Exercise 1.

For non-parametric density estimation , by kernel method

我們有

.

這裡選擇Eparechinlcov kernel,

經過計算(附錄), , 代入MISE中,

得到不同樣本數

觀察到, , 同樣如上

故,

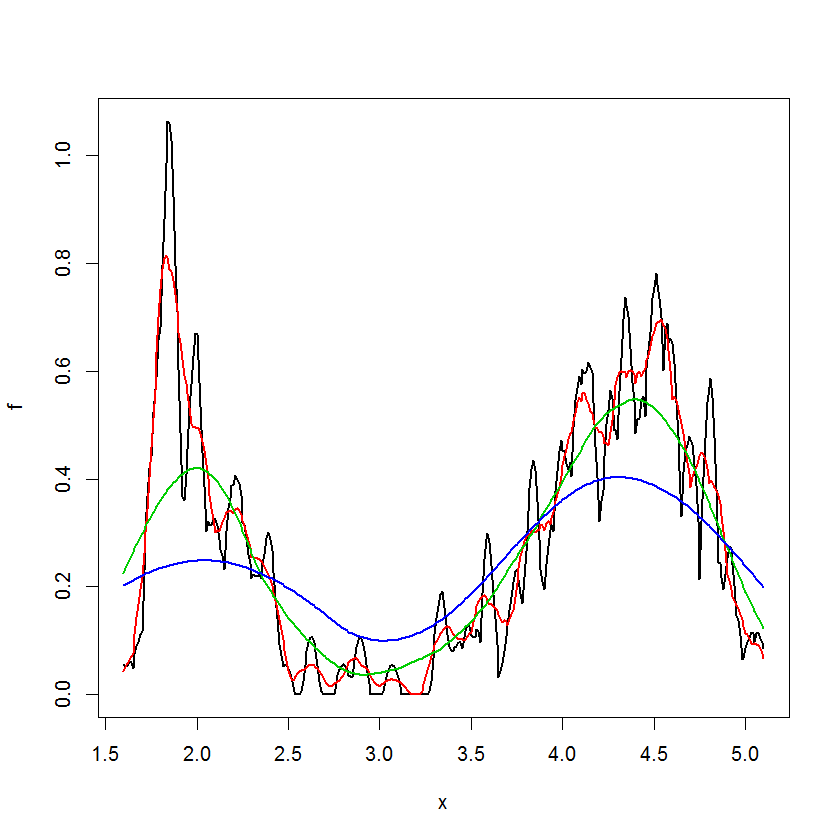
Exercise 2.

Data “faithful” 為黃石公園的噴泉噴發時間(X1)與等待時間(X2)，我們想要找到X1和X2的邊際密度函數和聯合密度函數.

利用kernel method, 這裡選擇Eparechinlcov kernel,

For ,

(i)我們有, 考慮估計量 , 對於不同的估計畫圖如下:

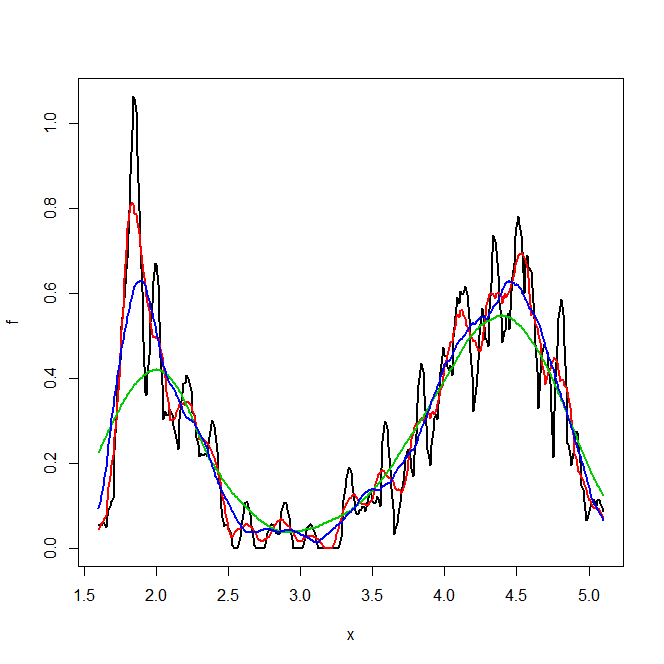


(黑色 ;紅色 ;綠色 ;藍色)

發現愈小則圖形越曲折複雜, 愈大則圖形越平滑.

