# Entropy in Mesoscopic Circuits

by

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# Abstract

We have tested whether the entropy of a quantum system can be measured nonlocally, using a capacitively coupled quantum dot as a sensor. We have extended upon the research already done in mesoscopic circuits showing that the entropy of the first few electron ground states in a quantum dot can be locally measured directly. To demonstrate whether this local measurement can be extended to measure the entropy of a nonlocal quantum system, we have probe a simple two state system made up of an electron in a superposition between a quantum dot and a reservoir. Entropy measurements were completed by capacitively coupling this two-state system to a probe quantum dot whose occupation affects the degeneracy of the two-state system and by extension, its entropy. In the dot acting as the probe dot, change in entropy of the entire system was measured by measuring shifts in the occupancy transition from 0 to 1 electrons as a function of temperature. By showing that we can measure the entropy of a quantum system nonlocally — or by measuring only the properties of a nearby, coupled quantum dot — we provide a path to distinguish more novel entangled states with unique entropies.

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# Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

# Acknowledgments

I would first like to thank all the students in Josh Folk's lab for all they have done for this project and for all they have taught me about being an experimentalist. Christian Olsen, who was patient while I was first learning how to do anything useful in the lab. Manab Kuiri, who discussed physics with me everyday, and is always up to date on all the newest results. And of course, Tim Child, without whom this project would not have been possible.

More here.

# Chapter 1

# Introduction

In the past few decades, significant advances in the field of quantum transport have yielded a large number of interesting quantum systems and effects including Majorana bound states [7], the 2-channel Kondo effect [9], and the $\nu = 5/2$ fractional quantum hall state [13]. All of these systems have been well characterized using traditional transport techniques. However, if we were able to measure the entropy of mesoscopic quantum systems like these, we would be able to more clearly distinguish them from trivial states, and perhaps detect deviations from theory in ways which traditional transport measurements do not allow. Of particular interest is the Majorana bound state whose characteristics make it especially well suited to the field of quantum computing [1, 8], but whose transport signature is suspiciously close to that of the much less interesting (and less useful) Andreev bound state [12]. It has been proposed that the entropy of such a Majorana bound state would significantly differ from that of an Andreev bound state [11]. However, in the past, entropy measurements of systems like these were never possible because of limitations of old techniques which rely on heat capacity and other macroscopic quantities.

A few years ago, Hartman et al. [5] showed that it is possible to measure the entropy a single spin $\frac{1}{2}$ particle in a quantum dot, opening the possibility of introducing entropy as a new technique for characterization of more interesting mesoscopic quantum systems, like those mentioned above.

To show that the protocol that Hartman et al. used to measure the entropy of a single spin $\frac{1}{2}$ particle can be extended into a regime where the the quantum dot is capacitively coupled to an external system. Here, we propose a mesoscopic circuit to investigate the effects of capacitively coupling an external quantum system to the measurement scheme of Hartman et al. as a proof that the technique can be extended to the measurement of more interesting quantum systems.

# Chapter 2

# Entropy in mesoscopic systems

## 2.1 From a Maxwell relation to entropy

To measure the entropy of a system using a mesoscopic circuit, we use the Maxwell relation and resulting integral.

$$\left(\frac{\partial \mu}{\partial T}\right)_{p,N} = -\left(\frac{\partial S}{\partial N}\right)_{p,T}, \quad \Delta S = \int_{\mu_1}^{\mu_2} \frac{dN(\mu)}{dT}\, d\mu \tag{2.1}$$

In other words, by measuring the occupation of a quantum dot as a function of the chemical potential, $N(\mu)$, and varying temperature, $T$, we can derive the change in entropy, $\Delta S$ over that change in occupation.

In systems with few degrees of freedom, the relevant discussion of entropy comes in the form of Boltzmann entropy, $S = k_b \ln \Omega$ with $\Omega$ being the number of available microstates [10]. In Hartman et al.'s experiment, it was shown that the change in entropy as a quantum dot goes from an occupation of $0 \to 1$ electrons was $\Delta S = k_b \ln 2 - k_b \ln 1 = k_b \ln 2$ as the dot went from only having one possible state to having two possible spin states (spin up and spin down). In addition, it was shown that by applying a large magnetic field, Zeeman splitting of the energy levels in the dot eliminated this degeneracy causing $\Delta S = k_b \ln 1 - k_b \ln 1 = 0$.

