

# **Sensitivity of the Hadley cells with respect to changes in CO<sub>2</sub> concentrations**

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**Abstract**

The Hadley cells (HC) are a principal component of the general atmospheric circulation and are one mechanism for heat transport from the equator to the tropics. The cells consist of a convection current transporting hot humid air upwards at the equator and northwards to the subtropical ridge, which finally descends back to the surface after losing most of its moisture. An idealized ocean Earth symmetric about its axis is modeled considering atmospheric convection currents that assume conservation of momentum and losses of heat only to outer space. HC circulation is modelled to be driven exclusively by surface temperature difference inducing a pressure gradient along the surface between the subtropical ridge and equator. Model results confirm a reduction in strength of the HC with respect to increased CO<sub>2</sub> concentrations. We conclude that this relationship is logarithmic with respect to global mean temperature increase induced by [CO<sub>2</sub>] forcing. This study contributes to a more comprehensive understanding of HC dynamics influenced by atmospheric CO<sub>2</sub> fluxes and corroborates previously observed weakenings in the HC. The drying of the subtropical ridge driven by HC circulation underscores the importance of understanding the system's evolution in response to anthropogenic CO<sub>2</sub> emissions.

**(194 words)**

## 1. Introduction

The Hadley cell (HC) circulation plays a significant role in the global climate because it is a mechanism of heat transport from the equator to the subtropics and contributes to desertification at the subtropical ridge. It is also responsible for the formation of trade winds in the tropics and controls low-latitude weather patterns. The recent acceleration in global warming and the increase in atmospheric CO<sub>2</sub> concentrations underscore the importance of examining the HC and changes in circulation strength as demonstrated by changes in velocity in response to increased CO<sub>2</sub> fluxes.

Past model approaches include the two-column model of the tropical atmosphere, one of which characterizes a region of high humidity with deep convection and latent heating of the atmosphere, while the second is a region of dry descent (Pierrehumbert, 1995). Other analyses (Held and Soden, 2006; Lu et al., 2007) have used an archive of coupled climate model results from the Fourth Assessment Report (AR4) of the Intergovernmental Panel on Climate Change to identify weakening in the Hadley cell circulation as a result of increased greenhouse gas forcing. Coupled Model Intercomparison Project Phase 5 model projections have also demonstrated the effects of CO<sub>2</sub>-induced warming on changes in the HC circulation through strengthening and weakening components (Lau and Kim, 2015). The weakening of the HC was determined by the maximum value of the meridional mass streamfunction in the subtropics (Lau and Kim, 2015). In terms of simplifications, Pierrehumbert (1995) assumed clouds to have a negligible effect on atmospheric heating and circulation, as well as on sea surface temperature, which we also assume.

While there exists much research about HC expansion and weakening caused by global warming, such approaches predominantly use climate data such as global monthly temperature

and velocity, as well as model projection data from under steps increases in CO<sub>2</sub> emission scenarios, to assess the weakening rates of the HC circulation. Our paper uses a 4-box model to assess the strength of the HC in response to step increases in CO<sub>2</sub> ppm, rather than climate data and projections. The objective of this paper is to model the HC and identify the effect of the temperature difference between the subtropics and equator at equilibrium on the circulation strength. We hypothesize that the HC circulation strength will decrease in this model with respect to an increased global temperature since effects such as polar amplification create a decreased meridional temperature gradient between the subtropics and the equator under a global warming regime (Sellers 1969; Budyko 1969). This is because the HC circulation is modeled as being driven exclusively by the temperature difference between the equator and subtropics. The exact relationship can be determined by perturbing the emissivity of the atmosphere based on a log relationship with [CO<sub>2</sub>] in ppm as described by Huang and Shahabadi (2014).

## **2. Results and Discussion**

The results of modelling HC circulation strength with respect to atmospheric CO<sub>2</sub> concentrations are presented in this section.

