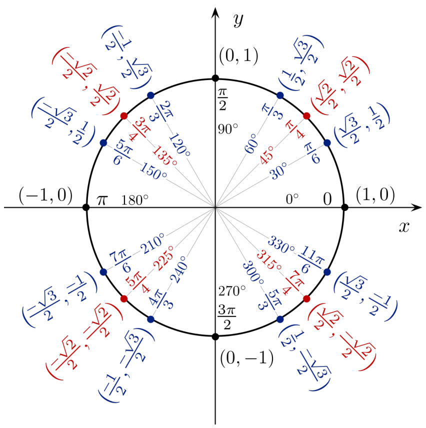
The only math appendix you'll need (and probably more) — painfully written by Laura Acinapura

Trigonometrie



Identitäten

$\cos(-x) = \cos(x)$
$\sin(-x) = -\sin(x)$
$\cos(x) = \sin(x + \frac{\pi}{2})$
$\sin(x) = \cos(x - \frac{\pi}{2})$
$\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$
$\cosh(x) = \frac{e^{x}+e^{-x}}{2}$
$\sin(x) = \frac{e^{ix}-e^{-ix}}{2i}$
$\sinh(x) = \frac{e^{x}-e^{-x}}{2}$
$e^{ix} = \cos(x) + i\sin(x)$
$e^{x} = \cosh(x) + \sinh(x)$
$\cos(ix) = \cosh(x)$
$\cosh(\pm ix) = \cos(x)$
$\sin(ix) = i\sinh(x)$
$\sinh(\pm ix) = \pm i\sin(ix)$
$\tan(x) = \frac{\sin(x)}{\cos(x)}$
$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
$\tan(-x) = -\tan(x)$
$\tan(x+\pi) = \tan(x)$
$\tan(x) = -i\tanh(ix)$
$\tanh(x) = -i\tan(ix)$
$\cot(x) = \frac{\cos(x)}{\sin(x)}$
$\coth(x) = \frac{\cosh(x)}{\sinh(x)}$
$\sec(x) = \frac{1}{\cos(x)}$
$\csc(x) = \frac{1}{\sin(x)}$

Addition

$\cos(x)^2 + \sin(x)^2 = 1$
$\cosh(x)^2 - \sinh(x)^2 = 1$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\tan(x \pm y) = \frac{\tan(x)\pm\tan(y)}{1\mp\tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x) = \frac{2\tan(x)}{1+\tan(x)^2}$
$\cos(2x) = 2\cos(x)^2 - 1 = \cos(x)^2 - \sin(x)^2$
$= 1 - 2\sin(x)^2 = \frac{1-\tan(x)^2}{1+\tan(x)^2}$
$\sin(x) + \sin(y) = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$
$\sin(x) - \sin(y) = 2\cos(\frac{x+y}{2})\sin(\frac{x-y}{2})$
$\cos(x) + \cos(y) = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$
$\cos(x) - \cos(y) = -2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})$

$\cos(x) + \sin(x) = \sqrt{2}\cos(\frac{\pi}{4} - x) = \sqrt{2}\sin(\frac{\pi}{4} + x)$
$\cos(x) + \sin(x) = \sqrt{2}\cos(\frac{\pi}{4} + x) = \sqrt{2}\sin(\frac{\pi}{4} - x)$
$\tan(x) \pm \tan(y) = \frac{\sin(x\pm y)}{\cos(x)\cos(y)}$
$1 + \tan(x)^2 = \frac{1}{\cos(x)^2}$
$1 - \tanh(x)^2 = \frac{1}{\cosh(x)^2}$
$1 + \tan(x) = \frac{\cos(x)+\sin(x)}{\cos(x)}$

Multiplication and Powers

$\sin(x)\sin(y) = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$
$\cos(x)\cos(y) = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$
$\sin(\alpha t)\cos(\beta t) = \frac{1}{2}(\sin((\alpha+\beta)t) + \sin((\alpha-\beta)t)$
$\sin(x)^2 = \frac{1}{2}(1 - \cos(2x))$
$\cos(x)^2 = \frac{1}{2}(1 + \cos(2x))$

$$\sin(x)^n = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\cos((n-2k)(x-\frac{\pi}{2}))$$

$$\cos(x)^n = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\cos((n-2k)x)$$

