Globally Anisotropic Quad-dominant Remeshing

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***Abstract*—In this paper, we proposed a novel anisotropic quad-dominant remeshing algorithm for the challenging problem of decimating meshes with arbitrary topology. Comparing the existing methods that are essentially based on local analysis to produce anisotropic principal curvature lines, our method is based on globally analysis to produce anisotropic principal curvature lines. Therefore, the mesh structure produced by our method can be significantly simplified while the geometry of the mesh is excellently preserved. Our method is straightforward in implementing and performances very well in applying to real-world models.**

I. Introduction

Well aligned quad meshes can represent local principal directions naturally and reflect better the symmetry of the modeled object. Classical results from differential geometry [1] also imply that these meshes provide a better surface approximation [2] and lesser normal noise [3]. These characteristics enable a number of important applications, including texturing, free-form surface modeling and finite element analysis.

In order to construct quad mesh with good trade-off between mesh quality and efficiency, the mesh faces should be anisotropic alignment. However, since the structured nature of the quadrilateral elements forcing global constraints on the mesh connectivity, anisotropic sampling is hard to achieved. This is the major challenge associated with improving the expression efficiency of quadrilateral mesh.

In this paper, we introduce an effective and flexible technique of quad-dominant remeshing. Figure 1 illustrates the main steps of our algorithm. The original model can be arbitrary topology. First we estimate principal curvatures per-vertex. Then we propose global parameterization[4] operators to obtain dense principal curvature-lines sampling. After this, we implement prioritization scheme of curvature-lines elimination. Finally, we take intersections of the curvature lines as vertices and define edges along the curve segments, and then the qualified quad-dominated mesh is generated.

A. Related Work

Generally, approaches of quadrilateral mesh generation can be roughly divided into two classes[5], [6]: parametrization based techniques and explicit quadrangulations.

Most structure aligned parametrization techniques are guided by vector or cross fields usually arising from estimated principal curvature directions [7]. Dong et al. used the Morse-Smale complex of Laplacian eigenfunctions[8] to derive patch

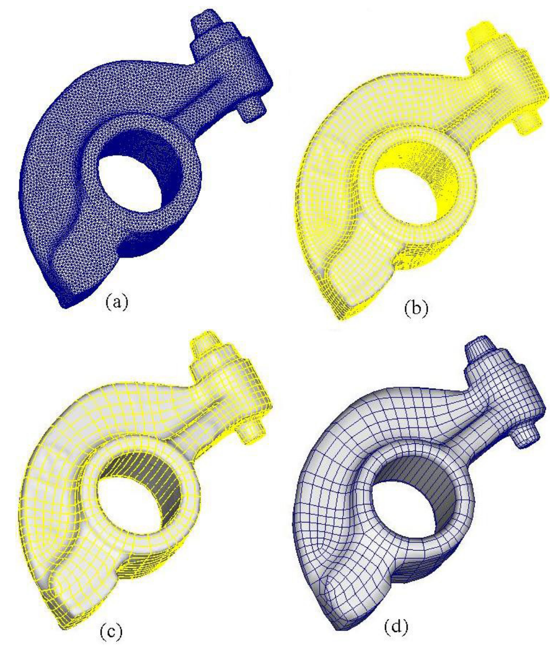

Fig. 1. Overview of our framework. (a)Input triangulated geometry,13954 vertices (b)Uniform and dense curvature lines sampling. (c)Anisotropic and effective curvature lines sampling.(d)Anisotropic quad-dominant mesh,1744 vertices

layouts. Their method was extended in[9] to enable the control over singularity positions, size, orientation and feature alignment. Tong et al. used user-designed singularity graphs to enrich the space of harmonic one-forms and compute globally smooth parametrizations[10]. By allowing affine transition functions and optimizing the charts, Bommes et al. [11] improved the distortion of the parametrization. Ray et. al proposed a fully automatic non-linear

parametrization technique which is guided by a vector field and assumes a single chart for each triangle[4]. Kalberer et al. developed a linear algorithm by mapping a cross field to a single vector field on a branched covering[12]. David Bommes et al.[13] formulate the quadrangulation problem as two mixed-integer problem.

Explicit approaches trace curves along the principal curvature directions or iteratively transform a triangular mesh into a quad-dominant mesh. The former such as [14]propose an anisotropic remeshing algorithm, based on the principal curvature directions estimation [7]. M.Marinov and L.

Kobbelt[15]extend this work by directly integrating curves on the input model. The latter such as Y.K.Lai etc[16]apply an iterative relaxation scheme which incrementally aligns the mesh edges to the principal directions.

