Raptures Mever ~~Dei~~ die

$x_1 = 50$

Auto-encoder $f(w, x) = \hat{x}$

$f^{-1}(\hat{x}\ \overline{w}) = x$

$$\begin{matrix} w \\ + \\ x \end{matrix} \xrightarrow{\quad f \quad} \begin{matrix} \overline{w} \\ + \\ \hat{x} \end{matrix} \xrightarrow{\quad f^{-1} \quad} x$$

As we know a differencial equation is to solve a function φ which envolve some derivative and other functions f as below:

$$\frac{dy}{dx} = f(x, y), \quad y|_{x=0} = y_0$$

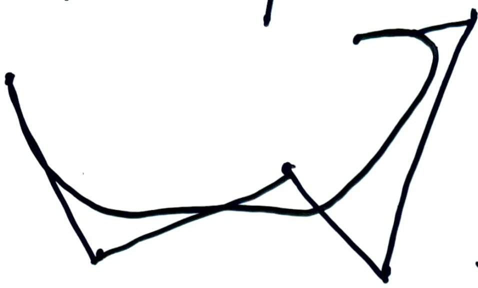
so maybe I can use a B-spline to Represent φ on a certain Range $[a, b]$.

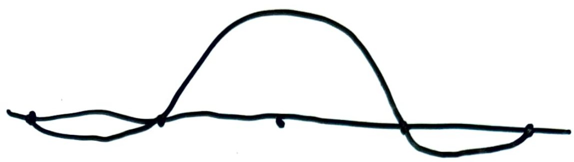
This B-spline is $\hat{\varphi} = P_i$, $P_i = (x_i, y_i)$

the derivative of a certain B-spline is $\dot{\hat{\varphi}}$

so we want $\dot{\hat{\varphi}} - f(x, \hat{\varphi}(x))$ to be small every where which $\| \dot{\hat{\varphi}} - f(x, \hat{\varphi}) \| < \varepsilon$ for any given $0 < \varepsilon < 1$ for all $x \in [a, b]$

B-spline Topics





Matrix form

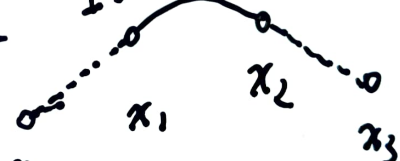
B-spline

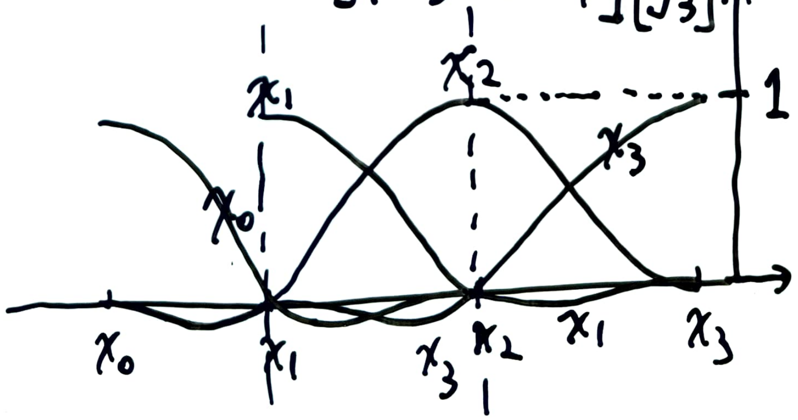
$$y(x) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Catmull-Rom Spline

$$y(x) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 0 & 2 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 2 & -5 & 4 & -1 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

