SIGNAL PROCESSING IN THE CONTEXT OF CHAOTIC SIGNALS

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ABSTRACT

Signals generated by chaotic systems represent a potentially rich class of signals both for detecting and characterizing physical phenomena and in synthesizing new classes of signals for communications, remote sensing and a variety of other signal processing applications. Since classical techniques for signal analysis do not exploit the particular structure of chaotic signals there is both a significant challenge and an opportunity in exploring new classes of algorithms matched to chaotic signals. In this paper we outline a variety of signal processing issues associated with the analysis and synthesis of chaotic signals. In addition we describe in some detail two examples illustrating some possible ways in which the characteristics of chaotic signals and systems can potentially be exploited. One example is a binary signalling scheme using chaotic signals.The second example is the use of synchronized chaotic systems for signal masking and recovery.

1. INTRODUCTION

In classical signal processing a rich set of tools has evolved for processing signals which are deterministic and predictable such as transient and periodic signals, and for processing signals that are stochastic. Chaotic signals associated with the homogeneous response of certain nonlinear dynamical systems do not fall in either of these classes. While they are deterministic, they are not predictable in any practical sense in that even with the generating dynamics known, estimation of prior or future values from a segment of the signal or from the state at a given time is highly ill-conditioned. In many ways these signals appear to be noise-like and can of course be analyzed and processed using classical techniques for stochastic signals. However, they clearly have considerably more structure than can be inferred from and exploited by traditional stochastic modeling techniques. Consequently it is important to develop new signal processing techniques which are matched to the special characteristics of this class of signals.

The basic structure of chaotic signals and the mechanisms through which they are generated are described in a variety of introductory books, e.g. [1] and summarized in [2]. Chaotic signals are of particular interest and importance in experimental physics because of the wide range of physical processes that apparently give rise to chaotic behavior. From the point of view of signal processing, the detection, analysis and characterization of signals of this type present a significant challenge and an opportunity to explore and develop completely new classes of algorithms for signal processing. In addition, chaotic systems provide a potentially rich mechanism for signal design and generation for a variety of communications and remote sensing applications.

In this paper we outline a variety of signal processing issues associated with the analysis and synthesis of chaotic signals. In addition, we illustrate our perspective and approach with two examples suggestive of some possible ways in which the characteristics of chaotic signals and systems can be exploited. Specifically, in section 4 we propose and illustrate a binary signalling scheme based on the use of chaotic signals and in section 5 we propose and illustrate the use of chaotic signals for masking. These represent only two of many possible directions in which chaotic signals can be exploited for communications and remote sensing.

2. MODELING AND REPRESENTATION OF CHAOTIC SIGNALS

The state evolution of chaotic dynamical systems is typically described in terms of the nonlinear state equation $\dot{\mathbf{x}}(t) = \mathbf{F}[\mathbf{x}(t)]$ in continuous time or $\mathbf{x}[n] = \mathbf{F}(\mathbf{x}[n-1])$ in discrete time. In a signal processing context we assume that the observed chaotic signal is a nonlinear function of the state and would typically be a scalar time function. In discrete-time, for example, the observation equation would be $y[n] = \mathbf{G}(\mathbf{x}[n])$ [1]. Frequently the observation $y[n]$ is also distorted by additive noise, multipath effects, fading etc.

Modeling a chaotic signal can be phrased in terms of determining from clean or distorted observations, a suitable state space and mappings $\mathbf{F}(\cdot)$ and $\mathbf{G}(\cdot)$ that capture the aspects of interest in the observed signal y.

The problem of determining from the observed signal a suitable state space in which to model the dynamics is referred to as the embedding problem. While there is, of course, no unique set of state variables for a system, some choices may be better suited than others. The most commonly used method for constructing a suitable state space for the chaotic signal is the method of delay coordinates in which a state vector is constructed from a vector of successive observations.

This work was done in the RLE Digital Signal Processing Group, M.I.T. This work was funded in part by a subcontract from Lockheed Sanders, Inc., under ONR Contract No. N00014-91-C-0125, in part by the Defense Advanced Research Projects Agency monitored by ONR under Grant No. N00014-89-J-1489, and in part by the U.S. Air Force-Office of Scientific Research under Grant No. AFOSR-91-0034.

[1] For convenience and brevity we will sometimes phrase our discussion only in terms of discrete time systems or continuous time systems although the essential points will generally apply equally to both classes.

