

# 20240614 $\lambda$ -Calculus with Typing in C or C++ Topics

P0

## Play ground & Examples:

Resources Link : ① [http://youtube/ViPNHMSUwog?si=wqa\\_Ao-qFzUoV8TB](http://youtube/ViPNHMSUwog?si=wqa_Ao-qFzUoV8TB)  
[Eyesomorphic]

### Case 1: Boolean

set  $TRUE = \lambda x \lambda y. x$

$FALSE = \lambda x. \lambda y. y$

$NOT = \lambda b. b \text{ FALSE } TRUE$

① " $\lambda x. \lambda y. x$ "  $\rightarrow$  " $\lambda x$ " " $\lambda y$ " " $x$ "

" $\lambda x. \lambda y. x$ "  $\beta$  " $K$ " find all  $x$  in the original string THEN Replace all word " $x$ " with " $K$ "

$\lambda "x" . " \lambda y. x "$   $\rightarrow L_1 \Rightarrow L_2 L_1 \quad L_1 = "x"$

now set  $L_1 = "K"$

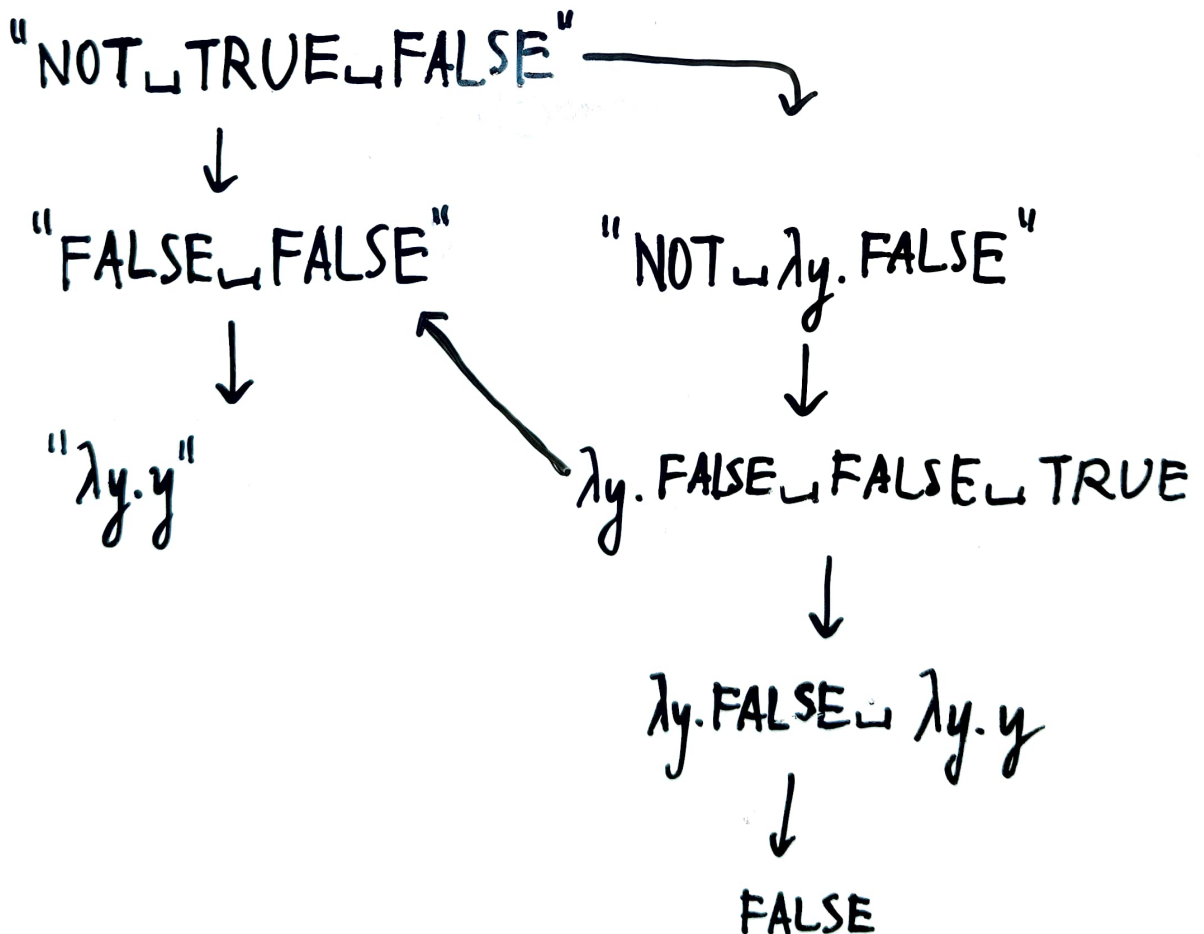
and return " $\lambda y. K$ " As new output

## Sidetrail Hulting Problem

set " $A$ " =  $\lambda x. \lambda y. A_{x,y}$

$A_{2,3} \rightarrow A_{4,2,3} \rightarrow A_{4,2,3} \rightarrow \dots$

Back continue Case 1



Case 2: Num

$$3 = \lambda f. \lambda x. f(f(f \_ x))$$

$$2 = \lambda f. \lambda x. f(f \_ x)$$

$$5 = \lambda f. \lambda x. f(f(f(f(f \_ x))))$$

$$+ = \lambda x. \lambda y. x$$

$$3 \_ f \_ (2 \_ f \_ x)$$

$$=$$

$$3 \_ f = \lambda x. f(f(f \_ x))$$

$$2 \_ (3 \_ f) = \lambda x. f(f(f \_ (f(f \_ x))))$$

$$2 \_ 3$$

↓

$$* / x : \lambda x. \lambda y. x \_ y$$

$$3 \_ 2 = \lambda y. 2(2(2 \_ y))$$

$$= \lambda y. 2(2(\lambda x. y(y \_ x)))$$

$$\lambda x. (\lambda x. y(y \_ x)) (\lambda x. y(y \_ x) \_ x)$$

$$\lambda x. y(y(y \_ x))$$

So As Above we found multiplication.

Case 2: Num

$$3 = \lambda f. \lambda x. f(f(f \sqcup x))$$

$$2 = \lambda f. \lambda x. f(f \sqcup x)$$

$$5 = \lambda f. \lambda x. f(f(f(f(f \sqcup x))))$$

$$+ = \lambda x. \lambda y. x$$

$$3 \sqcup f \sqcup (2 \sqcup f \sqcup x)$$

=

$$3 \sqcup f = \lambda x. f(f(f \sqcup x))$$

$$2 \sqcup (3 \sqcup f) = \lambda x. f(f(f \sqcup f(f(f \sqcup x))))$$

$$2 \sqcup 3$$

↓

$$* / x : \lambda x. \lambda y. x \sqcup y$$

$$3 \sqcup 2 = \lambda y. 2(2(2 \sqcup y))$$

$$= \lambda y. 2(2(\lambda x. y(y \sqcup x)))$$

$$\lambda x. (\lambda x. y(y \sqcup x)) (\lambda x. y(y \sqcup x) \sqcup x)$$

$$\lambda x. y(y(y \sqcup x))$$

So As Above we found multiplication.