The model was integrated for 100 years to reach a stable velocity and surface/ atmospheric temperatures at the subtropics and equator using a 1,000 second time-step. After running the model to equilibrium for 100 years, the two column, 4-box model described in the methods section was perturbed by a step increase in CO<sub>2</sub> concentration for 15 different values of CO<sub>2</sub> ppm between 280 and 560 ppm. These trials are shown influencing absolute velocity of surface winds and steady state meridional surface temperature difference in Figures 1 and 2. As expected, the temperature difference between 0 and 30 degrees latitude decreases with increased

CO<sub>2</sub> ppm as the absolute temperature increases due to the greenhouse effect, as demonstrated in Figure 3. The temperature gradient is the first order forcing that principally drives the circulation in our model, so the model predicts a weakening in the HC with respect to increased atmospheric CO<sub>2</sub> ppm. This is exhibited by the negative linear relationship between HC strength at equilibrium and atmospheric CO<sub>2</sub> concentrations in Figure 4. Following these results, the relationship between meridional surface temperature difference and HC circulation strength can be seen to be directly proportional by Figure 5.

The relationship between atmospheric emissivity and CO<sub>2</sub> ppm is modelled as logarithmic where the emissivity constant of the atmosphere is directly proportional to doublings of [CO<sub>2</sub>] in ppm. In the model, this relationship is carried through to HC circulation strength, so the average velocity of air in surface currents flowing south from the subtropical ridge is inversely proportional to doublings of [CO<sub>2</sub>] in ppm. This is exhibited by the approximately equivalent slope values of velocity shown between doublings of [CO<sub>2</sub>] ppm in Figure 6.

The decreased velocity in response to increased CO<sub>2</sub> ppm signifies the HC's reduction in strength, as anticipated by results of previous literature that identify a robust weakening and poleward expansion of the HC circulation in response to increased greenhouse gas forcings (Pierrehumbert 1995; Held and Soden 2006; Lu et al. 2007). Lu et al. (2007) concluded the HC to weaken at rates of 0-4%/K, with a mean of 1.2%/K. Also, models in Lu et al. (2007) projected a slowdown of the whole tropical overturning circulation. This is consistent with Held and Soden (2006)'s results determined from the scaling of the Clausius-Clapeyron expression for the saturation vapor pressure, in which the decrease in convective mass fluxes was a robust response to the increase in temperature and increase in saturation vapor pressure.

Shortcomings in the model include the idealistic fixed temperature profile in each box of the two-box model. Including the effects of continuous radiation and convection would allow for more accurate and dynamic temperature values. Additionally, the density of air and width of the HC remain unrealistically fixed in each box of the model, at  $1.225 \text{ kg/m}^3$  and  $1,500 \text{ km}$ , respectively. Future study results could be made more accurate by accounting for changes in these variables, especially as HC expansion has been identified as a result of increased  $\text{CO}_2$  concentrations.

### **3. Conclusion**

The results of this model suggest that the HC circulation strength should weaken under a global warming scenario. This model demonstrates that as meridional temperature declines, the pressure gradient induced by a temperature difference also declines, which also reduces the velocity of the surface winds that drive the convection currents. This study produces results showing that the relationship is logarithmic when HC circulation is modeled to be driven exclusively by a surface temperature difference-induced pressure gradient. Future work should examine whether this relationship remains logarithmic when other first order or additional second order terms, or if a different, nonlinear function emerges. Other forcing terms that have been examined by existing literature include baroclinic eddy fluxes (Kim and Lee, 2001). As well, many modern approaches to modeling HC circulation assume angular momentum is not conserved. Lu et al. (2007) demonstrates that the increase in the subtropical static stability has a significant effect on poleward expansion of HC circulation, which is another important aspect of the HC circulation that lies outside of this paper's scope.

