Coefficient of Thermal Expansion Lab

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1 Abstract

The purpose of this lab was to investigate the linear thermal expansion coefficient of a copper rod. A final value for the linear thermal expansion coefficient, α_L, was derived to be 16.819E-6 $\pm$ 0.241E-6 K^{-1}. This was found to be the case for the range of temperatures between 300K and 320K. This value is well within agreement with the results from the literature. The expected results were $\alpha_L = 16.759$E-6 at 300K and $\alpha_L = 16.960$E-6 at 300K. As expected, the value for the linear thermal expansion coefficient for temperatures between 300K and 320K fell between those two values.

2 Introduction

The thermal expansion of a pure copper rod was investigated using an interferometer and a heat source to vary the temperature of the rod and measure the change in length due to these temperature changes. The general principles behind this measurement come from the following partial differential for volumetric thermal expansion.

$$\alpha_v = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \tag{1}$$

The subject of this experiment was the coefficient of linear thermal expansion which is just a specific case of the general principle of volumetric thermal expansion (at constant pressure). Intensity data collected from the interferometer provided the basis for measuring changes in length while a temperature probe located on the interior of the copper rod allowed for temperature measurements. Fundamentally, the linear thermal expansion coefficient describes the way that the object's length changes as its temperature changes, therefore, by collecting data on both of these quantities, the coefficient can be calculated.

3 Methods

The copper rod was placed along one of the arms of the interferometer. This meant that as the rod expanded, the rejoining light beams went in and out of phase. A more specific explanation can be found in the caption of figure 1.

Over the course of this lab, measurements of changes in the copper rods dimensions as the rod heated up or cooled down were only measured along one axis, in contrast to the partial differential stated in the introduction (for volumetric thermal expansion). The following formula was used instead.

$$\alpha_L = \frac{1}{L}\frac{dL}{dT} \tag{2}$$

This formula holds true for the specific case where thermal expansion is an isotropic process, that is that the proportions of the object do not change during the process of expansion. It is also necessary to assume constant external pressure. In addition to using this formula the following approximation was made.

$$\frac{\Delta L}{L} = \alpha_L \Delta T \tag{3}$$

This approximation was made assuming (1) that the thermal expansion coefficient does not change over temperature [1] and (2) that $\Delta L << L$.

The intensity data provided sufficient information to calculated changes in length because, as the rod either expanded or contracted, the light beams went in and out of phase. The reason for these changes in phase is that only one of the two arms of the interferometer was changing in length. Notable to this technique is the fact that for every small change in length of the bar ϵ the path of the light beam changes by 2ϵ. Before measurement began, the laser was carefully calibrated to ensure that the center of the circular interference pattern fell on the center of the detector such that maximal intensity differences were observed.

To measure the changes in length as the temperature changed, it was most pertinent to heat the rod as regularly as possible using a heat gun, then measure the changes in length as the rod is cooled. This method ensures that (as best as possible) the rod is at a uniform temperature. Note that if the rod is not at a uniform temperature, then the assumption of isotropic expansion no longer applies and the system does not behave according to the linear expansion formula being

[1] This is not quite true but close enough for this experiment. In fact the linear thermal expansion coefficient is expected to change by about 1.1% in the temperature range investigated here - 300K to 320K (Kroeger). This change is small enough that we will disregard it.

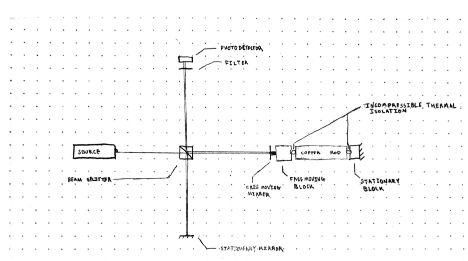

Figure 1: This diagram shows a simplified depiction of the set up used to measure changes in the length of the rod. As the rod cooled, intensity data was collected from the photo detector providing sufficient information to extrapolate the change in length of the bar over a given temperature change.

used. Copper is a very good thermal conductor so during the cooling, its temperature is fairly uniform. Length changes were measured knowing the wavelength of the light beam and that for each time that the beams go from being in phase (high intensity) to out of phase (low intensity) the length of the rod had changed 1 wavelength.

A simple algorithm was utilized to map all peaks and troughs in the photo detector data set to the corresponding length (we know the length because the initial length can be measured and the ΔL between each peak and trough is one wavelength). This data was then fit to a length vs temperature graph. Based on the definition of α_L stated in equation (2) and the approximation stated in equation (3), the slope of the (linear) fit should be equal to $\alpha_L L_0$.

$$\alpha_L L_0 = \frac{\Delta L}{\Delta T} \tag{4}$$

4 Results and Analysis

Data was collected on four different trials in the range of 300K to 320K. As shown in figure 5, temperature was plotted as a function of length and then fitted linearly to produce the experimentally determined value for α_L. Because only one or two trials could reasonably be completed per day, and techniques were refined over the course of the lab, the first trial lacked a lot of the rigor of the late trials, so it has been omitted from the final results shown in figure 4.

For trial 1, included below (figure 2) are some examples of what the raw temperature vs time and raw potential vs time graphs look like. Note that although the photo detector output a potential, this value was proportional to the intensity. As seen in the plot of potential vs time, the effect discussed in section 3 - as the copper rod expands, the two light beams go into and out of phase - is the effect that was observed.

