Thin plate spline exercise

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1 Small example

1.1 Interpolating TPS

After computing the interpolating TPS corresponding to the given data, we plot the function f in the region $[-1.5, 1.5] \times [-1.5, 1.5]$ sampled with a step of 0.1 in both dimensions. Cf. figure 1

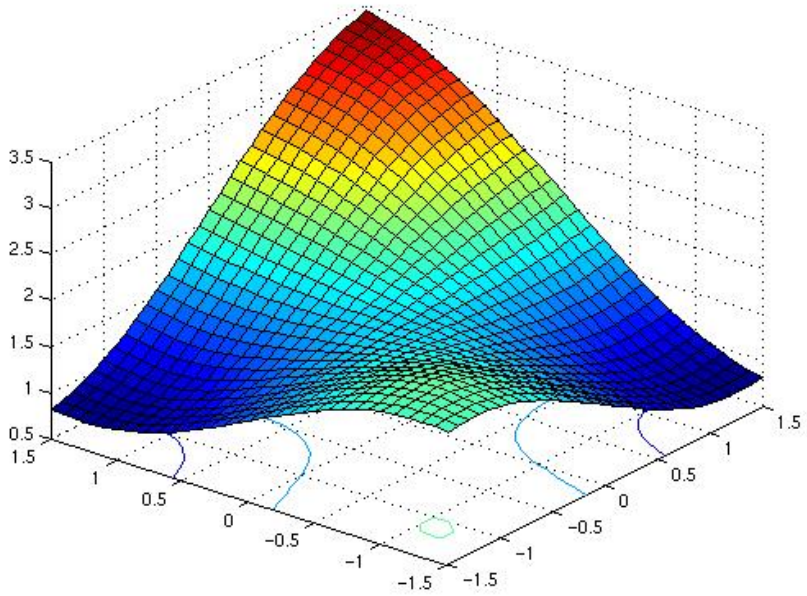


Figure 1: Function f corresponding to the smoothing TPS

1.2 Smoothing TPS

For each $\lambda = 0.1, 1, 10, 100$ we compute the smoothing TPS and plot f. To evaluate the equivalent degrees of freedom d, we compute the trace of the hat matrix. Cf. table 1.2

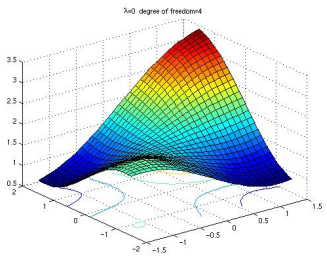
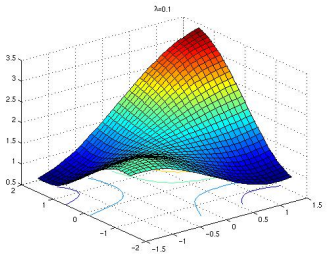
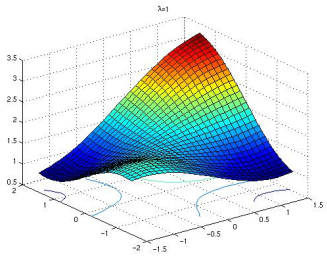
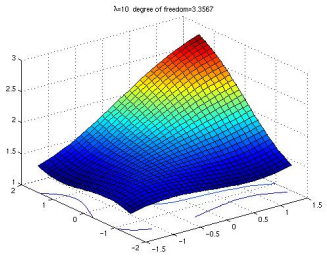
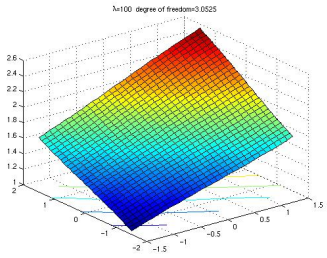
$\lambda = 0, d = 4$	
	
$\lambda = 0.1, d = 3.9823$	$\lambda = 1, d = 3.8472$
	
$\lambda = 10, d = 3.3567$	$\lambda = 100, d = 3.0525$
	

Table 1: Smoothing TPS and equivalent degrees of freedom for different values of λ

We see that for $\lambda = 0$ the degrees of freedom is maximum; it is equal to the number of points as this case is equivalent to pure interpolation. However, when $\lambda \to \infty$, the spline becomes a plan and the degrees of freedom tends to 3, which corresponds to the number of points that define a plan.

