SDM273: Intelligent Sensing and Signal Processing

2: Sensor Characteristics

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Sensors

"device that receives and responds to a signal or stimulus"

The quantity, property, or condition that is received and converted into electrical signal

"a translator of a generally nonelectrical value into an electrical value"

Voltage, current, or charge

Sensor Characteristics

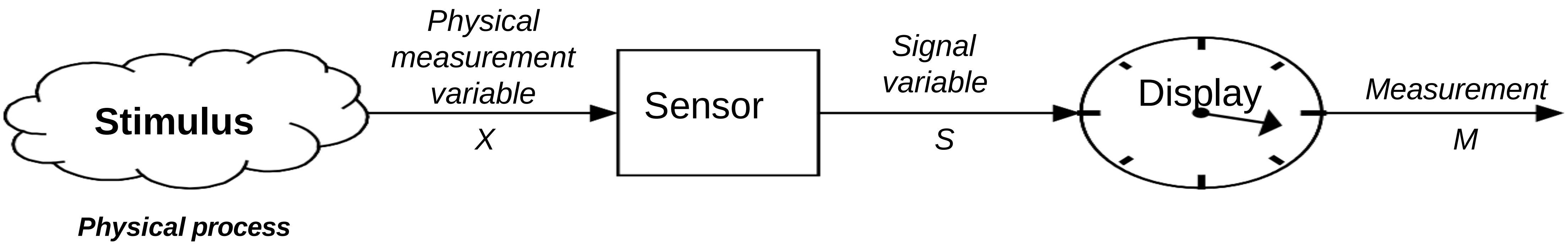
Sensor characteristics refer to the relationship between *Output* and *Input*

When the input is constant or changes very slowly, this relationship is called *Static characteristics*

When the amount of input changes rapidly over time, this relationship is called *Dynamic characteristics*

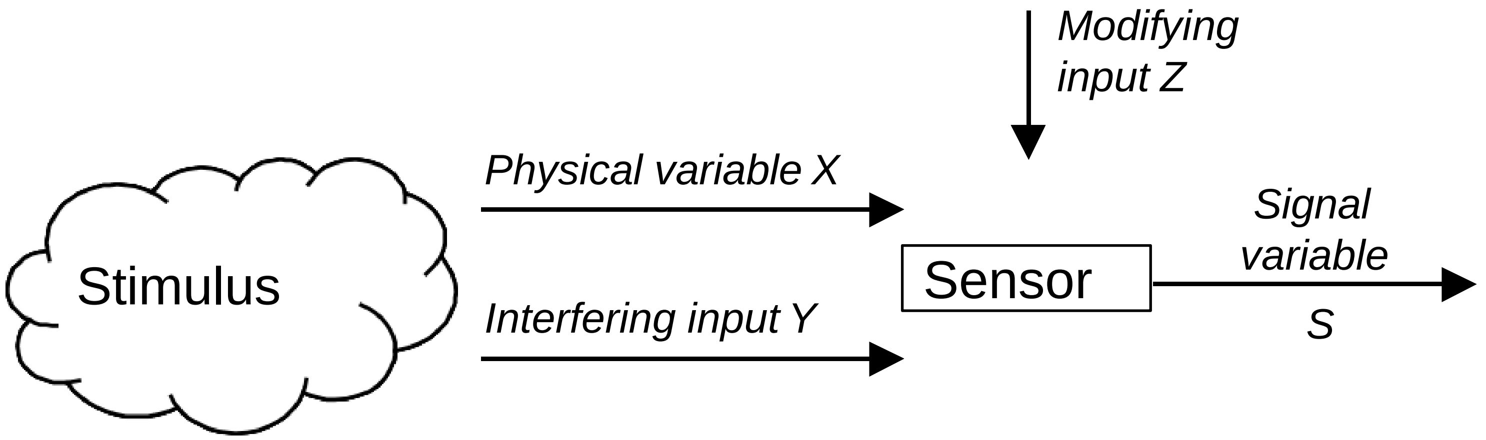
The relationship between sensor output and input can be described by *differential equations*. Theoretically, when the differential term of the first order and above in the differential equation is taken as zero, the static characteristic is obtained. Therefore, the static characteristics of the sensor are only a special case of the dynamic characteristics

A Simple Instrument Model

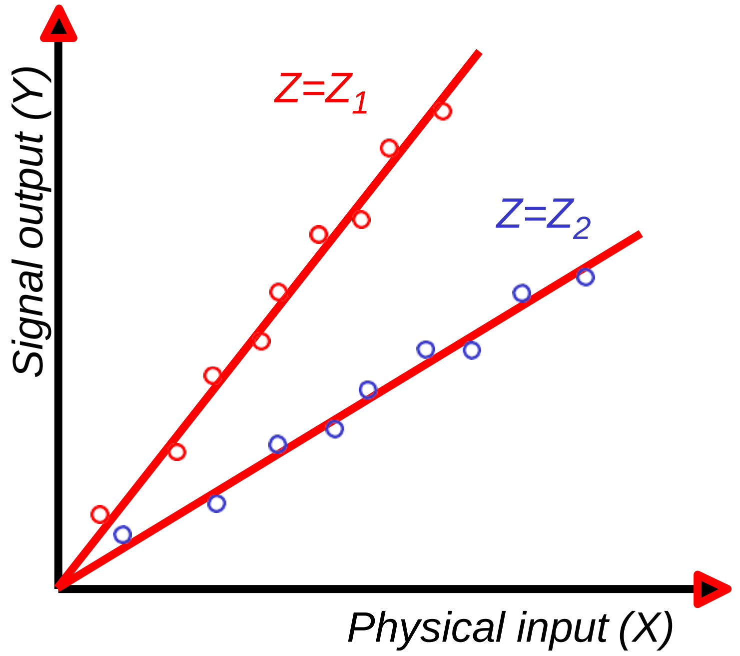


- **An observable variable X is obtained from the stimulus**
  - X is related to the stimulus in some KNOWN way (i.e., measuring mass)
- **The sensor generates a signal variable that can be manipulated:**
  - Processed, transmitted or displayed
- **In the example above the signal is passed to a display, where a measurement can be taken**