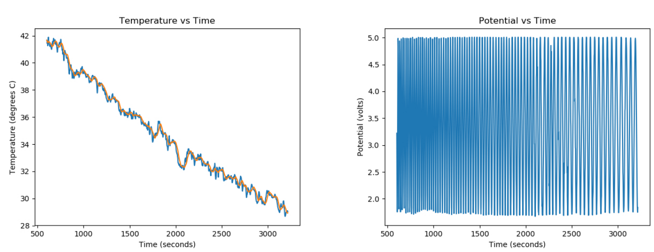

Figure 2: The above two graphs show the raw data as it is collected from the sensors during trial 1. Notably, a smoothing algorithm was applied to the first temperature vs time graph to produce the orange line.

For trial 2, included in figure 3 is what the raw temperature vs time looked like for all the other, more successful trials. This plot is fairly regular with a fair amount of noise (however nothing near the amount of noise in the plots from trial 1). Interestingly, on figure 3, it is possible to see the way that the rate of cooldown decreases as the copper rod nears room temperature. This is illustrated in the decreasing magnitude of the slope as time moves forward.

The results produced during this experiment fell fairly in line with what was expected. In the literature, the

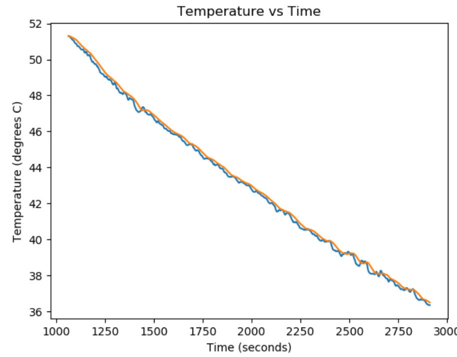

Figure 3: Although this plot includes temperature vs time data for the second trial, it looks essentially exactly the same as the temperature vs time plot for the rest of the data sets. The orange line is a smoothing algorithm run on the data.

Linear Thermal Expansion Coefficient — Copper

Trial	α_L (1/K)	u[α_L]
2	1.7080E-05	2.400E-07
3	1.6758E-05	2.410E-07
4	1.662E-05	2.20E-07
Average	1.6819E-05	2.41E-07

Figure 4: This chart gives the results from each of the trials 2 through 4 as well as their uncertainties. Finally, it also shows the average linear thermal expansion coefficient along with is associated uncertainty. All values were experimentally derived using the same technique as in figure 5.

value for the linear thermal expansion coefficient is 16.759E-6 $\pm$0.00670E-6 K $^{-1}$ for 300K and 16.960E-6 $\pm$0.00678E-6 K $^{-1}$ for 320K (Kroeger). The method employed in this lab to determine α_L does not allow for precise measurement at a given temperature, instead, it requires data over a larger temperature range. Therefore, it is difficult to compare perfectly to the values for α_L found in the literature. If we conduct a t-test on the value for α_L at 300K and the value for $\alpha_L =$ 16.819E-6 $\pm$0.241E-6 K $^{-1}$ experimentally determined in this lab over the range 305K to 320K, we get $t = 0.6$. This indicates that the two values are in agreement.

By far the largest source of error during this lab was the temperature probe used. The uncertainty in data taken from the probe is $\pm$0.5K, which is the reason so much noise is seen in the linear fit of temperature vs length. With a more precise temperature probe, the thermal expansion coefficient could be determined to a much higher degree of certainty. Additionally, with less noise, fits of the data could be performed on much smaller temperature ranges, allowing an approximation for α_L at different temperatures. The only other large source of error during the lab was the measurement of the initial length of the copper bar (at room temperature). This measurement was done using a pair of plastic calipers and could have easily been far more precise by simply using an improved pair of

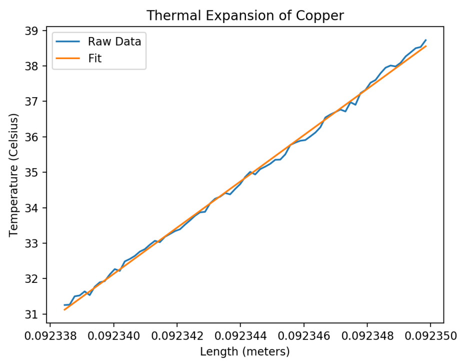

Figure 5: A plot of the length of a copper rod as a function of temperature fitted linearly for trial 4. The fit function was defined as $T = 652397L + 60210$ and achieved a chi squared of 1.5. Using this slope, a value for alpha was derived as $16.616 \times 10^{-5} \pm 0.238 \times 10^{-5} K^{-1}$ based on the fact that α_L is equal to 1/slope divided by L_0. See equation (4).

metal calipers.

5 Conclusion

Over the course of this experiment, the linear thermal expansion coefficient was experimentally derived based on the expansion of copper as it cooled down from being heated up using a heat gun. The length changes were measured using the changes in the intensity of the interferometer as the two intersecting beams went into and out of phase due the length of one of the arms of the interferometer changing.

The results of this experiment fell right in line with what was expected according to the literature. As expected, the average value for the linear thermal expansion of copper was measured as 16.819E-6 ± 0.241E-6 somewhere between 16.759E-6 K $^{-1}$ and 16.960E-6 K $^{-1}$.

6 References

- Kroeger, Frederick Robert Jr.,"The absolute thermal expansion of copper and aluminum between 5 K and 330 K" (1974). Retrospective Theses and Dissertations. 5151. `https://lib.dr.iastate.edu/rtd/5151` (unpublished)

7 Appendix

7.1

I thought it may be useful to include a link to all the scripts we wrote during this lab for data analysis. Here it is on my github. It is all in Python and not too long. Basically the script just searches for peaks and troughs in the data then matches them up with the correct temperatures in the temperature data to create a data set containing only temperature and length data.