# Analytic Geometry

April 4, 2023

## 1 Orthonormal Basis

Basis of a vector space, where the basis vectors are orthogonal to each other and their length is 1.

**Definition - Orthonormal Basis**
Consider an $n$-dimensional vector space $V$ and a basis $\{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n\}$ of $V$. If

$$\langle \boldsymbol{b}_i, \boldsymbol{b}_j \rangle = 0 \quad for i \neq j \tag{1}$$

$$\langle \boldsymbol{b}_i, \boldsymbol{b}_j \rangle = 1 \tag{2}$$

for all $i, j = 1, \ldots, n$ then the basis is called an *orthonormal basis* (ONB). If only Eq. 1 is satisfied, then the basis is called an *orthogonal basis*. Note that Eq. 2 implies that every vector has length/norm 1.

**Example - Orthonormal Basis**
In $\mathbb{R}^2$, the vectors

$$\boldsymbol{b}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{b}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \tag{3}$$

form an orthonormal basis since $\boldsymbol{b}_1^T \boldsymbol{b}_2 = 0$ and $\|\boldsymbol{b}_1\| = 1 = \|\boldsymbol{b}_2\|$.