Additional Inputs

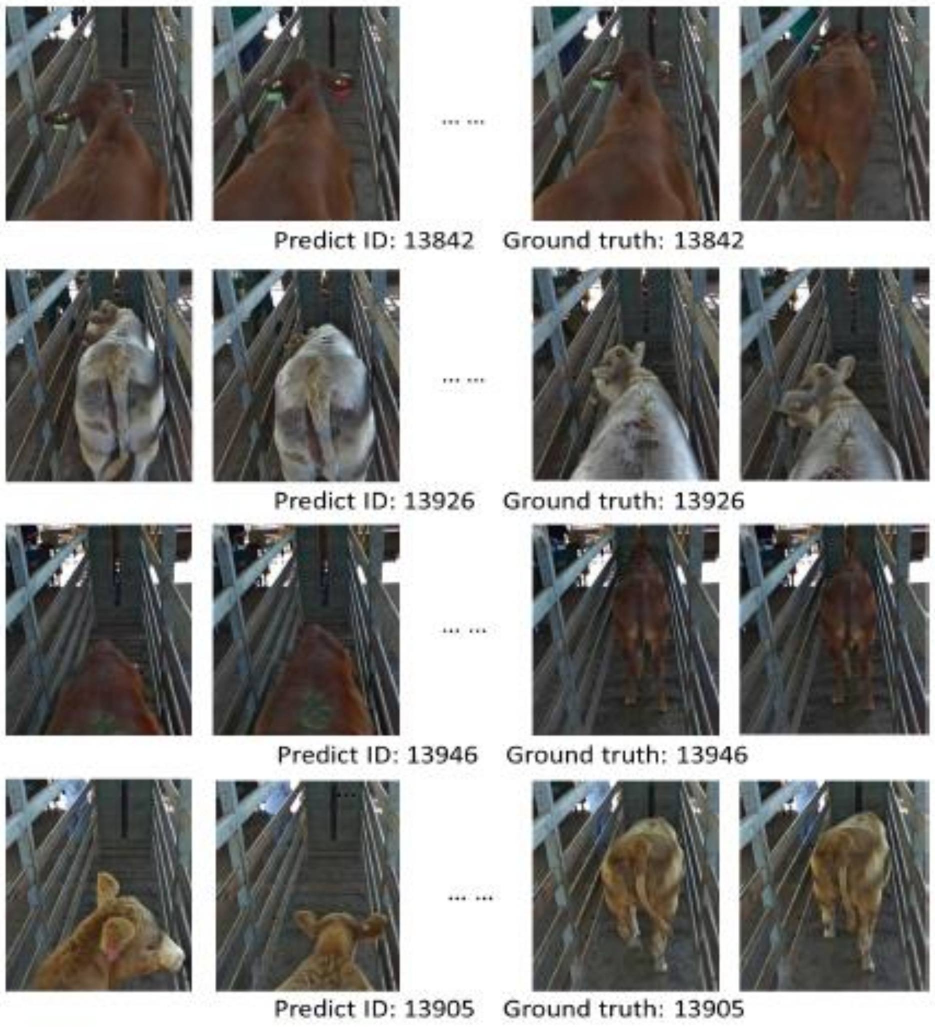


- **Interfering inputs (Y)**
  - Those that the sensor to respond as the linear superposition with the stimulus variable X
  - Linear superposition assumption: S(aX+bY)=aS(X)+bS(Y)
- **Modifying inputs (Z)**
  - Those that change the behavior of the sensor and, hence, the calibration curve
  - Temperature is a typical modifying input



A scenario in livestock farming

Fig. 4: Experimental setup at feedlot showing rear view camera (clamped to overhead bar) and side view camera (on tripod).



(a) True identification examples

2019 IFAC AgriControl, 2020 IEEE CASE

Transfer Functions

$$E=f(s)$$

Electrical signal

Input stimulus

From the measurement system, an inverse transfer function is employed to obtain value of the stimulus

$$s=f^{-1}(E)=F(E)$$

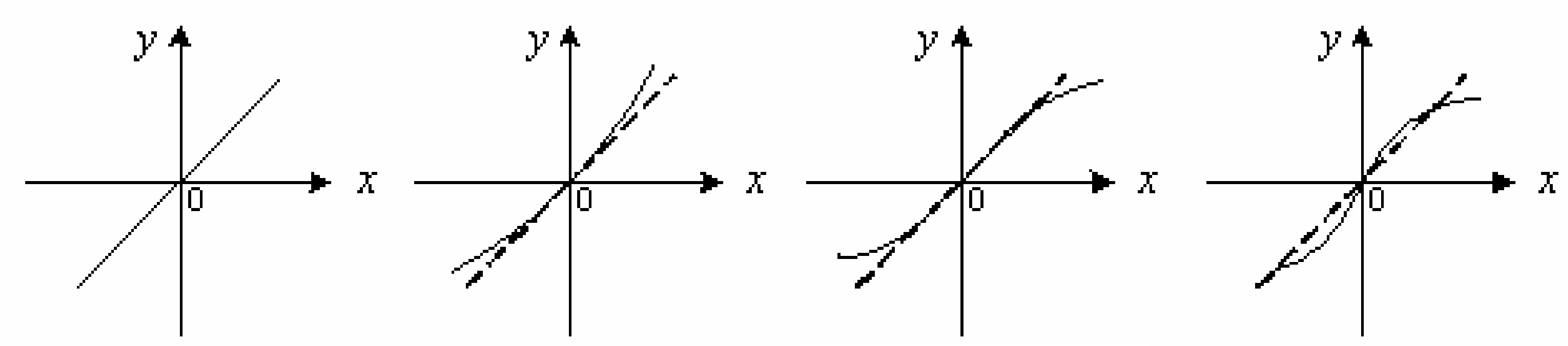
Static Characteristics of Sensor

Static model: Without considering external factors such as hysteresis, creep and friction, the static characteristics of the output and input of the sensor can be represented by a polynomial algebraic equation

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$$

式中，y——输出量；x——输入量；

a_0——零位输出；a_1——传感器的线性灵敏度，常用K或S表示；a_2，a_3，…，a_n——非线性项的待定常数。



Ideal Linearity

Only even-order nonlinear terms

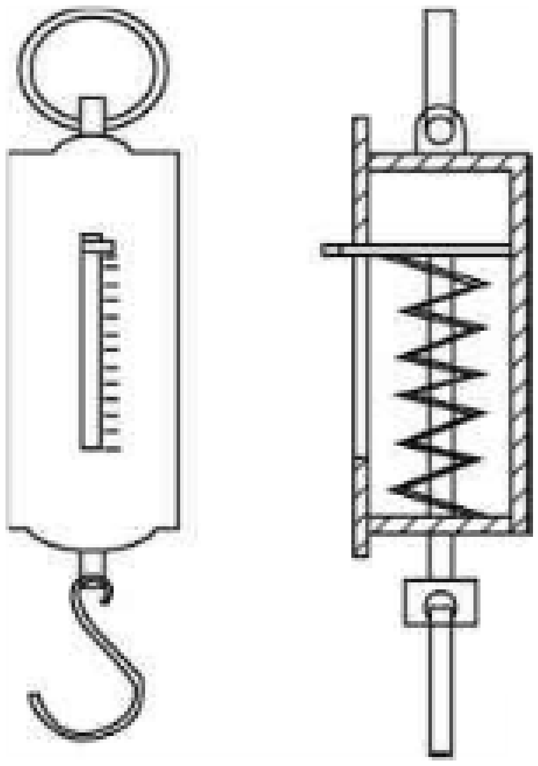
Only odd-order nonlinear terms

Actual characteristic curve

Ideal linear relationship： $y= a_1 x$

Case Study

Find the static characteristic expression of the tension spring gauge?