$\cosh(x)^2 = \frac{\cosh(2x)+1}{2}$
$\sinh(x)^2 = \frac{\cosh(2x)-1}{2}$

Inverse of Trigonometric Functions

$\cos(\arcsin(x)) = \sin(\arccos(x)) = \sqrt{1-x^2}$
$\cos(\arcsin(x)) = \sin(\arccos(x)) = \sqrt{1-x^2}$
$\cosh(\operatorname{arcsinh}(x)) = \sqrt{x^2+1}$
$\sinh(\operatorname{arccosh}(x)) = \sqrt{x^2-1}$
$\cos(\arctan(x)) = \frac{1}{\sqrt{x^2+1}}$
$\tan(\arccos(x)) = x^{-1}(1-x)^{1/4}$
$\tan(\arcsin(x)) = x(1-x)^{-1/4}$
$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2-1})$
$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2-+1})$

Limits

$\ln(\ln x) \ll \ln(x) \ll x^p \ll x^q \ll a^x \ll b^x \ll x! \ll x^x$

$\lim\limits_{x\to 0}\frac{\sin(x)}{x} = 1$
$\lim\limits_{x\to 0}\frac{\arcsin(x)}{x} = 1$
$\lim\limits_{x\to 0}\frac{\tan(x)}{x} = 1$
$\lim\limits_{x\to 0}\frac{\arctan(x)}{x} = 1$
$\lim\limits_{x\to 0}\frac{e^x-1}{x} = 1$
$\lim\limits_{x\to 0}\frac{a^x-1}{x} = \ln(a)$
$\lim\limits_{x\to 0}\frac{\ln(x+1)}{x} = 1$
$\lim\limits_{x\to 0}\frac{\ln(x+a)}{x} = a$
$\lim\limits_{x\to 0}\frac{1-\cos(x)}{x^2} = \frac{1}{2}$
$\lim\limits_{x\to\infty}(1+\frac{k}{x})^x = e^k$
$\lim\limits_{x\to\infty}(1-\frac{1}{x+1})^x = e^{-1}$
$\lim\limits_{x\to\infty}\left(\frac{x-1}{x+1}\right)^x = e^{-2}$

Some Algebra

Exponential and Logarithms

$e^a \cdot e^b = e^{a+b}$
$\ln(a) + \ln(b) = \ln(ab)$
$\frac{e^a}{e^b} = e^{a-b}$
$\ln(a) - \ln(b) = \ln(d\frac{a}{b})$
$(e^a)^b = e^{ab}$
$\ln(x^a) = a\ln(x)$
$e^0 = 1,\ e^{-\infty} = 0$
$\ln(1) = 0,\ \ln(e) = 1$
$x^{\frac{m}{n}} = \sqrt[n]{x^m}$
$\log_b(a) = \frac{\log_c(a)}{\log_c(b)}$

$$\boxed{\begin{aligned}\ln(x) = a &\iff x = e^a\\ e^x = a &\iff x = \ln(a)\\ e^{\ln(a)} = a &\implies a^x = e^{\ln(a)x}\end{aligned}}$$

Factorising Polynoms

$$ax^2 + bx + c \implies x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$x^3 + 1 = (x+1)(x^2 - x + 1)$
$x^3 - 1 = (x-1)(x^2 + x + 1)$
$x^3 \pm a^3 = (x \pm a)(x^2 \mp ax + a^2)$
$x^2 - a^2 = (x-a)(x+a)$

Given the polynom $f(x) = a_n x^n + \cdots + a_0$ all the possible roots of $f(x)$ are in the form $\pm\frac{\text{Factor of } a_0}{\text{Factor of } a_n}$

Complex roots $z = |z|e^{i\varphi} \to w_k = |z|^{\frac{1}{n}} e^{i(\frac{\varphi}{n} + \frac{2\pi k}{n})}$

Zeros of a Polynom 3rd Degree

$x^3 + Bx^2 + Cx + D = (x+a)(x+b)(x+c)$

$$\begin{cases} a+b+c = B \\ ab+bc+ac = C \\ abc = D \end{cases} \implies \text{Zeros are factors of } D$$