(ii)Choose  by peseudo likelihood,

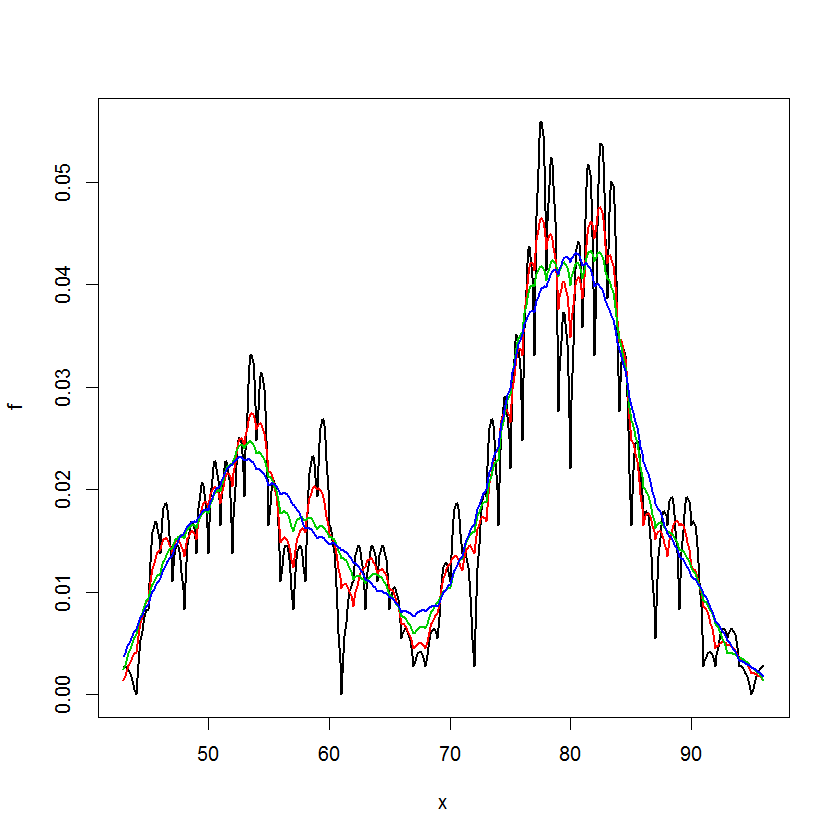
考慮 畫圖入下:



(黑色為, 紅色為 綠色為, 藍色為)

For ,

我們有, 考慮估計量 , 對於不同的估計畫圖如下:



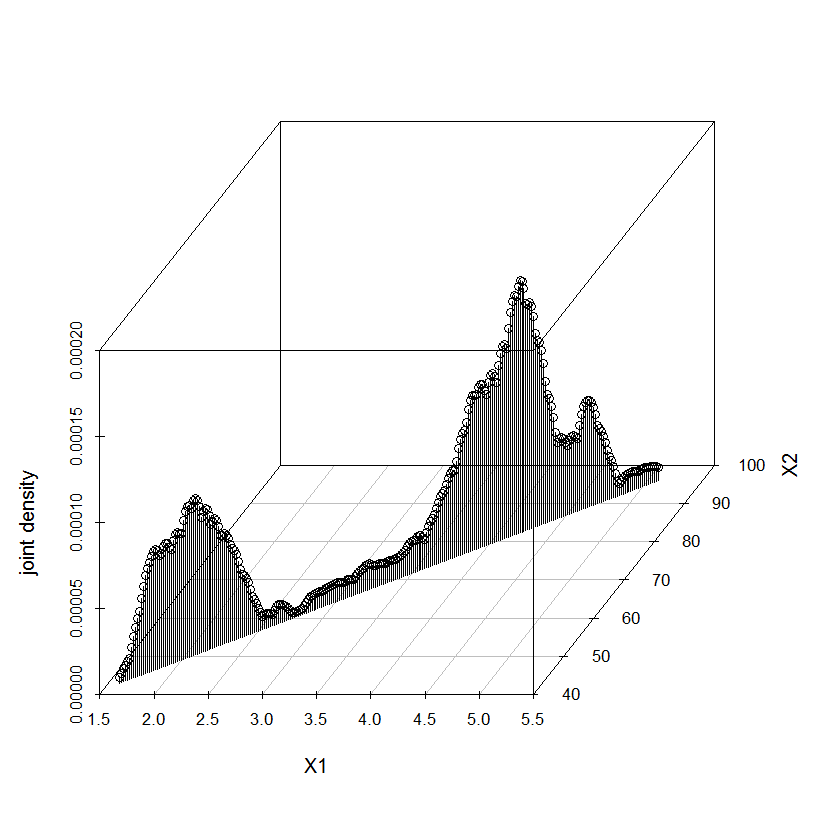
(黑色 ;紅色 ;綠色 ;藍色)