In practice, to measure the entropy of a small system using a mesoscopic circuit and the integral from Eqn. 2.1 we have a few requirements. First, we assumed constant pressure in the Maxwell relation. In the context of a 2-dimensional electron gas (2DEG) with which our measurements are conducted, the dominating pressure at temperatures below the Fermi temperature, $T_F \approx 100$K is the degeneracy pressure [2], an incompressibility emerging from the Pauli exclusion principle disallowing fermions from occupying the same quantum state. In addition, by keeping energy fluctuations due to thermal energy, $k_b T$, much smaller than the spacing between energy levels in the dot, we ensure that random temperature fluctuations do not produce unpredictable energy level occupation.

## 2.2 Free energy explanation

To get a physical intuition for the thermodynamics at work to make this measurement possible, it may be useful to consider the Maxwell relation in Eqn.2.1 in terms of free energy of the system.

# Chapter 3

# Capacitively coupled quantum dot

## 3.1 Quantum dots in GaAs/AlGaAs heterostructures

## 3.2 The device

## 3.3 Results

## 3.4 Comparison to theory

## 3.5 Conclusion

To carry out the integration from Eqn. **??** in practice, it is necessary to have an accurate way of measuring the occupancy, $N$, of the probe dot. We measure $N$ by measuring the conductance $G_{sens}$ through a charge sensing quantum point contact (QPC) seen in Fig. 3.1.

Because of the proximity of this QPC, referred to as the charge sensor, to the probe dot very small electrostatic changes in the probe dot affect the conduction across the charge sensor [4]. As such, a larger $G_{sens}$ indicates fewer electrons in the probe dot, while a smaller $G_{sens}$ indicates more electrons in the probe dot. In effect, this means that $G_{sens}$ can be used to directly measure the occupancy of the dot as a function of various other quantities like chemical potential, $\mu$, or temperature, $T$. We use $V_{plunger}$ ($V_p$) shown in in Fig. 3.1 to locally control the chemical potential of the dot. Varying the potential applied to this gate $V_p$ - and by extension the chemical potential in the dot - is our primary technique to control the occupancy of the dot. Based on this protocol, we can decompose Eqn. **??** into the following quantities which can be determined experimentally.

$$\Delta S = \int_{\mu_1}^{\mu_2} dG_{sens} \frac{dN}{dG_{sens}} \frac{1}{dT}\, d\mu \tag{3.1}$$

This integral tells us that we can measure the change in entropy between two chemical potentials in the dot by measuring three quantities: $dN/dG_{sens}$, $dT$, and $dG_{sens}$ as a function of chemical potential. The first two quantities $dN/dG_{sens}$ and $dT$ are scaling factors that can be independently experimentally determined but do not depend on $\mu$ however the final quantity $dG_{sens}$ does depend on $\mu$ and so must be measured as $\mu$ is changed. Intuitively, $dG_{sens}$ is a measure of the difference between the occupancy of the dot at higher $T$ and lower $T$ - this is illustrated by the shading on the plots in Fig. 3.3.

Using $V_{degeneracy}$ ($V_d$) the quantum system composed of the two dots (highlighted in yellow in Fig. 3.1) with a single electron confined within the system can be tuned between non-degenerate and doubly-degenerate. This works by first suppressing spin degrees of freedom with a large magnetic field then slowly changing the shape of the potential function separating the two dots using $V_d$. Once the potential barrier between the dots is large enough, the tunneling rate will become negligible.

Capacitive coupling between the probe dot and the pair of dots can be tuned such that occupation of the probe dot suppresses the degeneracy of the two-dot system. This is because the lowest energy state will occur when the two electrons are farthest apart. Thus, we can tune the system to a state where there is an entropy change of the entire thermodynamic system independent (while no change in entropy of the probe dot itself) as the probe dot changes occupation. In this way, we will be able to detect a non zero value of entropy if we are able to detect changes in entropy of a capacitively coupled system. Specifically, by tuning the upper two dot system between non-degenerate and doubly degenerate, we will be able to see the change as entropy as the probe dot transitions $0 \rightarrow 1$ electrons vary from $\Delta S = 0$ to $\Delta S = -\ln 2$, respectively.