4points only define 1 section





solution steps for B-spline

$$\text{set } y(x) = [1 \;\; t \;\; t^2 \;\; t^3] \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 & c_7 \\ c_8 & c_9 & c_{10} & c_{11} \\ c_{12} & c_{13} & c_{14} & c_{15} \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$[0 \;\; 1 \;\; 2t \;\; 3t^2]$$

$$t = \frac{x - x_1}{x_2 - x_1} \qquad b_{ij} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 & c_7 \\ c_8 & c_9 & c_{10} & c_{11} \\ c_{12} & c_{13} & c_{14} & c_{15} \end{bmatrix}$$

$$[0 \;\; 0 \;\; 2 \;\; 6t]$$

(1) $B_0(1) = 0$ $\quad b_{i0} \cdot 1^i = 0 \quad b_{00} + b_{10} + b_{20} + b_{30} = 0$

(2) $B_0'(1) = 0$ $\quad (b_{i0}) \cdot (i t^{i}) = 0 \quad b_{10} + 2b_{20} + 3b_{30} = 0$

(3) $B_0''(1) = 0$ $\quad 2b_{20} + 6b_{30} = 0$

(4) $B_0(0) = B_1(1)$ $\quad b_{00} = b_{01} + b_{11} + b_{21} + b_{31}$

(5) $B_0'(0) = B_1'(1)$ $\quad b_{10} = b_{11} + 2b_{21} + 3b_{31}$

(6) $B_0''(0) = B_1''(1)$ $\quad 2b_{20} = 2b_{21} + 6b_{31}$

(7) $B_1(0) = B_2(1)$

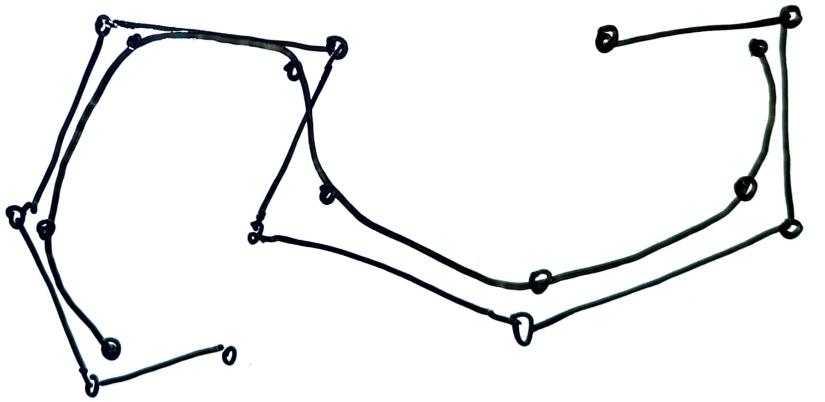
(8) $B_1'(0) = B_2'(1)$

(9) $B_1''(0) = B_2''(1)$

(10) $B_2(0) = B_3(1)$

(11) $B_2'(0) = B_3'(1)$

(12) $B_2'(0) = B_3'(1)$



(13) $B_3(0) = 0$

(14) $B_3'(0) = 0$

(15) $B_3''(0) = 0$

(16) $B_0(t) + B_1(t) + B_2(t) + B_3(t) = 1$

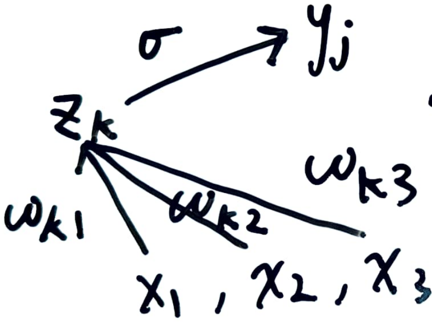
KAN Topics

Kolmogorov-Arnold Network

normal MLP

$x_i \in \mathbb{R}^n$ $\omega_{ji} \in \mathbb{R}^{m\times n}$ $z_j \in \mathbb{R}^m$ $y_j \in \mathbb{R}^m$