## Appendix

### Methods

This paper's model of the HC circulation consists of two air columns at the equator and subtropical ridge. Temperature is modelled in each column at the surface and in the atmosphere. The earth is modelled as uniformly covered in a surface ocean of fixed mixed layer depth and the atmosphere is assumed to be of constant density scaled to an appropriate scale height such that total mass is equivalent to that of earth. The governing equations for surface net wind velocity from equator to subtropical ridge and energy balance between earth's surface and atmosphere, are derived from the conservation laws of momentum and energy respectively. We make the assumption that the momentum of air is conserved as the circulation flows, so particles of air flow south from the subtropical ridge along the surface and flow north at the top of the tropopause such that the net transfer of mass between both columns is zero. It is further assumed that the dominant first order term determining flux in momentum at the surface is the temperature induced pressure gradient between the subtropical ridge and earth's equator according to the ideal gas law and that density plays a negligible role. The momentum equation for the column atmosphere can be written as

$$(1) M (dv/dt) = -V*\Delta P/L - F_f(v)$$

$$(2) \rho_a*L*H_a*(dv/dt) = -H_a*L*\Delta P/L - \rho_a*C_d*v^2*L$$

where  $M$  (kg) is mass of air,  $v$  (m/s) is northward velocity of air at the surface,  $V$  ( $m^3$ ) is the volume of air in the column,  $P$  ( $N/m^2$ ) is atmospheric surface pressure, and  $L$  (m) is half the distance from the equator to the subtropics. We model restoring friction force  $F_f(v)$  as a surface drag between the current of air and the earth's surface which is quadratic with velocity. We write  $F_f(v)$  as  $\rho_a*C_d*v^2*L$  where ( $C_d = 0.05$ ). Typical coefficients of friction are an order of magnitude

lower but since the friction term acts across the entire scale height of the atmosphere the coefficient is adjusted such that terminal velocity of air is close to the expected value. Going from (1) to (2), the longitudinal dimension is divided out so  $V \cdot \Delta P / L$  is expressed as  $H_a \cdot L \cdot \Delta P / L$  where  $H_a$  (=10,000 m) is the scale height of the atmosphere.  $\rho_a$  (kg/m<sup>3</sup>) is the density of air at earth's surface which we assume is negligibly influential on the circulation so for both the equator and subtropics it is fixed to 1.225 kg/m<sup>3</sup>.

Since we expect temperature delta to be a small fraction of absolute temperatures in degrees Kelvin, we make the approximation

$$\Delta P = H_a \cdot (P_0 / R) \cdot (1/T_{st} - 1/T_{eq}) \cdot g \approx \rho_a \cdot 2 \cdot (\Delta T / T_0) \cdot g \cdot H_a$$

where  $g$  (m/s<sup>2</sup>) is the gravitational constant and  $T_{eq}$  and  $T_{st}$  (K) are surface temperatures in equatorial and subtropical columns respectively. From this we can obtain

$$(3) \rho_a \cdot L \cdot H_a \cdot (dv/dt) = -H_a \cdot \rho_a \cdot (T_{eq} - T_{st}) / T_0 \cdot g \cdot H_a - F_f(v)$$

By simplifying further, we obtain the expression

$$(4) (dv/dt) = -H_a \cdot (T_{eq} - T_{st}) / T_0 \cdot g \cdot L^{-1} - Cd \cdot v^2 \cdot H_a^{-1}.$$

Energy balance between the surface ocean and atmosphere is modelled assuming that the surface acts as a perfect blackbody with an albedo  $\alpha$  of 0.3 and the atmosphere as having an emissivity constant  $\epsilon$  of 0.8. The flux of solar radiation received by the surface at the equator and subtropical ridge is scaled according to the ratio of cross sectional area over surface area between the latitudes 0-15 degrees and 15-30 degrees, respectively. Using the conservation of energy at the equator the governing equation of surface temperature can be written

$$(5) \rho_w \cdot H_o \cdot C_w \cdot (dT_{eq}/dt) = (A_{\perp eq} \cdot S_o(1-\alpha)) / A_{eq} - \sigma T_{eq}^4 + \epsilon \sigma T_a^4$$