2 Bias field correction

After recording the observation series, we determine the values of λ corresponding to the degrees of freedom wanted, d. To do that, we start by trying random values of λ until we find a d higher than the one want and a smaller. Then we proceed by dichotomy. Finally, we get:

d	λ
20	2750
10	23000
5	230000

For these three values we compute and plot the smoothing spline and the corrected image. Cf. table 2

As a result, it seems like the best results are obtained for a degrees of freedom not too low, otherwise the correction is very homogeneous over the picture while our bias is very localized in this case.

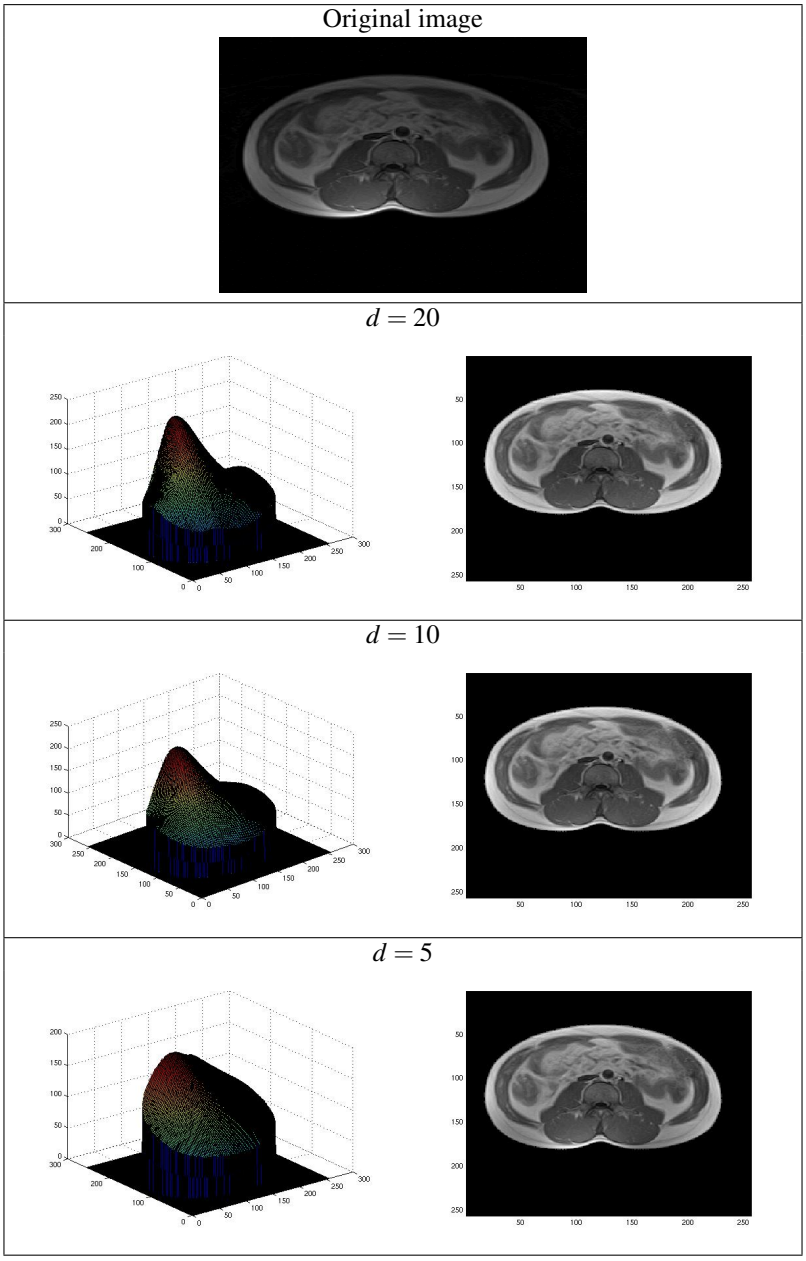


Table 2: Smoothing TPS and corrected images for different degrees of freedom