Rapport de laboratoire 3

MTH8408

NAMES

```
using Pkg
Pkg.activate("projet_env")
Pkg.add("ADNLPModels")
Pkg.add("NLPModels")
Pkg.add("Krylov")
Pkg.add("LinearOperators")

Pkg.add("OptimizationProblems")  # collection + outils pour sélectionner les problèmes
# TODO: add CUTest

using LinearAlgebra, NLPModels , ADNLPModels, Printf, Krylov, LinearOperators

using OptimizationProblems, OptimizationProblems.ADNLPProblems
```

```
function lbfgs_trust_region(nlp; eps_a=1.0e-5, eps_r=1.0e-5, mem=5, max_iter_ratio=10)

    xk = nlp.meta.x0
    dim = length(xk)

    fk = obj(nlp, xk)

    gk = grad(nlp, xk)
    grad_norm = norm(gk)

    # grad0 = deepcopy(grad)
    g0_norm = grad_norm


    # initial trust region radius
    deltak = 1.0

    pk = similar(gk)
    sk = similar(gk)
    gk_new = similar(gk)
    yk = similar(gk)

    # Hk = I(dim)
    Hk = LBFGSOperator(dim, mem = mem, scaling = true)  # par défaut, mem = 5

    max_iter = max_iter_ratio*dim

    for k in 1:max_iter
```

```
        # compute search direction using Hessian approx
        pk .= Hk*gk

        # compute step size using exact line search (for quad obj)
        # TODO: use Cauchy point?
        alphak = -dot(gk, pk)/dot(pk, Hk*pk)

        sk .= alphak*pk

        # clip step size to trust region radius
        sk_norm = norm(sk)
        if sk_norm > deltak
            sk .= sk/sk_norm*deltak
        end

        # predicted objective reduction
        red_pred = -dot(gk, sk)

        x_new = xk .+ sk
        f_new = obj(nlp, x_new)

        # actual obj reduction
        red_act = fk - f_new

        # actual obj reduction to predicted obj red ratio
        rho = red_act / red_pred

        # if reduction is greater than tolerance, accept step size. Otherwise,
        # reject step size
        if rho >= 1e-4
            # update new point
            xk .= x_new
            fk = f_new

            gk_new .= grad(nlp, xk)
            yk .= gk_new .- gk

            # update Hessian approximation
            push!(Hk, sk, yk)

            gk .= gk_new
            grad_norm = norm(gk)
        end

        # increase / decrease trust region radius
        if rho >= 0.75
            deltak *= 3
        elseif rho < 0.25
            deltak /= 3
        end

        # print optimization status
        @printf("|  k = %3d  |  grad_norm = %10.3e  |  fx = %10.3e  |  rho= %10.2f  |  deltak= %10.3e \n
```

```
        # check convergence crieteria
        if grad_norm <= eps_a + eps_r*g0_norm
            println("Convergence reached! ||gk|| = ", grad_norm)
            break
        end

    end

    return xk

end
```

```
lbfgs_trust_region (generic function with 1 method)
```

```
meta = OptimizationProblems.meta
problem_list = meta[(meta.ncon.==0).&.!meta.has_bounds.&(meta.nvar.==100), :name]
problems = (OptimizationProblems.ADNLPProblems.eval(Meta.parse(problem))() for problem   problem_list)
```

```
models = []
push!(models, OptimizationProblems.ADNLPProblems.chainwoo())
push!(models, OptimizationProblems.ADNLPProblems.errinros_mod())
push!(models, OptimizationProblems.ADNLPProblems.freuroth())

# Validation of the modified newton method
x_star = lbfgs_trust_region(models[2], mem=5)
```

```
|  k =   1  |  grad_norm =  5.440e+04  |  fx =  8.425e+04  |  rho=       0.77  |  deltak=  3.000e+00
|  k =   2  |  grad_norm =  2.607e+04  |  fx =  3.827e+04  |  rho=       0.67  |  deltak=  3.000e+00
|  k =   3  |  grad_norm =  1.236e+04  |  fx =  1.693e+04  |  rho=       0.70  |  deltak=  3.000e+00
|  k =   4  |  grad_norm =  6.000e+03  |  fx =  7.943e+03  |  rho=       0.69  |  deltak=  3.000e+00
|  k =   5  |  grad_norm =  2.886e+03  |  fx =  3.765e+03  |  rho=       0.70  |  deltak=  3.000e+00
|  k =   6  |  grad_norm =  1.424e+03  |  fx =  1.871e+03  |  rho=       0.70  |  deltak=  3.000e+00
|  k =   7  |  grad_norm =  7.078e+02  |  fx =  9.461e+02  |  rho=       0.70  |  deltak=  3.000e+00
|  k =   8  |  grad_norm =  3.537e+02  |  fx =  4.845e+02  |  rho=       0.70  |  deltak=  3.000e+00