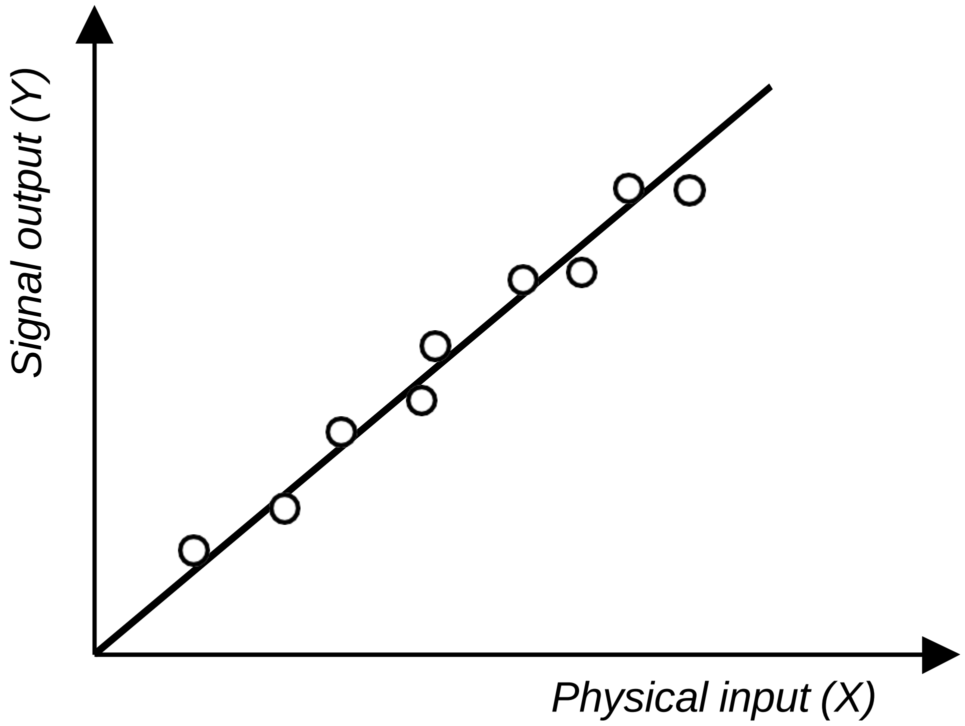
The mass is slowly hung on the hook without causing spring vibration, which realizes a static condition of the input quantity x (mass)

According to the Hooke's law of the spring, the static model of the spring gauge is:

$$y = Kx$$

Calibration

- **The relationship between the physical measurement variable (X) and the signal variable (S)**
  - A sensor or instrument is calibrated by applying a number of KNOWN physical inputs and recording the response of the system



Ways of Calibration

- Modifying the transfer function or its approximation to fit the experimental data
- Adjustment of the data acquisition system to modify its output by making the outputs signal to fit into a normalized or “ideal” transfer function
- Modification the sensor’s properties to fit the predetermined transfer function, thus the sensor itself is modified
- Creating the sensor-specific reference device with the matching properties at particular calibrating points

An example in robot audition

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Necessary and Sufficient Conditions for Observability of SLAM-Based TDOA Sensor Array Calibration and Source Localization

Daobilige Su, He Kong, Salah Sukkarieh, *Member, IEEE*, and Shoudong Huang, *Senior Member, IEEE*

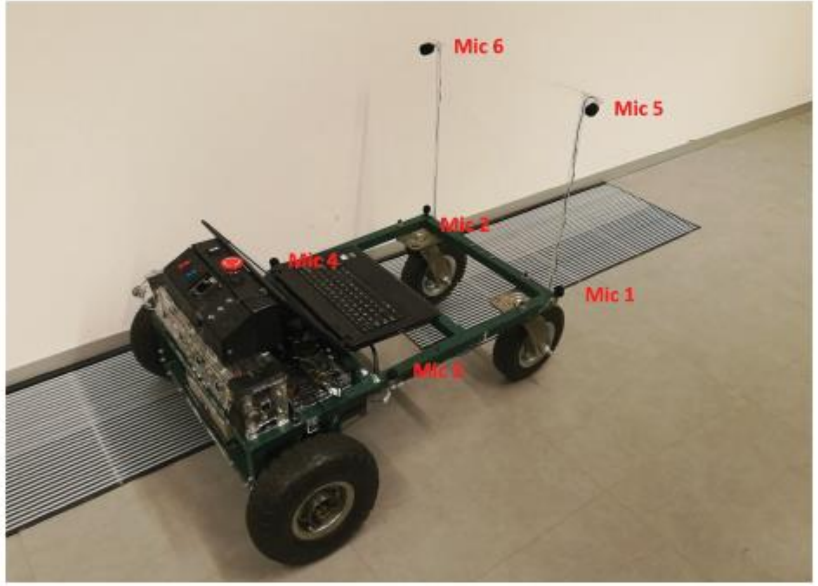

Fig. 7. Setup of a 3-D asynchronous microphone array on a mobile robot.

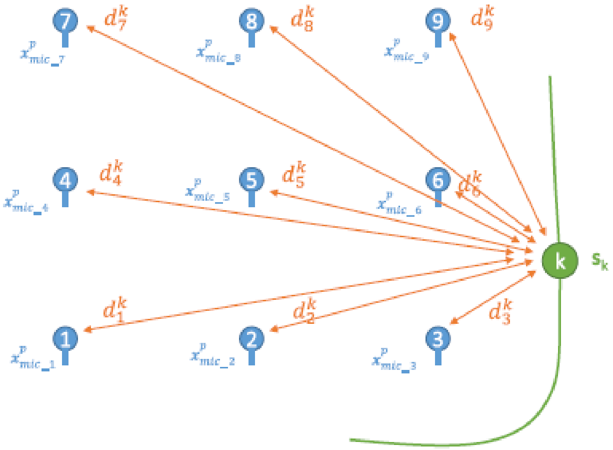

Fig. 1. Geometry of the problem setup and analytical TDOA.

$$T_{i1}^k = \frac{d_i^k}{c} - \frac{d_1^k}{c} + x^{\tau}_{\mathrm{mic}\_i} + k\Delta_t x^{\delta}_{\mathrm{mic}\_i}$$