It is frequently convenient to view the problem of identifying the map associated with a given chaotic signal in terms of an interpolation problem. Specifically, from a suitably embedded chaotic signal it is possible to extract a codebook consisting of state vectors and the states to which they subsequently evolve after one iteration. This codebook then consists of samples of the function $\mathbf{F}$ spaced, in general, non-uniformly throughout state space. A variety of both parametric and nonparametric methods for interpolating the map between the sample points in state space have emerged in the literature, and the topic continues to be of significant research interest. In this section we briefly comment on several of the approaches currently used. These and others are discussed and compared in more detail in the companion paper by Sidorowich [3].

One approach is based on the use of locally linear approximations to $\mathbf{F}$ throughout the state space [12, 13] This approach constitutes a generalization of autoregressive modeling and linear prediction and is easily extended to locally polynomial approximations of higher order. Another approach is based on fitting a global nonlinear function to the samples in state space [4].

A fundamentally rather different approach to the problem of modeling the dynamics of an embedded signal involves the use of hidden Markov models [5, 7]. With this method, the state space is discretized into a large number of states, and a probabilistic mapping is used to characterize transitions between states with each iteration of the map. Furthermore, each state transition spawns a state-dependent random variable as the observation $y[n]$. This framework can be used to simultaneously model both the detailed characteristics of state evolution in the system and the noise inherent in the observed data. While algorithms based on this framework have proved useful in modeling chaotic signals, they can be expensive both in terms of computation and storage requirements due to the large number of discrete states required to adequately capture the dynamics.

While many of the above modeling methods exploit the existence of underlying nonlinear dynamics, they do not explicitly take into account some of the properties peculiar to *chaotic* nonlinear dynamical systems. For this reason, in principle, the algorithms may be useful in modeling a broader class of signals. On the other hand, when the signals of interest are truly chaotic, the special properties of chaotic nonlinear dynamical systems ought to be taken into account, and, in fact, may often be exploited to achieve improved performance. For instance, because the evolution of chaotic systems is acutely sensitive to initial conditions, it is often important that this numerical instability be reflected in the model for the system. One approach to capturing this sensitivity is to require that the reconstructed dynamics exhibit Lyapunov exponents consistent with what might be known about the true dynamics. The sensitivity of state evolution can also be captured using the hidden Markov model framework since the structural uncertainty in the dynamics can be represented in terms of the probabilistic state transitions. In any case, unless sensitivity of the dynamics is taken into account during modeling, detection and estimation algorithms involving chaotic signals often lack robustness.

Another aspect of chaotic systems that can be exploited is that the long term evolution of such systems lies on an attractor whose dimension is not only typically non-integral, but occupies a small fraction of the entire state space. This has a number of important implications both in the modeling of chaotic signals and ultimately in addressing problems of estimation and detection involving these signals. For example, it implies that the nonlinear dynamics can be recovered in the vicinity of the attractor using comparatively less data than would be necessary if the dynamics were required everywhere in state space.

Identifying the attractor, its fractal dimension, and related invariant measures governing, for example, the probability of being in the neighborhood of a particular state on the attractor, are also important aspects of the modeling problem. Furthermore, we can often exploit various ergodicity and mixing properties of chaotic systems. These properties allow us to recover information about the attractor using a single realization of a chaotic signal, and assure us that different time intervals of the signal provide qualitatively similar information about the attractor.

3. ESTIMATION AND DETECTION

A variety of problems involving the estimation and detection of chaotic signals arises in potential application contexts. In some scenarios, the chaotic signal is a form of noise or other unwanted interference signal. In this case, we are often interested in detecting, characterizing, discriminating, and extracting known or partially known signals in backgrounds of chaotic noise. In other scenarios, it is the chaotic signal that is of direct interest and which is corrupted by other signals. In these cases we are interested in detecting, discriminating, and extracting known or partially known chaotic signals in backgrounds of other noises or in the presence of other kinds of distortion.

The channel through which either natural or synthesized signals are received can typically be expected to introduce a variety of distortions including additive noise, scattering, multipath effects, etc. There are, of course, classical approaches to signal recovery and characterization in the presence of such distortions for both transient and stochastic signals. When the desired signal in the channel is a chaotic signal, or when the distortion is caused by a chaotic signal, many of the classical techniques will not be effective and do not exploit the particular structure of chaotic signals.

The specific properties of chaotic signals exploited in detection and estimation algorithms depends heavily on the degree of *a priori* knowledge of the signals involved. For example, in distinguishing chaotic signals from other signals, the algorithms may exploit the functional form of the map, the Lyapunov exponents of the dynamics, and/or characteristics of the chaotic attractor such as its structure, shape, fractal dimension and/or invariant measures.

