※2 (i) Assume that $d_1, d_2, \dots, d_n$ are $Q_{n\times n}$ - conjugate, where Q is positive definite matrix. Please show $d_1, d_2, \dots, d_n$ are linear independent.

pf: Suppose $\{d_1, d_2, d_3, \dots, d_n\}$ are linear dependent.

i.e. $\exists$ some $a_i \neq 0$ s.t.

$$a_1 d_1 + a_2 d_2 + \dots + a_n d_n = 0_{n\times 1}$$

$$\Rightarrow \sum_{k=1}^{n} a_k d_k = 0$$

$$\overset{\times Q}{\Longrightarrow} \sum_{k=1}^{n} a_k Q d_k = 0$$

$$\overset{\times d_i^T}{\Rightarrow} \sum_{\substack{k=1 \\ k\neq i}}^{n} a_k d_i^T Q d_k + a_i d_i^T Q d_i = 0$$

$\because$ $d_1, \dots, d_n$ are Q-conjugate $\therefore \sum_{\substack{k=1 \\ k\neq i}}^{n} a_k d_i^T Q d_k = 0$

$\Rightarrow$ $a_i d_i^T Q d_i = 0$ and $a_i \neq 0$

$\Rightarrow$ $d_i^T Q d_i = 0$ $\longrightarrow\!\!\!\!\!\times$ ($\because$ Q is positive definite).

Hence, $\{d_1, d_2, \dots, d_n\}$ are linear independent.