|  k =   9  |  grad_norm =  1.766e+02  |  fx =  2.511e+02  |  rho=       0.71  |  deltak=  3.000e+00
|  k =  10  |  grad_norm =  8.438e+01  |  fx =  1.352e+02  |  rho=       0.70  |  deltak=  3.000e+00
|  k =  11  |  grad_norm =  3.995e+01  |  fx =  8.478e+01  |  rho=       0.70  |  deltak=  3.000e+00
|  k =  12  |  grad_norm =  2.016e+01  |  fx =  6.335e+01  |  rho=       0.71  |  deltak=  3.000e+00
|  k =  13  |  grad_norm =  1.130e+01  |  fx =  5.277e+01  |  rho=       0.73  |  deltak=  3.000e+00
|  k =  14  |  grad_norm =  1.130e+01  |  fx =  5.277e+01  |  rho=      -4.75  |  deltak=  1.000e+00
|  k =  15  |  grad_norm =  1.130e+01  |  fx =  5.277e+01  |  rho=      -4.75  |  deltak=  3.333e-01
|  k =  16  |  grad_norm =  4.208e+01  |  fx =  5.171e+01  |  rho=       0.28  |  deltak=  3.333e-01
|  k =  17  |  grad_norm =  9.291e+00  |  fx =  5.009e+01  |  rho=       0.40  |  deltak=  3.333e-01
|  k =  18  |  grad_norm =  8.364e+00  |  fx =  4.998e+01  |  rho=       0.85  |  deltak=  1.000e+00
|  k =  19  |  grad_norm =  7.986e+00  |  fx =  4.989e+01  |  rho=       0.65  |  deltak=  1.000e+00
|  k =  20  |  grad_norm =  7.867e+00  |  fx =  4.987e+01  |  rho=       0.76  |  deltak=  3.000e+00
|  k =  21  |  grad_norm =  8.144e+00  |  fx =  4.973e+01  |  rho=       0.84  |  deltak=  9.000e+00
|  k =  22  |  grad_norm =  7.861e+00  |  fx =  4.955e+01  |  rho=       0.77  |  deltak=  2.700e+01
|  k =  23  |  grad_norm =  7.704e+00  |  fx =  4.947e+01  |  rho=       0.70  |  deltak=  2.700e+01
|  k =  24  |  grad_norm =  7.879e+00  |  fx =  4.940e+01  |  rho=       0.64  |  deltak=  2.700e+01
|  k =  25  |  grad_norm =  7.622e+00  |  fx =  4.933e+01  |  rho=       0.58  |  deltak=  2.700e+01
|  k =  26  |  grad_norm =  7.876e+00  |  fx =  4.926e+01  |  rho=       0.59  |  deltak=  2.700e+01
|  k =  27  |  grad_norm =  7.437e+00  |  fx =  4.914e+01  |  rho=       0.74  |  deltak=  2.700e+01
|  k =  28  |  grad_norm =  7.660e+00  |  fx =  4.903e+01  |  rho=       0.78  |  deltak=  8.100e+01
```

| k = 29 | grad_norm = 7.339e+00 | fx = 4.896e+01 | rho= 0.68 | deltak= 8.100e+01
| k = 30 | grad_norm = 7.427e+00 | fx = 4.886e+01 | rho= 0.76 | deltak= 2.430e+02
| k = 31 | grad_norm = 7.154e+00 | fx = 4.881e+01 | rho= 0.69 | deltak= 2.430e+02
| k = 32 | grad_norm = 7.742e+00 | fx = 4.870e+01 | rho= 0.71 | deltak= 2.430e+02
| k = 33 | grad_norm = 7.059e+00 | fx = 4.863e+01 | rho= 0.67 | deltak= 2.430e+02
| k = 34 | grad_norm = 7.128e+00 | fx = 4.849e+01 | rho= 0.91 | deltak= 7.290e+02
| k = 35 | grad_norm = 7.005e+00 | fx = 4.846e+01 | rho= 0.70 | deltak= 7.290e+02
| k = 36 | grad_norm = 7.098e+00 | fx = 4.837e+01 | rho= 0.78 | deltak= 2.187e+03
| k = 37 | grad_norm = 6.854e+00 | fx = 4.833e+01 | rho= 0.69 | deltak= 2.187e+03
| k = 38 | grad_norm = 6.903e+00 | fx = 4.825e+01 | rho= 0.87 | deltak= 6.561e+03
| k = 39 | grad_norm = 6.883e+00 | fx = 4.822e+01 | rho= 0.67 | deltak= 6.561e+03
| k = 40 | grad_norm = 6.814e+00 | fx = 4.810e+01 | rho= 0.88 | deltak= 1.968e+04
| k = 41 | grad_norm = 6.780e+00 | fx = 4.807e+01 | rho= 0.75 | deltak= 5.905e+04
| k = 42 | grad_norm = 6.760e+00 | fx = 4.800e+01 | rho= 0.81 | deltak= 1.771e+05
| k = 43 | grad_norm = 6.699e+00 | fx = 4.797e+01 | rho= 0.69 | deltak= 1.771e+05
| k = 44 | grad_norm = 6.651e+00 | fx = 4.788e+01 | rho= 0.86 | deltak= 5.314e+05
| k = 45 | grad_norm = 6.698e+00 | fx = 4.785e+01 | rho= 0.70 | deltak= 5.314e+05
| k = 46 | grad_norm = 6.634e+00 | fx = 4.776e+01 | rho= 0.84 | deltak= 1.594e+06
| k = 47 | grad_norm = 6.598e+00 | fx = 4.772e+01 | rho= 0.77 | deltak= 4.783e+06
| k = 48 | grad_norm = 6.610e+00 | fx = 4.765e+01 | rho= 0.79 | deltak= 1.435e+07
| k = 49 | grad_norm = 6.507e+00 | fx = 4.762e+01 | rho= 0.70 | deltak= 1.435e+07
| k = 50 | grad_norm = 6.489e+00 | fx = 4.753e+01 | rho= 0.87 | deltak= 4.305e+07
| k = 51 | grad_norm = 6.488e+00 | fx = 4.749e+01 | rho= 0.71 | deltak= 4.305e+07
| k = 52 | grad_norm = 6.465e+00 | fx = 4.742e+01 | rho= 0.81 | deltak= 1.291e+08
| k = 53 | grad_norm = 6.440e+00 | fx = 4.738e+01 | rho= 0.78 | deltak= 3.874e+08
| k = 54 | grad_norm = 6.443e+00 | fx = 4.732e+01 | rho= 0.79 | deltak= 1.162e+09