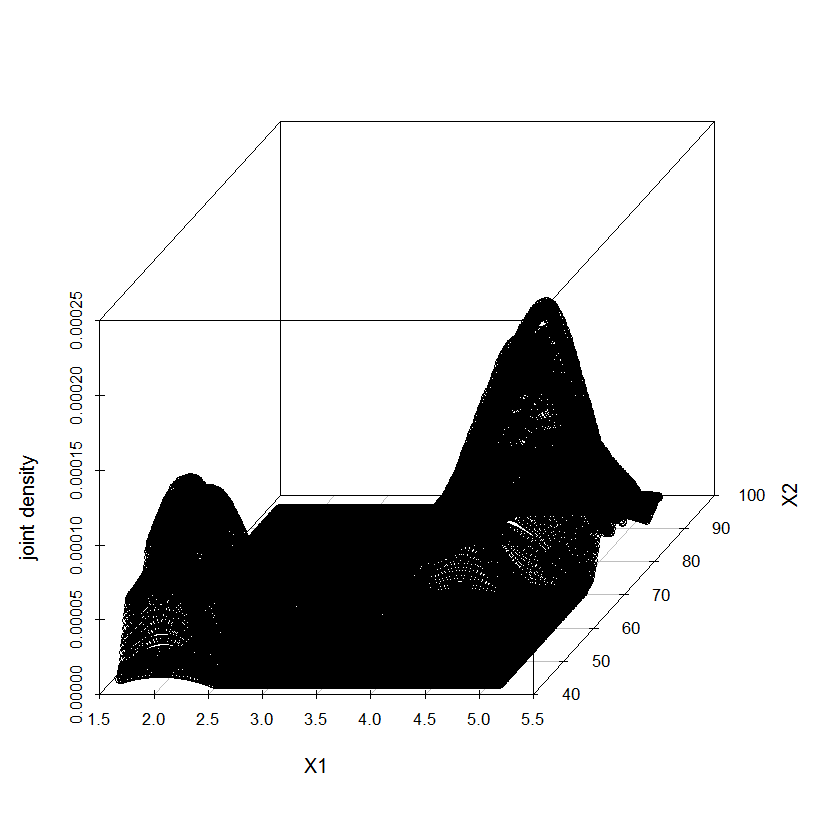
發現當圖形才比較平滑, 應該是原始資料X2等待時間的尺度和X1噴發時間不同, 可以看到不同樣本的等待時間差異較大. 故對於bandwith的選擇數字相對要大一些才趨於平滑.

For ,

有, 考慮估計量

, 畫出的3維圖如下:



(只畫出對角線部分的)

(所有的的)

#Exercise 1.

|  |
| --- |
| #non-parametric density est of kernal method  #R(f'')  R\_f<-function(x){ 1/pi\* (x^2-1)^2 \*exp(-x^2) }  I\_Simpson<-function(a,b,l){  #a<-0;b<-20;l<-50  h<-seq(a,b,(b-a)/l)  if(l%%2==0){  z1<-0;z2<-0  for(i in 2:(l/2)){ z1<-z1+2\*R\_f(h[2\*i-1]) }  for(i in 1:(l/2)){ z2<-z2+4\*R\_f(h[2\*i]) }  I\_S<-(R\_f(h[1])+R\_f(h[l+1])+z1+z2)\*(b-a)/l/3  } else {print("n need to be even")} }  Rf<-I\_Simpson(0,20,50)  #R(k) with Eparechinlcov kernal  Rk<-3/5  #optimal bandwith b\_n  bandwidth<-function(n){ ( Rk / n \*(1/25) \*Rf )^(1/5) }  #MISE  MISE<-function(n){ bandwidth(n)^4 \*(1/25) \*Rf /4 + 1/(n\*bandwidth(n)) \*Rk }  a<-MISE(10^6) |

#Exercise 2.

