Modeling Camphor Boat Speed at the Air-Water Interface

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Introduction

A camphor "boat" is propelled by the Marangoni force due to the ability of camphor to produce a surface tension gradient at the air-water interface. Its instantaneous speed at varying times throughout the life of the camphor boat reveals three distinct modes of motility; high speed motion with harmonic fluctuations, constant speed with small insignificant perturbations, and lastly relaxation oscillations characterized by long periods of little to no motion with sharp peaks in speed. Our model yields equilibrium points in the parameter space of the ODE system consistent with the experimental data.

Experimental Results

In a glass petri dish filled with distilled water we introduce a circular camphor boat 3mm in diameter with 1mm thickness.

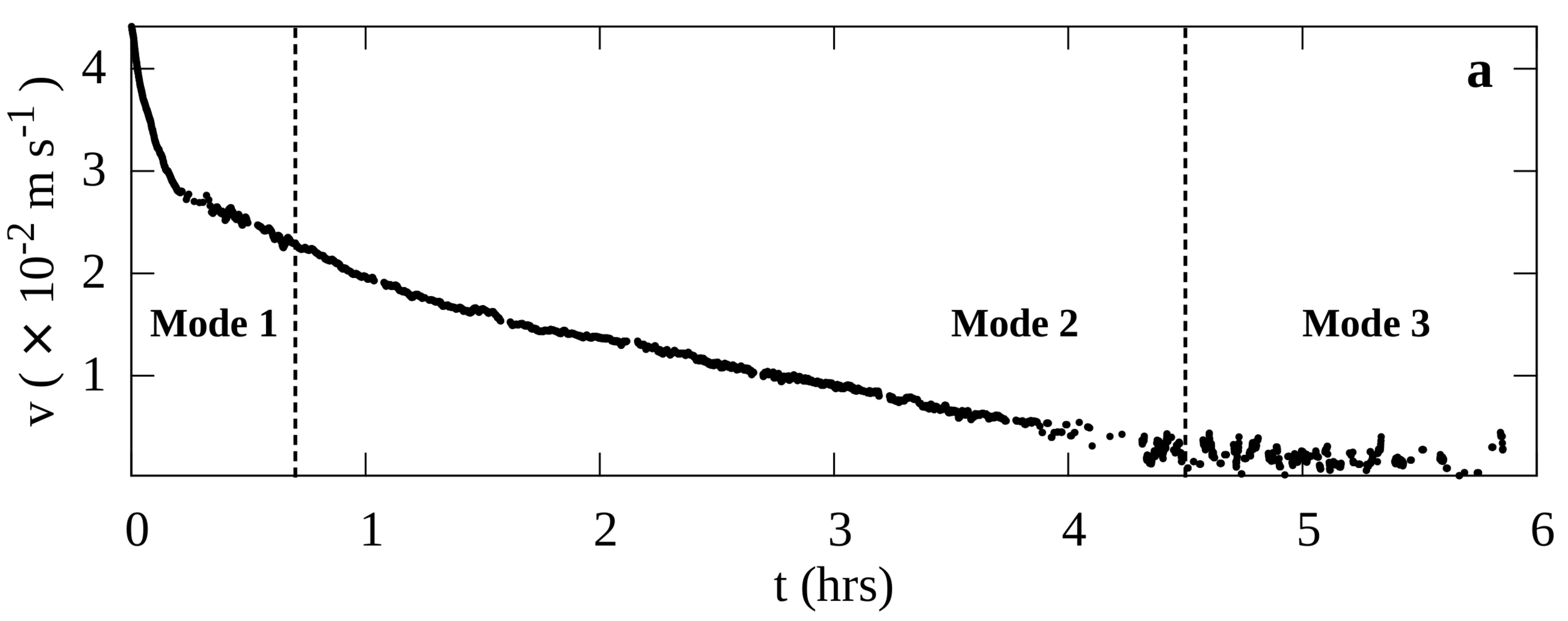


Figure 1: Average speed of a camphor boat at the air-water interface until consistent motion is no longer seen. Modes 1-3 refer to the local fluctuations in speed which correspond to varying dynamics in the system.

Model of Camphor Depletion

From Fig. 1 we infer that the concentration of camphor in the boat decreases exponentially. For given fitting parameters α and κ we integrate this concentration and set it equal to a constant κ:

$$\int_t^{t+\Delta t} e^{-\alpha t}\mathrm{d}t = \frac{1}{\alpha}\left[e^{-\alpha t} - e^{-\alpha(t+\Delta t)}\right] = \kappa \qquad (1)$$

we find an expression for the time it takes κ amount of camphor to leech onto the surface:

$$\Delta t = -\frac{1}{\alpha}\ln\left(1 - \alpha\kappa \cdot e^{\alpha t}\right) \qquad (2)$$

Defining a recursive series based on the above equation starting at $t = 0$ yields:

$$\Delta t_{n+1} = -\frac{1}{\alpha}\ln\left(1 - \alpha\kappa \cdot \exp\left(\alpha \sum_{i=0}^{n} \Delta t_i\right)\right) \qquad (3)$$

The sequence Δt_n gives the times at which the Marangoni force can act on the boat. To match the experimental data we set $\alpha = 0.65$ and $\kappa = 0.00015$.