| k = 55 | grad_norm = 6.351e+00 | fx = 4.729e+01 | rho= 0.72 | deltak= 1.162e+09
| k = 56 | grad_norm = 6.330e+00 | fx = 4.720e+01 | rho= 0.87 | deltak= 3.487e+09
| k = 57 | grad_norm = 6.324e+00 | fx = 4.716e+01 | rho= 0.71 | deltak= 3.487e+09
| k = 58 | grad_norm = 6.313e+00 | fx = 4.710e+01 | rho= 0.80 | deltak= 1.046e+10
| k = 59 | grad_norm = 6.305e+00 | fx = 4.706e+01 | rho= 0.76 | deltak= 3.138e+10
| k = 60 | grad_norm = 6.289e+00 | fx = 4.701e+01 | rho= 0.79 | deltak= 9.414e+10
| k = 61 | grad_norm = 6.221e+00 | fx = 4.697e+01 | rho= 0.75 | deltak= 9.414e+10
| k = 62 | grad_norm = 6.210e+00 | fx = 4.689e+01 | rho= 0.87 | deltak= 2.824e+11
| k = 63 | grad_norm = 6.164e+00 | fx = 4.686e+01 | rho= 0.72 | deltak= 2.824e+11
| k = 64 | grad_norm = 6.200e+00 | fx = 4.679e+01 | rho= 0.82 | deltak= 8.473e+11
| k = 65 | grad_norm = 6.168e+00 | fx = 4.675e+01 | rho= 0.74 | deltak= 8.473e+11
| k = 66 | grad_norm = 6.162e+00 | fx = 4.670e+01 | rho= 0.80 | deltak= 2.542e+12
| k = 67 | grad_norm = 6.104e+00 | fx = 4.667e+01 | rho= 0.77 | deltak= 7.626e+12
| k = 68 | grad_norm = 6.109e+00 | fx = 4.659e+01 | rho= 0.86 | deltak= 2.288e+13
| k = 69 | grad_norm = 6.030e+00 | fx = 4.656e+01 | rho= 0.73 | deltak= 2.288e+13
| k = 70 | grad_norm = 6.091e+00 | fx = 4.649e+01 | rho= 0.85 | deltak= 6.863e+13
| k = 71 | grad_norm = 6.040e+00 | fx = 4.646e+01 | rho= 0.72 | deltak= 6.863e+13
| k = 72 | grad_norm = 6.051e+00 | fx = 4.641e+01 | rho= 0.81 | deltak= 2.059e+14
| k = 73 | grad_norm = 6.001e+00 | fx = 4.637e+01 | rho= 0.78 | deltak= 6.177e+14
| k = 74 | grad_norm = 6.006e+00 | fx = 4.631e+01 | rho= 0.84 | deltak= 1.853e+15
| k = 75 | grad_norm = 5.928e+00 | fx = 4.628e+01 | rho= 0.75 | deltak= 1.853e+15
| k = 76 | grad_norm = 5.975e+00 | fx = 4.621e+01 | rho= 0.87 | deltak= 5.559e+15
| k = 77 | grad_norm = 5.933e+00 | fx = 4.618e+01 | rho= 0.72 | deltak= 5.559e+15
| k = 78 | grad_norm = 5.951e+00 | fx = 4.613e+01 | rho= 0.82 | deltak= 1.668e+16
| k = 79 | grad_norm = 5.916e+00 | fx = 4.609e+01 | rho= 0.78 | deltak= 5.003e+16
| k = 80 | grad_norm = 5.910e+00 | fx = 4.604e+01 | rho= 0.82 | deltak= 1.501e+17
| k = 81 | grad_norm = 5.849e+00 | fx = 4.601e+01 | rho= 0.76 | deltak= 4.503e+17
| k = 82 | grad_norm = 5.878e+00 | fx = 4.594e+01 | rho= 0.88 | deltak= 1.351e+18

| k = 83 | grad_norm = 5.834e+00 | fx = 4.591e+01 | rho= 0.73 | deltak= 1.351e+18
| k = 84 | grad_norm = 5.868e+00 | fx = 4.585e+01 | rho= 0.83 | deltak= 4.053e+18
| k = 85 | grad_norm = 5.842e+00 | fx = 4.582e+01 | rho= 0.76 | deltak= 1.216e+19
| k = 86 | grad_norm = 5.826e+00 | fx = 4.578e+01 | rho= 0.81 | deltak= 3.647e+19
| k = 87 | grad_norm = 5.783e+00 | fx = 4.575e+01 | rho= 0.77 | deltak= 1.094e+20
| k = 88 | grad_norm = 5.799e+00 | fx = 4.567e+01 | rho= 0.87 | deltak= 3.283e+20
| k = 89 | grad_norm = 5.743e+00 | fx = 4.565e+01 | rho= 0.75 | deltak= 3.283e+20
| k = 90 | grad_norm = 5.796e+00 | fx = 4.559e+01 | rho= 0.85 | deltak= 9.848e+20
| k = 91 | grad_norm = 5.773e+00 | fx = 4.555e+01 | rho= 0.74 | deltak= 9.848e+20
| k = 92 | grad_norm = 5.756e+00 | fx = 4.551e+01 | rho= 0.79 | deltak= 2.954e+21
| k = 93 | grad_norm = 5.724e+00 | fx = 4.548e+01 | rho= 0.78 | deltak= 8.863e+21
| k = 94 | grad_norm = 5.726e+00 | fx = 4.542e+01 | rho= 0.85 | deltak= 2.659e+22
| k = 95 | grad_norm = 5.667e+00 | fx = 4.539e+01 | rho= 0.76 | deltak= 7.977e+22
| k = 96 | grad_norm = 5.726e+00 | fx = 4.533e+01 | rho= 0.86 | deltak= 2.393e+23
| k = 97 | grad_norm = 5.701e+00 | fx = 4.530e+01 | rho= 0.72 | deltak= 2.393e+23
| k = 98 | grad_norm = 5.701e+00 | fx = 4.525e+01 | rho= 0.79 | deltak= 7.179e+23
| k = 99 | grad_norm = 5.667e+00 | fx = 4.522e+01 | rho= 0.78 | deltak= 2.154e+24
| k = 100 | grad_norm = 5.654e+00 | fx = 4.517e+01 | rho= 0.82 | deltak= 6.461e+24
| k = 101 | grad_norm = 5.608e+00 | fx = 4.514e+01 | rho= 0.77 | deltak= 1.938e+25
| k = 102 | grad_norm = 5.657e+00 | fx = 4.507e+01 | rho= 0.86 | deltak= 5.815e+25
| k = 103 | grad_norm = 5.624e+00 | fx = 4.505e+01 | rho= 0.73 | deltak= 5.815e+25
