ES-C-New York University-Team-3

Statement of Problem: Develop a mathematic model to determine the interaction within *P. brassicae* (a kind of butterfly) species as well as between *P. brassicae* and the parasitic wasps.

Based on given information, we know that when female butterfly secrets a kind of chemical signal to attract male butterfly to mate. The male who mates the female will then generate anti-aphrodisiacs to dissuade other males from mating with this female. This anti chemical is exerted on the female and as the parasitic wasps can sense this special chemical, the wasps then ride on female butterfly's back and lay eggs to where the female butterfly is going to spawn. The key problem then is to determine the trade-offs brought by releasing the anti-aphrodisiacs. We set up an ideal region with 3 assumptions for butterflies and wasps to inhabit.

(1) Within the given region, there are only the two species *P. brassicae* butterfly and parasitic wasps involving in the ongoing interaction.
(2) The food sources for wasps' larvae only depends on eggs of *P. brassicae.*
(3) *P. brassicae* butterfly has abundant food in this given area.

We feature the Lotka–Volterra equations to build the relationship between *P. brassicae* and parasitic wasps and we use W(t) to represent the population of wasps (1) and B(t) to represent the population of *P. brassicae* butterfly (2).

rate of change eggs deposited deaths of wasps

$$\frac{d\boldsymbol{W}(t)}{dt} = \lambda k\boldsymbol{W}(t)\boldsymbol{B}(t) - \beta\boldsymbol{W}(t) \qquad (1)$$

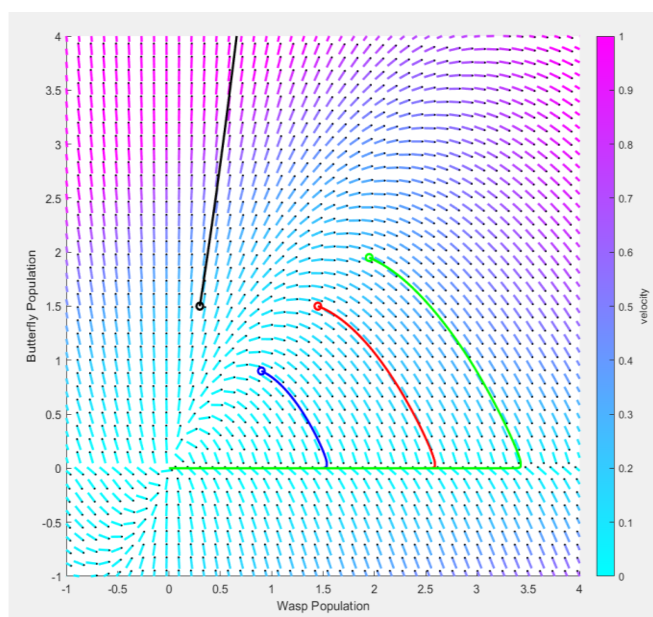
$$\frac{d\boldsymbol{B}(t)}{dt} = l\mu P\boldsymbol{B}(t)(1-P)\boldsymbol{B}(t) - \alpha k\boldsymbol{W}(t)\boldsymbol{B}(t) - \gamma\boldsymbol{B}(t) \qquad (2)$$

rate of change eggs deposited eggs being eaten deaths of butterflies

k is the interaction rate (i.e. the overlapping rate of inhabitants of these two species) between *P. brassicae* butterfly and parasitic wasps, and we keep it variable to see how the interaction rate affects the long-term balance within the two species. **λ = 30** is the average number of eggs laid by wasps attracted by one female butterfly. **β = 0.4** is the death rate of wasps. ***l*** is the interaction rate between male and female butterfly, and we keep it variable also to see how the rate makes influence. **P = 0.6** is the ratio of female butterfly to the total number of butterflies. **μ = 200** is the average number of eggs laid per female butterfly. **α = 60** is the average number of butterfly's eggs being eaten by wasps' larvae being generated with one female butterfly's attraction. **γ = 0.2** is the death rate of butterfly.

The two key variables **k** and ***l*** are somehow related because there is a trade-off with respect to the "anti-aphrodisiacs", released by male butterflies to successful fertilize the eggs but at the same time it attracts wasps that would result in butterfly-egg-destroy by wasps' larvae. Therefore, if **k** goes high, ***l*** must increase automatically.

We use MATLAB to draw the phase plane of this differential equation system and keep all the constants set, modifying the value of **k** (interaction rate in two species) and ***l*** (interaction rate within butterfly). Here is one sample with k = 0.4, ***l*** = 0.4 with four different initial cases.



The plot is plausible because if we start with far more butterflies than wasps, and then even if a large amount of eggs are eaten by wasps' larvae, since the population base of butterfly is large, the population of two species would grow up together. But if the starting amount of butterflies and wasps is similar, due to our settings of strong-egg-consumption of wasps' larvae, the butterfly population goes down sharply whereas the wasp population experiences a slow growth and then drops as well because there is no food sources for wasp larvae (since butterfly dies out).

As we test more situations of k and ***l*,** we find that based on the constants we set up, no matter what value of interaction rate we put, there are only two possible endings, both dying out or both boosting up. But we one forgot to think about the interaction rate ***l*** within the butterfly species, the outcome was amazing, the two species end up in a dynamic balance.