$$z_j = (\omega_{ji})\cdot(x_i) = \bigoplus_i \omega_{ji} x_i$$



$$y_j = \sigma(z_j) = \sigma\left(\bigoplus_i \omega_{ji} x_i\right)$$

$n = 3$

KAN

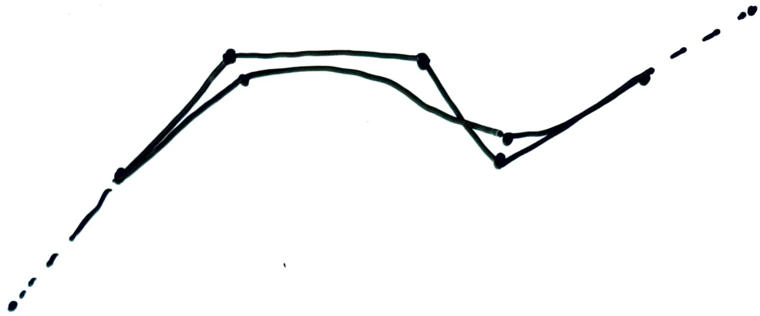
$\varphi_{ji} \in (\mathbb{R}^{\mathbb{R}})^{m\times n}$ $x_i \in \mathbb{R}^n$ $y_j \in \mathbb{R}^m$

$$y_j = \varphi_{ji} \circ x_i$$ $\circ$ means act on

$$y_j = \bigoplus_i \varphi_{ji}(x_i)$$

B-spline for KAN

$Y_1 \cdots\cdots Y_K$



$Y_k \in R^n$ for all $k \le K$

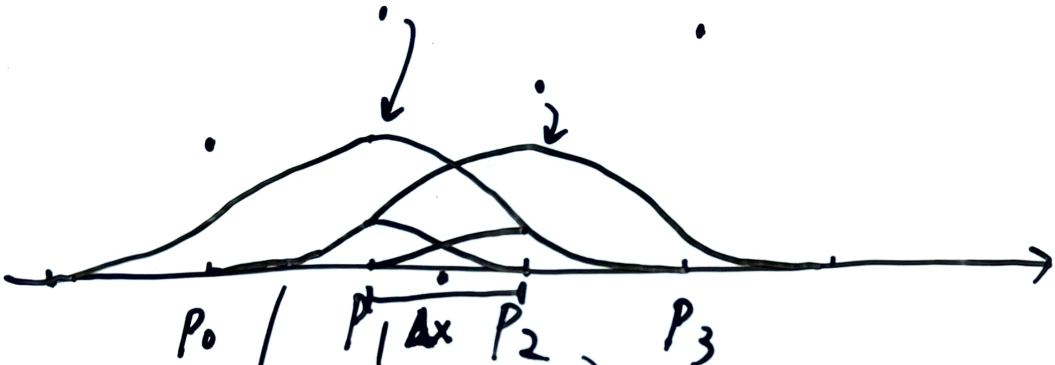
$$\varphi_{ki}(x_i) \overset{(t)}{=} \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \begin{bmatrix} \text{B-spine} \\ \text{Spectral} \end{bmatrix} \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$$

$$\phi(x) = w(b(x) + \mathrm{spline}(x))$$

$$b(x) = \mathrm{silu}(x) = \frac{x}{1+e^{-x}}$$

$$\mathrm{spline}(x) = \sum_i c_i B_i(x)$$

$$P(t) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 1 & 4 & 1 & 0 \\ -3 & 0 & 3 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$



$n = \lfloor t \rfloor = 1$

$\bar{t} = t - \lfloor t \rfloor$

$\frac{1}{6}(1-3t+3t^2-t^3)\circ P_0$

$\frac{1}{6}\bar{t}^3 \circ P_3$

$\frac{1}{6}(4-6t^2+3t^3)\circ P_1$

$\frac{1}{6}(1+3t+3t^2-3t^3)\circ P_2$

① set Δx

② get $\hat{x}$ as a input

③ $n = \left\lfloor \frac{\hat{x}}{\Delta x} \right\rfloor \quad t = \frac{\hat{x}}{\Delta x} - n$

④ $P_{n-1} \quad P_n \quad P_{n+1} \quad P_{n+2}$

⑤ $y(\hat{x}) = \sum_{i=-1}^{2} P_{n+i} \cdot B_{n,i}(t)$