Polynom Division $P(x)/Q(x)$

1. Divide the biggest term of P with the biggest term of Q
2. Multiply the result for $-Q$, write it under P and sum them
3. Repeat 1. and 2. until you can't divide anymore

$$\begin{array}{rl|l} 6x^3 - 2x^2 & +x+3 & x^2 - x + 1 \\ \cline{3-3} \underline{-6x^3 + 6x^2 - 6x} & & 6x+4 \\ 4x^2 - 5x + 3 & & \\ \underline{-4x^2 + 4x - 4} & & \\ -x-1 & & \end{array}$$

$$\implies \frac{6x^3 - 2x^2 + x + 3}{x^2 - x + 1} = \frac{-x-1}{x^2-x+1} + 6x + 4$$

Partialbruchzerlegung $P(x)/Q(x)$

1. Polynom Division mit Rest falls Grad $P \geq$ Grad Q
2. Nullstellen von Q berechnen
3. Nullstellen ihrem Partialbruch zuordnen:
   - reelle m-fache Nullstelle x_0

     $$\frac{A_1}{(x-x_0)} + \frac{A_2}{(x-x_0)^2} + \cdots + \frac{A_m}{(x-x_0)^m}$$

   - complexe Nullstelle x_0

     $$\frac{Ax+B}{x^2+2ax+b}$$
4. Unbekannte bestimmen mit Expansion + Koeffizientenvergleich

   Beispiel: $f(x) = \frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

   $$x(A+B) - A - 2B = x \implies \begin{cases} A+B = 1 \\ A+2B = 0 \end{cases}$$

Symmetries

Even function: $f(-x) = f(x)$

- y-axis symmmetric
- $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$
- Fourier Series has only **cosine** $\to b_n = 0$
- Laurent Series in 0 has only **even powers**
- Common even functions: $\cos x, x^{2n}, k$

Odd function: $f(-x) = -f(x)$

- Origin symmmetric
- $\int_{-a}^{a} f(x)dx = 0$
- Fourier Series has only **sine** $\to a_n = 0$
- Laurent Series in 0 has only **odd powers**
- Common odd functions: $\sin x, \tan x, \arctan x, x^{2n+1}$

even · even = even
odd · odd = even
even · odd = odd

$f(x) = f_e(x) + f_o(x)$
$f_e(x) = \frac{1}{2}(f(x) + f(-x))$
$f_o(x) = \frac{1}{2}(f(x) - f(-x))$

Series

Arithmetic Series $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$

Geometric Series $\sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$

Binomial Formula $(a+b)^n = \sum_{k=0}^{n} k = \binom{n}{k} a^k b^{n-k}$

Alternating Harmonic Series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n} = -\ln(2)$

Leibniz Formula $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = \frac{\pi}{4}$ | $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

Zeta-Function $\sum_{n=1}^{\infty}\frac{1}{n^s} = \begin{cases}\text{converges for} & s > 1\\ \text{diverges for} & s \leq 1\end{cases}$

Power Series

Die Entwicklung ist diesselbe im komplexen Fall

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad |x| < 1$$

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n} \quad |x| < 1$$

$$\frac{1}{a-x} = \frac{1/a}{1-x/a} \sum_{n=0}^{\infty} \frac{1}{a}\left(\frac{x}{a}\right)^n \quad |x| < a$$

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} (n+1) x^n \quad |x| < 1$$

$$\frac{1}{(1-x)^3} = \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n \quad |x| < 1$$

$$(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \cdots$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \cdots$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos(x)^2 = 1 - \frac{2}{2!}x^2 + \frac{2^3}{4!}x^4 - \frac{2^5}{6!}x^6 + \cdots$$

$$\sin(x)^2 = \frac{2}{2!}x^2 - \frac{2^3}{4!}x^4 + \frac{2^5}{6!}x^6 - \cdots$$

$$\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\sinh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\arcsin(x) = \sum_{n=0}^{\infty} \frac{(2n)! \cdot x^{2n+1}}{2^{2n}(n!)^2(2n+1)}$$

$$\arccos(x) = \frac{\pi}{2} - \arcsin(x)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$\ln(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$$

Inequalities

$|a+b| \leq |a| + |b|$
$|a-b| \geq ||a| - |b|| \geq |a| - |b|$
$|a| \leq b \to -b < a < b$
$2ab \leq a^2 + b^2$
$|a_n b_n| \leq \frac{1}{2}(|a|^2 + |b|^2)$
$|ab - cd| \leq |a| \cdot |b-d| + |d| \cdot |a-c|$
$|f_n g_n - fg| \leq |f_n| \cdot |g_n - g| + |g| \cdot |f_n - f|$

