(1) (Due on March 13) (a) Please develop the programming code for the Rectangle rule, Trapezoidal rule and Simpson's rule on the integration approximation. (b) Apply your code on the following integration problem ($\int_0^1 \exp(-x^2)dx$) under $n = 2, 4, 6, 8, 16, 32, 64, 128$ and compare the convergence rate if $\int_0^1 \exp(-x^2)dx \simeq 0.746824132812427$.

The following problem will be due on March 20.

(2) For the integration of a function $f(x)$ between [a,b] denoted by I, we can approximate it by the Trapezoidal rule. Assume the Trapezoidal approximate area defined $\hat{I}_T$ is calculated based on the grid points $\{x_0 = a, x_1, , .., x_n = b\}, x_i - x_{-1} = (b-a)/n$.

Please show that $|\hat{I}_R - I| \leq \frac{(b-a)^3 f''(\eta)}{12n^2} + o(n^{-2})$, $\eta \in [a, b]$ when $f \in C^2[a, b]$.

(Hint: For the function $p_i(x) = f(x_i) + (x - x_i)\frac{f(x_{i+1})-f(x_i)}{x_{i+1}-x_i}$, consider $p_i(x) - f(x)$ by Taylor expansion, and its integration over $[x_i, x_{i+1}]$.)

(3) Consider the function $f(x) = \exp(-x)(\cos(x^2))^2, 0 \leq x < \infty$. Please apply the above information to find the numerical integration (I_{nt}) such that $|I_{nt} - \int_0^\infty f(x)dx| < 10^{-3}$. It is necessary to show the detailed procedure and discuss for the required number of the grid points.