| k = 104 | grad_norm = 5.655e+00 | fx = 4.500e+01 | rho= 0.80 | deltak= 1.744e+26
| k = 105 | grad_norm = 5.611e+00 | fx = 4.496e+01 | rho= 0.76 | deltak= 5.233e+26
| k = 106 | grad_norm = 5.593e+00 | fx = 4.492e+01 | rho= 0.81 | deltak= 1.570e+27
| k = 107 | grad_norm = 5.560e+00 | fx = 4.489e+01 | rho= 0.78 | deltak= 4.710e+27
| k = 108 | grad_norm = 5.588e+00 | fx = 4.483e+01 | rho= 0.86 | deltak= 1.413e+28
| k = 109 | grad_norm = 5.555e+00 | fx = 4.480e+01 | rho= 0.75 | deltak= 1.413e+28
| k = 110 | grad_norm = 5.607e+00 | fx = 4.475e+01 | rho= 0.81 | deltak= 4.239e+28
| k = 111 | grad_norm = 5.557e+00 | fx = 4.472e+01 | rho= 0.75 | deltak= 4.239e+28
| k = 112 | grad_norm = 5.546e+00 | fx = 4.468e+01 | rho= 0.81 | deltak= 1.272e+29
| k = 113 | grad_norm = 5.514e+00 | fx = 4.464e+01 | rho= 0.78 | deltak= 3.815e+29
| k = 114 | grad_norm = 5.522e+00 | fx = 4.459e+01 | rho= 0.85 | deltak= 1.145e+30
| k = 115 | grad_norm = 5.494e+00 | fx = 4.456e+01 | rho= 0.76 | deltak= 3.434e+30
| k = 116 | grad_norm = 5.552e+00 | fx = 4.451e+01 | rho= 0.83 | deltak= 1.030e+31
| k = 117 | grad_norm = 5.501e+00 | fx = 4.448e+01 | rho= 0.74 | deltak= 1.030e+31
| k = 118 | grad_norm = 5.506e+00 | fx = 4.443e+01 | rho= 0.81 | deltak= 3.090e+31
| k = 119 | grad_norm = 5.466e+00 | fx = 4.440e+01 | rho= 0.77 | deltak= 9.271e+31
| k = 120 | grad_norm = 5.465e+00 | fx = 4.435e+01 | rho= 0.84 | deltak= 2.781e+32
| k = 121 | grad_norm = 5.434e+00 | fx = 4.432e+01 | rho= 0.77 | deltak= 8.344e+32
| k = 122 | grad_norm = 5.493e+00 | fx = 4.427e+01 | rho= 0.84 | deltak= 2.503e+33
| k = 123 | grad_norm = 5.442e+00 | fx = 4.424e+01 | rho= 0.74 | deltak= 2.503e+33
| k = 124 | grad_norm = 5.465e+00 | fx = 4.419e+01 | rho= 0.82 | deltak= 7.509e+33
| k = 125 | grad_norm = 5.417e+00 | fx = 4.416e+01 | rho= 0.75 | deltak= 2.253e+34
| k = 126 | grad_norm = 5.416e+00 | fx = 4.411e+01 | rho= 0.83 | deltak= 6.759e+34
| k = 127 | grad_norm = 5.375e+00 | fx = 4.409e+01 | rho= 0.78 | deltak= 2.028e+35
| k = 128 | grad_norm = 5.432e+00 | fx = 4.403e+01 | rho= 0.85 | deltak= 6.083e+35
| k = 129 | grad_norm = 5.381e+00 | fx = 4.400e+01 | rho= 0.74 | deltak= 6.083e+35
| k = 130 | grad_norm = 5.416e+00 | fx = 4.395e+01 | rho= 0.84 | deltak= 1.825e+36
| k = 131 | grad_norm = 5.367e+00 | fx = 4.392e+01 | rho= 0.75 | deltak= 1.825e+36
| k = 132 | grad_norm = 5.369e+00 | fx = 4.388e+01 | rho= 0.83 | deltak= 5.474e+36
| k = 133 | grad_norm = 5.323e+00 | fx = 4.385e+01 | rho= 0.78 | deltak= 1.642e+37
| k = 134 | grad_norm = 5.370e+00 | fx = 4.380e+01 | rho= 0.86 | deltak= 4.927e+37
| k = 135 | grad_norm = 5.321e+00 | fx = 4.377e+01 | rho= 0.75 | deltak= 1.478e+38
| k = 136 | grad_norm = 5.362e+00 | fx = 4.372e+01 | rho= 0.85 | deltak= 4.434e+38

| k = 137 | grad_norm = 5.315e+00 | fx = 4.369e+01 | rho= 0.74 | deltak= 4.434e+38
| k = 138 | grad_norm = 5.322e+00 | fx = 4.365e+01 | rho= 0.82 | deltak= 1.330e+39
| k = 139 | grad_norm = 5.275e+00 | fx = 4.362e+01 | rho= 0.77 | deltak= 3.991e+39
| k = 140 | grad_norm = 5.307e+00 | fx = 4.357e+01 | rho= 0.86 | deltak= 1.197e+40
| k = 141 | grad_norm = 5.262e+00 | fx = 4.354e+01 | rho= 0.76 | deltak= 3.592e+40
| k = 142 | grad_norm = 5.306e+00 | fx = 4.349e+01 | rho= 0.87 | deltak= 1.078e+41
| k = 143 | grad_norm = 5.261e+00 | fx = 4.346e+01 | rho= 0.74 | deltak= 1.078e+41
| k = 144 | grad_norm = 5.275e+00 | fx = 4.342e+01 | rho= 0.82 | deltak= 3.233e+41
| k = 145 | grad_norm = 5.228e+00 | fx = 4.339e+01 | rho= 0.76 | deltak= 9.698e+41
| k = 146 | grad_norm = 5.245e+00 | fx = 4.334e+01 | rho= 0.86 | deltak= 2.909e+42
| k = 147 | grad_norm = 5.203e+00 | fx = 4.332e+01 | rho= 0.77 | deltak= 8.728e+42
| k = 148 | grad_norm = 5.249e+00 | fx = 4.326e+01 | rho= 0.87 | deltak= 2.618e+43
| k = 149 | grad_norm = 5.202e+00 | fx = 4.324e+01 | rho= 0.74 | deltak= 2.618e+43
| k = 150 | grad_norm = 5.226e+00 | fx = 4.319e+01 | rho= 0.82 | deltak= 7.855e+43
| k = 151 | grad_norm = 5.179e+00 | fx = 4.317e+01 | rho= 0.74 | deltak= 7.855e+43
| k = 152 | grad_norm = 5.182e+00 | fx = 4.312e+01 | rho= 0.84 | deltak= 2.357e+44