Derivatives

Basic Derivatives

$f(x)$	$f'(x)$
c	0
x^n	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
e^{ax}	ae^{ax}
b^x	$\ln(b)b^x$
$\ln(x)$	$\frac{1}{x}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos(x)^2}$

$f(x)$	$f'(x)$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sin(x)$
$\tanh(x)$	$\frac{1}{\cosh(x)^2}$
$\arctan(x)$	$\frac{1}{1+x^2}$
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arctanh}(x)$	$\frac{1}{1-x^2}$
$\operatorname{arcsinh}(x)$	$\frac{1}{\sqrt{x^2+1}}$
$\operatorname{arccosh}(x)$	$\frac{1}{\sqrt{x^2-1}}$

Integrals

Basic Integral Forms

$f(x)$	$F(x)$
c	cx
x^n	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x-c}$	$\ln(\lvert x-c \rvert)$
$\frac{1}{x^n}$	$-\frac{1}{(n-1)x^{n-1}}$
$\sqrt{x}$	$\frac{2}{3}\sqrt{x^3}$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
$\frac{1}{1+x^2}$	$\arctan(x)$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(x)$
$\frac{1}{\sqrt{1+x^2}}$	$\operatorname{arcsinh}(x)$
$\frac{1}{\sqrt{x^2-1}}$	$\operatorname{arccosh}(x)$
$\frac{1}{(x+k)^2+m^2}$	$\frac{1}{m}\arctan\left(\frac{x+k}{m}\right)$

$f(x)$	$F(x)$
$g'(x)$	$g(x)$
$g^n(x)g'(x)$	$\frac{g^{n+1}(x)}{n+1}$
$\frac{g'(x)}{g(x)}$	$\ln(\lvert g(x) \rvert)$
$\frac{g'(x)}{g^n(x)}$	$-\frac{1}{(n-1)g^{n-1}(x)}$
$\sqrt{g(x)}g'(x)$	$\frac{2}{3}\sqrt{(g^3(x)}$
$e^{g(x)}g'(x)$	$e^{g(x)}$
$\sin(g(x))g'(x)$	$-\cos(g(x))$
$\cos(g(x))g'(x)$	$\sin(g(x))$
$\frac{g'(x)}{1+g^2(x)}$	$\arctan(g(x))$
$\frac{g'(x)}{\sqrt{1-g^2(x)}}$	$\arcsin(g(x))$
$\frac{g'(x)}{\sqrt{1+g^2(x)}}$	$\operatorname{arcsinh}(g(x))$
$\frac{g'(x)}{\sqrt{g^2(x)-1}}$	$\operatorname{arccosh}(g(x))$

Partial Integration

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)}$$

Integrals by Substitution

$$\int \frac{f'(x)}{f(x)}dx \to f(x) = t \mid dx = \frac{dt}{f'(x)}$$

$$\int f(g(x))g'(x)dx \to g(x) = t \mid dx = \frac{dt}{g'(x)}$$

$$\int F(e^x, \sinh(x), \cosh(x))dx \to e^x = t \mid dx = \frac{dt}{e^x} = \frac{1}{t}dt$$

$$\int F(\ln(x))dx \to \ln(x) = t \mid dx = xdt = e^t dt$$

$$\int F(\sqrt[\alpha]{Ax+B})dx \to \sqrt[\alpha]{Ax+B} = t$$

$$\int F(\cos(x)^{2n}, \sin(x)^{2n})dx \to \tan(x) = t$$

$$dx = \frac{1}{t^2+1}dt \mid \sin(x)^2 = \frac{t^2}{t^2+1} \mid \cos(x)^2 = \frac{t}{t^2+1}$$

$$\int F(\cos(x)^{2n+1}, \sin(x)^{2n+1})dx \to \tan(\frac{x}{2}) = t$$

$$dx = \frac{2}{t^2+1}dt \mid \sin(x) = \frac{2t}{t^2+1} \mid \cos(x) = \frac{1-t^2}{t^2+1}$$

$$\int F(\sqrt{1-x^2})dx \to \sin(x) = t \mid dx = \cos(t)dt$$

$$\int F(\sqrt{1+x^2})dx \to \sinh(x) = t \mid dx = \cosh(t)dt$$

$$\int F(\sqrt{x^2-1})dx \to \cosh(x) = t \mid dx = \sinh(t)dt$$

$$\int F\left(\frac{1}{\sqrt{Ax^2+Bx+C}}\right)dx \to \text{Complete the Square}$$

and try to get back to one of this forms:

