b. Adaptive Control based on Computed Torque Method

Robot Model: $\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta) = Y(\Theta,\dot{\Theta},\ddot{\Theta})\cdot\varphi$

Computed Torque Method: $\tau = \alpha\tau' + \beta$

$\Rightarrow \tau = M(\Theta)\ddot{\Theta}^* + V(\Theta,\dot{\Theta}) + G(\Theta)$ (C.T.M.)

where the servo control part is

$$\tau' = \ddot{\Theta}^* = \ddot{\Theta}_d + K_V\dot{E} + K_P E \quad \text{where } E = \Theta_d - \Theta$$

If the robot dynamic model is NOT precisely known, then the computed torque method gives the control law:

$$\tau = \hat{M}(\Theta)\ddot{\Theta}^* + \hat{V}(\Theta,\dot{\Theta}) + \hat{G}(\Theta) = Y(\Theta,\dot{\Theta},\ddot{\Theta})\hat{\varphi}$$

where $\ddot{\Theta}^* = \ddot{\Theta}_d + K_V\dot{E} + K_P E$ with $E = \Theta_d - \Theta$

and $(\hat{\ })$ is an estimate (model) of ( ).

Now we can write the above control law as:

$$\tau = \hat{M}(\Theta)[\underbrace{\ddot{\Theta}_d}_{-\ddot{\Theta}} + K_V\dot{E} + K_P E] + \hat{V}(\Theta,\dot{\Theta}) + \hat{G}(\Theta)$$

$$= \hat{M}(\Theta)[\ddot{E} + K_V\dot{E} + K_P E] + \hat{M}(\Theta)\ddot{\Theta} + \hat{V}(\Theta,\dot{\Theta}) + \hat{G}(\Theta)$$

$$= \hat{M}(\Theta)[\ddot{E} + K_V\dot{E} + K_P E] + Y(\Theta,\dot{\Theta},\ddot{\Theta})\hat{\varphi}$$

Then the error dynamics is

$$\underbrace{Y(\Theta,\dot{\Theta},\ddot{\Theta})\varphi}_{\text{model}} = \underbrace{\hat{M}(\Theta)[\ddot{E} + K_V\dot{E} + K_P E] + Y(\Theta,\dot{\Theta},\ddot{\Theta})\hat{\varphi}}_{\text{Control}}$$

model + Control = Error Dynamics

$$\Rightarrow \hat{M}(\Theta)[\ddot{E} + K_V\dot{E} + K_P E] = Y(\Theta,\dot{\Theta},\ddot{\Theta})(\varphi - \hat{\varphi})$$

$$= Y(\Theta,\dot{\Theta},\ddot{\Theta})\tilde{\varphi} \qquad \varphi = \hat{\varphi}$$

multiply $\hat{M}^{-1}(\Theta)$ from left side on both sides of above eqn.

$$\boxed{\ddot{E} + K_V\dot{E} + K_P E = \underbrace{\hat{M}^{-1}(\Theta)\cdot Y(\Theta,\dot{\Theta},\ddot{\Theta})\cdot\tilde{\varphi}}_{0} \approx 0 \quad (*)}$$

If $\hat{\varphi} = \varphi$ (dynamic model is precisely known), $\tilde{\varphi} = 0$

then eqn. (*) becomes

$$\ddot{E} + K_V\dot{E} + K_P E = 0 \quad \text{(Computed Torque Method)}$$

Now since $\hat{\varphi} \neq \varphi$, since dynamic model is NOT precisely known, then we have to find a new approach.