To recover chaotic signals in the presence of additive noise, some of the most effective noise reduction techniques proposed to date take advantage of the nonlinear dependence of the chaotic signal by constructing accurate models for the dynamics. A number of these techniques are reviewed and summarized in several companion ICASSP papers [5, 6]. Multipath and other types of convolutional distortion can best be described in terms of an augmented state space system. Convolution or filtering of chaotic signals can change many of the essential characteristics and parameters of chaotic signals. Effects of convolutional distortion and approaches to compensating for it are discussed in the companion paper by Isabelle, Oppenheim and Wornell [10].

4. CHAOTIC SWITCHING

There appears to be considerable potential in the use of chaotic signals as modulating waveforms in a variety of

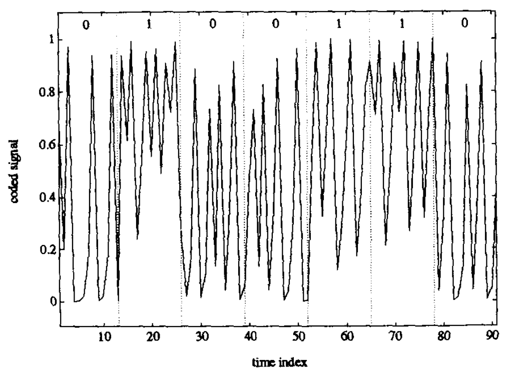

Figure 1: Typical Chaotic Switching Waveform

communication-based contexts due to their ease of generation, constant-amplitude characteristics, and broadband spectra. As a particular example, in this section we consider the transmission of a binary data stream by switching between two first-order chaotic signals with distinct dynamics during each signaling interval [2]. A typical transmission is shown in figure 1. We refer to the resulting scheme as "chaotic switching."

The associated hypothesis test for this problem involves determining which of two sets of chaotic dynamics corresponds to a given finite sequence of noisy observations $y[n]$. Specifically, under hypotheses H_0 and H_1 we observe a chaotic signal $x[n]$ of length K in stationary white Gaussian noise $w[n]$, where the associated maps are $\mathbf{F}_0(\cdot)$ and $\mathbf{F}_1(\cdot)$, respectively. Hence,

$$\mathbf{y} = \{y[n] = x[n] + w[n], \quad n = 0, 1, \ldots, K-1\}$$

where

$$\begin{aligned} H_0 : x[n] &= \mathbf{F}_0(x[n-1]) \\ H_1 : x[n] &= \mathbf{F}_1(x[n-1]). \end{aligned}$$

¿From the associated likelihood ratio test (LRT), for equally likely hypotheses, the minimum probability of error $\Pr(e)$ solution to this problem, in principle, involves iterating each of the two maps $\mathbf{F}_0$ and $\mathbf{F}_1$ from the initial condition $x[0]$ to generate the candidate clean signals $x_0[n]$ and $x_1[n]$, and determining which is closer to $y[n]$ in a least-squares sense. When the initial condition is unknown for all practical purposes, we may model $x[0]$ as a random variable, and derive a suitably modified LRT.

Although in some sense theoretically optimal, such receivers are inherently impractical because they fail to take into account the numerical instability characteristic of chaotic dynamical maps. A number of practical algorithms for solving this detection problem can be obtained, however. Here we describe a particularly simple heuristic algorithm whose performance suggests the basic feasibility of chaotic keying. The basic strategy is to exploit the fact that state pairs $(x[n], x[n+1])$ will fall on a curve in the plane whose characteristics depends on the valid hypothesis. Because each of the data pairs $(y[n], y[n+1])$ provides a noisy estimate of the state pair $(x[n], x[n+1])$, a reasonable

[2] We wish to thank Mr. Haralabos Papadopoulos for his assistance in generating some the results reported in this section.

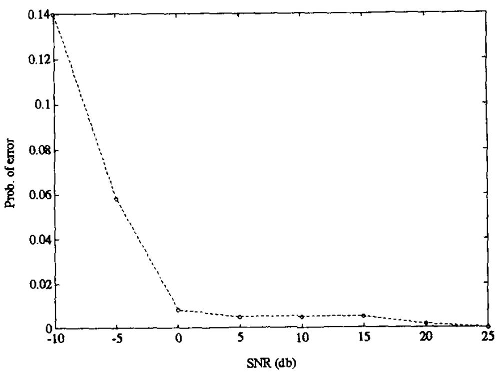

Figure 2: Bit Error Probability vs SNR

decision criterion is based on the total geometric distance between these data pairs and their nearest points on each of the two candidate curves. Figure 2 illustrates the empirical bit-error rate performance of this scheme as a function of SNR using $K = 13$ iterates per bit. In this case, the two maps are of the respective forms

$$\begin{aligned} \mathbf{F}_0(x) &= a_0 x(1-x)^2 \\ \mathbf{F}_1(x) &= a_1 x(1-x)^{0.5} \end{aligned}$$

with a_0 and a_1 chosen so that $\mathbf{F} : [0,1] \rightarrow [0,1]$. More generally the selection of maps for this application constitutes an interesting signal design problem which is currently being addressed. Furthermore, although the decoding scheme illustrated is certainly suboptimal—indeed both the time-ordering of the data pairs and the statistical dependence between noise pairs is ignored—the performance suggests that the scheme may be viable. We are currently exploring these and a number of other more optimal schemes.