where  $\rho_w$  (kg/m<sup>3</sup>) is the density of water,  $H_o$  (m) is the mixed layer depth of the ocean,  $C_w$  (= 4180 J/kg/K) is the specific heat capacity of water,  $A_{\perp_{eq}}$  (m<sup>2</sup>) is the cross sectional area of the earth's sphere between 0 and 15 degrees latitude,  $S_o$  (= 1360 W/m<sup>2</sup>) is the solar constant,  $A_{eq}$  (m<sup>2</sup>) is the surface area of earth between 0 and 15 degrees latitude, and  $\sigma$  (= 5.67x10<sup>-8</sup> W/m<sup>2</sup>/K<sup>4</sup>) is the Stefan-Boltzmann constant. The first term is solar flux heating the surface and the final two terms are heat lost through long wave radiation and heat gained from the atmosphere. The conservation of energy for the surface at the subtropics can be written in the same way.

$$(6) \rho_w * H_o * C_w * (dT_{s_{st}}/dt) = (A_{\perp_{st}} * S_o(1-a))/A_{st} - \sigma T_{s_{st}}^4 + \epsilon \sigma T_{a_{st}}^4$$

To model the heat flux resulting from convection currents the velocity of northward air obtained from (4) is used in the energy balance of atmospheres. The governing equation for the temperature of atmosphere in the equatorial column can be written

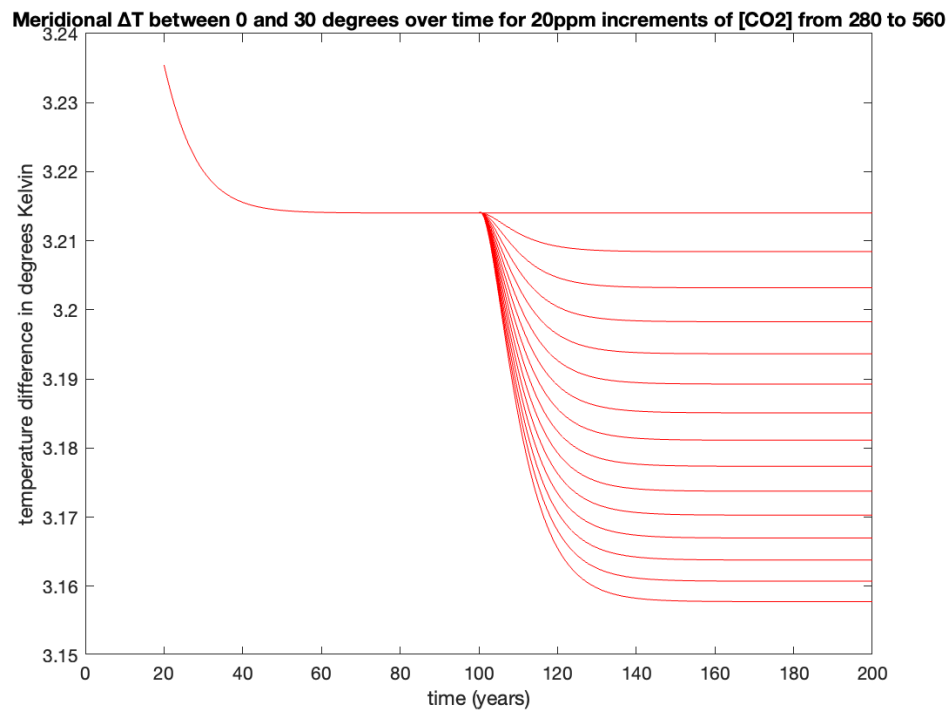
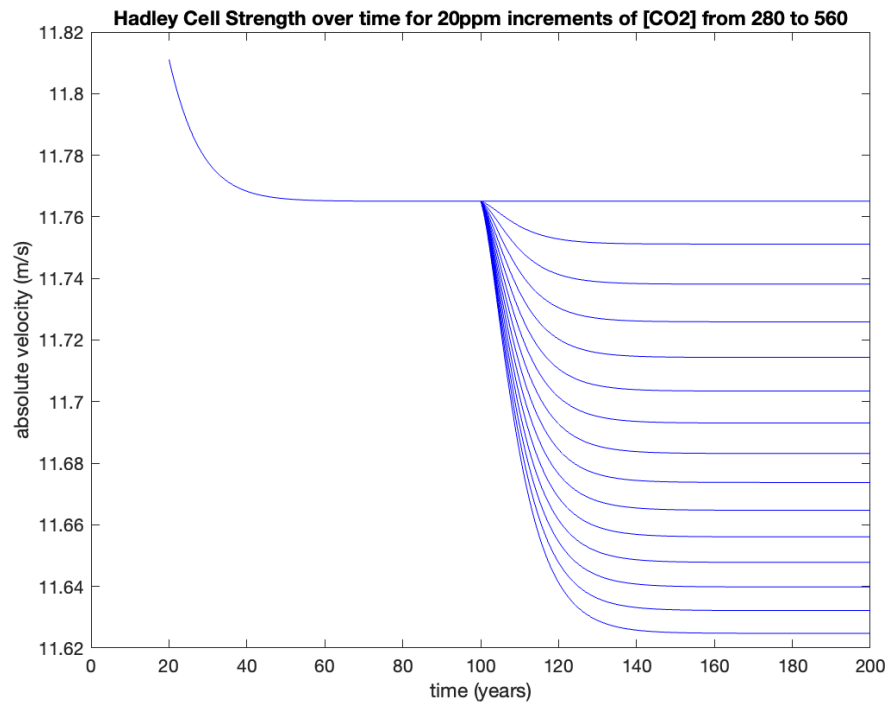
$$(7) \rho_a * H_a * C_a * (dT_{a_{eq}}/dt) = \epsilon \sigma T_{s_{eq}}^4 - 2\epsilon \sigma T_{a_{eq}}^4 - (\rho_a * H_a * v * C_a * (T_{a_{st}} - T_{a_{eq}}))/L$$