| k = 153 | grad_norm = 5.144e+00 | fx = 4.310e+01 | rho= 0.78 | deltak= 7.070e+44
| k = 154 | grad_norm = 5.189e+00 | fx = 4.304e+01 | rho= 0.87 | deltak= 2.121e+45
| k = 155 | grad_norm = 5.138e+00 | fx = 4.301e+01 | rho= 0.75 | deltak= 2.121e+45
| k = 156 | grad_norm = 5.171e+00 | fx = 4.297e+01 | rho= 0.83 | deltak= 6.363e+45
| k = 157 | grad_norm = 5.125e+00 | fx = 4.294e+01 | rho= 0.73 | deltak= 6.363e+45
| k = 158 | grad_norm = 5.117e+00 | fx = 4.290e+01 | rho= 0.82 | deltak= 1.909e+46
| k = 159 | grad_norm = 5.083e+00 | fx = 4.287e+01 | rho= 0.78 | deltak= 5.726e+46
| k = 160 | grad_norm = 5.121e+00 | fx = 4.282e+01 | rho= 0.86 | deltak= 1.718e+47
| k = 161 | grad_norm = 5.072e+00 | fx = 4.280e+01 | rho= 0.76 | deltak= 5.154e+47
| k = 162 | grad_norm = 5.108e+00 | fx = 4.275e+01 | rho= 0.83 | deltak= 1.546e+48
| k = 163 | grad_norm = 5.062e+00 | fx = 4.272e+01 | rho= 0.73 | deltak= 1.546e+48
| k = 164 | grad_norm = 5.053e+00 | fx = 4.268e+01 | rho= 0.81 | deltak= 4.638e+48
| k = 165 | grad_norm = 5.019e+00 | fx = 4.265e+01 | rho= 0.78 | deltak= 1.392e+49
| k = 166 | grad_norm = 5.042e+00 | fx = 4.260e+01 | rho= 0.84 | deltak= 4.175e+49
| k = 167 | grad_norm = 5.001e+00 | fx = 4.258e+01 | rho= 0.76 | deltak= 1.252e+50
| k = 168 | grad_norm = 5.037e+00 | fx = 4.253e+01 | rho= 0.84 | deltak= 3.757e+50
| k = 169 | grad_norm = 4.988e+00 | fx = 4.250e+01 | rho= 0.73 | deltak= 3.757e+50
| k = 170 | grad_norm = 4.987e+00 | fx = 4.247e+01 | rho= 0.80 | deltak= 1.127e+51
| k = 171 | grad_norm = 4.949e+00 | fx = 4.244e+01 | rho= 0.77 | deltak= 3.381e+51
| k = 172 | grad_norm = 4.956e+00 | fx = 4.239e+01 | rho= 0.83 | deltak= 1.014e+52
| k = 173 | grad_norm = 4.925e+00 | fx = 4.237e+01 | rho= 0.77 | deltak= 3.043e+52
| k = 174 | grad_norm = 4.956e+00 | fx = 4.232e+01 | rho= 0.84 | deltak= 9.130e+52
| k = 175 | grad_norm = 4.905e+00 | fx = 4.229e+01 | rho= 0.74 | deltak= 9.130e+52
| k = 176 | grad_norm = 4.914e+00 | fx = 4.225e+01 | rho= 0.80 | deltak= 2.739e+53
| k = 177 | grad_norm = 4.872e+00 | fx = 4.222e+01 | rho= 0.76 | deltak= 8.217e+53
| k = 178 | grad_norm = 4.866e+00 | fx = 4.218e+01 | rho= 0.82 | deltak= 2.465e+54
| k = 179 | grad_norm = 4.839e+00 | fx = 4.216e+01 | rho= 0.78 | deltak= 7.395e+54
| k = 180 | grad_norm = 4.866e+00 | fx = 4.211e+01 | rho= 0.83 | deltak= 2.219e+55
| k = 181 | grad_norm = 4.814e+00 | fx = 4.209e+01 | rho= 0.75 | deltak= 2.219e+55
| k = 182 | grad_norm = 4.830e+00 | fx = 4.204e+01 | rho= 0.81 | deltak= 6.656e+55
| k = 183 | grad_norm = 4.786e+00 | fx = 4.202e+01 | rho= 0.74 | deltak= 6.656e+55
| k = 184 | grad_norm = 4.772e+00 | fx = 4.198e+01 | rho= 0.81 | deltak= 1.997e+56
| k = 185 | grad_norm = 4.743e+00 | fx = 4.195e+01 | rho= 0.79 | deltak= 5.990e+56
| k = 186 | grad_norm = 4.765e+00 | fx = 4.191e+01 | rho= 0.83 | deltak= 1.797e+57
| k = 187 | grad_norm = 4.714e+00 | fx = 4.189e+01 | rho= 0.76 | deltak= 5.391e+57
| k = 188 | grad_norm = 4.733e+00 | fx = 4.184e+01 | rho= 0.82 | deltak= 1.617e+58
| k = 189 | grad_norm = 4.688e+00 | fx = 4.182e+01 | rho= 0.73 | deltak= 1.617e+58
| k = 190 | grad_norm = 4.673e+00 | fx = 4.178e+01 | rho= 0.80 | deltak= 4.852e+58

| k = 191 | grad_norm = 4.636e+00 | fx = 4.175e+01 | rho= 0.78 | deltak= 1.456e+59
| k = 192 | grad_norm = 4.655e+00 | fx = 4.171e+01 | rho= 0.82 | deltak= 4.367e+59
| k = 193 | grad_norm = 4.605e+00 | fx = 4.169e+01 | rho= 0.76 | deltak= 1.310e+60
| k = 194 | grad_norm = 4.624e+00 | fx = 4.165e+01 | rho= 0.83 | deltak= 3.930e+60
| k = 195 | grad_norm = 4.576e+00 | fx = 4.162e+01 | rho= 0.73 | deltak= 3.930e+60
| k = 196 | grad_norm = 4.568e+00 | fx = 4.159e+01 | rho= 0.80 | deltak= 1.179e+61
| k = 197 | grad_norm = 4.522e+00 | fx = 4.156e+01 | rho= 0.78 | deltak= 3.537e+61
| k = 198 | grad_norm = 4.535e+00 | fx = 4.152e+01 | rho= 0.82 | deltak= 1.061e+62
| k = 199 | grad_norm = 4.486e+00 | fx = 4.150e+01 | rho= 0.77 | deltak= 3.183e+62
| k = 200 | grad_norm = 4.505e+00 | fx = 4.146e+01 | rho= 0.84 | deltak= 9.550e+62
| k = 201 | grad_norm = 4.452e+00 | fx = 4.143e+01 | rho= 0.73 | deltak= 9.550e+62