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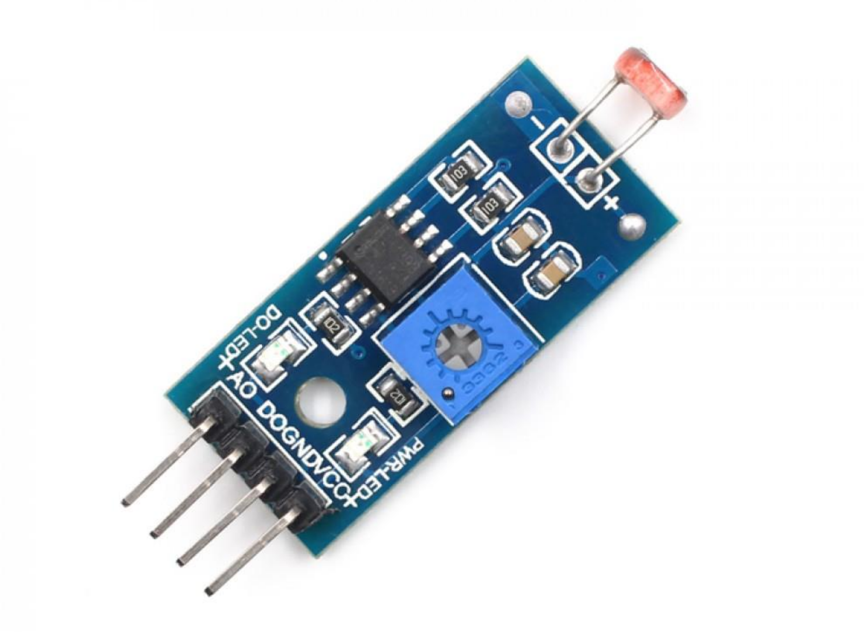
The relationship between sensor output and input can be described by differential equations. Theoretically, when the differential term of the first order and above in the differential equation is taken as zero, the static characteristic is obtained. Therefore, the static characteristics of the sensor are only a special case of the dynamic characteristics

Sensors Requirements for Mobile Devices

- Intelligent sensor: built-in signal conditioner and DSP
- Built-in communication circuit (I_2C, NFC, Bluetooth, etc.)
- Integrated supporting components (optics, thermostat, blower, etc.)



Light Sensor Module

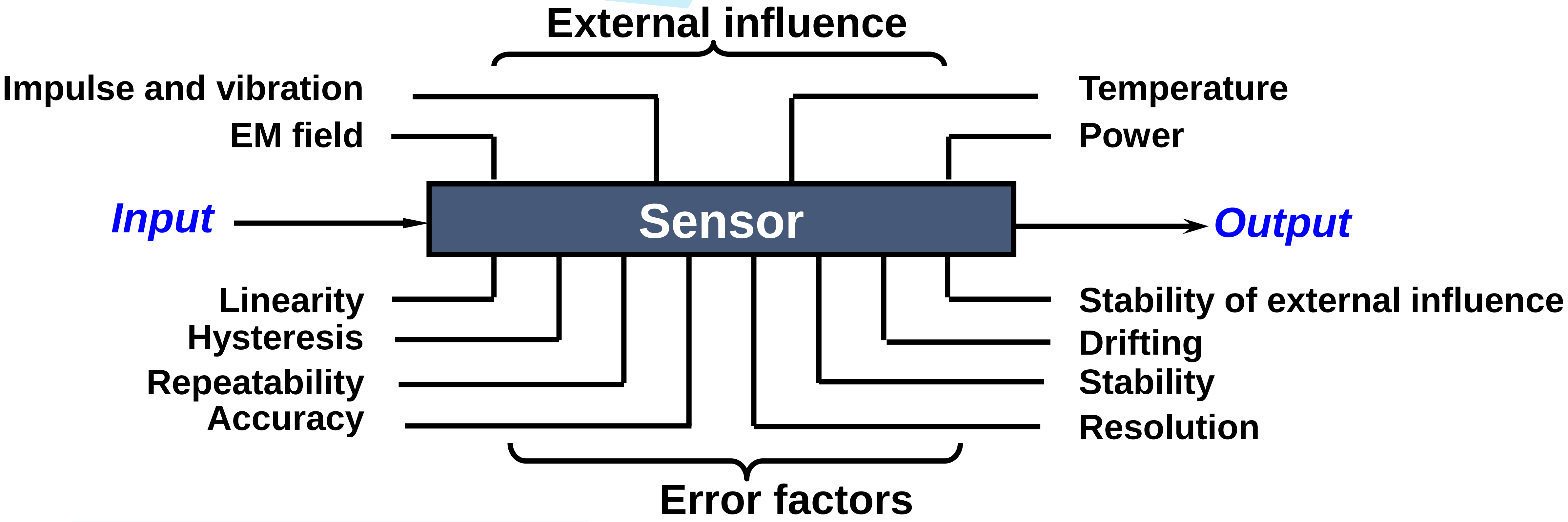
Temperature Sensor Module

Sensors Requirements for Mobile Devices

- High selectivity of the sensed signal (reject interferences)
- Fast response
- Miniature size to fit a mobile device
- Low power consumption
- High stability in changing environment
- Lifetime stability: no periodic recalibration or replacement
- Low cost at sufficiently high volumes

Sensor Characteristics

It can be reduced by the improvement of the sensor quality, or optimizing the external conditions