where  $C_a$  (=1000 J/kg/K) is the heat capacity of air. The energy balance of atmosphere in the subtropical column can be expressed as

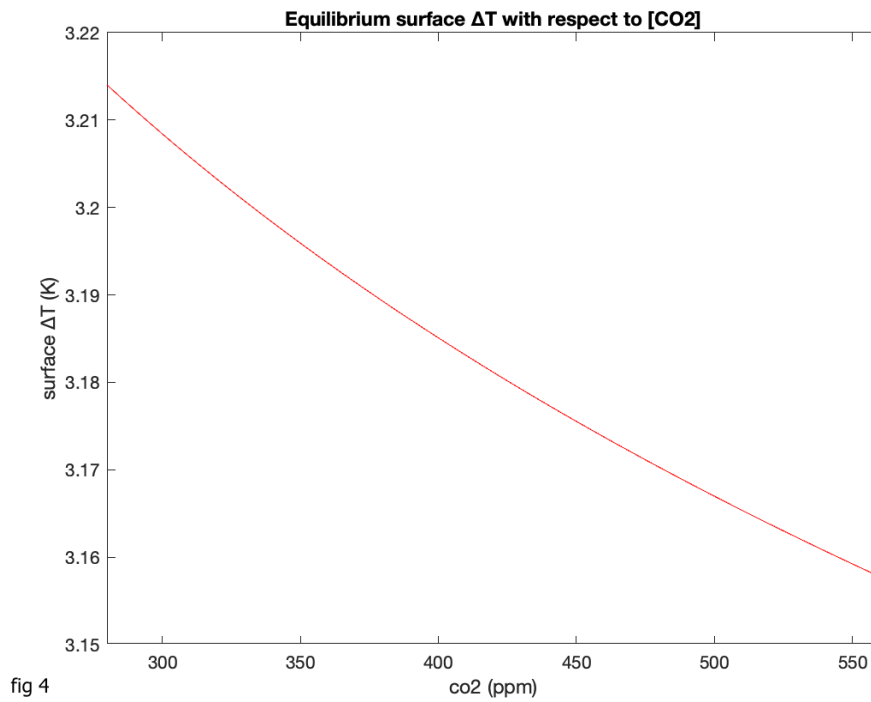
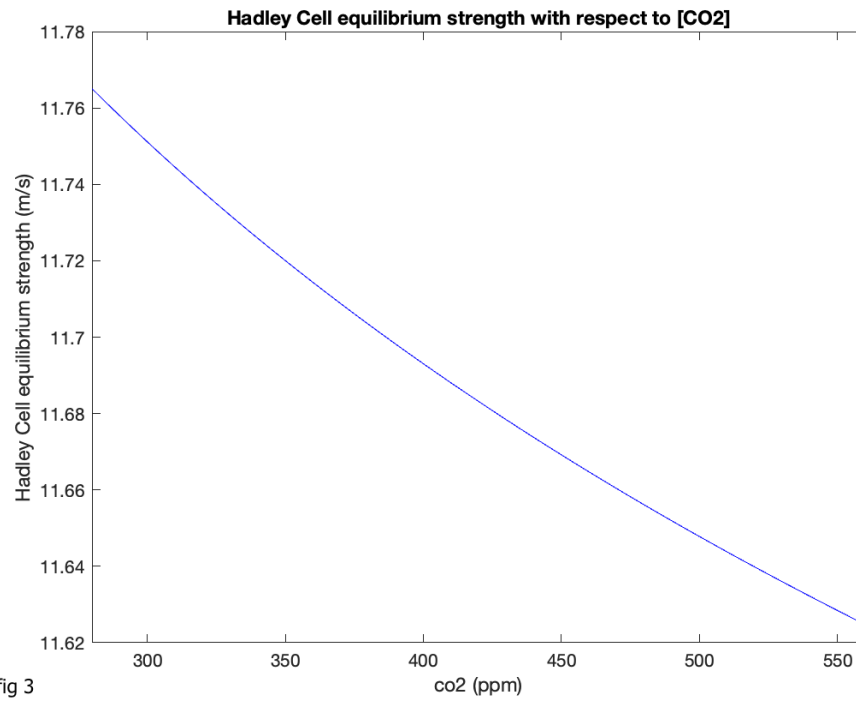
$$(8) \rho_a * H_a * C_a * (dT_{a_{st}}/dt) = \epsilon \sigma T_{s_{st}}^4 - 2\epsilon \sigma T_{a_{st}}^4 - (\rho_a * H_a * v * C_a * (T_{a_{eq}} - T_{a_{st}}))/L$$

To solve the model equations are discretized using an explicit numerical scheme and time marched forwards without any smoothing or stabilization schemes. Experiments that were performed to determine the effect on HC circulation strength with respect to changing CO<sub>2</sub> concentration in the atmosphere were done by modelling atmospheric emissivity by the following formula derived from Huang and Shahabadi (2014).

$$\epsilon = \epsilon_0 * (1 + a * \ln([CO_2] / 280))$$



The curves in descending order in Figures 1 and 2 show trials for [CO<sub>2</sub>] = 280, 300... 540, 560 ppm (incremented by 20 ppm).



Figures 3 and 4 show  $\Delta T$  between 0 and 30 degrees latitude at equilibrium for varying levels of [CO<sub>2</sub>] ppm.