| k = 202 | grad_norm = 4.453e+00 | fx = 4.140e+01 | rho= 0.80 | deltak= 2.865e+63
| k = 203 | grad_norm = 4.400e+00 | fx = 4.137e+01 | rho= 0.76 | deltak= 8.595e+63
| k = 204 | grad_norm = 4.407e+00 | fx = 4.134e+01 | rho= 0.81 | deltak= 2.579e+64
| k = 205 | grad_norm = 4.355e+00 | fx = 4.132e+01 | rho= 0.77 | deltak= 7.736e+64
| k = 206 | grad_norm = 4.376e+00 | fx = 4.127e+01 | rho= 0.85 | deltak= 2.321e+65
| k = 207 | grad_norm = 4.317e+00 | fx = 4.125e+01 | rho= 0.74 | deltak= 2.321e+65
| k = 208 | grad_norm = 4.325e+00 | fx = 4.122e+01 | rho= 0.81 | deltak= 6.962e+65
| k = 209 | grad_norm = 4.269e+00 | fx = 4.120e+01 | rho= 0.75 | deltak= 6.962e+65
| k = 210 | grad_norm = 4.269e+00 | fx = 4.116e+01 | rho= 0.81 | deltak= 2.089e+66
| k = 211 | grad_norm = 4.214e+00 | fx = 4.114e+01 | rho= 0.77 | deltak= 6.266e+66
| k = 212 | grad_norm = 4.235e+00 | fx = 4.110e+01 | rho= 0.85 | deltak= 1.880e+67
| k = 213 | grad_norm = 4.172e+00 | fx = 4.108e+01 | rho= 0.74 | deltak= 1.880e+67
| k = 214 | grad_norm = 4.183e+00 | fx = 4.105e+01 | rho= 0.83 | deltak= 5.639e+67
| k = 215 | grad_norm = 4.128e+00 | fx = 4.103e+01 | rho= 0.73 | deltak= 5.639e+67
| k = 216 | grad_norm = 4.122e+00 | fx = 4.100e+01 | rho= 0.80 | deltak= 1.692e+68
| k = 217 | grad_norm = 4.065e+00 | fx = 4.098e+01 | rho= 0.78 | deltak= 5.075e+68
| k = 218 | grad_norm = 4.084e+00 | fx = 4.094e+01 | rho= 0.85 | deltak= 1.523e+69
| k = 219 | grad_norm = 4.020e+00 | fx = 4.092e+01 | rho= 0.75 | deltak= 1.523e+69
| k = 220 | grad_norm = 4.028e+00 | fx = 4.088e+01 | rho= 0.84 | deltak= 4.568e+69
| k = 221 | grad_norm = 3.977e+00 | fx = 4.087e+01 | rho= 0.72 | deltak= 4.568e+69
| k = 222 | grad_norm = 3.966e+00 | fx = 4.084e+01 | rho= 0.79 | deltak= 1.370e+70
| k = 223 | grad_norm = 3.910e+00 | fx = 4.082e+01 | rho= 0.78 | deltak= 4.111e+70
| k = 224 | grad_norm = 3.923e+00 | fx = 4.078e+01 | rho= 0.85 | deltak= 1.233e+71
| k = 225 | grad_norm = 3.860e+00 | fx = 4.077e+01 | rho= 0.75 | deltak= 3.700e+71
| k = 226 | grad_norm = 3.864e+00 | fx = 4.073e+01 | rho= 0.85 | deltak= 1.110e+72
| k = 227 | grad_norm = 3.816e+00 | fx = 4.071e+01 | rho= 0.72 | deltak= 1.110e+72
| k = 228 | grad_norm = 3.804e+00 | fx = 4.069e+01 | rho= 0.78 | deltak= 3.330e+72
| k = 229 | grad_norm = 3.748e+00 | fx = 4.067e+01 | rho= 0.77 | deltak= 9.990e+72
| k = 230 | grad_norm = 3.753e+00 | fx = 4.064e+01 | rho= 0.83 | deltak= 2.997e+73
| k = 231 | grad_norm = 3.693e+00 | fx = 4.062e+01 | rho= 0.76 | deltak= 8.991e+73
| k = 232 | grad_norm = 3.694e+00 | fx = 4.059e+01 | rho= 0.85 | deltak= 2.697e+74
| k = 233 | grad_norm = 3.644e+00 | fx = 4.057e+01 | rho= 0.72 | deltak= 2.697e+74
| k = 234 | grad_norm = 3.637e+00 | fx = 4.055e+01 | rho= 0.78 | deltak= 8.092e+74
| k = 235 | grad_norm = 3.579e+00 | fx = 4.053e+01 | rho= 0.76 | deltak= 2.427e+75
| k = 236 | grad_norm = 3.575e+00 | fx = 4.051e+01 | rho= 0.82 | deltak= 7.282e+75
| k = 237 | grad_norm = 3.521e+00 | fx = 4.049e+01 | rho= 0.77 | deltak= 2.185e+76
| k = 238 | grad_norm = 3.519e+00 | fx = 4.046e+01 | rho= 0.85 | deltak= 6.554e+76
| k = 239 | grad_norm = 3.464e+00 | fx = 4.044e+01 | rho= 0.73 | deltak= 6.554e+76
| k = 240 | grad_norm = 3.461e+00 | fx = 4.042e+01 | rho= 0.78 | deltak= 1.966e+77
| k = 241 | grad_norm = 3.405e+00 | fx = 4.040e+01 | rho= 0.75 | deltak= 1.966e+77
| k = 242 | grad_norm = 3.390e+00 | fx = 4.038e+01 | rho= 0.80 | deltak= 5.899e+77
| k = 243 | grad_norm = 3.341e+00 | fx = 4.037e+01 | rho= 0.77 | deltak= 1.770e+78
| k = 244 | grad_norm = 3.337e+00 | fx = 4.034e+01 | rho= 0.84 | deltak= 5.309e+78

| k = 245 | grad_norm = 3.279e+00 | fx = 4.033e+01 | rho= 0.74 | deltak= 5.309e+78
| k = 246 | grad_norm = 3.277e+00 | fx = 4.030e+01 | rho= 0.79 | deltak= 1.593e+79
| k = 247 | grad_norm = 3.226e+00 | fx = 4.029e+01 | rho= 0.73 | deltak= 1.593e+79
| k = 248 | grad_norm = 3.201e+00 | fx = 4.027e+01 | rho= 0.78 | deltak= 4.778e+79
| k = 249 | grad_norm = 3.156e+00 | fx = 4.025e+01 | rho= 0.78 | deltak= 1.433e+80
| k = 250 | grad_norm = 3.150e+00 | fx = 4.023e+01 | rho= 0.82 | deltak= 4.300e+80
