Matrix Decompositions

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1 Cholesky Decomposition

The *Cholesky Decomposition* is a matrix decomposition that provides a square root equivalent operation on symmetric, positive definite matrices.

Theorem - Cholesky Decomposition
A symmetric, positive definite matrix $\boldsymbol{A}$ can be factorized into a product $\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^T$, where $\boldsymbol{L}$ is a lower triangular matrix with positive diagonal elements.

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} l_{11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ l_{n1} & \dots & l_{nn} \end{bmatrix} \begin{bmatrix} l_{11} & \dots & l_{n1} \\ \vdots & \ddots & \vdots \\ 0 & \dots & l_{nn} \end{bmatrix} \tag{1}$$

$\boldsymbol{L}$ is the Cholesky factor of $\boldsymbol{A}$, and $\boldsymbol{L}$ is unique.

The Cholesky Decomposition is an important tool for numerical optimizations in Machine Learning. It also allows us to efficiently calculate determinants

$$det(\boldsymbol{A}) = det(\boldsymbol{L})det(\boldsymbol{L}^T) = det(\boldsymbol{L})^2 \tag{2}$$

Since $\boldsymbol{L}$ is a triangular matrix, the determinant is simply the product of the diagonal elements.