5. CHAOTIC MASKING AND MODULATION

Because chaotic signals are typically broadband and noise-like, they potentially provide a class of signals which can be utilized in various communications, radar and sonar contexts for masking information-bearing signals and as modulating waveforms in spread spectrum systems. A particularly intriguing approach is suggested by exploiting the synchronizing characteristics of certain classes of chaotic dynamical systems. This property of chaotic systems was identified and demonstrated by Pecora and Carroll [8] and is described in their companion paper in this session [9]. We are actively exploring a number of ways in which this synchronization property can be used in spread spectrum modulation and demodulation and in signal masking [11]. We illustrate the approach here with one example of the use of synchronized chaotic systems for masking.

A noise-like masking signal is added at the transmitter to the information-bearing signal $s(t)$ and at the receiver the masking is removed. Abarbanel [2] and others [6] have been considering the use of noise reduction algorithms as an approach to retrieving information masked by chaotic signals. A very different approach which we are exploring is to use the received signal to regenerate the masking signal at the receiver and subtract it from the received signal to recover $s(t)$. Surprisingly, this can be done with synchronized chaotic systems since at least for some systems the

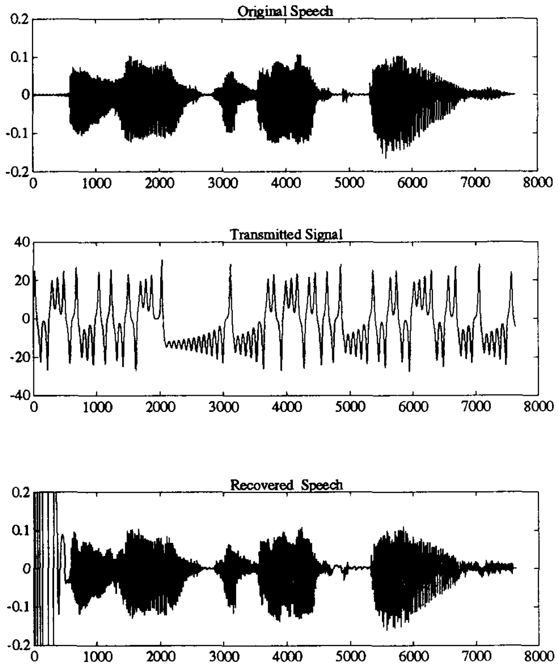

Figure 3: Chaotic Signal Masking

ability to synchronize is robust, i.e. is not highly sensitive to perturbations in the synchronizing drive and thus can be done with the masked signal. While there are many possible variations, consider, for example, the chaotic Lorenz system represented by the dynamical equations

$$\begin{aligned} \dot{x} &= 16(y-x) \\ \dot{y} &= 45.92x - y - xz \\ \dot{z} &= xy - 4z. \end{aligned}$$

Both the (x,z) and (y,z) subsystems are stable and consequently either x or y can be used as the synchronizing drive. Choosing x as the drive and the chaotic masking, the transmitted signal is $r(t) = x(t) + s(t)$ and it is assumed that for masking the power level of $s(t)$ is significantly lower than that of $x(t)$. The basic strategy then is to exploit the robustness of the synchronization using $r(t)$ as the synchronizing drive at the receiver. The dynamical system implemented at the receiver is

$$\begin{aligned} \dot{x}_1 &= 16(y_1 - x_1) \\ \dot{y}_1 &= 45.92r - y_1 - rz_1 \\ \dot{z}_1 &= ry_1 - 4z_1. \end{aligned}$$

If the receiver has synchronized with $r(t)$ as the drive, then $x_1 = x$ and consequently $s(t)$ is recovered as $\hat{s}(t) = r(t) - x_1(t)$.

We illustrate the performance of this system in figure 3 with a segment of speech from the sentence "He has the bluest eyes". Figure 3 (a),(b) and (c) show the original speech, the transmitted signal and the recovered speech respectively. The power spectra of the chaotic masking signal and the speech are highly overlapping and the overall signal-to-masking ratio is approximately -20dB. Clearly, the speech signal has been recovered. Also evident in figure 3(c) is the relatively rapid synchronization at the onset of the received signal.

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