| k = 251 | grad_norm = 3.093e+00 | fx = 4.022e+01 | rho= 0.74 | deltak= 4.300e+80
| k = 252 | grad_norm = 3.085e+00 | fx = 4.019e+01 | rho= 0.81 | deltak= 1.290e+81
| k = 253 | grad_norm = 3.041e+00 | fx = 4.018e+01 | rho= 0.72 | deltak= 1.290e+81
| k = 254 | grad_norm = 3.011e+00 | fx = 4.016e+01 | rho= 0.76 | deltak= 3.870e+81
| k = 255 | grad_norm = 2.966e+00 | fx = 4.015e+01 | rho= 0.79 | deltak= 1.161e+82
| k = 256 | grad_norm = 2.958e+00 | fx = 4.013e+01 | rho= 0.81 | deltak= 3.483e+82
| k = 257 | grad_norm = 2.905e+00 | fx = 4.012e+01 | rho= 0.75 | deltak= 1.045e+83
| k = 258 | grad_norm = 2.891e+00 | fx = 4.010e+01 | rho= 0.81 | deltak= 3.135e+83
| k = 259 | grad_norm = 2.850e+00 | fx = 4.009e+01 | rho= 0.71 | deltak= 3.135e+83
| k = 260 | grad_norm = 2.823e+00 | fx = 4.007e+01 | rho= 0.75 | deltak= 9.405e+83
| k = 261 | grad_norm = 2.774e+00 | fx = 4.006e+01 | rho= 0.78 | deltak= 2.821e+84
| k = 262 | grad_norm = 2.767e+00 | fx = 4.004e+01 | rho= 0.80 | deltak= 8.464e+84
| k = 263 | grad_norm = 2.716e+00 | fx = 4.003e+01 | rho= 0.75 | deltak= 2.539e+85
| k = 264 | grad_norm = 2.698e+00 | fx = 4.001e+01 | rho= 0.82 | deltak= 7.618e+85
| k = 265 | grad_norm = 2.657e+00 | fx = 4.000e+01 | rho= 0.72 | deltak= 7.618e+85
| k = 266 | grad_norm = 2.636e+00 | fx = 3.999e+01 | rho= 0.75 | deltak= 2.285e+86
| k = 267 | grad_norm = 2.584e+00 | fx = 3.998e+01 | rho= 0.77 | deltak= 6.856e+86
| k = 268 | grad_norm = 2.577e+00 | fx = 3.996e+01 | rho= 0.80 | deltak= 2.057e+87
| k = 269 | grad_norm = 2.526e+00 | fx = 3.995e+01 | rho= 0.75 | deltak= 6.170e+87
| k = 270 | grad_norm = 2.508e+00 | fx = 3.994e+01 | rho= 0.82 | deltak= 1.851e+88
| k = 271 | grad_norm = 2.464e+00 | fx = 3.993e+01 | rho= 0.72 | deltak= 1.851e+88
| k = 272 | grad_norm = 2.450e+00 | fx = 3.991e+01 | rho= 0.76 | deltak= 5.553e+88
| k = 273 | grad_norm = 2.394e+00 | fx = 3.990e+01 | rho= 0.76 | deltak= 1.666e+89
| k = 274 | grad_norm = 2.390e+00 | fx = 3.989e+01 | rho= 0.80 | deltak= 4.998e+89
| k = 275 | grad_norm = 2.336e+00 | fx = 3.988e+01 | rho= 0.75 | deltak= 1.499e+90
| k = 276 | grad_norm = 2.321e+00 | fx = 3.987e+01 | rho= 0.83 | deltak= 4.498e+90
| k = 277 | grad_norm = 2.274e+00 | fx = 3.986e+01 | rho= 0.72 | deltak= 4.498e+90
| k = 278 | grad_norm = 2.264e+00 | fx = 3.985e+01 | rho= 0.77 | deltak= 1.349e+91
| k = 279 | grad_norm = 2.208e+00 | fx = 3.984e+01 | rho= 0.74 | deltak= 1.349e+91
| k = 280 | grad_norm = 2.205e+00 | fx = 3.983e+01 | rho= 0.80 | deltak= 4.048e+91
| k = 281 | grad_norm = 2.150e+00 | fx = 3.982e+01 | rho= 0.75 | deltak= 1.215e+92
| k = 282 | grad_norm = 2.135e+00 | fx = 3.981e+01 | rho= 0.85 | deltak= 3.644e+92
| k = 283 | grad_norm = 2.091e+00 | fx = 3.980e+01 | rho= 0.72 | deltak= 3.644e+92
| k = 284 | grad_norm = 2.079e+00 | fx = 3.979e+01 | rho= 0.78 | deltak= 1.093e+93
| k = 285 | grad_norm = 2.027e+00 | fx = 3.979e+01 | rho= 0.73 | deltak= 1.093e+93
| k = 286 | grad_norm = 2.022e+00 | fx = 3.978e+01 | rho= 0.81 | deltak= 3.279e+93
| k = 287 | grad_norm = 1.971e+00 | fx = 3.977e+01 | rho= 0.76 | deltak= 9.838e+93
| k = 288 | grad_norm = 1.955e+00 | fx = 3.976e+01 | rho= 0.86 | deltak= 2.951e+94
| k = 289 | grad_norm = 1.918e+00 | fx = 3.975e+01 | rho= 0.72 | deltak= 2.951e+94
| k = 290 | grad_norm = 1.903e+00 | fx = 3.975e+01 | rho= 0.78 | deltak= 8.854e+94
| k = 291 | grad_norm = 1.855e+00 | fx = 3.974e+01 | rho= 0.73 | deltak= 8.854e+94
| k = 292 | grad_norm = 1.847e+00 | fx = 3.973e+01 | rho= 0.81 | deltak= 2.656e+95
| k = 293 | grad_norm = 1.803e+00 | fx = 3.973e+01 | rho= 0.76 | deltak= 7.968e+95
| k = 294 | grad_norm = 1.787e+00 | fx = 3.972e+01 | rho= 0.86 | deltak= 2.391e+96
| k = 295 | grad_norm = 1.753e+00 | fx = 3.971e+01 | rho= 0.72 | deltak= 2.391e+96
| k = 296 | grad_norm = 1.738e+00 | fx = 3.970e+01 | rho= 0.78 | deltak= 7.172e+96
| k = 297 | grad_norm = 1.691e+00 | fx = 3.970e+01 | rho= 0.73 | deltak= 7.172e+96
| k = 298 | grad_norm = 1.683e+00 | fx = 3.969e+01 | rho= 0.80 | deltak= 2.151e+97

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|  k = 299  |  grad_norm =  1.644e+00  |  fx =  3.969e+01  |  rho=       0.76  |  deltak=  6.454e+97