Define $\tilde{E}_{2n\times1} = \begin{bmatrix} E_{n\times1} \\ \dot{E}_{n\times1} \end{bmatrix}_{2n\times1}$ and $B_{2n\times n} = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \end{bmatrix}_{2n\times n}$,

and $A_{2n\times2n} = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -K_{P\,n\times n} & -K_{V\,n\times n} \end{bmatrix}_{2n\times2n}$

State Space Approach

Then we can rewrite (*) in state space as follows:

$$\dot{\tilde{E}}_{2n\times1} = \underbrace{A_{2n\times2n}\tilde{E}_{2n\times1}}_{2n\times1} + \underbrace{\underbrace{B_{2n\times n}\hat{M}^{-1}_{n\times n}(\Theta)}_{2n\times n}\cdot\underbrace{Y_{n\times r}(\Theta,\dot{\Theta},\ddot{\Theta})\tilde{\varphi}_{r\times1}}_{n\times1}}_{2n\times1}$$

$2n\times1$

Error dynamic equation in state space

Apply Lyapunov stability analysis to show that this error vector $\mathbb{E}$ is asymptotically stable.

Select the Lyapunov function candidate as:

$$\underset{1\times1}{V} = \underbrace{\underset{1\times2n}{\mathbb{E}^T}\,\underset{2n\times2n}{\mathbb{P}}\,\underset{2n\times1}{\mathbb{E}}}_{>0} + \underbrace{\underset{1\times r}{\tilde\varphi^T}\,\underset{r\times r}{\Gamma^{-1}}\,\underset{r\times1}{\tilde\varphi}}_{>0} > 0 \quad \text{except } V(0)=0.$$

where $\mathbb{P}$ is a $2n\times2n$ positive definite, constant, symmetric matrix and Γ is a diagonal, positive definite $r\times r$ matrix (constant). That is Γ can be written as

$$\Gamma = \begin{bmatrix} \gamma_1 & & & 0 \\ & \gamma_2 & & \\ & & \ddots & \\ 0 & & & \gamma_r \end{bmatrix}_{r\times r} = \mathrm{diag}(\gamma_1, \gamma_2, \dots \gamma_r)$$

where γ_i's are positive scalar constants.

Then $\dot V = \dot{\mathbb{E}}^T\mathbb{P}\mathbb{E} + \mathbb{E}^T\mathbb{P}\dot{\mathbb{E}} + \dot{\tilde\varphi}^T\Gamma^{-1}\tilde\varphi + \tilde\varphi^T\Gamma^{-1}\dot{\tilde\varphi}$

$\Rightarrow \dot V = [\mathbb{A}\mathbb{E} + \mathbb{B}\hat m^{-1}Y\tilde\varphi]^T\mathbb{P}\mathbb{E} + \mathbb{E}^T\mathbb{P}[\mathbb{A}\mathbb{E} + \mathbb{B}\hat m^{-1}Y\tilde\varphi] + 2\tilde\varphi^T\Gamma^{-1}\dot{\tilde\varphi}$

$\Rightarrow \dot V = -\mathbb{E}^T\mathbb{Q}\mathbb{E} + 2\tilde\varphi^T[\Gamma^{-1}\dot{\hat\varphi} + Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\mathbb{E}] \quad = 0 \quad (**)$

where $\mathbb{Q}$ is a positive definite, symmetric matrix that satisfies the Lyapunov equation:

$$\underset{2n\times2n}{\mathbb{A}^T}\underset{2n\times2n}{\mathbb{P}} + \underset{2n\times2n}{\mathbb{P}}\underset{2n\times2n}{\mathbb{A}} = -\underset{2n\times2n}{\mathbb{Q}}$$

If we want $\dot V = -\mathbb{E}^T\mathbb{Q}\mathbb{E} < 0$, $V(0) = 0$

then $2\tilde\varphi^T[\Gamma^{-1}\dot{\hat\varphi} + Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\mathbb{E}] = 0$

$\Rightarrow \Gamma^{-1}\dot{\hat\varphi} + Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\mathbb{E} = 0$

$\times\Gamma$ from left $\Rightarrow \dot{\hat\varphi} = -\Gamma\cdot Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\mathbb{E}$

Let $\dot{\hat\varphi} \triangleq \lim_{\Delta t\to0}\dfrac{\hat\varphi(t+\Delta t) - \hat\varphi(t)}{\Delta t} = -\Gamma\cdot Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\mathbb{E}$