B. Contributions

The main contribution of this paper is the novel idea of globally establishing the anisotropic distribution of mesh edges. While most relative anisotropic remeshing approaches using local geometry property to indicate where the curvature lines should be traced on the surface, we globally process the anisotropic sampling problem by resampling the whole surface with dense principal curvature lines first, and then globally analysis the principal curvature lines to indicate which curvature line should be removed.

II. Principle Curvature lines Sampling

In this section we propose a robust method to adequately sample the original surface with dense principle curvature lines.

Practically all curvature estimation techniques[17] have as an explicit user tunable parameter: the size of a neighborhood over which the estimate is computed. Usually, large neighborhood makes these algorithms stable, while results in blurred curvature estimates at the same time. In order to ensure high accuracy, we extract the curvature tensor field of the input mesh M using the smallest possible neighborhood, namely a 1-ring of faces around each vertex. The estimating approach is inspired by the technique described in [18].

For anisotropic regions, where the principal curvatures are well defined, we can find principal curvatures vector $\vec{K}$, $\vec{K^{\perp}}$ by computing eigenvalues and eigenvectors of II_v. For isotropic regions, where the principal curvature directions are undefined, we extrapolates the vector fields in a consistent manner from the anisotropic zones as used in[4].

Then ,we use the periodic global parameterization method to trace principal curvature lines. With the two principal curvatures vector fields $\vec{K}$, $\vec{K^{\perp}}$ as input, periodic global parameterization method constructing a complex manifold $\{\varphi^T\} = \{(\theta^T, \phi^T)\}$ so that principal directions aligned with coordinate axes in the parameter domain:

$$\nabla\theta^T = \omega\vec{K}; \nabla\phi^T = \omega\vec{K^{\perp}} \tag{1}$$

Due to using the natural periodicity of the sine and cosine function, the iso-lines for each triangle will correspond to the iso-lines of the global parameterization. A user prescribed sampling density parameter ω controls the period of the θ and ϕ functions which determine the distance of two principle curvatures next to each other.

III. Anisotropic distribution of curvature lines

Curvature lines are not local, in fact they extend and wind quite long over the surface. Supposing a curvature line crosses a highly curved surface region but also a rather flat region, it's difficult to get a reasonable result by the local resolution used by [14] and [15].

So we propose a global solution to this question. Enlightened by[19], we achieve the prioritization of the collapse operations by queuing the curvature lines based on the impact of the deletion on the resulting mesh. To define and quantize this impact, we characterize the error metric E of each principle curvature line. The error metric E assigned to a curvature lines L is:

$$E(L) = \alpha_n(1 - e^{-E_n(L)}) + \alpha_v(1 - e^{-E_v(L)}) \tag{2}$$

The function $E_n(L)$ returns the average value of normal variations over all of the vertexes on principle curvature line L. As shown in Figure 2, $v_0, v_1, v_2, v_3, v_4$are five vertexes on a principle curvature line $L^{'}$, and vertex v_2 is also on the curvature line L which is about to be eliminated. Since

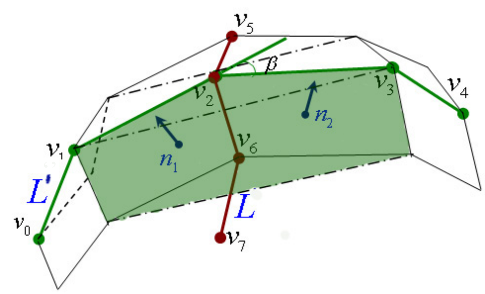

Fig. 2. Normal and volume variation related to curvature line L

curvature line L and $L^{'}$ are mutually perpendicular at vertex v_2, the normal mutation related to v_2 can be expressed as:

$$E_n(v_2) = \arccos\frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|} \approx \beta = \arccos\frac{\overrightarrow{v_1v_2}\cdot\overrightarrow{v_2v_3}}{|\overrightarrow{v_1v_2}|\cdot|\overrightarrow{v_2v_3}|} \tag{3}$$

and

$$E_n(L) = \sum_{i=0}^{m} E_n(v_i)/(m+1); (v_i \in L) \tag{4}$$

The value of $E_n(L)$ indicate the corresponding principle curvature line perpendicular to slower or steeper normal variation direction. The weighting term $E_v(L)$ measures the sum change in volume due to the deletion of the principle curvature line L. The volume change related to v_2 approximate to volume of the green triangular prism shown in Figure 2.