$\int \frac{1}{\sqrt{1-x^2}} = \arcsin x$
$\int \frac{1}{\sqrt{x^2-1}} = \operatorname{arccosh} x$
$\int \frac{1}{\sqrt{1+x^2}} = \operatorname{arcsinh} x$

Definite Integral of trigonometrics

$f(x)$	$\int_0^{\frac{\pi}{4}}$	$\int_0^{\frac{\pi}{2}}$	$\int_0^{\pi}$	$\int_0^{2\pi}$	$\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}$	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{4}}$	$\int_{-\pi}^{\pi}$
$\sin x$	$\frac{\sqrt{2}-1}{\sqrt{2}}$	1	2	0	0	0	0
$\sin^2 x$	$\frac{\pi-2}{8}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{\pi-2}{4}$	$\frac{\pi}{2}$	π
$\sin^3 x$	$\frac{8-5\sqrt{2}}{12}$	$\frac{2}{3}$	$\frac{4}{3}$	0	0	0	0
$\cos x$	$\frac{1}{\sqrt{2}}$	1	0	0	$\sqrt{2}$	2	0
$\cos^2 x$	$\frac{\pi+2}{8}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{\pi+2}{4}$	$\frac{\pi}{2}$	π
$\cos^3 x$	$\frac{5}{6\sqrt{2}}$	$\frac{2}{3}$	0	0	$\frac{5}{3\sqrt{2}}$	$\frac{4}{3}$	0
$\cos x \sin x$	$\frac{1}{4}$	$\frac{1}{2}$	0	0	0	0	0
$\sin^2 x \cos x$	$\frac{1}{6\sqrt{2}}$	$\frac{1}{3}$	0	0	$\frac{1}{3\sqrt{2}}$	$\frac{2}{3}$	0
$\sin x \cos^2 x$	$\frac{4-\sqrt{2}}{12}$	$\frac{1}{3}$	$\frac{2}{3}$	0	0	0	0

Numerics

$\sqrt{2} \approx 1.41421$	$7^2 = 49$	$25^2 = 625$
$\sqrt{3} \approx 1.73207$	$8^2 = 64$	$26^2 = 676$
$\sqrt{5} \approx 1.23607$	$9^2 = 81$	$27^2 = 729$
$\pi \approx 3.14159$	$11^2 = 121$	$28^2 = 784$
$e \approx 2.71828$	$12^2 = 144$	$29^2 = 841$
$e^2 \approx 7.38906$	$13^2 = 169$	$31^2 = 961$
$2! = 2$	$14^2 = 196$	$2^1 = 2$
$3! = 6$	$15^2 = 225$	$2^2 = 4$
$4! = 24$	$16^2 = 256$	$2^3 = 8$
$5! = 120$	$17^2 = 289$	$2^4 = 16$
$6! = 720$	$18^2 = 324$	$2^5 = 32$
$2^2 = 4$	$19^2 = 361$	$2^6 = 64$
$3^2 = 9$	$21^2 = 441$	$2^7 = 128$
$4^2 = 16$	$22^2 = 484$	$2^8 = 256$
$5^2 = 25$	$23^2 = 529$	$2^9 = 512$
$6^2 = 36$	$24^2 = 576$	$2^{10} = 1024$

Integrals formulas you'll hardly ever need

Integrals of Rational Functions

$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a}$

$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1$

$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)}$

$\int \frac{1}{1+x^2} dx = \tan^{-1} x$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$

$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2|$

$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a}$

$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2 \ln |a^2+x^2|$

$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$

$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x},\ a \neq b$

$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x|$

$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}}$

Integrals with Roots

$\int \sqrt{x-a}\ dx = \frac{2}{3}(x-a)^{3/2}$

$\int \frac{1}{\sqrt{x \pm a}}\ dx = 2\sqrt{x \pm a}$

$\int \frac{1}{\sqrt{a-x}}\ dx = -2\sqrt{a-x}$

$$\int x\sqrt{x-a}\ dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, \text{ or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, \text{ or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$

