Part I Foundations of Robotics

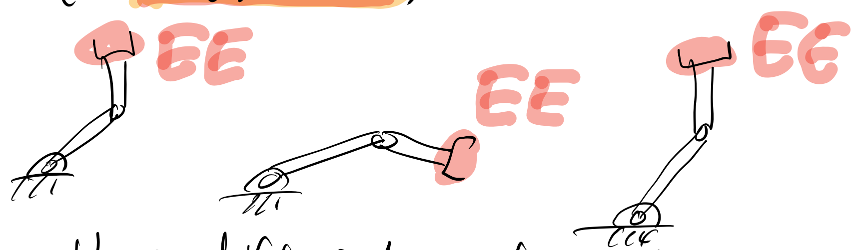
Chapter 1 Introduction

motion

A. Analysis

a. Zero-order analysis : Kinematics & Inverse Kinematics

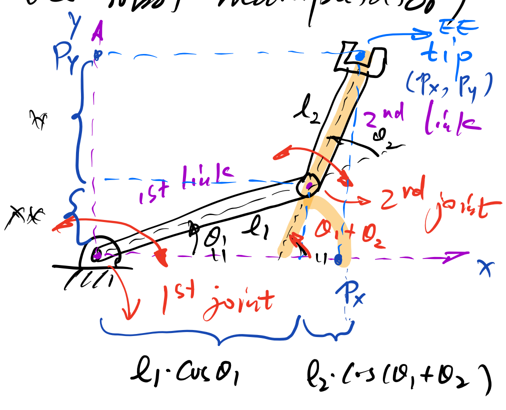
Kinematics : Given a robot manipulator (arm) with a specific configuration,



three different $\underbrace{\text{configurations}}_{\text{position \& orientation}}$

find the position and orientation of its end-effector (tip of the robot manipulator)

Example : The 2 degree-of-freedom (dof) planar robot as shown on the right.



P

$$\begin{cases} P_x = l_1\cdot\cos\theta_1 + l_2\cos(\theta_1+\theta_2) \\ P_y = l_1\cdot\sin\theta_1 + l_2\sin(\theta_1+\theta_2) \end{cases}$$

The above defines the kinematics of this robot manipulator

The orientation can be represented by $\theta_1+\theta_2$

$$\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} \underset{\text{Inverse Kinematics}}{\xrightarrow{\text{Kinematics}}} \begin{bmatrix}P_x\\ P_y\end{bmatrix}$$

Inverse Kinematics: Given desired position and orientation of the end-effector (e.e.), find the robot configurations that define such position and orientation of the e.e.

$$\begin{bmatrix} P_x \\ P_y \end{bmatrix} \longrightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

Example:

$$\begin{cases} P_x = l_1 \cos\theta_1 + l_2 \cdot \cos(\theta_1 + \theta_2) & (1) \\ P_y = l_1 \cdot \sin\theta_1 + l_2 \cdot \sin(\theta_1 + \theta_2) & (2) \end{cases}$$

$$\Rightarrow \begin{cases} [P_x - l_1 \cos\theta_1 = l_2 \cos(\theta_1 + \theta_2)]^2 + \\ [P_y - l_1 \sin\theta_1 = l_2 \cdot \sin(\theta_1 + \theta_2)]^2 \end{cases}$$

$$\Rightarrow (P_x - l_1 \cdot \cos\theta_1)^2 + (P_y - l_1 \sin\theta_1)^2 = l_2^2 \cdot 1$$

$$\Rightarrow \underbrace{\theta_1 = f(P_x, P_y) \longrightarrow \theta_2 = f(P_x, P_y)}_{\text{Inverse Kinematics}}$$

Joint Configuration $\qquad\qquad$ Tip P & O

Joint Space $\longleftrightarrow$ Cartesian Space

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \underset{\text{Inverse Kinematics}}{\overset{\text{Kinematics}}{\rightleftarrows}} \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \, \& \, \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

b. First-Order Analysis: Velocity relationship between joint space and Cartesian space

$$\dot{\Theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} \underset{\text{relationship}}{\overset{\text{Velocity}}{\rightleftarrows}} J \quad \dot{P} = \begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} \, \& \, \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

joint rotational velocity $\qquad$ linear Velocity & Angular velocity

Example: $p_x = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)$

$p_y = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)$

$$\Rightarrow \quad \dot{p}_x = -l_1 \sin\theta_1 \cdot \dot{\theta}_1 - l_2 \sin(\theta_1+\theta_2)\cdot(\dot{\theta}_1+\dot{\theta}_2)$$

$$\dot{p}_y = l_1 \cos\theta_1 \cdot \dot{\theta}_1 + l_2 \cos(\theta_1+\theta_2)\cdot(\dot{\theta}_1+\dot{\theta}_2)$$

$$\Rightarrow \quad \dot{p}_x = -[l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)]\cdot\dot{\theta}_1 - l_2 \sin(\theta_1+\theta_2)\cdot\dot{\theta}_2$$

$$\dot{p}_y = [l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)]\cdot\dot{\theta}_1 + l_2 \cos(\theta_1+\theta_2)\cdot\dot{\theta}_2$$

$$\Rightarrow \quad \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} -[l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)] & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\Rightarrow \quad \dot{\mathbb{P}} = \underbrace{J(\Theta)}_{\text{Jacobian matrix}} \; \dot{\Theta}$$

$$\Rightarrow \quad \dot{\Theta} = J^{-1}(\Theta) \cdot \dot{\mathbb{P}}$$

Condition: J must be invertible.