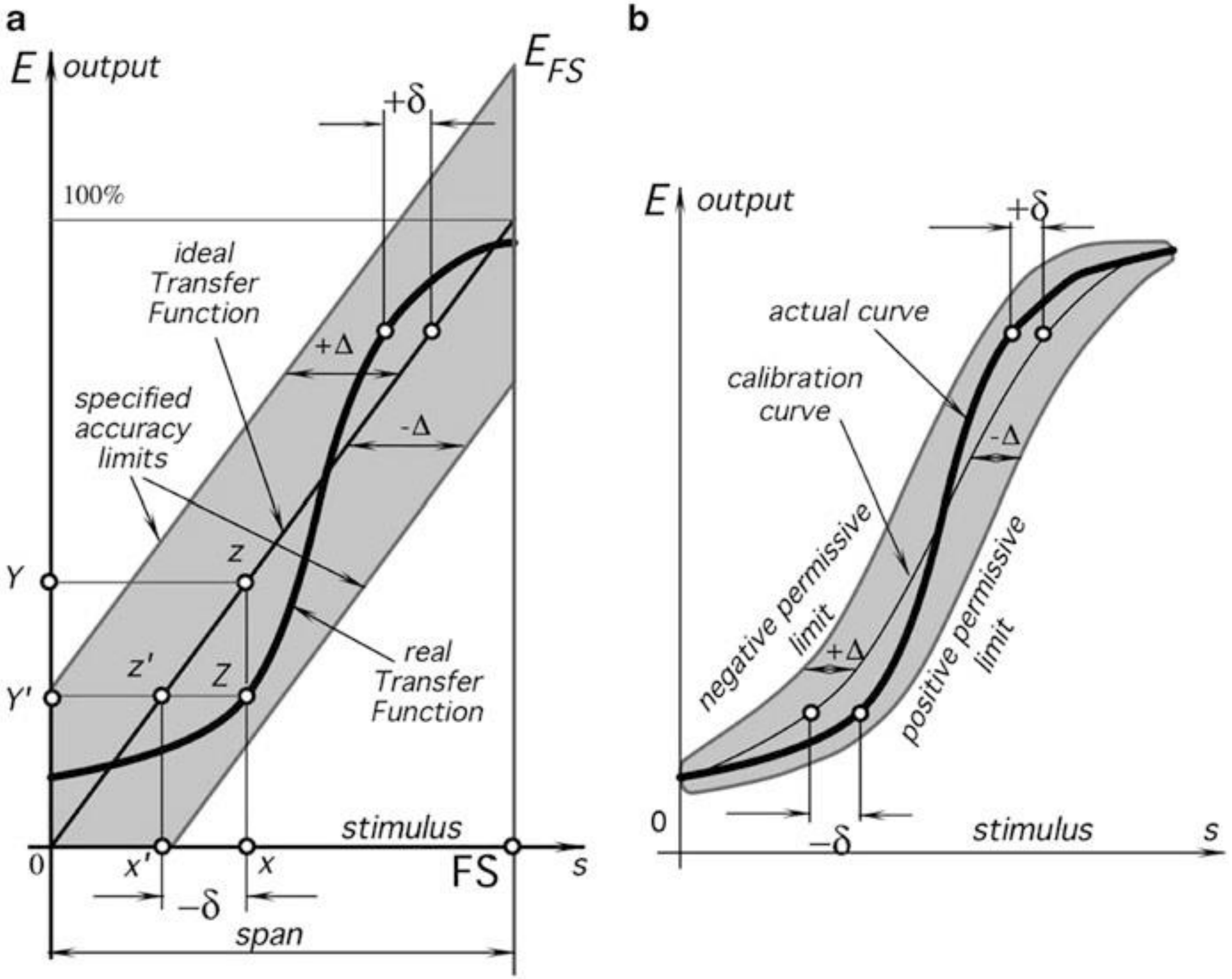


Main indicators of sensor characteristics

Sensor Characteristics

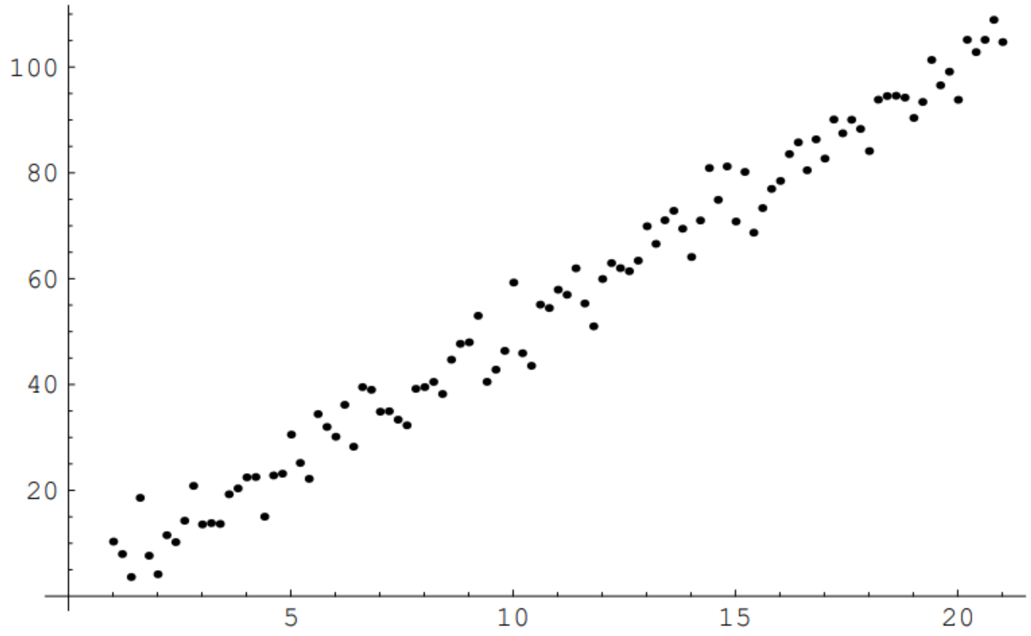
- **Span (Full-Scale Input)**
  - A dynamic range of stimuli that may be converted by a sensor
  - It represents the highest possible input value, which can be applied to the sensor without causing unacceptably large error
- **Full-Scale Output**
  - It represents the highest possible output value, which can be applied to the sensor without causing unacceptably large error
  - For an analog output, it is the algebraic difference between the electrical output signals measured with maximum input stimulus and the lowest input stimulus applied
  - For a digital output, it is the maximum digital count that the A/D convertor can resolve for the absolute maximum full scale input

Span and Full Scale Output



Static Characteristics of Sensor

The important indicators for evaluating the static characteristics of sensors are linearity, hysteresis (迟滞), repeatability, sensitivity, resolution, and drift.



Static Characteristics of Sensor

The important indicators for evaluating the static characteristics of sensors are linearity, hysteresis, repeatability, sensitivity, resolution, and drift.

- **Linearity**
  - Refers to the degree of linearity between the sensor output and the input
  - The static characteristic curve can be obtained through experimental tests
  - The output-input relationship of the sensor is more or less non-linear
  - For the convenience of calibration and data processing, a linear relationship is desired
  - In the case where the non-linear error is not too large, a ***linear fitting*** is always used for linearity

Static Characteristics of Sensor

- **Linearization**
  - When using a sensor with non-linear characteristics, a straight line such as a tangent line or a secant line is used to approximately represent a section of the actual curve under the tolerance of measurement error, so called the linearization of the non-linear characteristics of the sensor
  - **Advantages**
    - Can greatly simplify the theoretical analysis and design calculation of the sensor
    - Sensor calibration and data processing are convenient
    - Instrument scale can be uniformly scaled, easy to make, install and debug
    - Avoids non-linear compensation