$\int \sqrt{ax+b}\ dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right) \sqrt{ax+b}$

$\int (ax+b)^{3/2}\ dx = \frac{2}{5a}(ax+b)^{5/2}$

$\int \frac{x}{\sqrt{x \pm a}}\ dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a}$

$\int \sqrt{\frac{x}{a-x}}\ dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a}$

$\int \sqrt{\frac{x}{a+x}}\ dx = \sqrt{x(a+x)} - a \ln(\sqrt{x} + \sqrt{x+a})$

$\int x\sqrt{ax+b}\ dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b}$

$\int \sqrt{x(ax+b)}\ dx = \frac{1}{4a^{3/2}} \Big[(2ax+b)\sqrt{ax(ax+b)} -$
$\ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \Big]$

$\int \sqrt{x^3(ax+b)}\ dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right] \sqrt{x^3(ax+b)} +$
$\frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right|$

Integrals with Logarithms

$\int \ln ax\ dx = x \ln ax - x$

$\int x \ln x\ dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4}$

$\int x^2 \ln x\ dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9}$

$\int x^n \ln x\ dx = x^{n+1} \left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n \neq -1$

$\int \frac{\ln ax}{x}\ dx = \frac{1}{2} (\ln ax)^2$

$\int \frac{\ln x}{x^2}\ dx = -\frac{1}{x} - \frac{\ln x}{x}$

$\int \ln(ax+b)\ dx = \left(x + \frac{b}{a}\right) \ln(ax+b) - x, a \neq 0$

$\int \ln(x^2+a^2)\ dx = x \ln(x^2+a^2) + 2a \tan^{-1} \frac{x}{a} - 2x$

$\int \ln(x^2-a^2)\ dx = x \ln(x^2-a^2) + a \ln \frac{x+a}{x-a} - 2x$

$\int \ln(ax^2+bx+c)\ dx = \frac{1}{a}\sqrt{4ac-b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x +$
$\left(\frac{b}{2a} + x\right) \ln(ax^2+bx+c)$

$\int x \ln(ax+b)\ dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right) \ln(ax+b)$

$\int x \ln(a^2-b^2x^2)\ dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right) \ln(a^2-b^2x^2)$

$\int (\ln x)^2\ dx = 2x - 2x \ln x + x(\ln x)^2$

$\int (\ln x)^3\ dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x$

$\int x(\ln x)^2\ dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x$

$\int x^2(\ln x)^2\ dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3 \ln x$

$\int \sqrt{x^2 \pm a^2}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$

$\int \sqrt{a^2-x^2}\ dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2 \tan^{-1} \frac{x}{\sqrt{a^2-x^2}}$

$\int x\sqrt{x^2 \pm a^2}\ dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$

$\int \frac{1}{\sqrt{x^2 \pm a^2}}\ dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right|$

$\int \frac{1}{\sqrt{a^2-x^2}}\ dx = \sin^{-1} \frac{x}{a}$

$\int \frac{x}{\sqrt{x^2 \pm a^2}}\ dx = \sqrt{x^2 \pm a^2}$

$\int \frac{x}{\sqrt{a^2-x^2}}\ dx = -\sqrt{a^2-x^2}$

$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\ dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right|$

$\int \sqrt{ax^2+bx+c}\ dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c}$
$+ \frac{4ac-b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx^{+}c)} \right|$

$\int \frac{1}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx+c)} \right|$

$\int \frac{x}{\sqrt{ax^2+bx+c}}\ dx = \frac{1}{a}\sqrt{ax^2+bx+c}$
$-$
$frac b 2a^{3/2} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx+c)} \right|$

$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$

Integrals with Exponentials

$\int e^{ax}\ dx = \frac{1}{a}e^{ax}$

$\int \sqrt{x}e^{ax}\ dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}\left(i\sqrt{ax}\right),$
where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

