

Class (buys)

นางสาวณัฏฐาณว อัมฤตตานนท์

633020439-4

$$\begin{aligned}
 \text{Info}(D) &= \sum_{i=1}^n p_i \log_2(p_i) \\
 &= I(q, s) \\
 &= -\left(\frac{9}{14} \log_2 \frac{9}{14}\right) + \left(-\frac{5}{14} \log_2 \frac{5}{14}\right) \\
 &= -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} \\
 &= -\frac{9}{14} (-0.637) - \frac{5}{14} (-1.485) \\
 &= 0.940
 \end{aligned}$$

Feature

$$\begin{aligned}
 \text{Info}_{\text{age}}(D) &= \sum_{i=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{5}{14} \times (2, 3) + \frac{1}{14} \times (4, 0) + \frac{5}{14} \times (3, 2) \\
 &= \frac{5}{14} \left[-\frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{3}{5} \log_2 \left(\frac{3}{5} \right) \right] + \frac{1}{14} \left[-\frac{4}{4} \log_2 \left(\frac{4}{4} \right) - \frac{0}{4} \log_2 \left(\frac{0}{4} \right) \right] + \frac{5}{14} \left[-\frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{2}{5} \log_2 \left(\frac{2}{5} \right) \right] \\
 &= \frac{5}{14} (0.529 + 0.442) + \frac{1}{14} (0 + \text{ไม่ได้}) + \frac{5}{14} (0.442 + 0.529) \\
 &= \frac{5}{14} (0.971) + \frac{5}{14} (0.971) \\
 &= 0.347 + 0.347 \\
 &= 0.694
 \end{aligned}$$

$$\text{Gain}(\text{age}) = \text{Info}(D) - \text{Info}_{\text{age}}(D) = 0.940 - 0.699 = 0.291$$

$$\begin{aligned}
 \text{Info}_{\text{income}}(D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info}(D_j) \\
 &= \frac{4}{14} \times (2, 2) + \frac{6}{14} \times (4, 2) + \frac{4}{14} \times (3, 1) \\
 &= \frac{4}{14} \left[-\frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) \right] + \frac{6}{14} \left[-\frac{4}{6} \log_2 \left(\frac{4}{6} \right) - \frac{2}{6} \log_2 \left(\frac{2}{6} \right) \right] + \frac{4}{14} \left[-\frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right) \right] \\
 &= \frac{4}{14} (0.5 + 0.5) + \frac{6}{14} (0.390 + 0.528) + \frac{4}{14} (0.311 + 0.5) \\
 &= \frac{4}{14} + \frac{6}{7} (0.918) + \frac{4}{14} (0.811) \\
 &= 0.286 + 0.39 + 0.232 = 0.912
 \end{aligned}$$

$$\text{Gain (income)} = \text{Info (D)} - \text{Info}_{\text{income}} (D) = 0.940 - 0.912 = 0.028$$

$$\begin{aligned} \text{Info}_{\text{student}} (D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info} (D_j) \\ &= \frac{7}{14} \pm (3,4) + \frac{7}{14} \pm (6,1) \\ &= \frac{7}{14} \left[-\frac{3}{7} \log_2 \left(\frac{3}{7} \right) - \frac{4}{7} \log_2 \left(\frac{4}{7} \right) \right] + \frac{7}{14} \left[-\frac{6}{7} \log_2 \left(\frac{6}{7} \right) - \frac{1}{7} \log_2 \left(\frac{1}{7} \right) \right] \\ &= \frac{7}{14} (0.524 + 0.461) + \frac{7}{14} (0.191 + 0.401) \\ &= \frac{7}{14} (0.985) + \frac{7}{14} (0.592) \\ &= 0.493 + 0.296 = 0.789 \end{aligned}$$

$$\text{Gain (student)} = \text{Info (D)} - \text{Info}_{\text{student}} (D) = 0.940 - 0.789 = 0.151$$