Then $E_v(v_2)$ can be expressed as:

$$E_v(v_2) = \overrightarrow{v_1v_2}\cdot\overrightarrow{v_2v_3}\cdot|\overrightarrow{v_2v_6}| \tag{5}$$

and

$$E_v(L) = \sum_{i=0}^{m} E_v(v_i); (v_i \in L) \tag{6}$$

For the remeshing results shown in this paper, normal variation is the dominant factor of the sorting metrics ($\alpha_n = 0.8, \alpha_v = 0.2$). The small weights given to the volume metrics allow curvature lines with similar $E_n(L)$ to be sorted based on their geometric impact.

IV. Results and Discussion

A comparison between our approach and direct anisotropic[15] is carried out in Figure 3. Notice how our method obtain better anisotropic edge distribution around green regions and better mesh quantity around yellow region with fewer mesh faces.

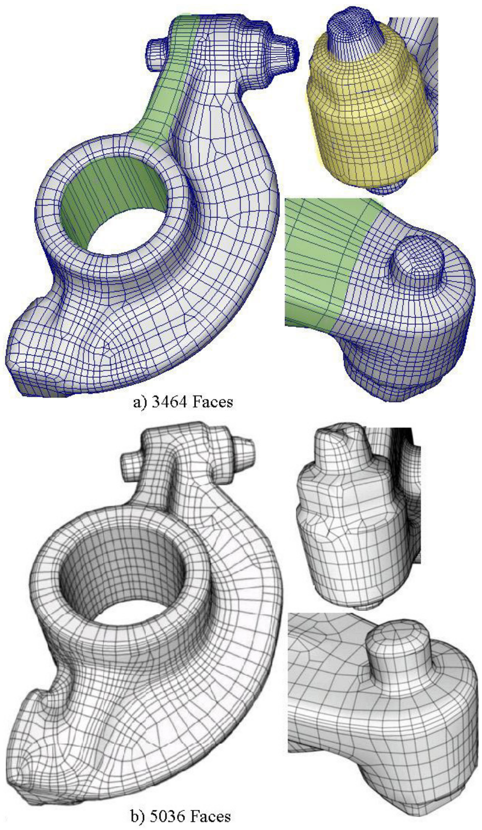

Fig. 3. A comparison between the technique described in this paper (top) and the Direct Anisotropic approach[15](bottom).

A comparison between our approach and Mixed-Integer Quadrangulation[13] is carried out in Figure 4. Mixed-integer quadrangulation produces oriented and aligned parametrizations with fewer singularities, but our method obtain more smooth surface with about the same mesh sizes because anisotropic sampling around green regions improved the efficiency of mesh expression.

All our experiments are carried out on a commodity PC with Intel Core2Duo 1.86 GHz processor and 512 MB RAM. Timings of anisotropic remeshing depend on the input mesh complexity and vary w.r.t. the output.

V. Conclusion

This paper presents an algorithm for exploiting the natural anisotropy of surfaces to construct anisotropic quad-dominant meshes. It yields high-quality quad dominant meshes with sampling elements according anisotropic distribution and edges aligned to the principal directions. Some limitations still exist. One is that there might remain some irregular polygons due to singularities in the tangent vector field. Another limitation is that small features may be smoothed since we rate the relevance of individual curvature lines by estimating the average curvature across the entire line.

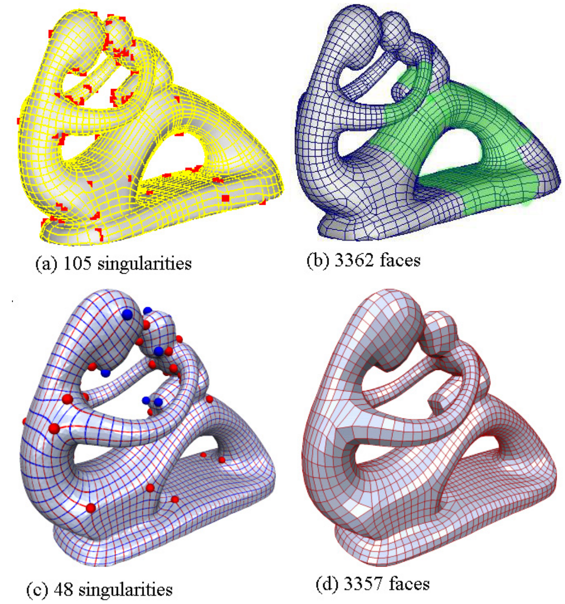

Fig. 4. A comparison between the technique described in this paper (top) and the Mixed-integer quadrangulation approach[13](bottom).

There are many avenues for future research. We plan to explore sampling and remeshing methods which producing quadrilaterial meshes with fewer singularities. We wish to investigate improvements of the simplification weighting to add robustness to sharp creases too. We are also interested in incorporating globally quad-dominant remeshing with graphics applications such as automatic measurement of the human body scanned data.

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