Find the minimum of $f(x) = x^4 - 14x^3 + 60x^2 - 70x$, $x \in [0,2]$.

1. Use Newton method to find x^* $s.t.$ $f'(x^*) = 0$.

   by Taylor expansion at a,

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$$

$$0 = f'(x) = f'(a) + f''(a)(x-a)$$

   That is , $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$, $n = 0,1,2,\ldots$

   So, Step 1. Choose initial value x_0

   Step 2. $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$, $n = 0,1,2,\ldots$

   Step 3. repeat step2. until $|x_{n+1} - x_n| < 10^{-5}$

   Case 1:

   The initial value $x_0 = 0.1$.

```
the  1 -th step, x is  0.1 , f(x) is  -6.4139
the  2 -th step, x is  0.6228762 , f(x) is  -23.55557
the  3 -th step, x is  0.7691724 , f(x) is  -24.36535
the  4 -th step, x is  0.7808111 , f(x) is  -24.3696
the  5 -th step, x is  0.7808835 , f(x) is  -24.3696
```

   and $f'(0.7808835) = -2.09610107 \times 10^{-7}$, $f(0) = 0$, $f(2) = 4$.

   Hence, the minimum is -24.3696 , x is 0.7808835.

   Case 2:

   The initial value $x_0 = 1.9$.

```
the  1 -th step, x is  1.9 , f(x) is  0.6061
the  2 -th step, x is  -7.252268 , f(x) is  11769.76
the  3 -th step, x is  -3.816278 , f(x) is  2131.211
the  4 -th step, x is  -1.602691 , f(x) is  330.537
the  5 -th step, x is  -0.2476316 , f(x) is  21.22985
the  6 -th step, x is  0.4755351 , f(x) is  -21.17379
the  7 -th step, x is  0.7413672 , f(x) is  -24.32074
the  8 -th step, x is  0.7800808 , f(x) is  -24.36958
the  9 -th step, x is  0.7808831 , f(x) is  -24.3696
```

   and $f'(0.7808830) = -2.882316607 \times 10^{-5}$, $f(0) = 0$, $f(2) = 4$.

   Hence, the minimum is -24.3696 , x is 0.7808830.

   And, I have tried different initial value , $x_0 = 0,0.1,0.2,0.3,\ldots,1.9,2$,

   All these value except $x_0 = 2$ will converge (i.e $x_0 = 2$ $diverge$).

2. Use golden-section method:

   To $find\ minimum\ of\ f(x) \Leftrightarrow find\ maximum\ of\ -f(x)$

   Let $\mathrm{g(x)} = -f(x)$,

   Step 1. choose $\mathrm{x_l} < x_m < x_r\ \ s.t.\ \ g(x_l) \le g(x_m)\ and\ g(x_r) \le g(x_m)$.

   Step 2. choose $\mathrm{y} \in \max\{(\mathrm{x}_l, \mathrm{x_m}), (\mathrm{x_m}, \mathrm{x_r})\}$.

   Step 3.

   In $(\mathrm{x}_l, \mathrm{x_m})$, If $\mathrm{g(y)} > \mathrm{g(x_m)}\ then\ x_r = x_m\ , \mathrm{x_m} = y$. Else $\mathrm{x_l} = y$.

   In $(\mathrm{x}_r, \mathrm{x_m})$, If $\mathrm{g(y)} > \mathrm{g(x_m)}\ then\ x_l = x_m\ , \mathrm{x_m} = y$. Else $\mathrm{x}_r = y$.

   Step 4. repeat step 3. until $|\mathrm{x_r} - \mathrm{x_l}| < 10^{-5}$

   <u>Simulation:</u>

   $\mathrm{x_l} = 0\ , x_m = 1\ , x_r = 2\ \ with\ g(x_l) = 0\ , g(x_m) = 23\ ,\ \mathrm{g(x_r)} = -4$,

   y is the mean point.

```
left g(x): 21.6875 , middle g(x): 23 , right g(x): -4
left g(x): 21.6875 , middle g(x): 23 , right g(x): 12.1875
left g(x): 21.6875 , middle g(x): 24.33984 , right g(x): 23
left g(x): 23.57788 , middle g(x): 24.33984 , right g(x): 23
left g(x): 23.57788 , middle g(x): 24.33984 , right g(x): 24.10522
left g(x): 24.09154 , middle g(x): 24.33984 , right g(x): 24.10522
left g(x): 24.09154 , middle g(x): 24.33984 , right g(x): 24.3391
left g(x): 24.24783 , middle g(x): 24.33984 , right g(x): 24.3391
left g(x): 24.33984 , middle g(x): 24.3696 , right g(x): 24.3391
left g(x): 24.36238 , middle g(x): 24.3696 , right g(x): 24.3391
left g(x): 24.36238 , middle g(x): 24.3696 , right g(x): 24.36175
left g(x): 24.36789 , middle g(x): 24.3696 , right g(x): 24.36175
left g(x): 24.36789 , middle g(x): 24.3696 , right g(x): 24.36754
left g(x): 24.36921 , middle g(x): 24.3696 , right g(x): 24.36754
left g(x): 24.36921 , middle g(x): 24.3696 , right g(x): 24.36904
left g(x): 24.36952 , middle g(x): 24.3696 , right g(x): 24.36904
left g(x): 24.36952 , middle g(x): 24.3696 , right g(x): 24.36944
left g(x): 24.36959 , middle g(x): 24.3696 , right g(x): 24.36944
left g(x): 24.36959 , middle g(x): 24.3696 , right g(x): 24.36955
left g(x): 24.36959 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
```

```
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left g(x): 24.3696 , middle g(x): 24.3696 , right g(x): 24.3696
left point: 0.78088 , middle point: 0.7808838 ,right point: 0.7808876
```

Hence, the minimum of f(x)= -24.3696 , and x is 0.7808838.

```
#newton method
f<-function(x){ x^4-14*x^3+60*x^2-70*x }
seed<-1.9
for(i in 1:20){
epslon<-10^-5 ;h<-10^-6
x1<-0;x2<-seed ;loop<-1

while(abs(x2-x1)>=epslon &&loop<500){
   cat("the ",loop,"-th step,","x is ",x2,", f(x) is ",f(x2),"\n")
   loop<-loop+1
   x1<-x2
   firdif<-(f(x1+h)-f(x1))/h
   secdif<-(f(x1+2*h)-2*f(x1+h)+f(x1) ) / h^2
   x2<-x1-firdif/secdif
}
seed<-seed+0.1
}
#golden-section
g<-function(x){ -(x^4-14*x^3+60*x^2-70*x) }
g(seq(0,100,1))
xl<-0 ;g(xl)
xm<-1 ;g(xm)
xr<-2 ;g(xr)
loop<-1
while((xr-xl)>=epslon){#f(xm)-f(xl)>=epslon && f(xm)-f(xr)>=epslon
   if(abs(xl-xm)>=abs(xr-xm)){                    #choose large interval
      x4<-(xl+xm)/2
      if(g(x4)>g(xm)){ xr<-xm ;xm<-x4 }else{ xl<-x4 }
```

```
        }else{
            x4<-(xr+xm)/2
            if(g(x4)>g(xm)){ xl<-xm ;xm<-x4 }else{ xr<-x4 }
        }
    loop<-loop+1
    cat("left g(x):",g(xl),", middle g(x):",g(xm),", right g(x):",g(xr),"\n")
};cat("left point:",xl,", middle point:",xm,", right point:",xr,"\n")
```