

115702295 - Donnacha Oisin Kidney - Thesis Submission

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Finiteness in Cubical Type Theory

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Recommendation: award pending minor revision.

Report

This thesis presents itself as an introduction to (cubical) dependently typed programming by working through the details of various different formulations of finiteness. The work relies on some extremely technical and subtle proof techniques that are developed throughout. As a final flourish, the thesis shows how an interface inspired by QuickCheck can be used to solve the classic countdown problem.

The scientific value of this thesis is in its presentation of the different notions of finiteness in Agda, both in terms of classifying and formalising these different notions of finiteness, but furthermore proving how they are connected to one another. Since the definitions, theorems and proofs of this thesis are written in Agda, we can be confident that they are correct.

This is by far the most advanced Master's projects I have encountered, and it is impressive that the background material of this thesis has been assimilated in the first place, since the techniques are advanced. The material that is tackled here is well beyond the scope of what I would normally expect from a student at this level, and is akin more to what a PhD student would produce after, say, two years of intense study. There has clearly been a good show of independence in this work. The amount of ground that is covered is quite vast, both in the depth of intricacy that is required for some of the very subtle arguments made, but also in the breadth of the material that is covered.

The aim of this research is to demonstrate some of the interesting things that can be done in the context of cubical type theory. This is a topic of very active research in the programming languages community, and so this thesis is surely a valuable and relevant contribution to that community.

The literature that is covered is somewhat broad, but here lies a criticism: not all of the references are clearly marked, and it is at times difficult to see where the results that are mentioned folklore, built on prior work, or original. There needs to be more care here.

In terms of the technical style itself, the writing is of a precise yet informal nature. I think this helps alleviate some of the heaviness of the material that is presented, and makes reading this work enjoyable.

Weaknesses of this thesis

Although this thesis is clearly very strong there are a few points of weakness that should be easily addressable:

1. The exact technical contributions of the thesis should be prominently listed in the first chapter. Presently the reader is forced to be a domain expert to appreciate what is novel and what is not. The contributions should be stated up-front by the author.

2. References to existing work should be included where possible. At points the thesis states that results exist elsewhere but do not point to exact sources.

This should be more carefully addressed. It would also be worth repeating some citations when they have been out of context for a while.

3. The conclusions are lacking. While there is a discussion of related work which helps to position the thesis in the context of what else has been done, there is a distinct lack of reflection of what has been achieved in the thesis itself. Such concluding remarks are currently missing. I would also expect to see some comments about future work and potential directions for what else could be explored.

The thesis discusses intuitionistic logic (constructivism), cubical Agda, finiteness predicates, topos, search and countably infinite types.

This is a solid piece of work in terms of reading and interpreting the relevant literature in an advanced area. It gives a good overview of recent results. The candidate undertook the study of challenging and interesting new aspects of type theory, topos theory and related subjects, and produced a clearly written thesis discussing these.

My main concerns, and something which needs to be addressed, are the introduction (and conclusion), the lack of an indication of which parts are novel and consistent accreditation of work.

The applicant has done a mostly thorough job of citing sources, but producing these references occurs in a haphazard way, sometimes with the theorems stated (as it should). At other times the origin of discussed work is mentioned in sections at the very end. At times it seems results are presented for which the origin is never made clear.

This needs to be addressed in a revision. The source needs to be stated with each theorem/lemma etc. An alternative could be to cite the sources at the start of each section (stating which theorem arises from which source ahead of introducing these).

Currently the reader needs to hunt for the origin of results. This means that original contributions remain hidden and there is a risk of confusing the stated results with results obtained by the applicant.

It is not clear if proofs have been adapted or changed in some way, which could also support evaluating the originality of the work.

Currently several of the finiteness-related proofs are given without reference to the original source, and this issue may also occur in other parts. It is essential that

this is addressed consistently throughout by the applicant and a revision produced which does not leave one to guess the originality of the work, but clearly outlines this from the start and throughout.

Overall this is an excellent MSc thesis. With appropriate revision in terms of accreditation and highlighting novelty of personal contributions, should be awarded on the back of the minor revisions.

Detailed comments

Introduction

On page 5: Constructively, though, proofs themselves are mathematical objects

Proofs are mathematical objects even in the context of (non-constructive) mathematical logic. Not clear what the point is of this sentence. Please clarify.

Section 1.3

Page 8: HoTT is a type theory and foundational theory for mathematics: it is designed to stand in for ZFC in mathematical terms, and things like Martin-Löf type theory computer science terms.

The latter part of the sentence makes no sense. It needs to be clarified. Could be a typo or it could be too vague. Hard to tell from the way it is written.

page 8: ZFC is used without mentioning what the acronym stands for. It is well-known to experts. It should be clarified on first mention rather than much later on in the thesis as is currently the case.

Page 9: The intuition underlying the notion of paths can be sketched in one sentence to give the reader guidance at the start. Instead it is introduced much later in the thesis. It is mentioned a few times in the introduction without motivation/guidance. It is a simple notion for which the intuition can easily be sketched.

Page 9: “It’s worth noting that the technique mentioned here is used quite pervasively in everyday mathematics: it’s just that foundational systems could not justify its use.”

Be more specific here as to which foundational systems you refer to.

Citations need to be consistent throughout

Some examples (where I the thesis needs to be reviewed as a whole once this issue is addressed for consistency):

Lemma 3.2 Split enumerability implies decidable equality, page 34

Lemma 3.1 A proof of split enumerability is equivalent to a split surjection from a finite preimage of the natural numbers.

Lemma 3.3 Any split enumerable set is manifest Bishopness
We will only sketch the proof here.

It is not clear whether this is a proof by the applicant, and if so why it is not provided in full. Is it due to another source? Which one? It seems that manifest Bishop finiteness is equivalent to split enumerable. This in turn is an equivalent version of this predicate was called Listable in (Firsov and Uustalu, 2015). It is not clear why predicates are renamed in the thesis, again making it harder to determine origin/originality. Also, one only discovers at the end of the thesis on page 71, that both the notions of manifest Bishop finiteness and split enumeration are already known (Firsov, Uustala), leading to confusion on originality of the work.

Page 71, and and the setting (repeated word)

Page 72: Exhaustibility The twin notions of and omniscience (word missing)

Page 72, first mention of original work; As far as we know, ours is the first paper to look at countdown from the dependently-typed, proof perspective.

This needs to come in the introduction, clearly outlining which parts are new and what parts are not. The part at the end discussing related work addresses this to some extent, but insufficiently so.