Step #1 Number the links and joint axes from the base to the tip of the robot manipulator.

Step #2 Attach link frames (coordinates) to each link as follows:

* Z_{i-1} - axis coincides with axis $i-1$ (joint $i-1$)
* X_{i-1} - axis coincides with the common normal between the joint axis $i-1$ and the joint axis i.
* Y_{i-1} - axis can be obtained by using the right hand rule.

Step #3. For each link, four parameters are assigned to describe the link itself and its connection to the neighboring links:

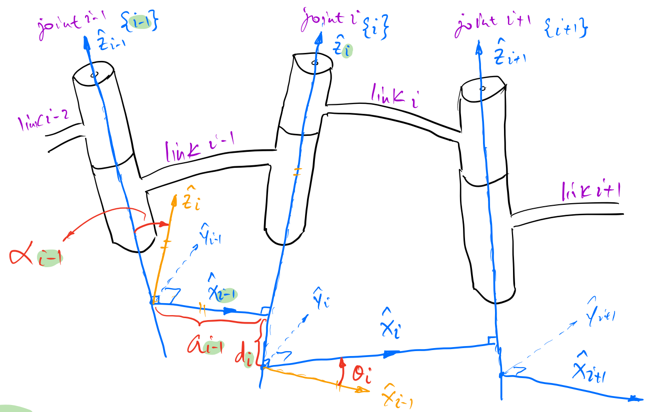
α_{i-1} - link twist (angle from $\hat{z}_{i-1}$ to $\hat{z}_i$ along $\hat{x}_{i-1}$)

a_{i-1} - link length (distance from $\hat{z}_{i-1}$ to $\hat{z}_i$ along $\hat{x}_{i-1}$)

d_i - link offset (distance from $\hat{x}_{i-1}$ to $\hat{x}_i$ along $\hat{z}_i$)

θ_i - joint angle (angle from $\hat{x}_{i-1}$ to $\hat{x}_i$ along $\hat{z}_i$)

Illustration:



Remarks:

#1. Each frame, say frame $\{i\}$, is attached rigidly to its link (i) and moves with the link.

#2. α_{i-1}, a_{i-1} are always fixed (robot parameters).
when joint i is rotational, θ_i is variable, d_i is fixed.
when joint i is translational, θ_i is fixed, d_i is variable.

#3. Frame $\{0\}$ is attached to the base and is always fixed with $\hat{z}_i$-axis aligned with joint axis i (i.e., $\hat{z}_i$-axis) and we have $\alpha_0 = 0$ and $a_0 = 0$.

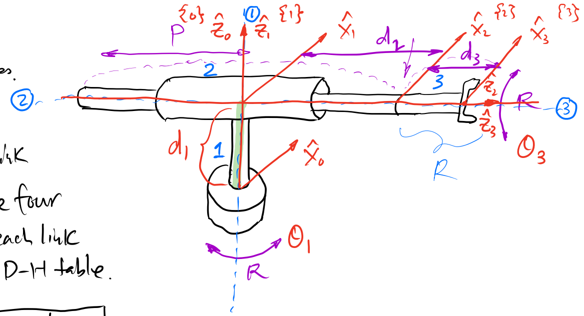
#4 In case there is no joint at the end-effector (tip) of the robot manipulator, a frame obtained by a translation may be attached for convenience.

Example. A 2R-1P non-planar robot arm as shown. Find the D-H table for the robot arm.

Step #1: Number the links and axes.

Step #2: Attach frames to each link

Step #3: Find the four parameters for each link and form the D-H table.



link	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	d_1	θ_1
2	90°	0	d_2	0°
3	0°	0	d_3	θ_3

R ← θ_1 is variable, d_1 is fixed

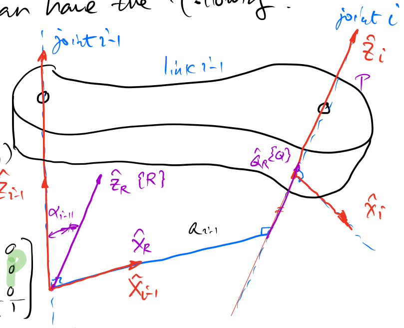
P ← θ_2 is fixed $= 0$, d_2 is variable

R ← θ_3 is variable, d_3 is fixed.

B. Robot Manipulator Kinematics

Let ${}^{i-1}_{i}T$ be the homogeneous transformation associated with frame $\{i\}$ and $\{i-1\}$ that are attached to link i and link $i-1$, respectively. Then, we can have the following:

$${}^{i-1}_{i}T = {}^{i-1}_{R}T \cdot {}^{R}_{Q}T \cdot {}^{Q}_{P}T \cdot {}^{P}_{i}T \text{, thus}$$



$${}^{i-1}_{i}T = \mathrm{ROT}(\hat{x}_{i-1}, \alpha_{i-1}) \cdot \mathrm{TRANS}(\hat{x}_{i-1}, a_{i-1}) \cdot \mathrm{ROT}(\hat{z}_i, \theta_i) \cdot \mathrm{TRANS}(z_i, \theta_i)$$

where
$$\mathrm{ROT}(\hat{x}_{i-1}, \alpha_{i-1}) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -\sin\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

R P

$$\mathrm{TRANS}(\hat{x}_{i-1}, a_{i-1}) = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

similarly,

$$\mathrm{ROT}(\hat{z}_{i-1}, \hat{\theta}_i) = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\{i-1\} \xrightarrow{R(\alpha_{i-1})} \{R\} \longrightarrow \{Q\} \longrightarrow \{P\} \longrightarrow \{i\}$$

rotation about $\hat{x}_{i-1}$ by α_{i-1} — $\mathrm{ROT}(\hat{x}_{i-1}, \alpha_{i-1})$

translation along $\hat{x}_{i-1}$ by a_{i-1} — $\mathrm{TRANS}(\hat{x}_{i-1}, a_{i-1})$

rotation about $\hat{z}_i$ by θ_i — $\mathrm{ROT}(\hat{z}_i, \theta_i)$

Translation along $\hat{z}_i$ by d_i — $\mathrm{TRAN}(\hat{z}_i, d_i)$

and
$$\mathrm{TRANS}(\hat{z}_i, d_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Then multiply together, we have

$${}^{i-1}_{i}T = \left[\begin{array}{ccc|c} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \cdot c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} \cdot d_i \\ s\theta_i \cdot s\alpha_{i-1} & c\theta_i \cdot s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} \cdot d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

R P

So the robot manipulator kinematics is given by

$${}^{0}_{n}T = \left[\begin{array}{c|c} {}^{0}_{n}R & P_{n\times g} \\ \hline 0\ 0\ 0 & 1 \end{array}\right].$$

Kinematics