Static Characteristics of Sensor

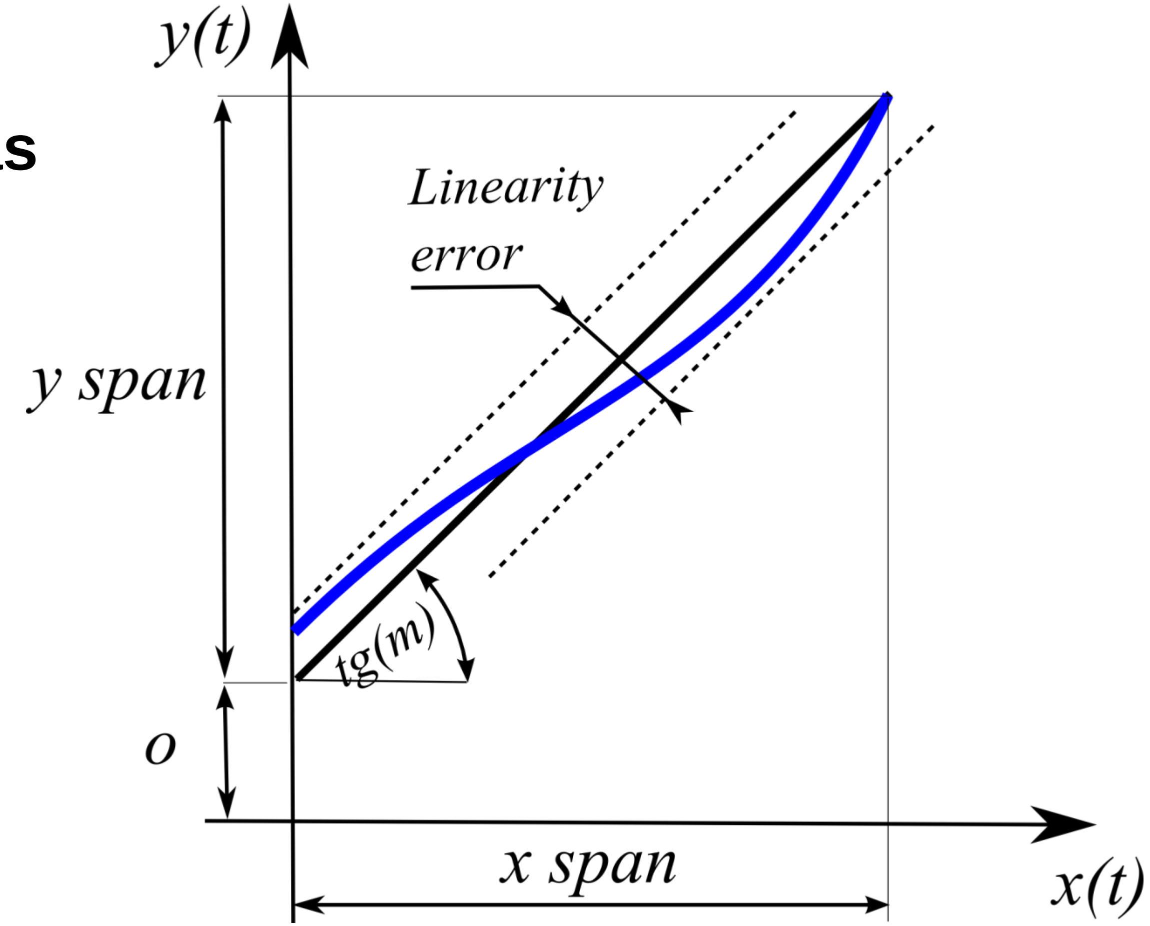
- **Nonlinearity=Linearity Error**
  - For a measuring system with linear output, the maximum deviation of the output of the measuring system from a specified straight line
  - 4 types of reference lines:
    - Terminal Line: A line drawn from the origin to the farthest data point
    - End Point Line: A line drawn between the end points of a data plot
    - Best Fit Line: The line midway between the two closest parallel straight lines that enclose all of the data points
    - Least Square Line: The line for which the sum of the squares of the deviations of the data points from the line being fit is minimized

Linearity Error

- Linearity errors e_L can be expressed as

$$e_L = \pm \frac{\Delta\text{max}}{y_{FS}} \times 100\%$$

  - **Δ*max*—max linearity error**
  - **y_{FS}—Full scale range**



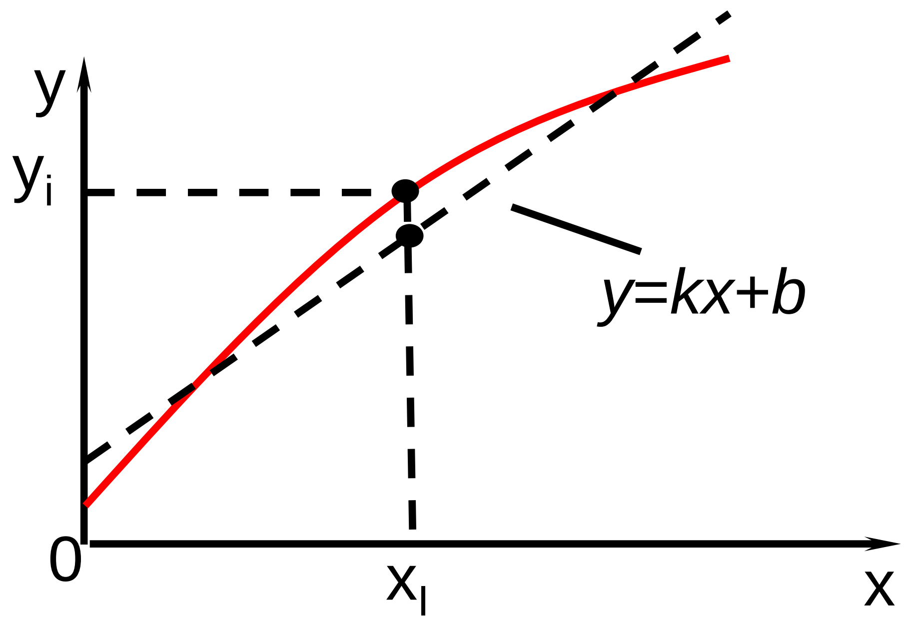
Least Square Fitting

$$(x_1, y_1)\,, (x_2, y_2)\,, (x_3, y_3)\,,\ \cdots\cdots, (x_n, y_n)$$

- **Minimize total square-error**
- Straight line approximation

$$f(x) = a_0 + a_1 x$$
$$y_i = f(x_i) = a_0 + a_1 x_i$$

- Not likely to pass all points if n > 2



Least square fitting

Least Square Fitting

$$(x_1, y_1)\ , (x_2, y_2)\ , (x_3, y_3)\ ,\ \cdots\cdots, (x_n, y_n)$$

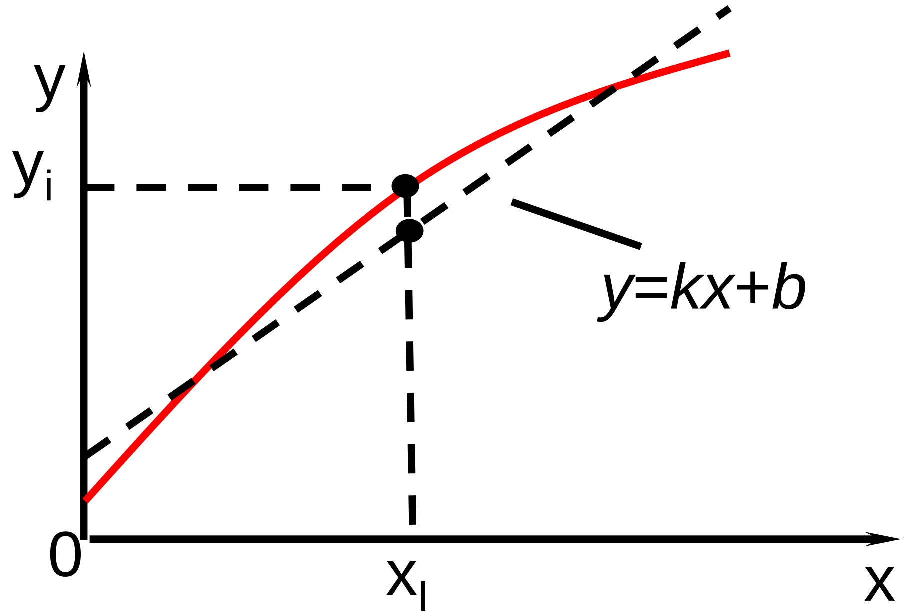
- **Total square-error function: sum of the squares of the residuals**

$$S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

- **Minimizing square-error** $S_r(a_0, a_1)$

$$\begin{cases} \dfrac{\partial S_r}{\partial a_0} = 0 \\ \dfrac{\partial S_r}{\partial a_1} = 0 \end{cases}$$