C. Second-Order Analysis: Acceleration & Forces/Torques

Given the desired dynamic motion profile $(\theta, \dot{\theta}, \ddot{\theta})$ in terms of $\Theta, \dot{\Theta}, \ddot{\Theta}$ in joint space of the robot manipulator, how to find the required joint torques/forces to make this happen?

dynamics

$f = ma$

Dynamics equation in joint space:

$$\boxed{\mathbb{M}(\Theta)\ddot{\Theta} + \mathbb{V}(\Theta,\dot{\Theta}) + \mathbb{G}(\Theta) = \tau}$$

where $\mathbb{M}(\Theta)$ – $n\times n$ mass matrix, symmetric, P.D
positive definite

$\mathbb{V}(\Theta)$ – $n\times 1$ vector of Centrifugal terms
friction/velocity related

$\mathbb{G}(\Theta)$ – $n\times 1$ vector of gravitational terms

τ – $n\times 1$ vector of joint torques / forces
rotational joint / translational joint

Popular dynamic equations:

* Newton–Euler formulation (force/moment based approach)
* Lagrange formation (energy based approach)
* Regressor formulation (linearized representation in terms of dynamic parameters)

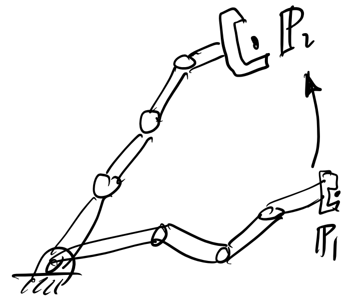
$$\underbrace{\mathbb{M}\ddot{\Theta} + \mathbb{V}(\Theta,\dot{\Theta}) + \mathbb{G}(\Theta)}_{\text{2nd-order Nonlinear differential equation in matrix form}} = \underbrace{\underbrace{\mathbb{Y}(\Theta,\dot{\Theta},\ddot{\Theta})}_{\text{regressor}} \cdot \underbrace{\rho}_{\text{dynamic parameter vector}}}_{\text{linearized equation in terms of dynamic Parameters}}$$

– N.E equation is iterative, suitable for computer algorithms.
– L-equation is a closed-form solution, suitable for analysis.

B. Control

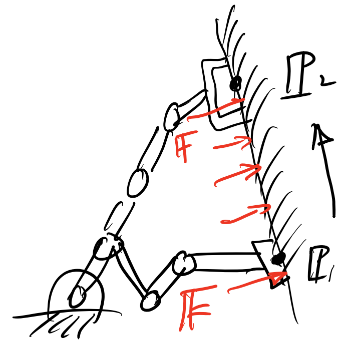
a. **Position control**: Regulation of the robot to track the desired trajectory $\Theta_d, \dot{\Theta}_d, \ddot{\Theta}_d$, to move from $\mathbb{P}_1$ to $\mathbb{P}_2$.

free space motion Control



b. Force control: Position control from $\mathbb{P}_1$ to $\mathbb{P}_2$ following a desired trajectory while maintaining a contact force $\mathbb{F}$ with the robot working environment.

Constraint motion Control



C. Notations and Conventions

* Upper case / Bold face : vector & matrices $\mathbb{M}_{n\times n}$, $\Theta_{n\times 1}$
* lower case : scalar, $a, b, c, l_1, l_2, m, \alpha, \beta$.
* Super scripts & Subscripts :

leading superscript: Reference frames, coordinates

Trailing superscript: math operation

$${}^{A}_{C}\mathbb{B}^{T}_{\alpha}$$

leading subscript: current frame/coordinates

Trailing subscript: Components/element

Example: ${}^{A}_{B}R$ - rotational relationship between $\{A\}$ and $\{B\}$, i.e, orientation of $\{B\}$ w.r.t. (with reference to) $\{A\}$.

* Trigonometric function simplification :

$\cos\theta_1 = C\theta_1 = C_1$

$\sin\theta_1 = S\theta_1 = S_1$

$\cos(\theta_1+\theta_2) = C\theta_{12} = C_{12}$

$\sin(\theta_1+\theta_2) = S\theta_{12} = S_{12}$

[DO NOT use C & S for variables in robotics. Reserve them for Sin and cos simplification]

* Vectorial Summation :

${}^0\omega_3 = {}^0\omega_1 + {}^0\omega_2 + {}^0\omega_3$ (doable, same reference frames)

${}^0\omega_3 \neq {}^0\omega_1 + {}^1\omega_2 + {}^2\omega_3$ (not doable, cannot be added since the reference frames are different!)

${}^0\omega_3 = {}^0\omega_1 + {}^0_1R \cdot {}^1\omega_2 + {}^0_1R \cdot {}^1_2R \cdot {}^2\omega_3$ (doable, can be added together since all the vectors are expressed with reference to the same frame {0})

rotational transform