Preliminary evidence that we will be able to see a change in the measured entropy based on this coupling between the probe dot and another system comes from data collected on a different device coupled to an impurity in the substrate. In Fig 3.4, data from this device shows that as $V_d$ (effective) is varied, the entropy measured changes significantly. Because no parallel field is applied and the probe dot transitions from $0 \rightarrow 1$ electron, the probe dot on its own causes a change in entropy of $\Delta S/k_b = \ln 2 - \ln 1 = \ln 2$. As such, changes in this entropy due to the impurity cause deviations in the entropy from this expected value. Clearly, this situation differs from the experiment we are proposing as there is either a positive or negative change in entropy depending on $V_d$ (effective).

The measurement protocol as laid out in Fig. 3.3 requires the ability to vary the temperature of the system. However, in practice, instability in the exact locations of $V_{mid}$ makes it difficult to accurately determine $\Delta S$ of the hot and cold curves independently. Because of this instability, it is necessary to oscillate between hot and cold as we measure over the transition. With this measurement scheme, any heating not localized on the device is not useful as it will take too long to equilibrate through the entirety of whatever larger system is being heated.

We are pursuing multiple possible solutions for fast localized heating. The first is the technique employed by Hartman et al. and used in previous device designs which involves directly injecting hot electrons into the electron reservoir coupled to the probe dot. This can be done by running a current through a QPC (tuned to be fairly resistive) pointed into the electron reservoir. When the current is turned off the heat in the electron reservoir is dissipated by coupling to phonons and connection to a 'cold' ground - i.e. a much larger bath of electrons at lower temperature. By turning on and off the current running through this resistive heater QPC, we can locally control the temperature of the electrons in the system. Another possible technique that could be employed to allow for fast localized heating is heating the crystalline lattice of the substrate. With this technique, the electron-phonon coupling in the electron reservoir would ensure that the electrons quickly thermalize. Resistive heating would most likely be used to heat the phonons, either by electron phonon coupling using a QPC which is electrically insulated from the device, or by building a resistor on top of the substrate using a very long wire with some finite resistance.

Devices to complete this experiment will be built on GaAs/AlGaAs heterostructure which hosts a 2DEG (see Fig 3.5). The 2DEG is electrostatically gated to allow for local control of the electron density i.e. by applying an electric field with an electrically isolated gate, the electron density of the 2DEG is controlled beneath this gate [6]. Ohmic contact is made with the 2DEG via a diffusive process of Ni/Au/Ge through the substrate. Gating structures will be defined using standard photolithography and electron beam lithography techniques.