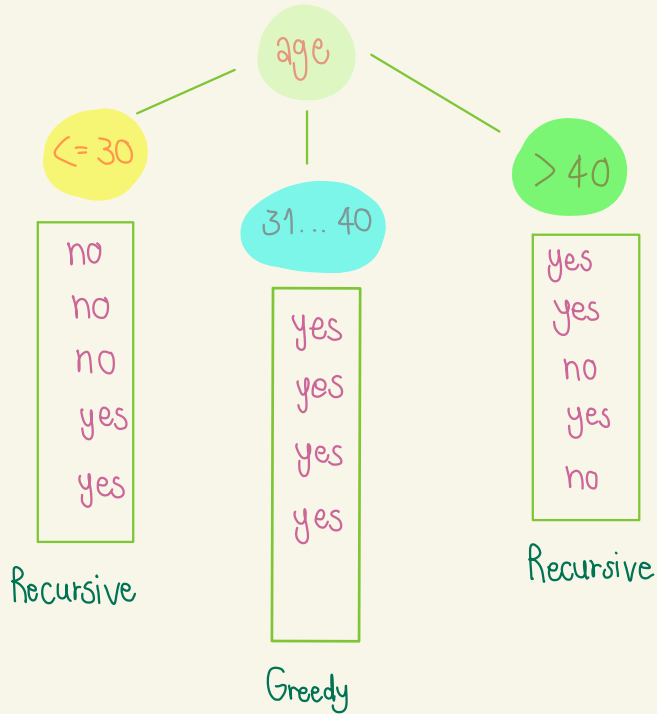
$$\begin{aligned} \text{Info}_{\text{credit}} (D) &= \sum_{j=1}^v \left| \frac{D_j}{D} \right| \times \text{Info} (D_j) \\ &= \frac{8}{14} \pm (6,2) + \frac{6}{14} \pm (3,3) \\ &= \frac{8}{14} \left[-\frac{6}{8} \log_2 \left(\frac{6}{8} \right) - \frac{2}{8} \log_2 \left(\frac{2}{8} \right) \right] + \frac{6}{14} \left[-\frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{3}{6} \log_2 \left(\frac{3}{6} \right) \right] \\ &= \frac{8}{14} (0.311 + 0.5) + \frac{6}{14} (0.5 + 0.5) \\ &= \frac{8}{14} (0.811) + \frac{6}{14} \\ &= 0.464 + 0.429 \\ &= 0.893 \end{aligned}$$

$$\begin{aligned} \text{Gain (credit - rating)} &= \text{Info (D)} - \text{Info}_{\text{credit}} (D) \\ &= 0.94 - 0.893 \\ &= 0.047 \end{aligned}$$

ดังนั้นเราจึงเลือก Gain (age) เพราะค่าที่ค่าที่เลือกซึ่งเปลี่ยนเป็นทางเลือกที่ดีที่สุด

Training data set: Who buys computer?

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no



F_1 age ≤ 30

age	income	student	credit	buys
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
≤ 30	medium	no	fair	no
≤ 30	low	yes	fair	yes
≤ 30	medium	yes	excellent	yes

$$\begin{aligned}
 \text{Info}(D) &= \sum_{i=1}^n p_i \log_2(p_i) \\
 &= I(2,3) \\
 &= -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \\
 &= 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info}_{\text{income}}(D) &= \frac{2}{5} \log_2(0,2) + \frac{2}{5} \log_2(1,1) + \frac{1}{5} \log_2(1,0) \\
 &= \frac{2}{5} \left[-\frac{0}{2} \log_2\left(\frac{0}{2}\right) - \frac{2}{2} \log_2\left(\frac{2}{2}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2}\right) \right] + \frac{1}{5} \left[-1 \log_2\left(\frac{1}{5}\right) - 0 \log_2(0) \right] \\
 &= 0.4
 \end{aligned}$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) = 0.971 - 0.4 = 0.571$$

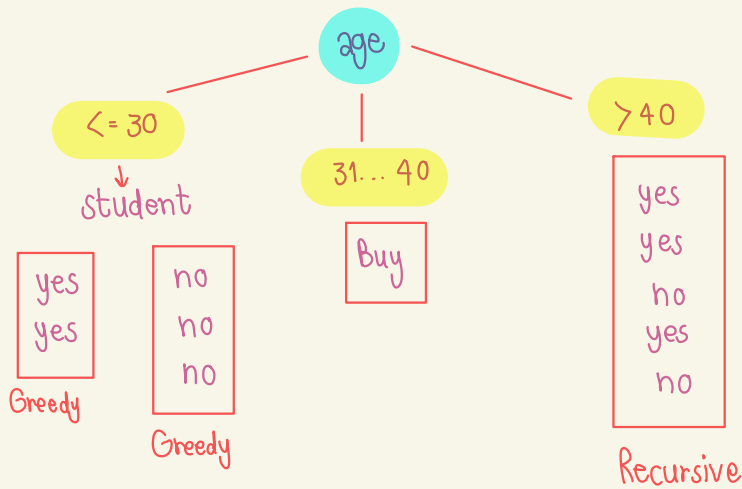
$$\begin{aligned}
 \text{Info}_{\text{student}}(D) &= \frac{3}{5} \log_2(0,3) + \frac{2}{5} \log_2(2,0) \\
 &= \frac{3}{5} \left[-\frac{0}{3} \log_2\left(\frac{0}{3}\right) - \frac{3}{3} \log_2\left(\frac{3}{3}\right) \right] + \frac{2}{5} \left[-\frac{2}{2} \log_2\left(\frac{2}{2}\right) - \frac{0}{2} \log_2\left(\frac{0}{2}\right) \right] \\
 &= 0
 \end{aligned}$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D) = 0.971 - 0 = 0.971$$