$\Rightarrow \hat\varphi(t+\Delta t) = \hat\varphi(t) - \Gamma\cdot Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\cdot\mathbb{E}\cdot\Delta t$

If we update (identify) $\hat\varphi$ using

$$\hat\varphi_{new} = \hat\varphi_{old} - \Gamma\cdot Y^T\hat m^{-1}\mathbb{B}^T\mathbb{P}\cdot\mathbb{E}\cdot 1$$

as the adaptation law, then

$\dot V = -\mathbb{E}^T\mathbb{Q}\mathbb{E} < 0$, $V(0) = 0$

$\Rightarrow \mathbb{E} \xrightarrow{t\to\infty} 0 \Rightarrow \begin{matrix} \mathbb{E} \xrightarrow{t\to\infty} 0 \\ \dot{\mathbb{E}} \xrightarrow{t\to\infty} 0 \end{matrix}$

The system is asymptotically stable, if we use the following adaptive control law:

$$\tau = \hat{M}(\Theta)\cdot\ddot{\Theta}^* + \hat{V}(\Theta,\dot{\Theta}) + \hat{G}(\Theta) = Y(\Theta,\dot{\Theta},\ddot{\Theta})\,\hat{\varphi}$$

and the following dynamic parameter adaptation law:

$$\dot{\hat{\varphi}} = -\Gamma Y^T(\Theta,\dot{\Theta},\ddot{\Theta})\,\hat{M}^{-1}(\Theta)\mathbb{B}^T\mathbb{P}.\mathbb{E}$$

which we can use to update/identify φ, the dynamic parameter vector $\Longrightarrow$ true parameters.

Conclusion: This makes it possible that we can use adaptive control based on the Computed Torque Method to stablize the robot system and at the same time to estimate/update/identify the robot dynamic parameters simultaneously?

II. Dynamic Simulation/Animation

Problem: Given control τ, how the robot moves?

The model: $M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta) = \tau$ (***)

For simulation/animation, we need to know that under the control τ, what are Θ, $\dot{\Theta}$, $\ddot{\Theta}$?

Let's rewrite eqn. (***) as

$$\ddot{\Theta} = M^{-1}(\Theta)\left[\tau - V(\Theta,\dot{\Theta}) - G(\Theta)\right] \quad (*\text{-}1)$$

This is the acceleration caused by the control τ.

We may use the acceleration to calculate the velocity and position as follows:

Given the initial conditions on the motion of the robot, $\Theta(0) = \Theta_0$, $\dot{\Theta}(0) = \dot{\Theta}_0$ (let $\Theta_0 = \dot{\Theta}_0 = 0$), we then numerically integrate (*-1) forward in time by steps of size Δt using Euler integration, start with $t=0$, iteratively compute as follows:

$$\dot{\Theta}(t+\Delta t) = \dot{\Theta}(t) + \ddot{\Theta}(t)\cdot\Delta t \quad (*\text{-}2)$$

$$\Theta(t+\Delta t) = \Theta(t) + \dot{\Theta}(t)\cdot\Delta t + \frac{1}{2}\ddot{\Theta}(t)\cdot\Delta t \quad (*\text{-}3)$$

Eqns (*-1) to (*-3) numerically compute the acceleration $\ddot{\Theta}$, velocity $\dot{\Theta}$ and position Θ of the robot dynamic motion caused by a control input vector τ.

Review:

1° Introduction & basic concepts.

2° Space description and transformation

Kinematics, inverse kinematics (two special approaches)

3° Jacobians: Singularities and workspace

4° Dynamics: Lagrangian dynamics formulation

Different representations of dynamic formula

Regressor formulation

5° Linear & Feedback Control: linear 2^{nd}-order system

6° Nonlinear & Adaptive Control: Computed Torque method, Lyapunov stability theory, adaptive control