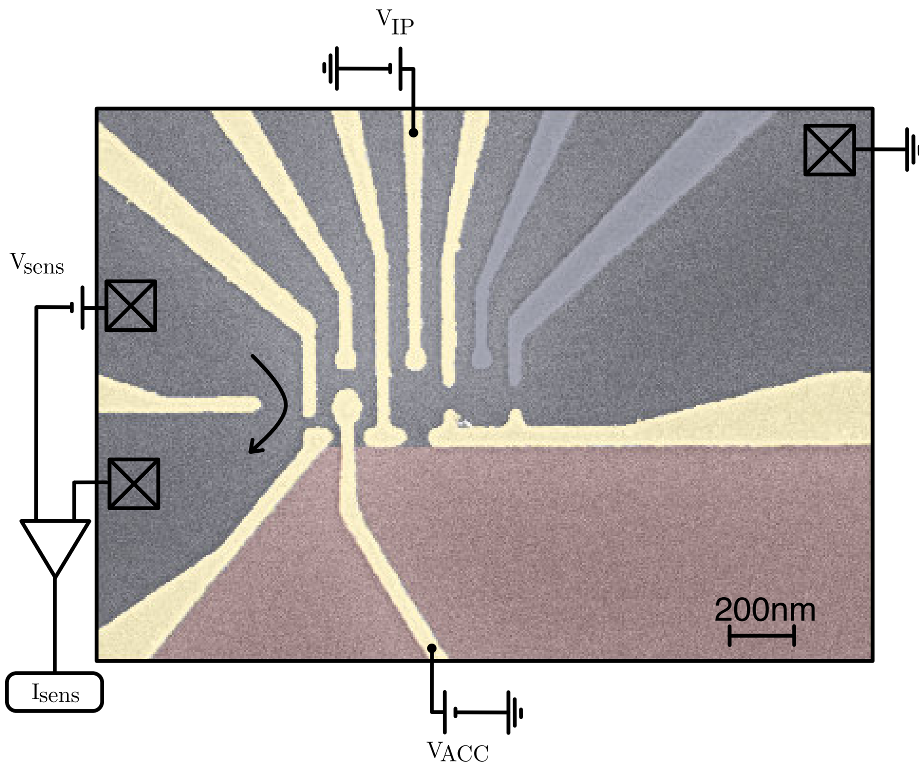


**Figure 3.1:** A top-down SEM image of the device measured in this experiment. Lighter gold regions are the gold gates, while darker regions are the GaAs substrate. A single quantum dot (right hand side) is probed by the leftmost dot whose occupation is measured using $I_{sens}$. In this device, temperature oscillations occur across electrons in a reservoir connected to the probe dot heated via a small thermocurrent through an adjacent quantum point contact. $V_{ACC}$ is used to control the chemical potential, $\mu$, in the probe dot. In the right hand of dot, similar gate structures, labelled $V_{IP}$ allow for the system to be tuned to be tuned degenerate with the probe dot. The X indicates ohmic contact to the 2DEG. The light red section indicates the thermal reservoir of the system. Greyed out gates were not used in this experiment.

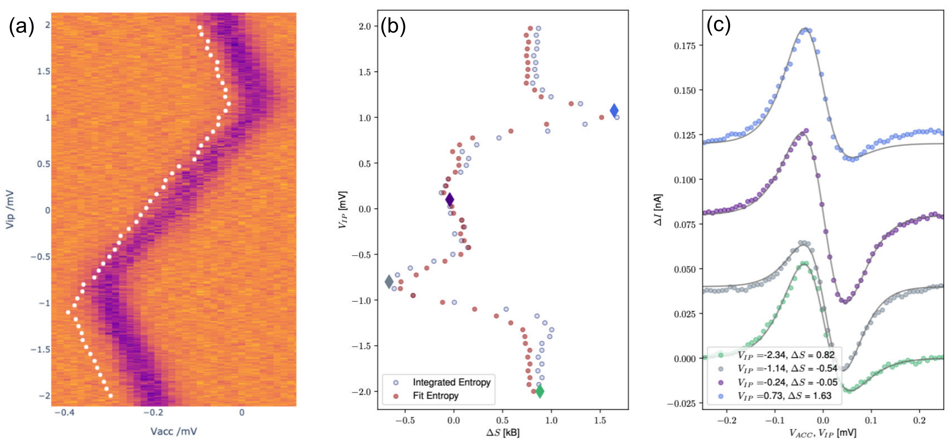


**Figure 3.2:** Entropy along the $0{,}1 \to 1{,}0$ transition in the pair of dots. In (a) the dark region indicates the transition in the main quantum dot. White dots indicate the locations where entropy scans were taken. In (b) the $\Delta S$ of the transition at each white point is plotted. In the $0{,}0 \to 1{,}0$ regime, the $\Delta S$ is found to be roughly $\ln 2 = 0.69$ as expected from the spin degeneracy in the probe dot. In the central regime, where there is a full $0{,}1 \to 1{,}0$ transition, entropy is found to be around $\ln 2 - \ln 2 = 0$ since the spin degeneracy of the impurity dot is simply replaced by the spin degeneracy in the probe dot. In the mixed regime, charge degeneracies of the transitions yield non-trivial $\Delta S$.