Solve for (a_0, a_1)



Least square fitting

Least Square Fitting

- **Minimize**

$$S_r(a_0, a_1) = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

$$\begin{cases} \dfrac{\partial S_r}{\partial a_0} = 0 = -2\displaystyle\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i) \\ \dfrac{\partial S_r}{\partial a_1} = 0 = -2\displaystyle\sum_{i=1}^{n}(y_i - a_0 - a_1 x_i)x_i \end{cases} \Rightarrow \begin{cases} na_0 + \left(\displaystyle\sum_{i=1}^{n} x_i\right)a_1 = \displaystyle\sum_{i=1}^{n} y_i \\ \left(\displaystyle\sum_{i=1}^{n} x_i\right)a_0 + \left(\displaystyle\sum_{i=1}^{n} x_i^2\right)a_1 = \displaystyle\sum_{i=1}^{n} x_i y_i \end{cases}$$

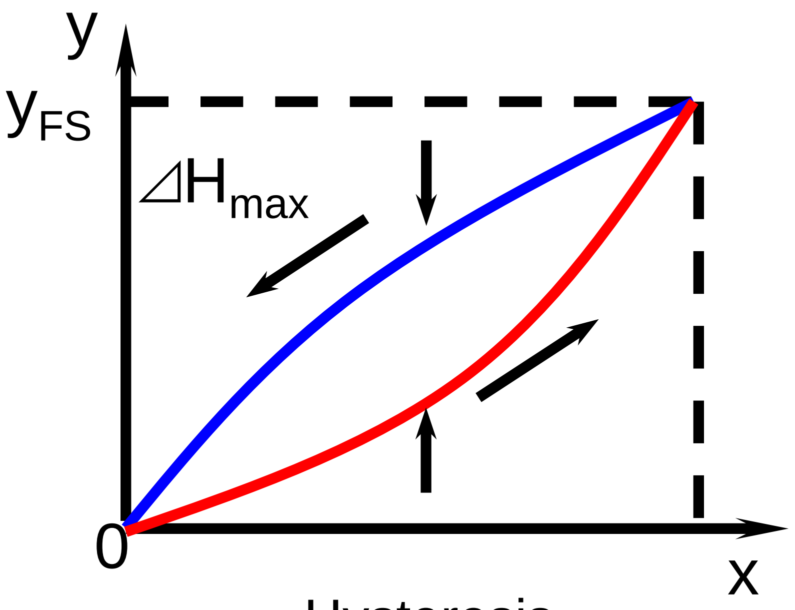
- **Normal equation $y = a_0 + a_1 x$**

$$a_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x} = \frac{\sum y_i}{n} - a_1 \frac{\sum x_i}{n}$$

Hysteresis

- When the sensor's output and input curves do not overlap during the positive (increase input) and reverse (decrease input) path
- The hysteresis is measured experimentally
- Hysteresis error is expressed as a percentage of full-scale output, i.e. For the convenience of calibration and data processing, a linear relationship is desired



Hysteresis

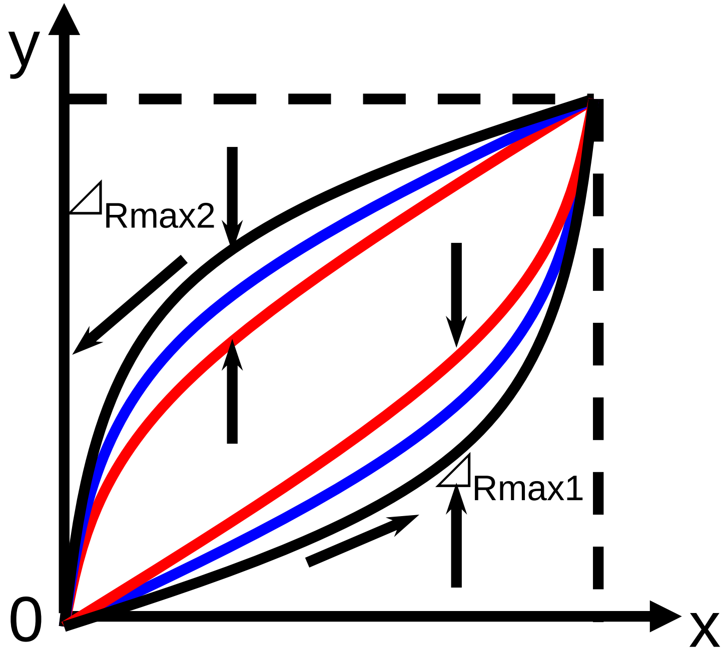
$$e_H = \pm \frac{\Delta \mathrm{H_{max}}}{y_{FS}} \times 100\%$$

Where △ Hmax—the maximum difference between the forward and reverse path of output

Repeatability

- refers to the degree of inconsistency in the characteristic curve obtained when the sensor changes the input in the same direction over multiple consecutive times

$$e_R = \pm \frac{\Delta R_{\max}}{y_{FS}} \times 100\%$$



$\triangle_{Rmax1}$ **The max difference during the positive path**

$\triangle_{Rmax2}$ **The max difference during the negative path**

Sensitivity

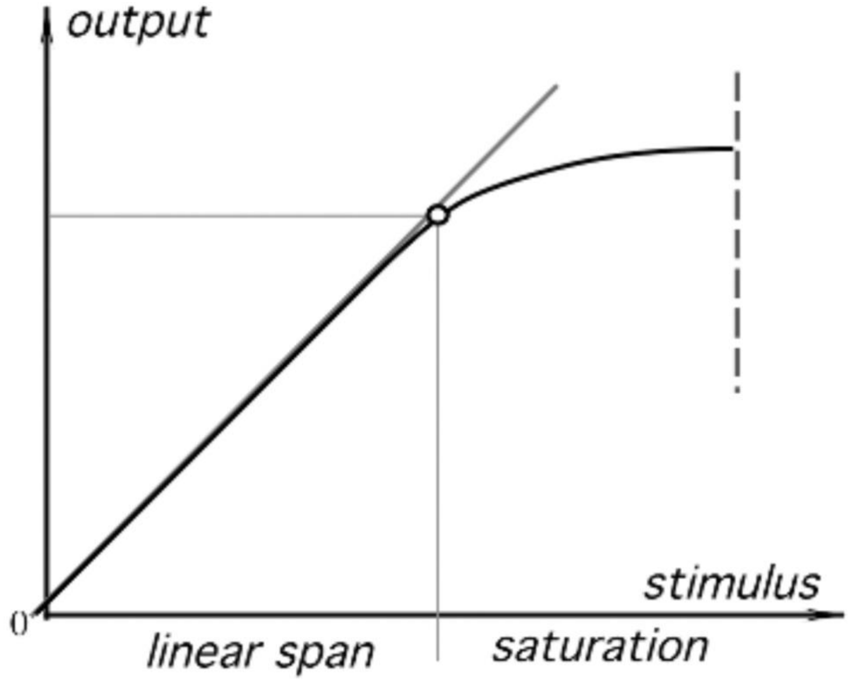
- The ratio of the change y of the sensor output to the change x of the input that caused the change is its static sensitivity, and its expression is

$$K=\Delta y/\Delta x$$

- It can be seen that the slope of the sensor output curve is its sensitivity. For sensors with linear characteristics, the slope of the characteristic curve is the same everywhere, and the sensitivity k is a constant regardless of the amount of input.
- It is generally required that the sensitivity of the sensor is high and constant over the full range
- For some reason, sensitivity changes may occur, resulting in ***sensitivity errors***