$$\begin{aligned}
 \text{Info}_{\text{credit}}(D) &= \frac{3}{5} \log_2(1,2) + \frac{2}{5} \log_2(1,1) \\
 &= \frac{3}{5} \left[-\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= 0.551 + 0.4 = 0.951
 \end{aligned}$$

$$\text{Gain}(\text{credit}) = \text{Info}(D) - \text{Info}_{\text{credit}}(D) = 0.971 - 0.951 = 0.020$$

ดังนั้น Gain ที่มากที่สุด คือ Gain(student)



F₂ age > 40

age	income	student	credit	buy
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
>40	medium	yes	fair	yes
>40	medium	no	excellent	no

$$\begin{aligned}
 \text{Info}(D) &= \pm(3,2) \\
 &= -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \\
 &= 0.971
 \end{aligned}$$

$$\begin{aligned}
 \text{Info income}(D) &= \frac{3}{5} \pm(2,1) + \frac{2}{5} \pm(1,1) \\
 &= \frac{3}{5} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] \\
 &= 0.551 + 0.4 = 0.951
 \end{aligned}$$

$$\text{Gain(income)} = \text{Info}(D) - \text{Info income}(D) = 0.971 - 0.951 = 0.020$$

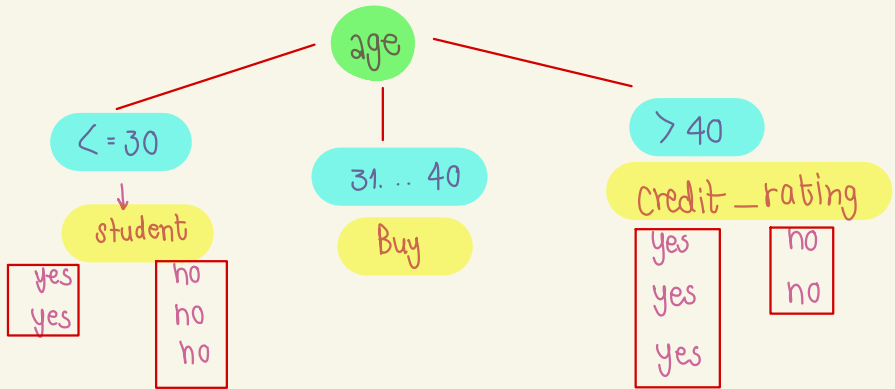
$$\begin{aligned}
 \text{Info student}(D) &= \frac{2}{5} \pm(1,1) + \frac{3}{5} \pm(2,1) \\
 &= \frac{2}{5} \left[-\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) \right] + \frac{3}{5} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right] \\
 &= 0.4 + 0.551 = 0.951
 \end{aligned}$$

$$\text{Gain(student)} = \text{Info}(D) - \text{Info student}(D) = 0.971 - 0.951 = 0.020$$

$$\begin{aligned} \text{Info credit (D)} &= \frac{3}{5} \pm (3,0) + \frac{2}{5} \pm (1,1) \\ &= \frac{3}{5} \left[-\frac{3}{5} \log_2 \left(\frac{2}{3} \right) - \frac{0}{3} \log_2 \left(\frac{0}{3} \right) \right] + \frac{2}{5} \left[-\frac{1}{2} \log_2 \left(\frac{1}{2} \right) - \frac{1}{2} \log_2 \left(\frac{1}{2} \right) \right] \\ &= 0.4 \end{aligned}$$

$$\begin{aligned} \text{Gain (Credit)} &= \text{Info (D)} - \text{Info credit (D)} = 0.971 - 0.4 \\ &= 0.571 \end{aligned}$$

ดังนั้น Gain ที่เลือกมาที่สุด คือ Gain (credit)



Decision Tree Induction