|  k = 300  |  grad_norm =  1.631e+00  |  fx =  3.968e+01  |  rho=       0.85  |  deltak=  1.936e+98
|  k = 301  |  grad_norm =  1.597e+00  |  fx =  3.968e+01  |  rho=       0.72  |  deltak=  1.936e+98
|  k = 302  |  grad_norm =  1.584e+00  |  fx =  3.967e+01  |  rho=       0.78  |  deltak=  5.809e+98
|  k = 303  |  grad_norm =  1.538e+00  |  fx =  3.967e+01  |  rho=       0.73  |  deltak=  5.809e+98
|  k = 304  |  grad_norm =  1.529e+00  |  fx =  3.966e+01  |  rho=       0.80  |  deltak=  1.743e+99
|  k = 305  |  grad_norm =  1.494e+00  |  fx =  3.966e+01  |  rho=       0.76  |  deltak=  5.228e+99
|  k = 306  |  grad_norm =  1.484e+00  |  fx =  3.965e+01  |  rho=       0.84  |  deltak= 1.568e+100
|  k = 307  |  grad_norm =  1.452e+00  |  fx =  3.965e+01  |  rho=       0.73  |  deltak= 1.568e+100
|  k = 308  |  grad_norm =  1.440e+00  |  fx =  3.964e+01  |  rho=       0.78  |  deltak= 4.705e+100
|  k = 309  |  grad_norm =  1.396e+00  |  fx =  3.964e+01  |  rho=       0.73  |  deltak= 4.705e+100
|  k = 310  |  grad_norm =  1.387e+00  |  fx =  3.963e+01  |  rho=       0.79  |  deltak= 1.412e+101
|  k = 311  |  grad_norm =  1.355e+00  |  fx =  3.963e+01  |  rho=       0.75  |  deltak= 1.412e+101
|  k = 312  |  grad_norm =  1.348e+00  |  fx =  3.963e+01  |  rho=       0.83  |  deltak= 4.235e+101
|  k = 313  |  grad_norm =  1.318e+00  |  fx =  3.962e+01  |  rho=       0.74  |  deltak= 4.235e+101
|  k = 314  |  grad_norm =  1.308e+00  |  fx =  3.962e+01  |  rho=       0.78  |  deltak= 1.270e+102
|  k = 315  |  grad_norm =  1.266e+00  |  fx =  3.962e+01  |  rho=       0.73  |  deltak= 1.270e+102
|  k = 316  |  grad_norm =  1.256e+00  |  fx =  3.961e+01  |  rho=       0.79  |  deltak= 3.811e+102
|  k = 317  |  grad_norm =  1.228e+00  |  fx =  3.961e+01  |  rho=       0.74  |  deltak= 3.811e+102
|  k = 318  |  grad_norm =  1.222e+00  |  fx =  3.960e+01  |  rho=       0.82  |  deltak= 1.143e+103
|  k = 319  |  grad_norm =  1.196e+00  |  fx =  3.960e+01  |  rho=       0.75  |  deltak= 1.143e+103
|  k = 320  |  grad_norm =  1.189e+00  |  fx =  3.960e+01  |  rho=       0.78  |  deltak= 3.430e+103
|  k = 321  |  grad_norm =  1.148e+00  |  fx =  3.960e+01  |  rho=       0.72  |  deltak= 3.430e+103
|  k = 322  |  grad_norm =  1.137e+00  |  fx =  3.959e+01  |  rho=       0.80  |  deltak= 1.029e+104
|  k = 323  |  grad_norm =  1.112e+00  |  fx =  3.959e+01  |  rho=       0.73  |  deltak= 1.029e+104
|  k = 324  |  grad_norm =  1.108e+00  |  fx =  3.959e+01  |  rho=       0.81  |  deltak= 3.087e+104
|  k = 325  |  grad_norm =  1.085e+00  |  fx =  3.958e+01  |  rho=       0.76  |  deltak= 9.261e+104
|  k = 326  |  grad_norm =  1.081e+00  |  fx =  3.958e+01  |  rho=       0.78  |  deltak= 2.778e+105
|  k = 327  |  grad_norm =  1.044e+00  |  fx =  3.958e+01  |  rho=       0.72  |  deltak= 2.778e+105
|  k = 328  |  grad_norm =  1.032e+00  |  fx =  3.958e+01  |  rho=       0.80  |  deltak= 8.335e+105
|  k = 329  |  grad_norm =  1.009e+00  |  fx =  3.957e+01  |  rho=       0.72  |  deltak= 8.335e+105
|  k = 330  |  grad_norm =  1.006e+00  |  fx =  3.957e+01  |  rho=       0.80  |  deltak= 2.501e+106
|  k = 331  |  grad_norm =  9.853e-01  |  fx =  3.957e+01  |  rho=       0.77  |  deltak= 7.502e+106
|  k = 332  |  grad_norm =  9.843e-01  |  fx =  3.957e+01  |  rho=       0.78  |  deltak= 2.251e+107
|  k = 333  |  grad_norm =  9.520e-01  |  fx =  3.957e+01  |  rho=       0.72  |  deltak= 2.251e+107
|  k = 334  |  grad_norm =  9.399e-01  |  fx =  3.956e+01  |  rho=       0.80  |  deltak= 6.752e+107
Convergence reached! ||gk|| = 0.9398855807368246

100-element Vector{Float64}:
 -0.5413679368720895
  0.013057644819587183
  0.06864746008922935
  0.20036004619054945
  0.3081539982140512
  0.13811936830491098
  0.05906802764504452
  0.043664306337813694
  0.060269004231416574
  0.14585366290112622

  0.20914491252584635
  0.3053198629557684
  0.1335590080863471
  0.05781960570586988
```

```
0.04369705813020619
0.061603928742426116
0.15155020307787012
0.31227420397021133
0.17017981800716891
```