Functional Analysis

Ojas G Bhagavath

Course	MSM421 Functional Analysis
Instructor	Prof. Rajeev Bhaskaran
Prerequisites	MSM321 Complex Analysis and MSM411 Measure Theory
Learning Outcomes	Based on core analysis courses and linear algebra, this course builds further on the study of Banach and Hilbert spaces. The theory and techniques studied in this course support, in a variety of ways, many advanced courses, in particular in analysis and partial differential equations, as well as having applications in mathematical physics and other areas
Syllabus	• Normed linear spaces, Riesz lemma, characterization of finite dimensional spaces, Banach spaces. Operator norm, continuity and boundedness of linear maps on a normed linear space. (6) • Fundamental theorems: Hahn-Banach theorems, uniform boundedness principle, divergence of Fourier series, closed graph theorem, open mapping theorem and some applications. (8) • Dual spaces and adjoint of an operator: Duals of classical spaces, weak and weak* convergence, adjoint of an operator. (6) • Hilbert spaces: Inner product spaces, orthonormal set, Gram-Schmidt orthonormalization, Bessel's inequality, orthonormal basis, separable Hilbert spaces. Projection and Riesz representation theorems: Orthonormal complements, orthogonal projections, projection theorem, Riesz representation theorem. (10) • Bounded operators on Hilbert spaces: Adjoint, normal, unitary, self-adjoint operators, compact operators. (5) • Spectral theorem: Spectral theorem for compact self adjoint operators, statement of spectral theorem for bounded self adjoint operators. (5)

Lecture 01: Revision of Linear Algebra 08 Jan 2024 11:30

Recall (Linear Algebra)***:***

- Vector spaces, the axioms, and examples.
- Linear dependence of subsets of a vector spaces (both finite and infinite), basis is a maximal linear independent subset of a vector space.
- Every vector space has a basis (Zorn's Lemma), every basis of a vector space has the same cardinality, hence, dimension of a vector space defined as the cardinality of any of its basis is well defined.

References

[1] R. Bhatia. *Notes on Functional Analysis*. 1st ed. Hindustan Book Agency, 2009.

[2] S. Kesavan. *Functional Analysis*. 2nd ed. Springer, 2023.