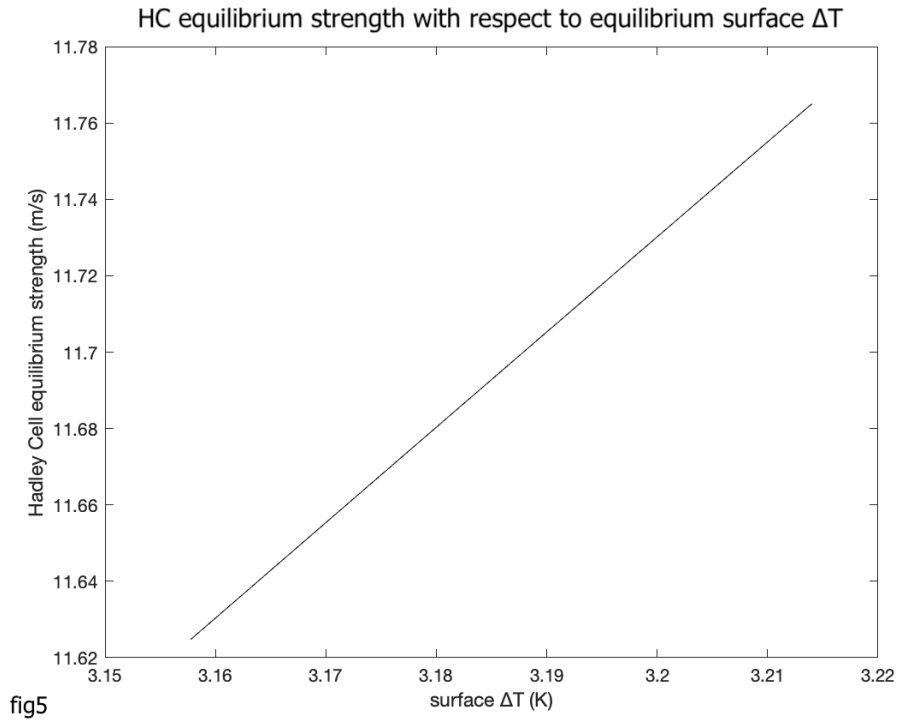
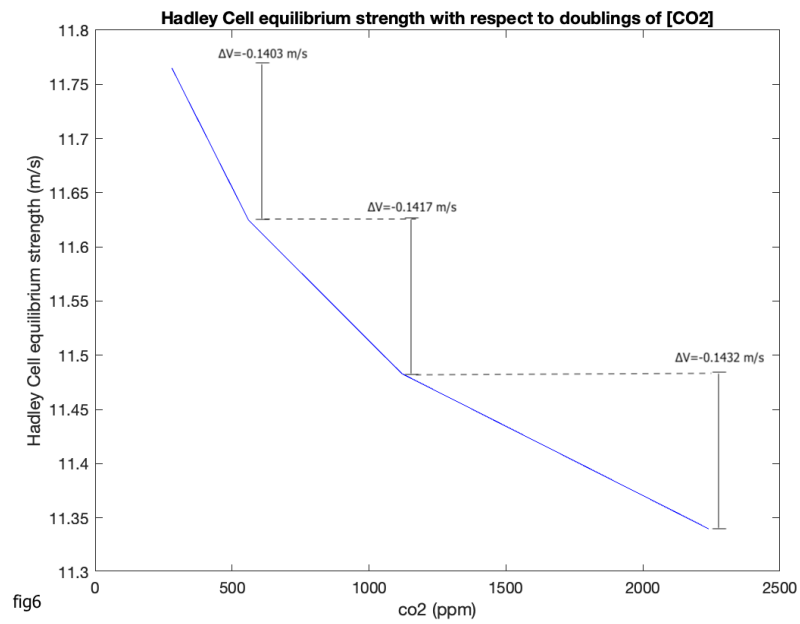


Figure 5 shows the direct relationship between surface temperature gradient and the HC strength.



The doublings in the Figure 6 trials represent  $CO_2$  concentrations at 280, 560, 1120, and 2240 ppm, respectively.

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