|  |
| --- |
| #Eparechinlcov kernel  K<-function(x){ if(x>-1 && x<1){ return(3/4\*(1-x^2)) }else{ return(0) } }  #Triangle kernel  K<-function(x){ if(x>-1 && x<1){ return(1-abs(x)) }else{ return(0) } }  #Uniform kernel  K<-function(x){ if(x>-1 && x<1){ return(1/2) }else{ return(0) } }  #Normal kernel  K<-function(x){ return(1/sqrt(2\*pi)\*exp(-x^2/2)) }  #g1 density  data<-faithful  n<-length(data[,1])  b\_n<-c(0.05,0.1,0.5,1)  for(k in 1:length(b\_n)){  x<-seq(min(data[,1]),max(data[,1]),0.01)  f<-rep(0,length(x))  for(l in 1:length(x)){  y<-rep(0,n)  for(i in 1:n){  y[i]<-K( (x[l]-data[i,1]) /b\_n[k] ) /n /b\_n[k]  f[l]<-sum(y)  }  }  if(k==1){ windows();plot(x,f,type = "l",lwd=2) }  else{ lines(x,f,col= k,lwd=2) }  }  #g2 density  data<-faithful  n<-length(data[,2])  b\_n<-c(1,2,3,4)  for(k in 1:length(b\_n)){  x<-seq(min(data[,2]),max(data[,2]),0.1)  f<-rep(0,length(x))  for(l in 1:length(x)){  y<-rep(0,n)  for(i in 1:n){  y[i]<-K( (x[l]-data[i,2]) /b\_n[k] ) /n /b\_n[k]  f[l]<-sum(y)  }  }  if(k==1){ windows();plot(x,f,type = "l",lwd=2) }  else{ lines(x,f,col= k,lwd=2) }  }  #g(x1,x2) density  data<-faithful  n<-272  #j=1  b\_n1<-0.5  x1<-seq(min(data[,1]),max(data[,1]), (max(data[,1])-min(data[,1]))/n )  #j=2  x2<-seq(min(data[,2]),max(data[,2]), (max(data[,2])-min(data[,2]))/n )  b\_n2<-2  #g(x1,x2) density 對角線部分  n<-272  x<-cbind(x1,x2)  f<-rep(0,length(x[,1]))  for(l in 1:length(x[,1])){  y<-rep(0,n)  for(i in 1:n){  y[i]<-K( (x[l,1]-data[i,1]) /b\_n1 ) /b\_n1 /n \*K( (x[l,2]-data[i,2]) /b\_n2 ) /b\_n2 /n  f[l]<-sum(y)  }  }  windows();plot(x[,1],f,type = "l",lwd=2)  windows();plot(x[,2],f,col = 2,type = "l",lwd=2)  library(scatterplot3d)  windows();scatterplot3d(x1,x2,f,color = ,type = "h",  xlab = "X1",  ylab = "X2",  zlab = "joint density")  library(plotly)  plot\_ly(data.frame(x=x1 ,y=x2 ,z=f)  , x = ~x ,y = ~y, z = ~z,color = , colors = ) %>%  add\_markers() %>%  layout(scene = list(xaxis = list(title = "X1"),  yaxis = list(title = "X2"),  zaxis = list(title = "joint density")))  #3維圖全部  n<-272  f<-matrix(0,length(x2),length(x1))  for(k in 1:length(x1)){  for(l in 1:length(x2)){  y<-rep(0,n)  for(i in 1:n){  y[i]<-K( (x1[k]-data[i,1]) /b\_n1 ) /b\_n1 /n \*K( (x2[l]-data[i,2]) /b\_n2 ) /b\_n2 /n  f[k,l]<-sum(y)  }  }  }  plot(x=1:9,y=matrix(11:19,3,3))  B<-c()  for(i in 1:273){  A<-cbind(x1,x2[i])  B<-rbind(B,A)  }  windows();scatterplot3d(B[,1],B[,2],f,color = ,type = "p",  xlab = "X1",  ylab = "X2",  zlab = "joint density")  plot\_ly(data.frame(x=B[,1] ,y=B[,2] ,z=as.vector(f) )  , x = ~x ,y = ~y, z = ~z,color = , colors = ) %>%  add\_markers() %>%  layout(scene = list(xaxis = list(title = "X1"),  yaxis = list(title = "X2"),  zaxis = list(title = "joint density"))) |