$\int xe^x\ dx = (x-1)e^x$

$\int xe^{ax}\ dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$

$\int x^2e^x\ dx = (x^2 - 2x + 2)\, e^x$

$\int x^2e^{ax}\ dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax}$

$\int x^3e^x\ dx = (x^3 - 3x^2 + 6x - 6)\, e^x$

$\int x^ne^{ax}\ dx = \frac{x^ne^{ax}}{a} - \frac{n}{a} \int x^{n-1}e^{ax}\, \mathrm{d}x$

$\int x^ne^{ax}\ dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax],$ where $\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\ dt$

$\int e^{ax^2}\ dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right)$

$\int e^{-ax^2}\ dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(x\sqrt{a}\right)$

$\int xe^{-ax^2}\ dx = -\frac{1}{2a}e^{-ax^2}$

$\int x^2e^{-ax^2}\ dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2}$

Integrals with Trigonometric Functions

$\int \sin ax\ dx = -\frac{1}{a} \cos ax$

$\int \sin^2 ax\ dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

$\int \sin^3 ax\ dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a}$

$\int \sin^n ax\ dx = -\frac{1}{a} \cos ax\ \ {}_2F_1 \left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$

$\int \cos ax\ dx = \frac{1}{a} \sin ax$

$\int \cos^2 ax\ dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a}$

$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1 \left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$

$\int \cos x \sin x\ dx = \frac{1}{2} \sin^2 x + c_1 = -\frac{1}{2} \cos^2 x + c_2 = -\frac{1}{4} \cos 2x + c_3$

$\int \cos ax \sin bx\ dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$

$\int \sin^2 ax \cos bx\ dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$

$\int \sin^2 x \cos x\ dx = \frac{1}{3} \sin^3 x$

$\int \cos^2 ax \sin bx\ dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$

$\int \cos^2 ax \sin ax\ dx = -\frac{1}{3a} \cos^3 ax$

$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$

$\int \sin^2 ax \cos^2 ax\ dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$

$\int \tan ax\ dx = -\frac{1}{a} \ln \cos ax$

$\int \tan^2 ax\ dx = -x + \frac{1}{a} \tan ax$

$\int \tan^n ax\ dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1 \left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right)$

$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax$

$\int \sec x\ dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2}\right)$

$\int \sec^2 ax\ dx = \frac{1}{a} \tan ax$

$\int \sec^3 x\ dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$

$\int \sec x \tan x\ dx = \sec x$

$\int \sec^2 x \tan x\ dx = \frac{1}{2} \sec^2 x$

$\int \sec^n x \tan x\ dx = \frac{1}{n} \sec^n x, n \neq 0$

$\int \csc x\ dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C$

$\int \csc^2 ax\ dx = -\frac{1}{a}\cot ax$
$\int \csc^3 x\ dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x|$
$\int \csc^n x \cot x\ dx = -\frac{1}{n}\csc^n x, n \neq 0$
$\int \sec x \csc x\ dx = \ln|\tan x|$

Products of Trigonometric Functions and Monomials

$\int x\cos x\ dx = \cos x + x\sin x$
$\int x\cos ax\ dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$
$\int x^2\cos x\ dx = 2x\cos x + (x^2-2)\sin x$
$\int x^2\cos ax\ dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax$
$\int x^n\cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1,-ix)\right.$
$\left.+(-1)^n\Gamma(n+1,ix)\right]$
$\int x^n\cos ax\ dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1,-iax) - \Gamma(n+1,ixa)\right]$
$\int x\sin x\ dx = -x\cos x + \sin x$
$\int x\sin ax\ dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$
$\int x^2\sin x\ dx = (2-x^2)\cos x + 2x\sin x$
$\int x^2\sin ax\ dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2}$
$\int x^n\sin x\ dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1,-ix) - (-1)^n\Gamma(n+1,-ix)\right]$
$\int x\cos^2 x\ dx = \frac{x^2}{4} + \frac{1}{8}\cos 2x + \frac{1}{4}x\sin 2x$
$\int x\sin^2 x\ dx = \frac{x^2}{4} - \frac{1}{8}\cos 2x - \frac{1}{4}x\sin 2x$
$\int x\tan^2 x\ dx = -\frac{x^2}{2} + \ln\cos x + x\tan x$
$\int x\sec^2 x\ dx = \ln\cos x + x\tan x$