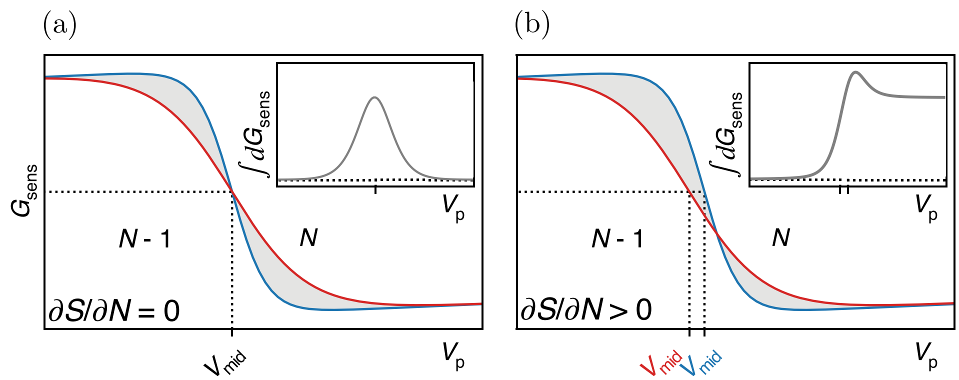


**Figure 3.3:** In (a) and (b) we show two examples of the measurement protocol where the occupancy of the probe dot is measured using $G_{sens}$. In each case, the occupancy is swept from $N-1$ to $N$ electrons both at a higher temperature (red) and a lower temperature (blue), however in (a) this change in $N$ does not correspond to an entropy change in the system whereas in (b) we see a positive change in entropy of the system due to this change in occupancy. The inlaid plots show the cumulative integral of $dG_{sens}$ – or the difference between hot and cold $G_{sens}$ curves. The entropy change of the system is measured by the value of this integral after the completion of this transition. $V_{mid}$ is labelled for each curve, notably, $V_{mid}$ is the same in the zero entropy case, but shifts in the finite entropy case. Figure from Hartman et al.

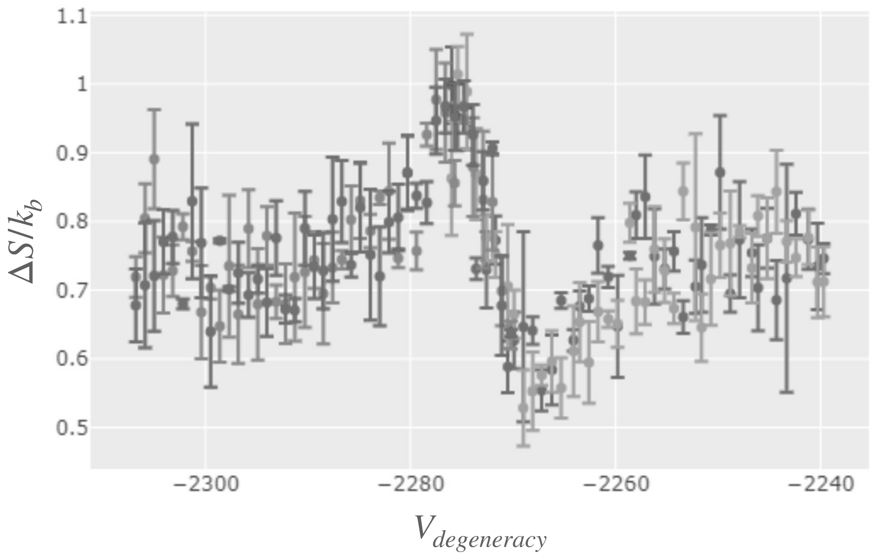


**Figure 3.4:** Data from an older device exhibits a change in entropy as degeneracy of an unknown external impurity is adjusted. The demonstrates the potential for measuring the entropy of capacitively coupled systems.

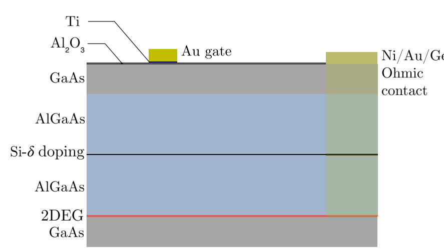


**Figure 3.5:** A cross section of the GaAs/AlGaAs heterostructure hosting a 2-dimensional electron gas (2DEG) formed at the boundary between an AlGaAs and GaAs layer where the conductance band briefly falls below the Fermi energy [3]. Gold gates allow local control of the electron density of the 2DEG. Ohmic contact to the 2DEG is established by a diffusion of a combination of Ni/Au/Ge from the surface to the 2DEG.

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## Appendix A

# Data Analysis

This section will include

- Detailed description of experimental techniques by which data are collected
- Samples of multiple levels of data through processing

# Appendix B

# Measurement technique

This section will include

- Detailed description of wiring implemented
- Noise measurements on this wiring, measurements of electron temperature

# Appendix C

# Device Fabrication

This section will include

- Processes which are followed for the production of devices on a GaAs substrate
- Various check lists for processes (space depending).