Camphor Boat Dynamics

Narrowing the time averaging period of Fig. 1, three distinct dynamics are seen

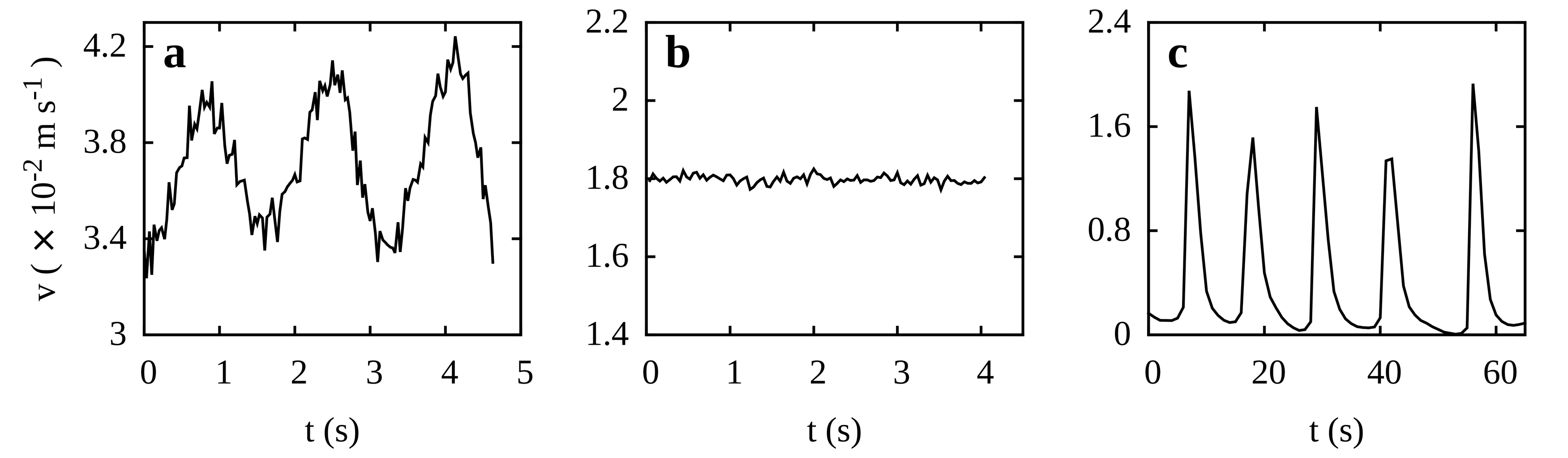


Figure 2: The instantaneous speed of the camphor boat shows the distinction between Modes 1-3 labeled in Fig. 1: (**a**) harmonic oscillations of speed, (**b**) a stable local speed, and (**c**) relaxation oscillations in speed.

Resulting Model of Camphor Speed

A logical starting point is a system of ordinary differential equations encompassing harmonic oscillations as well as relaxation oscillations.

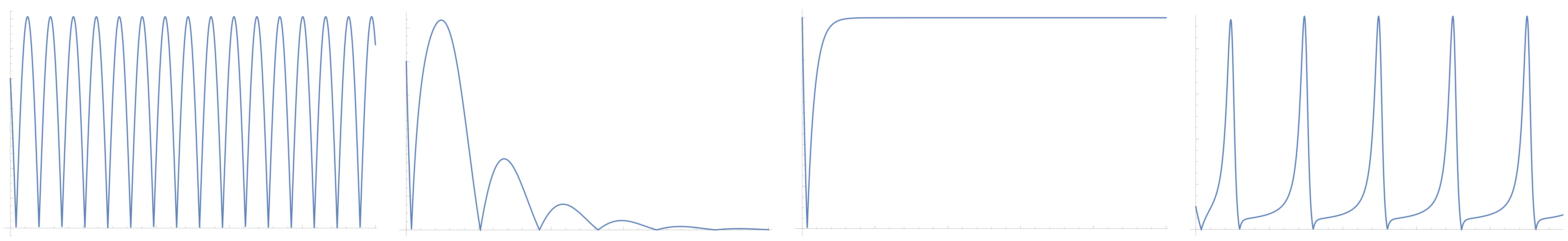
$$\begin{aligned}\frac{\mathrm{d}\nu}{\mathrm{d}t} &= \gamma\left(1-\chi^2\right)\nu - \chi \\ \frac{\mathrm{d}\chi}{\mathrm{d}t} &= \nu\end{aligned} \qquad (4)$$

For $\gamma = 0$ we see that the above system simplifies to a standard harmonic oscillator, and for large $\gamma > 1$ we find relaxation oscillations, corresponding to Mode 1 and 3 in Fig. 1, respectively. To enable the model to exhibit the steady state solution of Mode 2 we make the following modifications:

$$\begin{aligned}\dot{\nu} &= \gamma\left((\gamma-1)-\chi^2\right)\nu - \chi \\ \dot{\chi} &= \nu\frac{(1-\gamma)^2}{\gamma+1}\end{aligned} \qquad \text{given} \quad \gamma = \frac{\Delta t_n}{0.007} \qquad (5)$$

Using the sequence Δt_n from Eq. 3 we define γ as above, where 0.007 is the characteristic time of relaxation oscillations found in Mode 3.

Simulation Results for $\gamma = \Delta t_n/0.007$



a: $\gamma = 0$ **b:** $\gamma = 0.5$ **c:** $\gamma = 1$ **d:** $\gamma = 1.7$

Figure 3: Plots of speed ($|\nu|$) for varying values of γ. (**a**) exhibits harmonic oscillations with high frequency, (**b**) a damped oscillator settling to a stable equilibrium point, (**c**) nearly immediate convergence to stability, and (**d**) relaxation oscillations of decreasing frequency as $\gamma \to \infty$

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