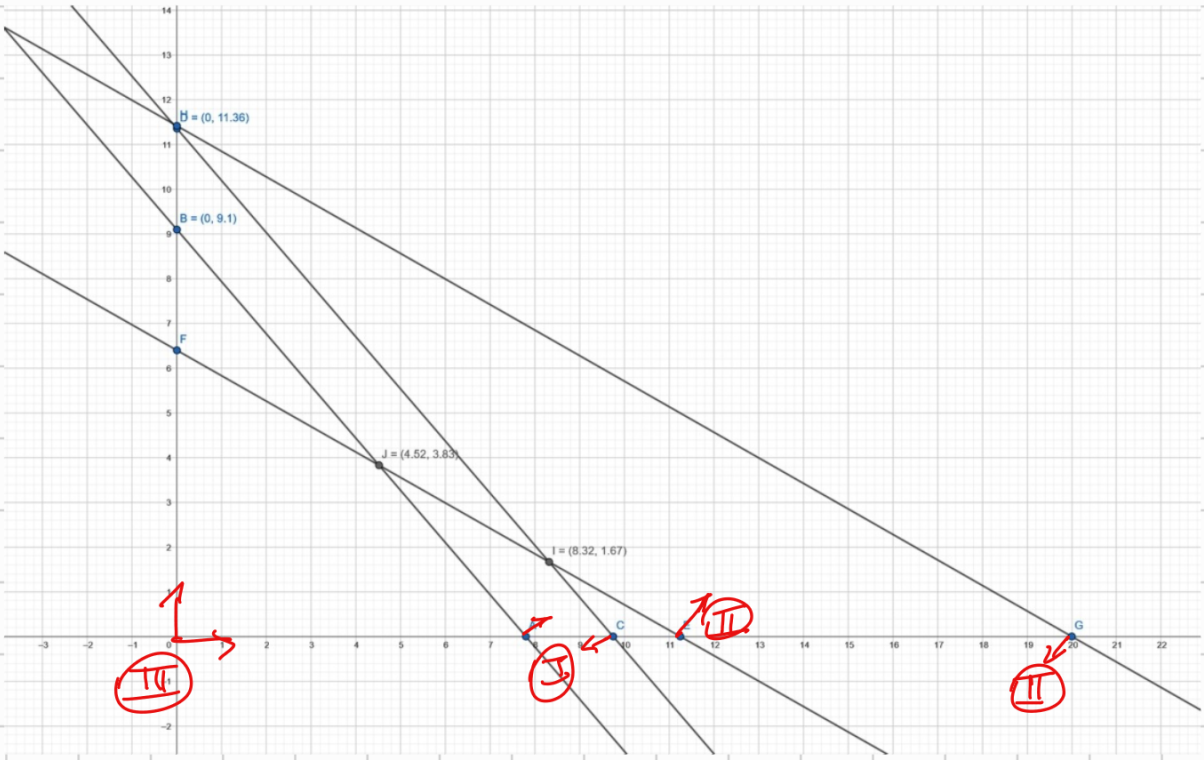
Cost: $X_1 \cdot 1,5 + X_2 \cdot 2,23$

Const.:

(I) Carbo.: $6 \leq X_1 \cdot 0,77 + X_2 \cdot 0,66 \leq 7,5$

(II) Prot.: $0,9 \leq X_1 \cdot 0,08 + X_2 \cdot 0,14 \leq 1,6$

(III) $X_1 ; X_2 \geq 0$



Sol.: $X_I = 0$ Cost: 25,33
$X_{II} = 11,36$

$X_I = 0$ Cost: 20,29
$X_{II} = 9,1$

$X_I = 4,52$ Cost: 15,32 *
$X_{II} = 3,83$

$X_I = 8,32$ Cost: 21,06
$X_{II} = 1,67$

Avengers Problem

Var.: $X_n \rightarrow$ nº of attacks by ...

$X_1 \rightarrow$ Strange

$X_2 \rightarrow$ Iron Man

$X_3 \rightarrow$ Thor

$X_4 \rightarrow$ Captain America

$X_5 \rightarrow$ Black Widow

$X_6 \rightarrow$ Spider Man

$X_7 \rightarrow$ Star-Lord

time span: 2 min
120 sec.

T.F.: maximize

$$X_1 \cdot 24 + X_2 \cdot 21 + X_3 \cdot 30 + X_4 \cdot 13 + X_5 \cdot 8 + X_6 \cdot 18 + X_7 \cdot 6$$

Const.:

$X_1 \cdot 33 \leq 250$
$X_2 \cdot 21 \leq 200$
$X_3 \cdot 35 \leq 300$
$X_4 \cdot 13 \leq 130$
$X_5 \cdot 11 \leq 90$
$X_6 \cdot 18 \leq 120$
$X_7 \cdot 7 \leq 50$

$X_1 \cdot 22 \leq 120$
$X_2 \cdot 20 \leq 120$
$X_3 \cdot 25 \leq 120$
$X_4 \cdot 10 \leq 120$
$X_5 \cdot 19 \leq 120$
$X_6 \cdot 24 \leq 120$
$X_7 \cdot 14 \leq 120$

$X_6 \geq 5$
$X_3 \leq 6$
$X_1 \leq 2X_2$

$X_2 + X_4 + X_5 + X_7 \geq X_1 + X_3$