Products of Trigonometric Functions and Exponentials

$\int e^x\sin x\ dx = \frac{1}{2}e^x(\sin x - \cos x)$
$\int e^{bx}\sin ax\ dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax)$
$\int e^x\cos x\ dx = \frac{1}{2}e^x(\sin x + \cos x)$
$\int e^{bx}\cos ax\ dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax)$
$\int xe^x\sin x\ dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x)$
$\int xe^x\cos x\ dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x$

Integrals of Hyperbolic Functions

$\int \cosh ax\ dx = \frac{1}{a}\sinh ax$

$$\int e^{ax}\cosh bx\ dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases}$$

$\int \sinh ax\ dx = \frac{1}{a}\cosh ax$

$$\int e^{ax}\sinh bx\ dx = \begin{cases} \dfrac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases}$$

$\int \tanh ax\ dx = \frac{1}{a}\ln\cosh ax$

$$\int e^{ax}\tanh bx\ dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}\,{}_2F_1\left[1+\dfrac{a}{2b}, 1, 2+\dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad -\dfrac{1}{a}e^{ax}\,{}_2F_1\left[1, \dfrac{a}{2b}, 1+\dfrac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases}$$

$\int \cos ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right]$
$\int \cos ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cos ax\cosh bx + a\sin ax\sinh bx\right]$
$\int \sin ax\cosh bx\ dx = \frac{1}{a^2+b^2}\left[-a\cos ax\cosh bx + b\sin ax\sinh bx\right]$
$\int \sin ax\sinh bx\ dx = \frac{1}{a^2+b^2}\left[b\cosh bx\sin ax - a\cos ax\sinh bx\right]$
$\int \sinh ax\cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right]$
$\int \sinh ax\cosh bx\ dx = \frac{1}{b^2-a^2}\left[b\cosh bx\sinh ax - a\cosh ax\sinh bx\right]$

Some Linalg

Determinanten

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\det\left(\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right) = aei + bfg + cdh - bdi - afh - ceg$$

$$\det\begin{pmatrix} a^+ & b^- & c^+ \\ d^- & e^+ & f^- \\ g^+ & h^- & i^+ \end{pmatrix} = +a\cdot\det\begin{pmatrix} e & f \\ h & i \end{pmatrix} - d\cdot\det\begin{pmatrix} b & c \\ h & i \end{pmatrix} + g\cdot\det\begin{pmatrix} b & c \\ e & f \end{pmatrix}$$

- $\det(A) = \det(A^\mathsf{T})$
- $\det(A\cdot B) = \det(A)\cdot\det(B)$
- $\det(A^{-1}) = \frac{1}{\det(A)}$
- Die Determinate einer Dreiecksmatrix ist das Produkt der Diagonalelemente

Inverse

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} = \frac{1}{\det(A)}\begin{pmatrix} ei-fh & ch-bi & bf-ce \\ fg-di & ai-cg & cd-af \\ dh-eg & bg-ah & ae-bd \end{pmatrix}$$

Eigenwerte

- $\lambda \in \mathbb{C}$ ist ein EW von A falls $\exists\ x \in \mathbb{C}^n, x \neq 0 : Ax = \lambda x$
- $\lambda \in \mathbb{C}$ ist ein EW von $A \iff \det(A - \lambda\mathbb{I}) = 0$
- Die nichttriviale Lösungen von $(A - \lambda_i\mathbb{I})x = 0$ sind die EV von A zu λ_i
- $\prod_{n=1}^{n}\lambda_i = \det(A) \quad \sum_{n=1}^{n}\lambda_i = \mathrm{Spur}(A) = \text{Summe der Diagonalelemente}$
- Die EW einer Dreiecksmatrix sind die Diagonalelemente
- A und A^T haben die selben EW

Diagonalisierbarkeit

AG = GM $\iff$ A diagonalisierbar mit

$$A = T^{-1}DT \quad D = diag(\lambda_1, \ldots \lambda_n) \quad T = \begin{pmatrix} | & \cdots & | \\ EV_1 & \cdots & EV_n \\ | & \cdots & | \end{pmatrix}$$

Bei **Symmetrische** Matrizen gilt:
- Alle EW sind **reell**
- EW zu verschiedene EV sind **orthogonal** $\implies T^{-1} = T^\mathsf{T}$
- A ist diagonalisierbar

Plots and Graphs