Sensor Characteristics

- **Saturation**
  - Every sensor has its operating limits. Even if it is considered linear, at some levels of the input stimuli, its output signal no longer will be responsive. A further increase in stimulus does not produce a desirable output. It is said that the sensor exhibits a span-end nonlinearity or saturation



Sensor Characteristics

- **Resolution**
  - Resolution is the smallest input increment that a sensor can detect
  - Resolution is expressed as an absolute value
  - The resolution of a sensor with a digital output format is given by the number of bits
- **Threshold (Dead Band)**
  - Threshold means that when the input of a sensor increases very slowly from zero, the change in output can only be measured after reaching a certain minimum value. This minimum value is called the sensor's threshold value.
  - In fact the threshold is the resolution of the sensor near zero.

Sensor Characteristics

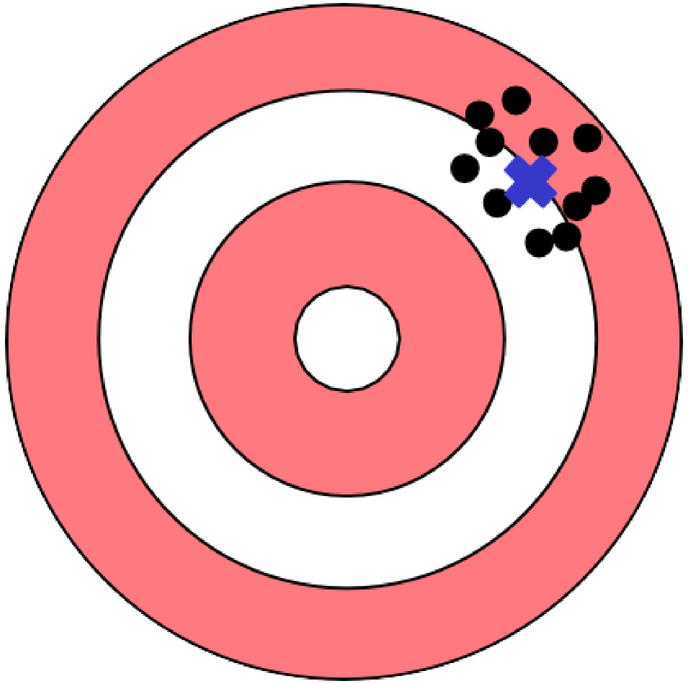
- **Drift**
  - Drift means that within a certain time interval, there is an unnecessary change in the output of the sensor that is independent of the input
  - It can be caused by several factors including environmental contamination, vibration or extreme **temperature** fluctuations.
- **Reproducibility**
  - Reproducibility is the precision of a set of measurements **BUT**:
    - taken over a long time interval or Performed by different operators
    - with different instruments or in different laboratories

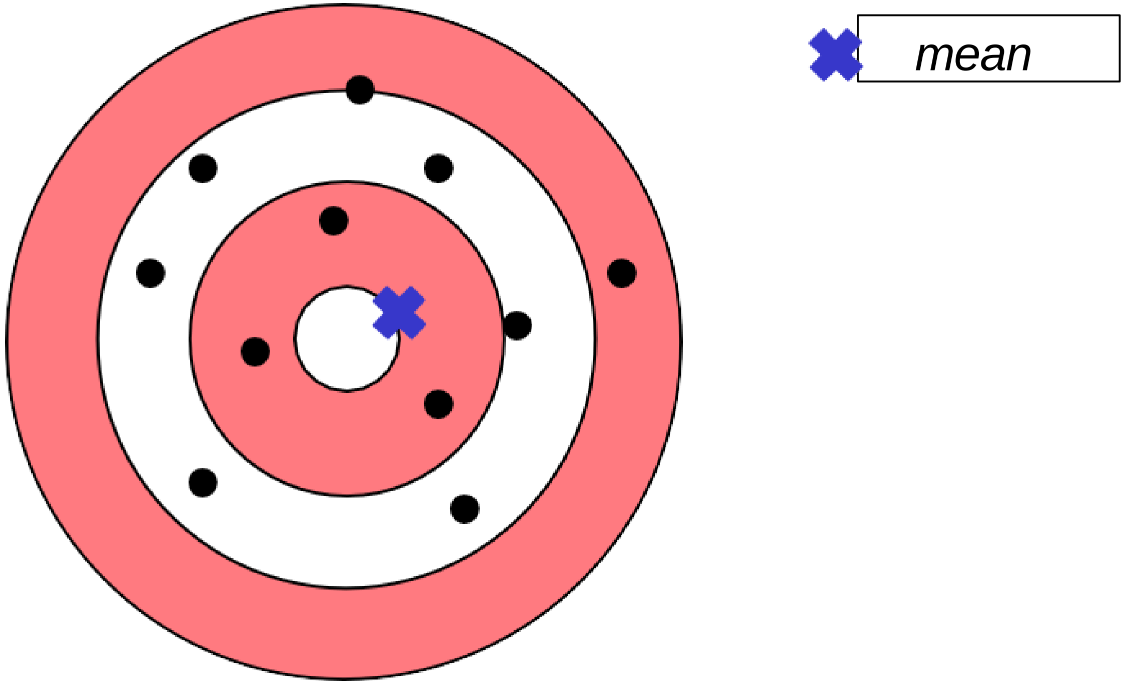
Sensor Characteristics

- **Accuracy**
  - The capacity of a measuring instrument to give RESULTS close to the TRUE VALUE of the measured quantity
- **Precision**
  - The capacity of a measuring instrument to give the same reading when repetitively measuring the same quantity under the same prescribed conditions
  - Precision is related to the variance of a set of measurements
  - Precision is a necessary but not sufficient condition for accuracy

Shooting Darts

- *Which shooter is more accurate (准确)?*
- *Which shooter is more precise (精确)?*





Summary

- **Transfer function**
- **Calibration**
- **Static characteristics of sensors**
  - Span
  - Resolution
  - Hysteresis
  - Nonlinearity
  - Saturation
  - Accuracy and Precision